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Praise Worthy Prize

Buckling and Post-Buckling of Cracked Plates under Axial Compression Load in Elastic-Plastic Mode

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Abstract – *Existence of cracks in industrial structures is one of the important causes of their failure. Therefore, it must be considered in stress analysis, designing and loading of such structures. In this paper, the buckling phenomenon in cracked plates is investigated. The effects of geometrical and mechanical parameters on deformation of plates and determination of buckling load are studied. The main purpose of this research is to investigate the buckling and post-buckling of cracked plates under axial compression loading in elastic-plastic materials, by consideration some parameters, such as crack length, crack angle, boundary conditions, imperfection and different material of plates (i. e. steel and aluminum).The numerical solution is carried out by the ABAQUS finite element software. The results show considerable effects of the above mentioned parameters on the buckling loads and post-buckling behavior of plates.*

Keywords: *Cracked Plate, Buckling, Post-buckling, Finite Element Method*

I. Introduction

Thin plates are among the widely-used structures in various industries such as aerospace and ship-building. As a result, the procedure of possible collapses for these structures is one of the important matters for engineers in field of mechanics, aerospace and ship-building. One of the usual phenomena in collapse of the plates is buckling. Buckling usually occurs due to presence of tension, compression and shear loading. Upon the increase of these forces and their arrival at the critical limit, the plate begins to buckle and this causes in-plane displacement and also out-of-plane displacement in the plates. It is worth mentioning that buckling occurs under tension in plates which have cracks or cut-outs.

In recent years one of the matters attracting the attention of researchers and engineers is the buckling of cracked plates but few articles have been presented in this field. Lee and Sih [1] studied the behavior of cracked plates under tension and compression loads and displayed the mode shapes related to these plates. They showed that increase of crack length results in the decrease of the quantity of critical load. Shaw & Huang [2] studied the behavior of cracked plates under uniaxial tension force using the finite element method and examined the effect of crack length, boundary condition and loading on buckling load. Riks et al. [3] studied the buckling and post-buckling of cracked plates under tensile load using the finite element method. Their results indicated that the stress intensity factor in post-buckling always adopts higher quantity in comparison to stress intensity factor in prebuckling. Guz and Dyshel [4] studied the two-layer metal plates, (steel-aluminum alloy) with crack under tension and studied the critical stresses in the onset of buckling and the concerned deformations. Satish Kumar and Paik [5] computed the buckling load for plates with central crack and edge crack under uniaxial compressive load, biaxial compressive load and in-plane shear load, using the hierarchical trigonometric functions. They proved the validity of their relations through finite element method. Brighetni [6]-[7] used the numerical finite element method and also analytical method to study the phenomenon of buckling in cracked plates under tensile and compressive load. He studied the effect of mechanical and geometrical variables such as Poisson's ratio, boundary conditions, crack length, and crack angle on buckling load. His results indicate that each of these parameters leave a considerable effect on buckling load. Alinia et al. [8]-[9] reviewed the cracked plates under shear load using the finite element method and also the effect of some of the mechanical and geometrical specifications in the behavior of these plates. They also studied sensitivity of the inferred results to the type of element-setting. Also Paik et al. in his extensive work [12] made a numerical/ experimental study on the collapse behaviour of plates with crack under both tensile and compressive loads. They finally derived the ultimate strength formulations for such cases. Besides, the problem of cracked plate elements under compression was the subject of few research studies [1], [5]-[7], [13], [14], [17]. Most of these studies were performed on the plates with either central or perpendicular-to-one-edge crack of varying crack length. Khedmati et al. addresses a finite element study on the buckling strength of a cracked plate with simple supports subjected to an axial

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compressive edge load.

The effects of crack location, crack orientation, crack length and plate aspect ratio are analysed [15]. Brighenti studied the effects of cracks' length and orientation on the buckling loads of rectangular elastic thin-plates characterised by different boundary conditions and by various Poisson's ratio under tension and compression [16].

A glance at the presented articles, we find out that the post-buckling of cracked plates under the compression load and the effect of such factors as crack length, crack angle, boundary conditions, the type of the material and imperfection have not been studied and reviewed in this phenomenon, while the effect of each of the mentioned factors and specially type of material and imperfection are very important and they are among the effective and determining factors in the buckling load.

Therefore, in this paper, we studied the behavior of post-buckling of cracked plates under axial compression load numerically and consider the effect of above mentioned parameters.

The numerical solution is carried out by the ABAQUS finite element software. The results show considerable effects of the above mentioned parameters on the buckling loads and post-buckling behavior of plates.

II. Method of Study

Using the analytical methods to solve these problems is very difficult and hard. Consequently most of researchers have resorted to numerical methods to find the response to these problems. In this paper, buckling and post-buckling of plates under axial compression load along with central crack have been studied using the numerical finite element method.

The quantities of the critical buckling load and also the graphs related to behavior of post-buckling of these concerned plates obtained using this method. The concerned analyses have been done using the finite element software, ABAQUS [10]. To reach the desired results a series of linear and nonlinear numerical computations are done by this software. ABAQUS software uses the general method of solving problems related to eigenvalue problems for assessment of buckling critical load and also for display of mode shape.

Post-buckling operations are carried out in two steps: First step is the same as initial buckling or ''Buckle'' step which is a linear perturbation procedure in ABAQUS software. At this step a preload is enforced on the plate and out of its results the eigenvalues are computed for various mode shapes, which are also used at the second step.

The second step is the post-buckling or ''Static riks'' step which predicts the post-buckling behavior of plates using the data of the ''Buckle'' step. It is worth mentioning that at this stage the quantity of the imperfection and also the data of plastic region related to the under-study material must be defined for the software.

III. Mechanical and Geometrical Conditions

In Fig. 1 we see the under-study geometry which is a rectangular plate under compression load along with central crack. In this figure t stands for thickness, 2W stands for width, and 2L stands for plate length. Their quantities are equal to 2mm, 100mm and 200mm respectively. θ and 2a are crack angle and crack length respectively adopting various quantities.

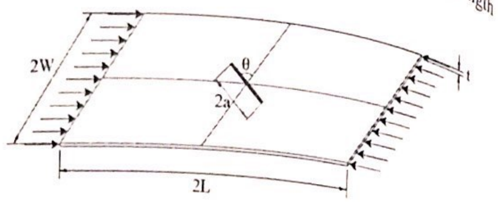


Fig. 1. Under-study geometry. A cracked rectangular plate under axial compression load

In the carried-out analyses the following parameters have been studied.

III.1. Boundary Conditions and Type of Material

The studies and investigations have been carried out for two opposite clamped edges and two opposite supported edges boundary conditions. Two different materials (Aluminum and Steel) are taken into consideration and the mechanical properties curves related to each of them have been obtained using the experimental method. The Figs. 2 indicate the stress-strain curves of these two materials for true cases and engineering cases. As pointed out earlier, we require the data of plastic region of the stress-strain curves for execution of post-buckling analysis in elastic-plastic case. The following properties have been found out of the curves of each of them: Mechanical properties of aluminum: $E = 72000MPa, \sigma_y = 145MPa, \nu = 0.33$ and Mechanical properties of steel: $E = 217000MPa, \sigma_y = 247MPa, \nu = 0.3$, where E is the Young's modulus, σ_y stands for yield strength and v is Poisson's ratio.

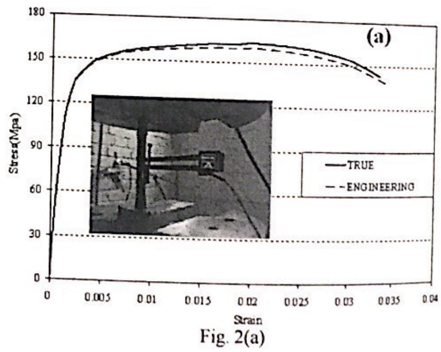


Fig. 2(a)

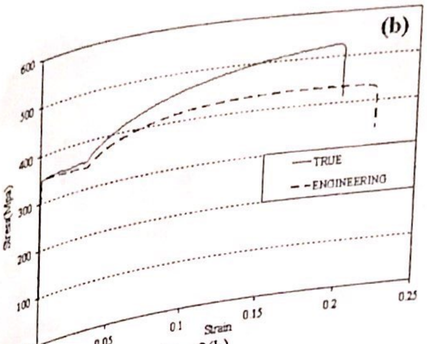


Fig. 2(b)

Figs. 2. Stress-strain curves for two under-study materials: (a) Aluminum; (b) Steel

III.2. *Dimensionless Crack Length and Crack Angle*

The analyses have been carried out for dimensionless crack lengths and with the following angles:

Dimensionless crack lengths:

$$\varphi = \frac{a}{W} = 0.1, 0.2, 0.3, 0.4, 0.5$$

Crack angles : $\theta = 0^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 90^{\circ}$

III.3. *Imperfection*

In the present paper, the effect of the imperfection has also been studied which is considered as one of the parameters being effective on critical buckling load in cracked plates and is regarded as percentage of the plate thickness. In the carried-out analyses the effect of some different quantities of the imperfection has been studied as follows:

$$\omega = t \,.\, SF = 5e-5, 1e-4, 2e-4, 4e-4, 8e-4 \ mm \qquad (1)$$

where ω is the quantity of the imperfection, t stands for the thickness and SF is a coefficient whose quantity varies from 2.5% up to 40%.

IV. Mesh Type

In ABAQUS software, the best type of element to mesh plates for analysis of buckling is S8R5 which is an 8-node element with 5 degree of freedom in each node. One of the important stages in analysis of cracked structures is the procedure of the mesh related to them. In the conducted analyses, the procedure of mesh is as displayed in Fig. 3 and we have tried to take into consideration around 1200 elements for each plate in all analyses in order that they can show an acceptable convergence.

V. Buckling of the Cracked Plates

We use the Buckling load multiplier for studying critical load and it is defined as follows:

$$\Psi = \sigma_{cr} / \sigma_0 \qquad (2)$$

in this relation, σ_{cr} is the critical buckling stress for cracked plates and σ_0 stands for Euler stress for uncracked plates and is defined as follows [11]:

$$\sigma_0 = k \, \frac{\pi^2}{4w^2} \frac{D}{t} \qquad (3)$$

where k is a numerical coefficient and its quantity is dependent on the proportion of W/L, D is the torsion rigidity of the plate and W, t and L are also geometric specifications of plate displayed in Fig. 1.

As we pointed out earlier, we can draw the concerned mode shape at the first stage.

In Fig. 4 and Fig. 5 we can see the first and second mode shapes related to cracked plates under compression load for two opposite clamped edges and two opposite supported edges boundary conditions.

VI. Post-Buckling and Effect of Imperfection

Here we deal with study of deformations caused by post-buckling and investigate the effect of the quantity of the imperfection in the cracked plates.

In Fig. 6 we can see the deformation caused by post-buckling in cracked plates for two opposite clamped edges and two opposite supported edges boundary conditions.

As you see the deformation complies with the changes of the first mode shape in Figs. 4 and Figs. 5 and it is an acceptable deformation because the deformation always follows the mode shape which has the least eigenvalue (critical load) and this value adopts the least quantity for the first mode shape.

Generally, post-buckling phenomenon is definable along with the imperfection and as a result its quantity must be taken into consideration and correctly chosen.

In Fig. 7 we can see the effects caused by the increase of the imperfection on the buckling load for steel cracked plate with two opposite clamped edges boundary conditions. As we clearly see, the increase of the quantity of the imperfection causes the decrease of the quantity of critical buckling load and it is also evident that a little time after buckling, as much as 0.7mm, the curves are made convergent to a definite extent.

This point reveal that imperfection mostly influence the critical buckling load than its post-buckling behavior. Fig. 8 presents the influence of increase of imperfection and crack length and variation of crack angle on critical buckling load. The results shown in Figs. 8 are obtained for a cracked plate with considered geometry and two opposite clamped edge boundary conditions.

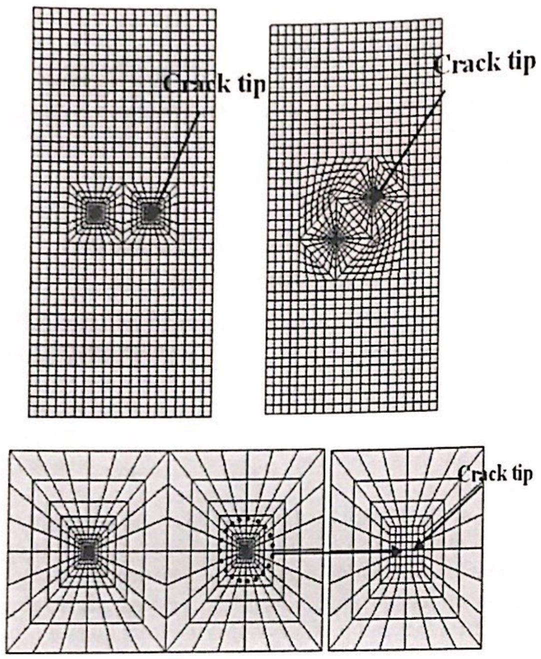
It is observed that as the crack length rise, the critical buckling load is decreased and increase in crack angle lessen the influences caused by crack.

Results reveal that enlargement or reduction of crack length and also variation of crack angle does not affect the imperfection and increase of imperfection, reduces the critical buckling load.

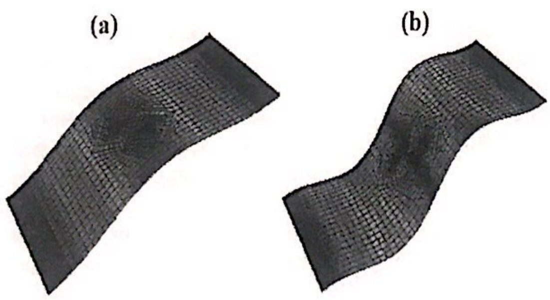
VII. Discussion and Results

We can reach considerable points from the results gathered from the numerical finite element method.

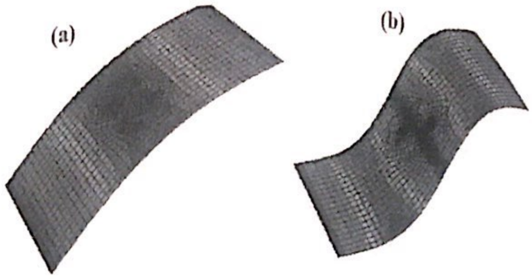
At first we study the quantities of the buckling load multiplier for two metals of steel and aluminum in proportion to the dimensionless crack length for various angles in two opposite clamped edges and two opposite supported edges boundary conditions (Figs. 9).



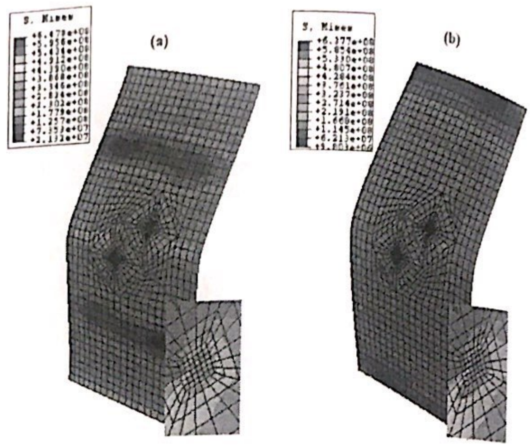
Figs. 3. Concerned mesh-setting for the cracked plate buckling analyses. (a) 0 degree and (b) 45 degree, (c) and (d) crack tip mesh detail



Figs. 4. The first and second mode shapes related to cracked plates (two opposite clamped edges), under axial compression load. (a) first mode shape. (b) second mode shape



Figs. 5. The first and second mode shapes related to cracked plates (two opposite supported edges), under axial compressive load. (a) first mode shape. (b) second mode shape



Figs. 6. The deformation caused by post-buckling in cracked plates. (a) two opposite clamped edges. (b) two opposite supported edges

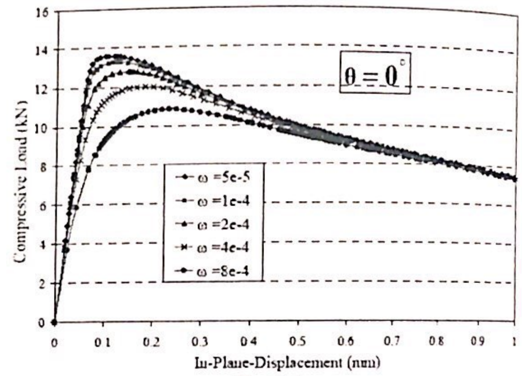


Fig. 7(a)

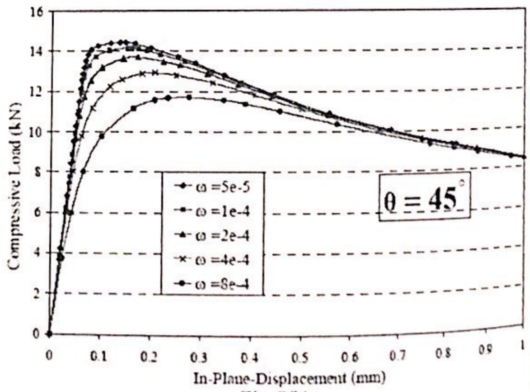


Fig. 7(b)

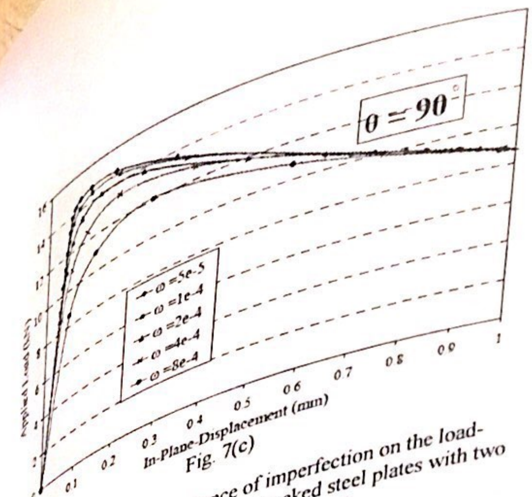


Fig. 7(c)

Figs. 7 The effect of the presence of imperfection on the load-displacement curve for buckling of the cracked steel plates with two opposite clamped edges (a=3mm). θ= (a) 0 degree, (b) 45 degree and (c) 90 degree

In view of the curves we can see that increase of the crack length causes the decrease of the quantity of the buckling load multiplier, while the increase of the crack angle causes the decrease of the effect of the crack presence, in such a way that for the 0-degree angle crack, the increase of the dimensionless crack length, causes a considerable decrease in the buckling load multiplier, while this decrease in 90-degree angle, is not so marked and perceptible. Here it is worth mentioning that the difference between the least quantity and the most quantity is related to the 0-degree angle whose difference reaches 16.47 % for two opposite clamped edges and 15.04 % for the two opposite supported edges and this phenomenon displays and indicates an acceptable compliance with the reference [6].

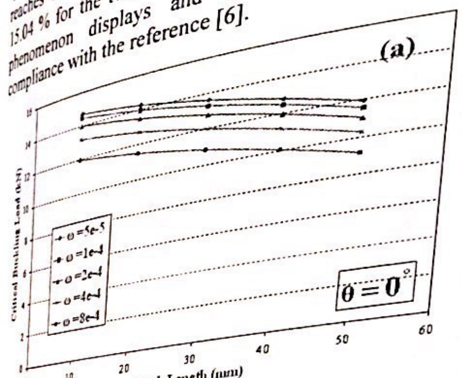


Fig. 8(a)

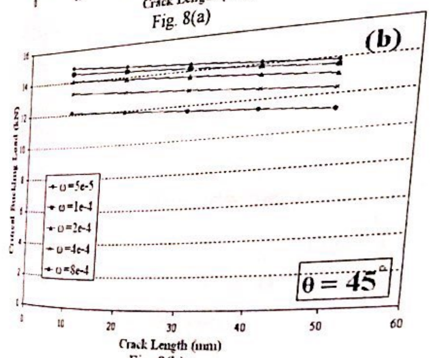


Fig. 8(b)

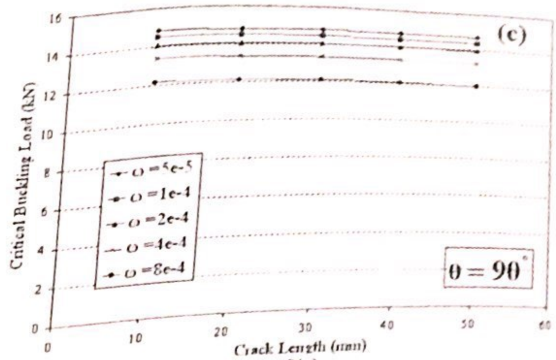


Fig. 8(c)

Figs. 8. Variations induced by increase in imperfection and enlargement of crack length on critical buckling load for steel cracked plates with two opposite clamped edge, for θ= (a) 0 degree, (b) 45 degree and (c) 90 degree

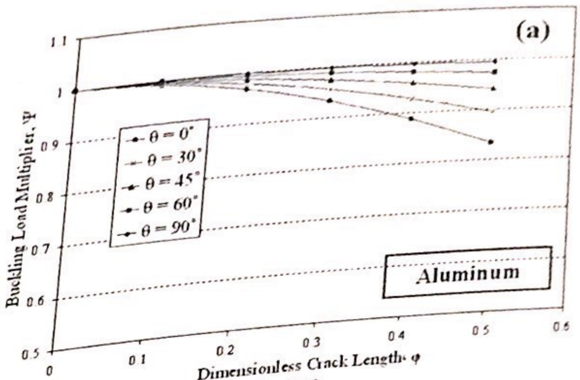


Fig. 9(a)

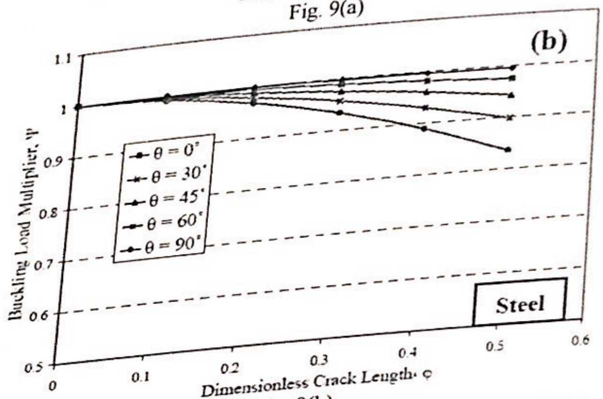


Fig. 9(b)

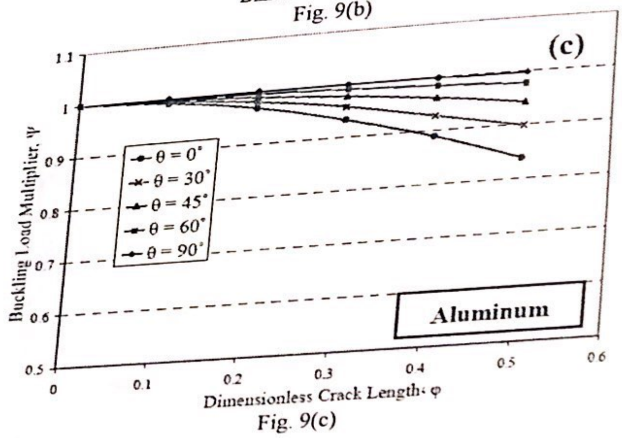


Fig. 9(c)

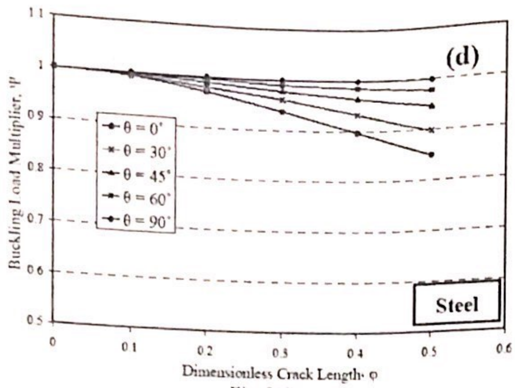


Fig. 9(d)

Figs. 9. The curves of the buckling load multiplier in proportion to the various quantities of the dimensionless crack length and the various crack angles for two metals of Steel and Aluminum.(a) and (b) two opposite clamped edges, (c) and (d) two opposite supported edges.

Furthermore, the trend of the decrease of buckling load multiplier is the same for both metals and the little difference between them (the most difference amounts to 0.3%) is due to difference in the Poisson's ratio related to each of them. In fact, we must say that change in Poisson's ratio creates little effect in the load coefficient of buckling of under-compression cracked plates. In continuation, we study post-buckling phenomenon for under axial compression load cracked plates for two metals of steel and aluminum. Figs. 10 and Figs. 11 display the load-displacement curves (in-plane and out-of-plane) for two metals of steel and aluminum in two opposite clamped edges boundary conditions with the imperfection of ω=5-e5. The concerned curves have been drawn for the uncrack plates and for plates with various crack lengths and we see that for each of these two metals, the increase of the crack length causes the decrease of the quantity of critical load buckling and generally the buckling load in the field of post-buckling. Maybe at the beginning, we are enticed to suppose that increase of crack length causes the decrease of only the critical load of buckling and the curves will be made convergent in a definite quantity, that is to say, the event which occurs while discussing the imperfection, but we can see that the increase of crack length, causes the decrease of the general buckling load and the curves go forward in an almost parallel way.

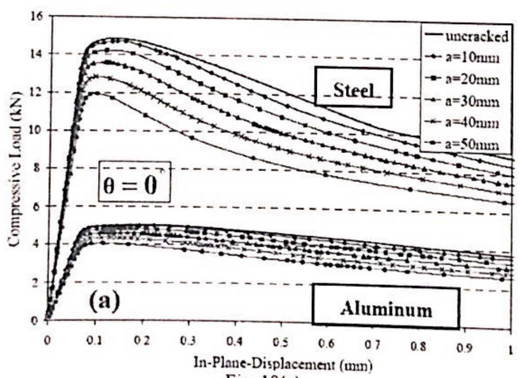


Fig. 10(a)

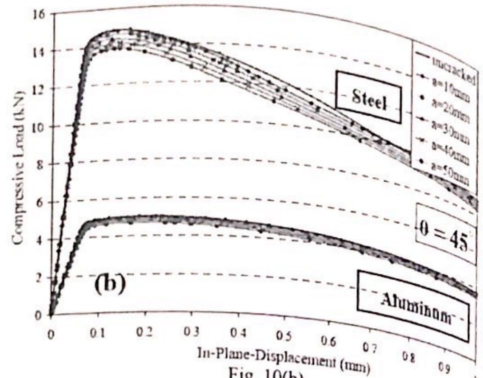


Fig. 10(b)

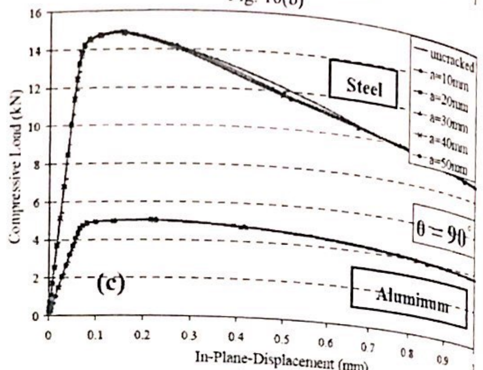


Fig. 10(c)

Figs. 10. The load-displacement curves (in-plane) for two metals of steel and aluminum in two opposite clamped edges boundary conditions with the imperfection of ω= 5-e5 and the crack angles of a: 0 degree, b: 45 degrees, and c: 90 degrees

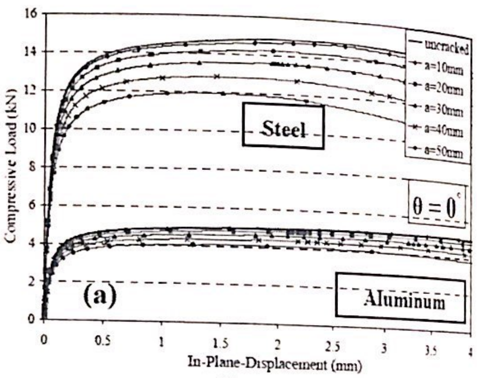


Fig. 11(a)

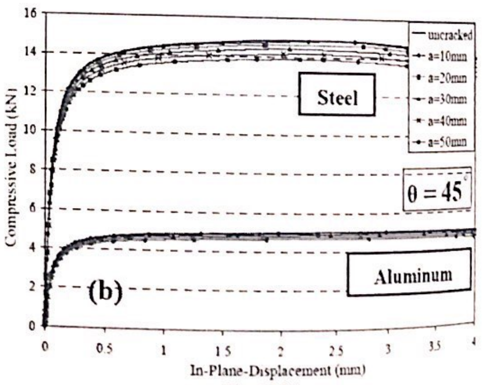


Fig. 11(b)

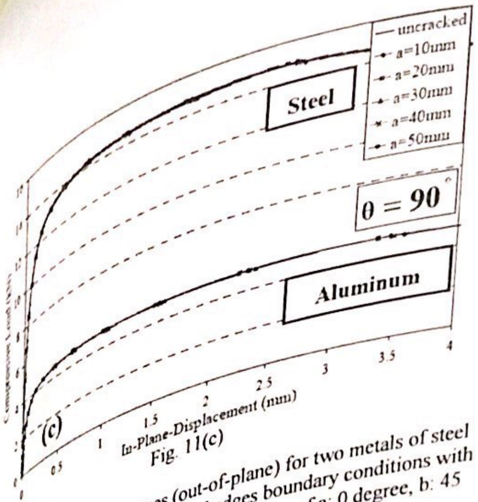


Fig. 11(c)

Fig 11. load-displacement curves (out-of-plane) for two metals of steel and aluminum in two opposite clamped edges boundary conditions with the imperfection of ω= 5-e5 and the crack angles of a: 0 degree, b: 45 degrees, and c: 90 degrees

Furthermore, we see that the increase of the crack angle, reduces the effect of the presence of the crack and if the crack length is increased a perceptible or tangible change is viewed in buckling load in such a way that the greatest difference of critical load of buckling amounts to 2780N in 0-degree angle and to 969N in 45-degree angle and to 66N in 90-degree angle which can be ignored. Moreover, we see in the curves that the critical buckling load for steel is three times as much as the critical load of buckling for aluminum. This is due to the difference in the quantities of their Young's modulus. Young's modulus of steel is around 3 times as much as Young's modulus of aluminum. In addition we see that the behaviors of their post-buckling are different from each other to some extent. This is due to difference in their yield strength and plastic behaviors. If we refer to the Fig. 2 we will see that the plastic behaviors of the two metals are different and this difference is displayed in the post-buckling behavior.

VIII. Conclusion

The results show that:

1- The increase of crack length, results in the decrease of the quantity of critical load of the buckling considerably.

2- The increase of the crack angle, decrease the degree of the effect of the crack presence and this effect can be ignored in the angles near to 90 degrees.

3- The critical buckling load in under axial compressive load cracked plates is directly related to the boundary conditions.

4- In under axial compressive load cracked plates, the increase of the imperfection causes the decrease of the critical buckling load only and the curves are made converged to a definite extent.

5- The increase of crack length causes the decrease of the general buckling load and the curves go forward in an almost parallel way.

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