Simultaneous fault detection and control design for robots with linear fractional-order model

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***Abstract*—The problems of simultaneous detection of fault and control (SFDC) for robots with linear fractional-order (FO) model or any system that can be modeled as a linear fractional-order are investigated in this study. The new rules in terms of LMI are displayed to build a SFDC unit. In essence, a Luenberger observer has been used as a fault explorer and the controller has been designed to form of an observer-based controller. This design is converted to H- /H∞ problem. In order to show the correctness of the design method and the resulting formulas, a numerical model simulated with MATLAB is presented in this article.**

Keywords*— *Fault Detection, linear fractional-order Systems, SFDC, LMI.

I. Introduction

Knowledge of fault diagnosis in practical systems can be considered as medical science for humans. With the rapid diagnosis of some diseases, medical science prevents the spread of disease and treats it. If not enough attention to medical science, what would happen to humans? Fault detection and isolation (FDI) Also, troubleshooting itself in robots is an emerging research field during the recent years Due to the increasing use of robots in various industries as well as in the galaxy. For an operating system that encounters an error, the following factors should be taken into consideration: detection and isolation; quantification, characterization, and identification; and the reaction to the fault [1]. Diagnosing a fault over a short period of time is a fundamental necessity to reduce the likelihood of hardware damage [2-6]. In recent decade, much attention in the field of reconnaissance of fault has been attracted to the model-based fault detection [8]. Authors [7] successfully demonstrated a model-based FD approach experimentally. Today's, the majority of the systems are a closed feedback system. Sometimes in these systems, it may be hidden faults by operation control. This caused the SFDC problem to attract much attention from researchers in recent decades [9-13]. The Simultaneous integrated design of control and reconnaissance units converts into a controller / detector unit. In [13-15], the SFDC problem is presented by technique multi-objective H∞/H- framework. The authors [16, 17] studied the problem of SCFD in mixed H2 /H∞ optimization technique.

Although the subject of the fractional calculus has been discussed since a few centuries ago, engineers have paid special attention to it in recent years. [18–20]. In addition to modeling dynamic systems using the means of the fractional calculus [20-29], the application of this science is seen in many engineering fields such as robotics [30, 31]. Moreover, in the control theory, It has been proven that for some systems, fractional controllers are more flexible and more flexibilities and robustness, e.g. To mention a few, designing FO PID controllers for a robotic tracking control can be found in [32]. Authors [33] present the fractional controllers in a hexapod robot. In [34], the trajectory control is investigated by the fractional calculus for planar manipulators.

In this paper, the issue of SFDC for robots with linear FO model or any system that can be modeled as a linear fractional-order are considered and via employing the generalized Kalman-Yakubovich-Popov (KYP) lemma, new rules are developed in the LMI form and design a SFDC unit such that probable faults are detected as well as the FO system is stable.

Notations: Throughout this paper, A^T refers the transpose, $\overline{A}$ is conjugate and A^* is conjugate transpose of matrix A . $Her(A)$ is short for $A + A^*$.

II. The Problem Statement and Definitions

Consider a robot modeled as the following FOS model

$$G:\begin{cases} D^{\alpha} x(t) = A\,x(\mathrm{t}) + B\,\mathrm{u(t)} + \mathrm{B}_d\, d(t) + B_f\, f(t) \\ y(t) = C\,x(t) + D\,u(t) + \mathrm{D}_d\, d(t) + D_f\, f(t) \\ \mathrm{z}(t) = E\,x(t) + F_d\, d(t) + F_f\, f(t) \\ x(t) = \phi(\mathrm{t}) \qquad \mathrm{t} \in [-\mathrm{h}, 0] \end{cases} \tag{1}$$

Where α is the fractional appropriate order and $0 < \alpha < 1$; $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^r$ and $z(t) \in \mathbb{R}^t$ are the pseudo state vector, measured output and the regulated output, respectively. Too, $u(t) \in \mathbb{R}^m$, $d(t) \in \mathbb{R}^p$ and $f(t) \in \mathbb{R}^q$ are the control input, the disturbance and the fault vector, respectively . $\phi(t)$ is initial function defined on [$-h$, 0] where h is known positive scalar. The appropriately dimensioned matrices A, B, B_d , B_f , C , C_1, D_f , D_d , F_d and are F_f real known constant matrices. The fractional differentiation operator of order α is D^{α} . Caputo defined derivative for fractional order as follows:

$$ {}_aD_t^{\alpha} f(t) \triangleq \frac{1}{\Gamma(k-\alpha)} \int_a^t \frac{f^{(k)}(\tau)}{(t-\tau)^{\alpha+1-k}} d\tau \quad (2) $$

Where $k = \{1,2,3,...\}$ and $k-1 \le \alpha < k$. The transfer function from fault input to output $G_{yf}(s)$ is

$$ G_{yf}(s) = C(S^{\alpha} I - A)^{-1} B_f + F_f \quad (3) $$

And the transfer function from disturbance input $G_{yd}(s)$ is

$$ G_{yd}(s) = C(S^{\alpha} I - A)^{-1} B_d + F_d \quad (4) $$

Controller that is state feedback and detector that is observer for system (1) Designed as a single unit, which is called controller/detector unit as:

$$ \begin{cases} D^{\alpha} \hat{x}(t) = A\hat{x}(t) + Bu(t) + Lr(t) \\ \hat{y}(t) = C\hat{x}(t) + Du(t) \\ r(t) = y(t) - \hat{y}(t) \\ u(t) = K\hat{x}(t) \end{cases} \quad (5) $$

Where $\hat{x}(t) \in \mathbb{R}^n$ the observer (detection filter) state vector is $\hat{y}(t) \in \mathbb{R}^r$ is the output estimation vectors, the residual is $r(t) \in \mathbb{R}^r$, $K \in \mathbb{R}^{m \times n}$ and $L \in \mathbb{R}^{n \times r}$ is the controller gain and the filter gain respectively.

By defining $e(t) = x(t) - \hat{x}(t)$ then:

$$ \begin{cases} D^{\alpha} \xi(t) = \tilde{A}(t)\xi(t) + \tilde{B}_d d(t) + \tilde{B}_f f(t) \\ r(t) = \tilde{C}\xi(t) + D_d d(t) + D_f f(t) \\ z(t) = \tilde{E}\xi(t) + F_d d(t) + F_f f(t) \end{cases} $$

(6)

Where

$$ \xi^T = [x(t)^T \quad e(t)^T]^T, $$

$$ \tilde{A} = \begin{bmatrix} A+BK & -BK \\ 0 & A-LC \end{bmatrix}, \quad \tilde{B}_d = \begin{bmatrix} B_d \\ B_d - LD_d \end{bmatrix}, $$

$$ \tilde{B}_f = \begin{bmatrix} B_f \\ B_f - LD_f \end{bmatrix}, \quad \tilde{C} = [0 \quad C], \quad \tilde{E} = [E \quad 0], $$

Now, the SFDC problem is the design of the control / detection unit for the FO system (1), must continue in a way

1 - The close loop system (6) becomes stable.

2. If fault occur in the system, alarms should be made immediately.

3 - Disturbances are not considered fault in order to confuse the alarm detector system.

The above rules can be expressed as a mixed H_- / H_∞ optimizations problem as follows:

(i). $\sup \dfrac{\|z(t)\|_2}{\|d(t)\|_2} < \gamma_1, \ \gamma_1 > 0$

(ii). $\inf \dfrac{\|r(t)\|_2}{\|f(t)\|_2} > \beta, \ \beta > 0$

(iii). $\sup \dfrac{\|r(t)\|_2}{\|d(t)\|_2} < \gamma_3, \ \gamma_3 > 0$

(iv). $\sup \dfrac{\|z(t)\|_2}{\|f(t)\|_2} < \gamma_2, \ \gamma_2 > 0$

(i), (iii), and (iv) are H_∞ problems. The performance indices (ii) are H_- optimizations problem. The following definitions and lemmas are presented for later developed.

Definition1. [35] Transfer $T_{yd}(s)$ H_∞ norm is:

$$ \|T_{yd}\|_{H_\infty} := \sup_{Re(s) \ge 0} \bar{\sigma}(T_{yd}(s)) \quad (7) $$

Lemma 2 (H-BR): [36] $\|G(s)\|_{H_\infty} < \gamma$ if $P > 0$ and $Q > 0$ such that the following LMI

$$ \begin{bmatrix} sym(AX) & * & * \\ CX & -\gamma I & * \\ B^T & D^T & -\gamma I \end{bmatrix} < 0 \quad (8) $$

Where

$$ X = \begin{cases} e^{j\theta} P + e^{-j\theta} Q, if \ 0 < \alpha < 1 \\ e^{j\theta} P \qquad if \ 1 \le \alpha < 1 \end{cases} \qquad \theta = \frac{\pi}{2}(1-\alpha) $$

Lemma 3: [37] Let matrices $\psi \in H_2$, $A \in \mathbb{R}^{n \times n}$, $\Phi \in H_2$, $B \in \mathbb{R}^{n \times m}$, and $\Theta \in H_{(n+m)}$, Set Λ is defined as

$$ \Lambda(\Phi, \Psi) \triangleq \left\{ \lambda \in C \left| \begin{bmatrix} \lambda \\ I \end{bmatrix}^* \Phi \begin{bmatrix} \lambda \\ I \end{bmatrix} = 0, \right| \begin{bmatrix} \lambda \\ I \end{bmatrix}^* \Psi \begin{bmatrix} \lambda \\ I \end{bmatrix} \ge 0, \right\} \quad (9) $$

1) For $H(\lambda) \triangleq (\lambda I_n - A)^{-1} B$

$$ \begin{bmatrix} H(\lambda) \\ I_m \end{bmatrix}^* \Theta \begin{bmatrix} H(\lambda) \\ I_m \end{bmatrix} < 0 \quad \forall \lambda \in \Lambda \quad (10) $$

(2) $P, Q \in H_n$ and $Q > 0$ such that

$$ \begin{bmatrix} A & B \\ I_n & 0 \end{bmatrix}^* (\Phi \otimes P + \Psi \oplus Q) \begin{bmatrix} A & B \\ I_n & 0 \end{bmatrix} + \Theta < 0 \quad (11) $$

Then "$2) \Rightarrow 1)$" always holds. But "$2) \Leftrightarrow 1)$" holds if Λ displays a curve in the complex plane.

Lemma 4: (Projection lemma) [38]. If U and V are two matrices of column dimension m and $Z \in S_m$ is a symmetric matrix; then

$$ U^T X V + V^T X^T U + Z < 0 \quad (12) $$

If and only if

$$ N_U^T Z N_U < 0 \quad (13\text{-a}) $$

$$ N_V^T Z N_V < 0 \quad (13\text{-b}) $$

III. MAIN RESULTS

As discussed in the previous section, the controller/detector unit should be designed such that all indexes (i)–(iii) are satisfied simultaneously and the close-loop system (6) is stable.

Theorem 1. The system (6) is stable and (i)–(iv) are guaranteed for given positive scalars $\gamma_1,\gamma_2,\gamma_3$, and β, if there exist positive definite symmetric matrices P_1,P_2,P_3,P_4,Q_1, Q_2,Q_3,Q_4 and matrices X_1, $X_2,\hat{X}_1,N$ and M such that the following optimization problem is solved:

$$\max_{X_1,X_2,\hat{X}P_i,P_1,P_2,P_3,P_4,Q_1,Q_2,Q_3,Q_4,N,M} \beta$$

Subject to:

$$\text{a)}\begin{bmatrix} Her(\varphi)+\Xi_1 & \Xi & \psi+\Xi_3 F_d \\ * & -\lambda(X+X^T) & \lambda\psi \\ * & * & F_d^T F_d-\gamma_1^2 I \end{bmatrix}<0$$

$$b)\begin{bmatrix} Her(\varphi)-\Xi_1 & \Xi & \psi+\Xi_3 D_f \\ * & -\lambda(X+X^T) & \lambda\psi \\ * & * & \beta^2 I-D_f^T D_f \end{bmatrix}<0 \qquad (14)$$

$$c)\begin{bmatrix} Her(\varphi)+\Xi_1 & \Xi & \psi+\Xi_3 D_d \\ * & -\lambda(X+X^T) & \lambda\psi \\ * & * & D_d^T D_d-\gamma_2^2 I \end{bmatrix}<0$$

$$d)\begin{bmatrix} Her(\varphi)+\Xi_1 & \Xi & \psi+\Xi_3 F_f \\ * & -\lambda(X+X^T) & \lambda\psi \\ * & * & F_f^T F_f-\gamma_3^2 I \end{bmatrix}<0$$

$$B^T X_1=\hat{X}_1 B^T$$

Where

$$\Xi_1=\begin{bmatrix} E^T & 0\end{bmatrix}\begin{bmatrix} E \\ 0\end{bmatrix},\ \Xi_3=\begin{bmatrix} E & 0\end{bmatrix}^T,$$

$$\Pi=\begin{bmatrix} A^T X_1+M^T B^T & 0 \\ -M^T B^T & A^T X_2-C^T N^T \end{bmatrix}$$

$$\Psi=\begin{bmatrix} X_1^T B_d \\ X_2^T B_d-N^T D_d \end{bmatrix},\ \Xi_{12}=\lambda\Psi-X^T+\overline{r}P+rQ$$

$$X=diag(X_1,X_2)$$

Where $r=e^{j\theta},\theta=(1-\alpha)\frac{\pi}{2}$ and

$$L=X_2^{-T}N$$

$$K=\hat{X}_1^{-T}M.$$

Proof: based on definition 1

$$\left\|T_{zd}\right\|_{H_\infty} := \sup_{Re(s)\ge 0} \overline{\sigma}(T_{zd}(s)) = \sup_{Re(s)\ge 0} \overline{\sigma}(C(s^{\alpha}I-A)B+D) \qquad (15)$$

also

$$\left\|G(s)\right\|_{H_\infty}<\gamma \Leftrightarrow G(s)G^*(s)-\gamma^2 I<0 \quad \forall\,\mathrm{Re}(s)\ge 0$$

$$\Leftrightarrow \begin{bmatrix} H(\lambda) \\ I_m\end{bmatrix}^* \Theta \begin{bmatrix} H(\lambda) \\ I_m\end{bmatrix}<0 \quad \forall\lambda\in\Lambda(\Phi,\Psi) \qquad (16)$$

Where $H(\lambda)\triangleq(\lambda I_n-\tilde{A})^{-1}\tilde{B}_d$ and $\Lambda(\Phi,\Psi)$ is defined in Eq. (9), also:

$$\Theta=\begin{bmatrix} \tilde{E}^T\tilde{E} & \tilde{E}^T F_d \\ F_d^T\tilde{E} & F_d^T F_d-\gamma_1^2 I\end{bmatrix} \qquad (17)$$

By invoking Lemma 3, $\exists P,Q\in H_n,, P>0$ and $Q>0$ such that:

$$\begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_d\end{bmatrix}^T(\Phi\otimes P+\Psi\otimes Q)\begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_{cli}\end{bmatrix}+\Theta<0 \qquad (18)$$

Similar to [37],

$$\Phi=\begin{bmatrix} 0 & \overline{r} \\ r & 0\end{bmatrix},\ \Psi=\begin{bmatrix} 0 & r \\ \overline{r} & 0\end{bmatrix}, \qquad (19)$$

Due to the (13-a) the inequality (18) can be written

$$Z=\begin{bmatrix} -\tilde{E}^T\tilde{E} & \overline{r}P+rQ & \tilde{E}F_d \\ rP+\overline{r}Q & 0 & 0 \\ F_d^T\tilde{E} & 0 & -\gamma_1^2 I+F_d^T F_d\end{bmatrix}, \qquad (20)$$

$$N_U=\begin{bmatrix} I & 0 \\ \tilde{A} & \tilde{B}_d \\ 0 & I\end{bmatrix},$$

By specifying the matrices N_V and V as follows:

$$N_V=\begin{bmatrix} \lambda I & 0 \\ -I & 0 \\ 0 & I\end{bmatrix}\rightarrow V=\begin{bmatrix} I & \lambda I & 0\end{bmatrix} \qquad (21)$$

Using lemma 4, it can be can be inferred that $N_U^T Z N_U < 0$ is equivalent to:

$$Z+\begin{bmatrix}\tilde{A}^T\\ -I\\ \tilde{B}_d^T\end{bmatrix}\begin{bmatrix}X & \lambda X & 0\end{bmatrix}+\begin{bmatrix}X^T\\ \lambda X^T\\ 0\end{bmatrix}\begin{bmatrix}\tilde{A} & -I & \tilde{B}_d\end{bmatrix}<0 \quad (22)$$

And if $X = diag(X_1, X_2), X_i \in R^{n\times n}, i = 1,2$

$$\begin{bmatrix}Her(\varphi)+\Xi_1 & \Xi & \psi+\Xi_3 Fd\\ * & -\lambda(X+X^T) & \lambda\psi\\ * & * & F_d^T F_d-\gamma_1 I\end{bmatrix}<0 \quad (23)$$

Where

$$\Xi = \bar{r}P + rQ + \lambda\varphi - X^T \quad (24)$$

$$\varphi=\begin{bmatrix}A^T X_1 + K^T B^T X_1 & 0\\ -K^T B^T X_1 & A^T X_2 - C^T N^T\end{bmatrix} \quad (25)$$

first inequality in Eq. (14) is obtained by substitute $L = X_2^{-T} N$ and $K = \hat{X}_1^{-T} M$ into the inequality (19),

By replacing Θ and Z as follows:

$$Z=\begin{bmatrix}-\tilde{E}^T\tilde{E} & \bar{r}P+rQ & -\tilde{E}^T D_f\\ rP+\bar{r}Q & 0 & 0\\ -D_f^T\tilde{E} & 0 & -\beta^2 I + D_f^T D_f\end{bmatrix}, \quad (26)$$

$$\Theta=\begin{bmatrix}-\tilde{E}^T\tilde{E} & -\tilde{E}^T D_f\\ -D_f^T\tilde{E} & \beta^2 I - D_f^T D_f\end{bmatrix}$$

(b) in Eq. (14) is satisfied.

The proof (c) and (d) in Eq. (14) can also be shown is similar to the above technique.

IV. NUMERICAL EXAMPLE

Consider the FOS (1) with parameters:

$$A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}, B_d=\begin{bmatrix}0.2\\ 0\end{bmatrix}, B_f=\begin{bmatrix}0.2\\ 0.1\end{bmatrix} \alpha = 0.5$$

$$C=\begin{bmatrix}0.1 & 0\end{bmatrix}, \quad D_d = D_f = 0.2, \; F_f = F_d = 0.1$$

$d(t) = 0.1\exp(-.04t)\cos(0.03\pi t)\,\mathrm{u(t)}$. The fault signal $f(t)$ has been shown in Fig. 1. For a given $\gamma_1 = 1.7017, \gamma_2 = 1.7874$, and $\gamma_3 = 1.1944$ optimization problem corollary 1, was solved and β is obtained as 0.0836. Furthermore, the observer gains were obtained as follows:

$$L_1=\begin{bmatrix}50.1036\\ 13.1357\end{bmatrix},$$

$$\mathrm{K}=\begin{bmatrix}-61.3024\\ -10\end{bmatrix}$$

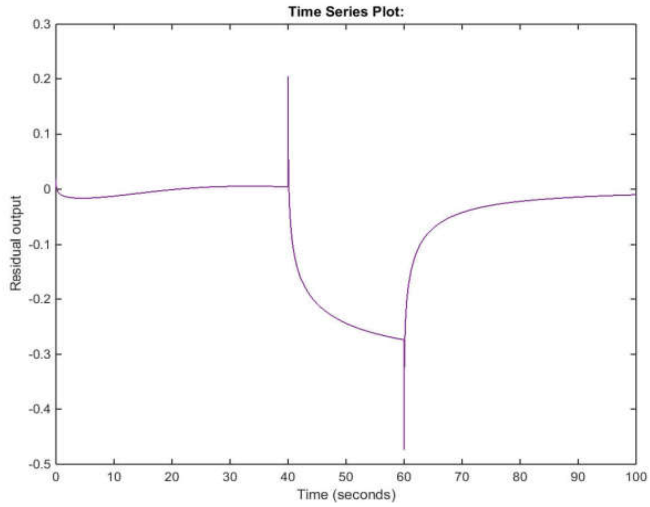

Figure 1. Residual signal r (t)

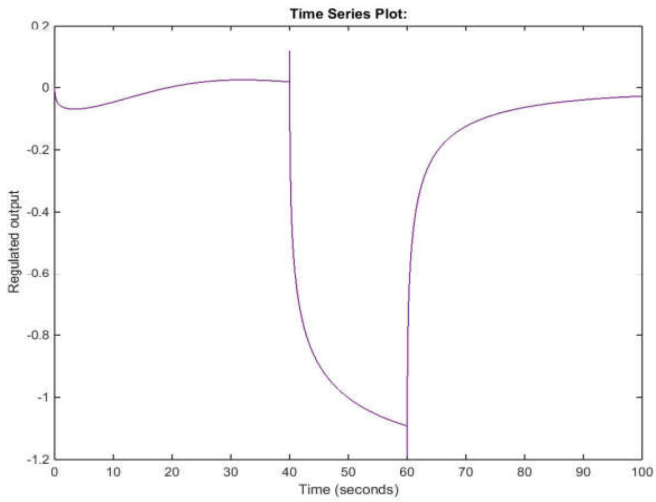

Figure 2. Regulated output z (t)

V. CONCLUSION

In this study, a robust distributed SFDC problem for robots with linear fractional-order model or any system that can be modeled as a linear fractional-order using observer detector and state feedback controller is proposed and developed. The new rules in terms of LMI are displayed to build a SFDC unit to warrant stability and the four mixed H_-/H_∞ performances. In order to prove the correctness of the design method and the resulting formulas, a numerical example simulated with MATLAB is presented in this article.

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