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Calculation of Stress Intensity Factor in Rotary Cracked Disks with Central Hole by Using Emulator Technology

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Abstract: In this paper, the crack growth and variations of stress intensity factor in rotary disks with central hole is studied. For modeling and analyzing samples, the FRANC2D software is used and to predict stress intensity factor, DACE emulator (Design & Analysis of Computer Experiments) is used and, at the end comparison between results of modeling and results of simulator has been done. The emulator predicts the stress intensity factors very accurate.

Key word: Rotating disks . emulator . simulator . crack . FRANC2D . stress intensity factor

INTRODUCTION

In crack growth with different geometries, a specified parameter is designated to show behavior of crack under hoop stresses. This specified parameter is stress intensity factor and it is very important because if the value of this factor to be unique in tip of two cracks with different geometries, then behavior of growth in two cracks are similar to each other.

According to equation (1), *Z(z)* is a complex function and refer to equation (2), f is the bi-harmonic Airy stress function, because we have, $\nabla^4 f = 0$.

For example, Fig. 1 shows a plate with a central crack under steady constant stress field. So boundary conditions for this geometry are written according to equation (3); therefore Z(z) is valid for these boundary conditions. By equation (4) we acn calculate the stresses in the tip of crack.

It is obvious that in these equations, for each variation in shape of crack, length of crack and loading on plate, only stress intensity factor will change. The general form of mathematical formulation of stress intensity factor is, $K_I = \beta s \sqrt{\pi a}$. Where β is a geometrical function, s is far field applied stress and a is crack length. According to geometry of Fig. 1, β equals to one.

$$Z(z) = \frac{\sigma z}{\sqrt{z^2 - a^2}} \qquad (1)$$

$$\phi = \mathrm{Re}\left(\iint Z(z).d^2z\right) + y.\mathrm{Im}\left(\int Z(z).dz\right) \qquad (2)$$

$$\begin{aligned} x = \pm\infty \quad &\rightarrow \quad \sigma_x = \sigma \\ y = \pm\infty \quad &\rightarrow \quad \sigma_y = \sigma \qquad (3) \\ -a < x < +a &\rightarrow \quad \sigma_y = 0 \end{aligned}$$

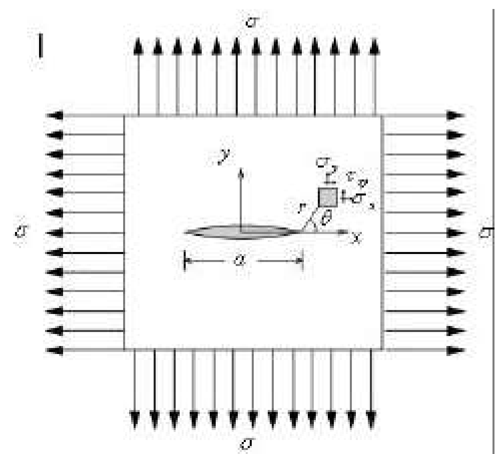

Fig. 1: An infinity plate under steady constant tension loads

$$\begin{aligned} \sigma_x &= \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\times\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \\ \sigma_y &= \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\times\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right] \qquad (4) \\ \tau_{xy} &= \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\times\sin\frac{\theta}{2}\times\cos\frac{3\theta}{2} \end{aligned}$$

Till now many researches were done to predict stress intensity factor during the crack growth in rotary disks and most of these researches had been done with analytical methods. Reference [1] has mentioned to one of these equations. In fact, this equation is a result of analytical methods and it has compared with numerical analysis results.

Another method to predict stress intensity factor is weighted functions [2, 3]. Also experimental methods

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Table 1: Comparison between an emulator and a simulator

	Speed of calculation	Accuracy	Optimization
Simulator	Slow	Very good	Local
Emulator	Quick	Good	Total

are used to predict stress intensity factor such as using standard samples that are used to predict a real value for stress intensity factor [4]. In crack growth in rotary disks, the positions and numbers of cracks are studied such as radial cracks and peripheral cracks [5-7]. In this paper we calculate stress intensity factors in cracked rotary disks by using DACE Emulator and statistical data in simulator.

STATISTICAL EMULATORS

Nowadays in optimization researches, reducing computational time and economic cost and having good accuracy is more important.Emulators technology is good example for these aims. So emulators are useful and quick methods in different optimization problems. In other words, emulators are statistical functions and display behavior of case study, so emulators can locate to predict a simulator's behavior. Thus, we can use the emulators as a useful substitution for prediction of behavior of a simulator. In the main, we can compare between an emulator and a simulator according to Table 1.

In this process to create an emulator at first step, we must select valid data, then we can make an emulator base on resultant data from sampling (design points). Each design point contains effective parameters in problem and equivalent output data from simulator. In this case, accuracy of the emulator is important. Indeed accuracy of an emulator depends on effective parameters in system and number of design points. To make use of emulators can be obvious optimum designing in complex systems, so decreases costs and computational time and risk in different problems.

SAMPLING METHODS

Generally in a DACE emulator, two sampling methods are used that they have been illustrated in Fig. 2 [8]:

First method: RG (Rectangular Grid)

Second method: LHS (Latin Hypercube Sampling)

RG method (Rectangular Grid method): Suppose x_j is a number that defines in this form: $l_j \le x_j \le u_j$ and m is numbers of designing points and n is numbers of effective parameters. Then RG distribution will define according to equation (5).

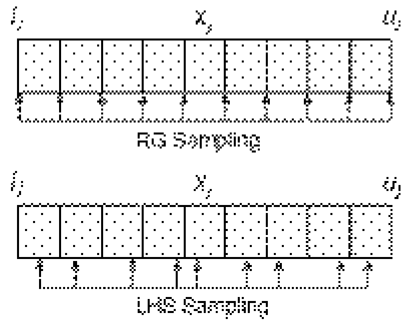

Fig. 2: Comparison between two sampling methods RG& LHS

$$S_j^{(i)} - \ell_j + k_j^{(i)} \frac{u_j - \ell_j}{v_j}, k_j^{(i)} = 0,1,....,v_j \qquad (5)$$

LHS method (Latin hypercube sampling method): According to this method, the inputs are selected randomly with special strategy which cover all of workspace. This process divides to three main stages:

First stage: Each domain for each parameter divides to m numbers of sub domain and probability of selection is similar in each sub domain. In this part, if distribution of data is unique, then length of these sub domains will equal to each other.

Second stage: Random sampling from sub domains were created at first stage and this sampling is based on density of probability.

Third stage: Creation the probability sampling from selected samples to produce independence variables input points.

DACE EMULATOR [8]

We suppose S as a set of m numbers of designing points and each point contains n numbers designing parameters also Y is a set of answers of simulator that q is number of derived answers. Function $Z_{:q}(X)$ was produced with zero mean values by answers of system in designing points $Y_{:q}(X)$.

$$Y_{:q}(X) = \beta_{:q}(X) + Z_{:q}(X) \qquad (6)$$

In equation (6), $\beta_{:q}$ is an unknown constant value and if $S_{i:}$ and $S_{i':}$ are input points, then covariance between these points is:

Table 2: Functions that are used in equation (9) [8]

Name	$\Psi(\theta_j, p_j, d_j)$	
EXP	$\exp(-\theta_j\|d_j\|)$	
GAUSS	$\exp(-\theta_j\|d_j\|^2)$	
EXPG+	$\exp(-\theta_j\|d_j\|^{p_j})$ $0<P_j\le 2$	
LIN	Max $\{0, 1-\theta_j\|d_j\|\}$	
SPHERICAL	$1-1.5x_j+0.5x_j^3$	$x_j = \mathrm{Min}\{1, \theta_j\|d_j\|\}$
CUBIC	$1-3x_j^2+2x_j^3$	
SPLINE	$1-1.5x_j^2+30x_j^3$	$0\le x_j\le 0.2$
	$1.25(1-x_j)^3$	$0.2<x_j<1$
	0	$1\le x_j$

$$\mathrm{Cov}\big(Z_{:q}(S_{i:}),\ Z_{:q}(S_{i':})\big) = \sigma^2 \Re(S_{ij} - S_{i'j}) \qquad (7)$$

In equation (7), s^2 is variance and other parameters are given below:

$$r_{1\times m},\ R_{m\times m},\ F_{m\times 1},\ S_{m\times n},\ Y_{m\times 1},\ X_{k\times n}$$

$$i \in \{1,2,\cdots,m\},\ j\in\{1,2,\cdots,n\} \qquad (8)$$

$$r_i = \Re(S_{ij}, X_j)$$

In this equation $\hat{F}$ is a converter vector also S is input matrix and each row contains different variables in problem (R_i, R_o, L,ν). Therefore $\hat{Y}$ is an answer vector that contains the values of stress intensity factor (K_I) for each point that is produced due to implementation of simulator. Equation (9) defines matrix R as a correlation matrix between designing points:

$$R_{ii'} = \Re(S_{ij}, S_{i'j}) = \prod_{j=1}^{n} \Psi(\theta_j, p_j, d_j),\ d_j = S_{ij} - S_{i'j} \qquad (9)$$

Where, $\psi(\theta_j, p_j, d_j)$ is a correlation function and can be a function from Table 2 that it relates to problem conditions and distribution of designing points.

Behaviors of some functions of Table 2 are shown in Fig. 3 and unknown values of θ_j and p_j are Emulator's indexes.These indexes are derived from minimizing the following correlation function.

$$\sigma^2 |R|^{\frac{1}{m}} \qquad (10)$$

Genetic algorithm has been used to find minimum value of the function and it is a quick method for optimization. This method increases speed of computation and it is basis of quick optimization problems.

Finally with regarding of above mentioned equations, answer of emulator calculates via equation (14). Fig. 2 contains four figures that show utilized functions in an Emulator.

$$\beta = (F^T R^{-1} F)^{-1} F^T R^{-1} Y \qquad (11)$$

$$\sigma^2 = \frac{1}{m}(Y - F\beta)^T R^{-1}(Y - F\beta) \qquad (12)$$

$$\delta = R^{-1}(Y - F\beta) \qquad (13)$$

$$\hat{Y}(x) = F\beta + r\delta \qquad (14)$$

DESIGNING AN EMULATOR

In this stage, the problem will solve via using DCEA Emulator. This Emulator answers real value of designing points which produce in simulator (Fig. 4). Therefore it is similar to interpolation problem in the algebraic analysis. But in algebraic analysis for obtaining an emulator, curve fitting is studied, because of maintain of function behavior and normalizing of variations derivatives of function.

The DCEA emulator in accuracy is similar to interpolation and in behavior of function variations is similar to curve fitting. And these are preference of the DCEA Emulator.

In this study an emulator to predict crack growth behavior in rotary disks with central hole is designed. We want to find equations for crack growth behavior in rotary disks with central hole. Before this research, many equations were designated [1] but these equations have derived from analytical methods. Initially for study in crack growth behavior in rotary disks with central hole, a model in FRANC2D software with three main variables has considered, also effective geometrical parameters show in Fig. 5 and Table 3. FRANC2D is skilled software for analyzing crack growth in 2-dimensional and 3-dimensional geometry. This software was made by mechanical department in Cornell University and it is free to use. This software is able to draft geometrical model, mesh generation, loading, creating crack and show crack growth. Also this software is able to draft crack growth graphs. Figure 6 shows of FRANC2D menus.

In crack growth study in rotary disks with central hole, 144 tests are sufficient for extend studies. Totally 144 different samples with specified effective parameters and equal distances have calculated (Table 2) and these samples have selected to analyze by FRANC2D software. The sample is a disk that it's radius and density is unique (i.e. one) and it's rotary

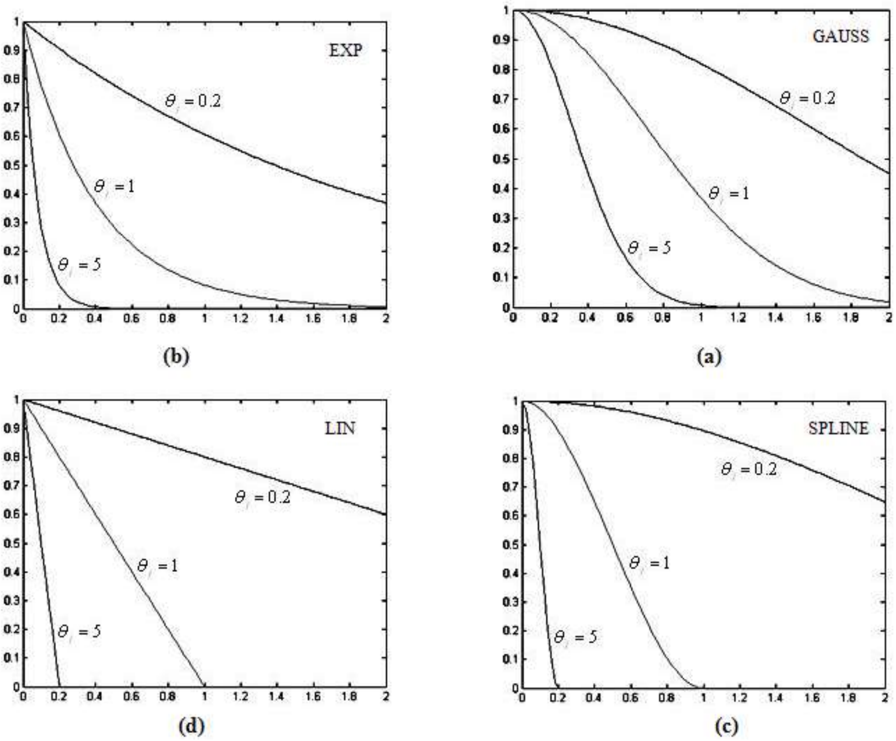

Fig. 3: The functions used in emulator as Table 2
Behavior of, (a): GAUSS function, (b): EXP function, (c): SPLINE function and (d): LIN function

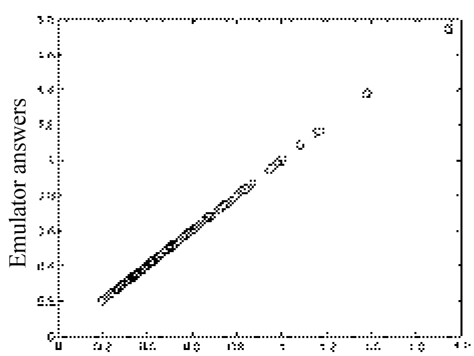

Fig. 4: Sufficient accuracy in sampling points to predict stress intensity factor values

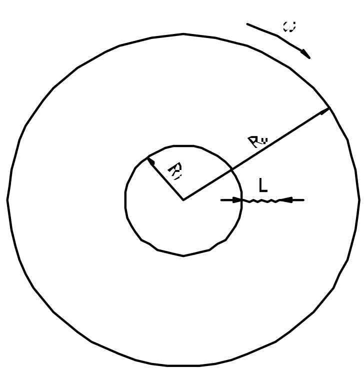

Fig. 5: Schematic view of a rotary disk and effective parameters in crack growth

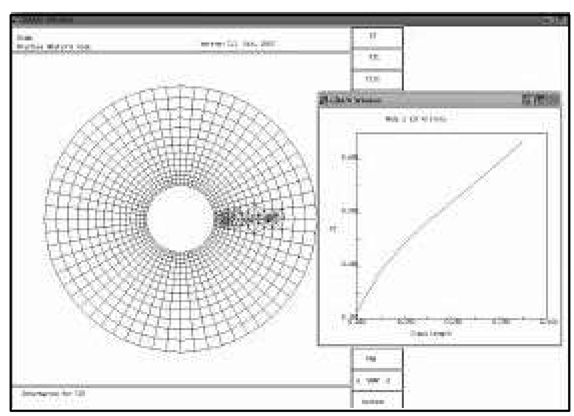

Fig. 6: A view of FRANC2D software

Table 3: Variation range for each parameter

Variable	Minimum	Maximum
Poisson Ratio (v)	0.20	0.40
R_i/R_o	0.05	0.55
L/R_o	0.05	0.40

velocity is 1 rad/s. Variation range of variables has shown in Table 3.

Where, v is the Poison's ratio, R_i, inner radius of disk, R_o, outer radius of disk and L is crack length.

At first 130 designing points are selected with LHS method then according to simulator's answers and LHS

Table 6: Comparison between DACE Emulator's answers and finite element results by FRANC2D software

$\frac{R_i}{R_o}$	$\frac{L}{R_o}$	v	FEM results K_I ($ksi\sqrt{in}$)	Emulator Results K_I ($ksi\sqrt{in}$)	Error values
0.05	0.1	0.3	0.2376	0.2357	-0.008
0.05	0.2	0.4	0.3006	0.2996	-0.003
0.15	0.3	0.2	0.4304	0.4320	-0.004
0.15	0.4	0.3	0.4937	0.5044	0.021
0.25	0.1	0.2	0.3718	0.3735	0.004
0.25	0.2	0.3	0.4732	0.4731	0.000
0.25	0.3	0.4	0.5569	0.5560	-0.002
0.35	0.4	0.2	0.7887	0.7926	0.005
0.35	0.1	0.4	0.4388	0.4392	0.001
0.45	0.2	0.2	0.6586	0.6582	-0.001
0.45	0.3	0.3	0.8566	0.8553	-0.001
0.45	0.4	0.4	1.092	1.0654	-0.024
0.55	0.1	0.3	0.5303	0.5269	-0.006
0.55	0.2	0.4	0.8435	0.8419	-0.002

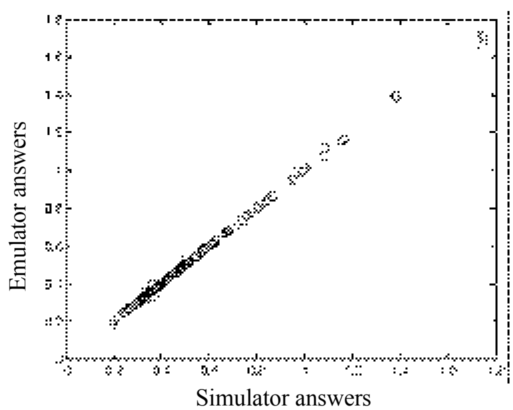

Fig. 6: Cross validation graph

method, a matrix S, is made in order to produce of a suitable emulator to prediction of behavior function for each reply. This matrix contains different values of problem variables and $\hat{Y}$ vector and answers of simulator in designing points. Specifications of Emulator are shown in Table 4, also Estimated Root Mean Square Error (ERMSE) has been estimated in equation (15) and this equation is a basic method to define error value in an Emulator.

$$ERMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left[\hat{Y}_i(X) - Y_i(X)\right]^2} \qquad (15)$$

Mean error in an Emulator for this case study is less than 1% that it is very good accuracy. The accuracy of emulator is shown in Table 5.

Figure 6 shows a graph relates to Emulator's accuracy. This graph defines cross validation and emulator accuracy in prediction of new points answers and it is a criterion of Emulator dependency to designing points.

Horizontal axis contains predictive values by 130 designing points and perpendicular axis contains predictive values by designated emulator with 129 primitive designing points in eliminated point.

Table 6 compares predictive values by Emulator with values have been obtained by finite element simulator. According to this table error is very small and accuracy of Emulator is high.

CONCLUSION AND DISCUSSION

In this paper we focus on stress intensity factor and effect of this parameter in growth of crack for different geometries. The DACE Emulator is used to solve problems with several variables. Also problems relate to growth of crack in rotary disks with central hole are studied. A DACE Emulator is made to predict stress intensity factor in rotary cracked disks. The FRANC2D software has been used to valid accuracy of Emulator so results of simulator (finite element software) and Emulator (DACE) have been compared with each other and have shown in a table. This table confirms high accuracy in DACE Emulator to predict stress intensity factors.

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