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Calculation of Stress Intensity Factor by Algebraic Emulator Based on Statistical Resultants of FRANC2D in Rotary Cracked Disks

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Abstract: The aim of this study is the determination of Stress Intensity Factor (SIF) and the crack growth behavior of a rotating disk with uniform thickness. The rotary disk with a central hole and radial crack has studied. Then, results of analysis of crack growth and effect of each effective parameters have been investigated at plane stress condition with a finite element analysis software (FRANC2D). Finally using obtained results from finite element method, we set out a mathematical emulator (a formula for determination of SIF) based on the statistical resultants that can predict crack behavior and stress intensity factor of a cracked rotating disk during crack growth for each model.

Key words: Rotating disks, emulator, simulator, crack, FRANC2D, stress intensity factor

INTRODUCTION

Till now many researches have accomplished to predict stress intensity factors during the crack growth in rotary disks and a lot of these researches base on analytical methods (Roos and Griesinger, 1987). This equation really is results due to analytical methods that are compared to numerical analysis results. Also, weighted function method is used to predict stress intensity factors (Lorenzo and Cartwright, 1994; Schineider and Danzer, 1989). Also, experimental methods are used by researches that it can mention to standard samples to predict real stress intensity factors (Charls *et al*., 1995). In this topic, crack position and number of cracks in a rotary disk are studied such as radial or peripheral cracks (Wilson and Meguid, 1995; Isida, 1995; Smith, 1985).

In this study, it will demonstrate how can achieve to algebraic equation based on statistical resultant with utilizing algebraic methods. This equation includes some effective parameters for determination of stress intensity factor of a rotating disk. In this research, we will use a numerical analysis simulator (FRANC2D software) to predict stress intensity factor by an emulator (an equation for calculation of SIF) in cracked rotary disks. Using analytical methods we can show that when we consider geometrical parameters with unique values, only peculiar physical effective parameter is Poisson's ratio.

MATERIALS AND METHODS

Nowadays in optimization researches, reducing computational time and economic cost and having good accuracy is more important. Emulators technology is good example for these aims. So, emulators are useful and quick methods in different optimization problems. In other words, emulators are statistical functions and display behavior of case study, so, emulators can locate to predict a simulator's behavior. Thus, we can use the emulators as a useful substitution for prediction of behavior of a simulator (Table 1).

In this process, to create an emulator at first step, it must select valid data, then we can make an emulator base on resultant data from sampling (design points). Each design point contains effective parameters in problem and equivalent output data from simulator. In this case, accuracy of the emulator is important. Indeed accuracy of an emulator depends on effective parameters in system and number of design points. To make use of emulators can be obvious optimum designing in complex systems, so decreases costs and computational time and risk in different problems.

Table 1: Comparison between an emulator and a simulator

	Speed of calculation	Accuracy	Optimization
Simulator	Slow	Very good	Local
Emulator	Quick	Good	Total

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CRACKED ROTARY DISKS WITH CENTRAL HOLE

Here, an algebraic emulator to predict crack growth behavior in rotary disks with central hole is considered. We want to find equations for crack growth behavior in rotary disks with central hole. Before this research, many equations were designated, but these equations have derived from analytical methods (Roos and Griesinger, 1987). Initially for study in crack growth behavior in rotary disks with central hole, a model in FRANC2D software with three main variables has considered, also effective geometrical parameters show in Fig. 1 and Table 2. FRANC2D is skilled software for analyzing crack growth in 2-dimensional and 3-dimensional geometry. This software was made by mechanical department in Cornell University and it is free to use. This software is able to draft geometrical model, mesh generation, loading, creating crack and show crack growth. Also this software is able to draft crack growth graphs. Figure 2 shows of FRANC2D menus.

In crack growth study in rotary disks with central hole, 144 tests are sufficient for extend studies. Totally 144 different samples with specified effective parameters and equal distances have calculated (Table 2) and these samples have selected to analyze by FRANC2D software. The sample is a disk that it's radius and density is unique (i.e., one) and it's rotary velocity is 1 rad sec^{-1}. Variation range of variables has shown in Table 2.

Where, ν is the Poisson's ratio, R_i, inner radius of disk, R_o, outer radius of disk and L is crack length.

In continue, behavior of each effective parameter is studied. Figure 3 shows extrapolate curve on many test points that stress intensity factor has shown in terms of radius of the hole variations, also stress intensity factor in the tip of crack has calculated in FRANC2D software by finite element method.

Results are derived from finite element analysis in FRANC2D software show that for $R_i/R_o>0.1$, Eq. 1 able to predict stress intensity factors in the tip of crack with good accuracy. The main aim is finding equations that

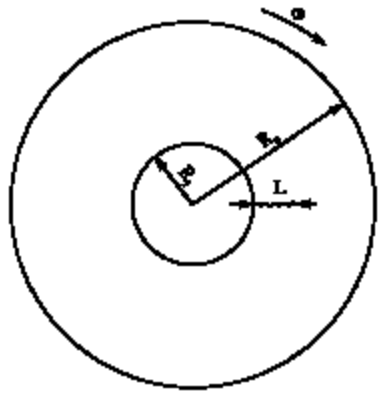

Fig. 1: Schematic view of a rotary disk and effective parameters in crack growth

Table 2: Variation range for each parameter

Variables	Minimum	Maximum
Poisson ratio (ν)	0.20	0.40
R_i/R_o	0.05	0.55
L/R_o	0.05	0.40

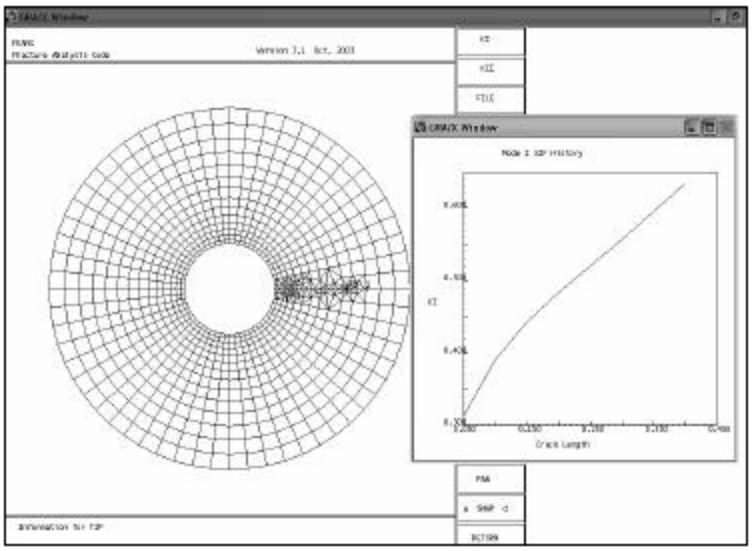

Fig. 2: A view of FRANC2D software

have minimum number of factors and maximum of accuracy. Therefore, CurveExpert 1.3 software was used because this software plot reputable curves with fitting curves on experimental data and measures errors and finally selects the best of all curves that has minimum error and minimum numbers of parameters (Table 2). Results are derived from this modeling show that Harris model (Eq. 1) is valid for each value of L/R_o. Also, Eq. 2-4 are derived with CurveExpert 1.3 software.

$$K_I = \frac{1}{a + b(\frac{R_i}{R_o})^c} \tag{1}$$

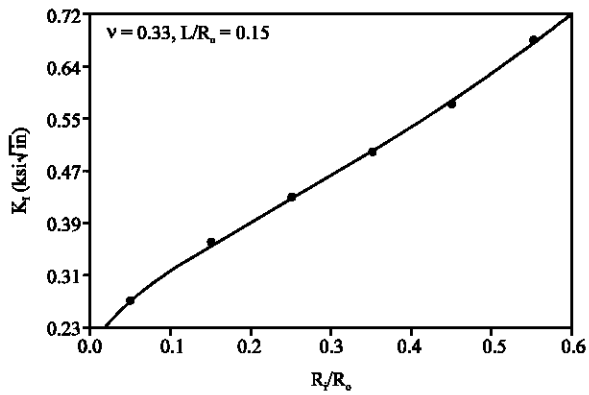

Fig. 3: Stress intensity factor in terms of radius of the hole variations

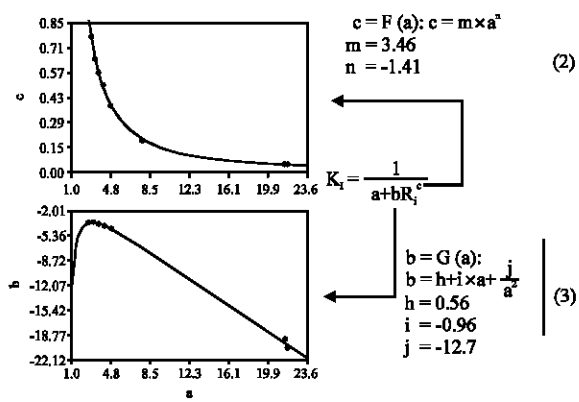

Fig. 4: Variations of parameters c and b in terms of parameter a

Table 3 shows variant values of coefficients for Eq. 1 due to L/R_o.

Equation 1 shows a relation between stress intensity factor and (R_i/R_o) and in next stage for minimizing effective independent variables in Eq. 1 we search a relation between a, b and c parameters. Thus, according

Table 3: Coefficients relate to Eq. 1 for ν = 0.33

L/R_o	a	b	c
0.1	21.75	-20.45	0.049
0.15	7.76	-6.99	0.183
0.2	4.78	-4.50	0.383
0.25	3.99	-4.03	0.501
0.3	3.60	-3.85	0.568
0.35	3.24	-3.71	0.649
0.4	2.89	-3.69	0.777

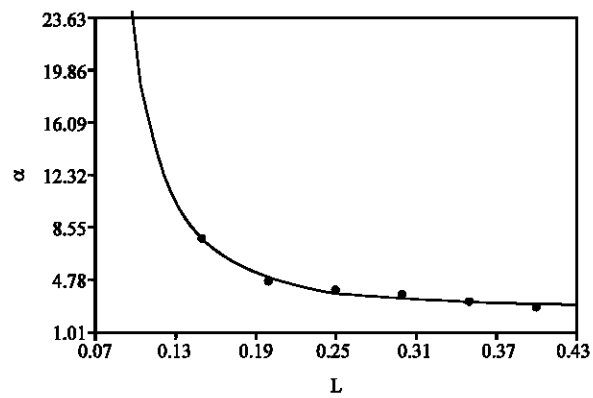

Fig. 5: Variations of parameter *a* curve in terms of crack length (L/R_o)

as Fig. 4 shows, variables b and c are obtained as functions of a.

Furthermore, according to Eq. 4, parameter a has considered like Fig. 5 as function of crack length (L/R_o). Also, Fig. 6 has illustrated stress intensity factor in terms of radius ratio (R_i/R_o) and crack length (L/R_o).

$$a = H(\frac{L}{R_o}) : a = e \times f^{(R_o/L)} \times (\frac{L}{R_o})^g \qquad e = 2.24,\ f = 1.49,\ g = 1.76 \tag{4}$$

When, Eq. 2, 3 and 4 act on Eq. 1 then Eq. 5 will get, where L is as (L/R_o) and R_i is as (R_i/R_o). This recent equation is valid for ν = 0.33 and it is obvious that for different non-unique values, term of $\rho\omega^2 R_o^{5/2}$ will multiply on Eq. 5 (Table 4).

$$K1 = \frac{{}_{e^2f}\left(2\frac{1}{L}\right)_L{}^{(2g)}}{\left[{}_{e^3f}\left(3\frac{1}{L}\right)_L{}^{(3g)}{}_{+Ri}\left({}_{m}\left({}_{ef}\left(\frac{1}{L}\right)_{L^g}\right)^n\right)_{he^2f}\left(2\frac{1}{L}\right)_L{}^{(2g)}{}_{+} {}_{Ri}\left({}_{m}\left({}_{ef}\left(\frac{1}{L}\right)_{L^g}\right)^n\right)_{ie^3f}\left(3\frac{1}{L}\right)_L{}^{(3g)}{}_{+j\,Ri}\left({}_{m}\left({}_{ef}\left(\frac{1}{L}\right)_{L^g}\right)^n\right)\right]} \tag{5}$$

The results obtained from emulator and finite element software FRSANC2D are compared in Table 5.

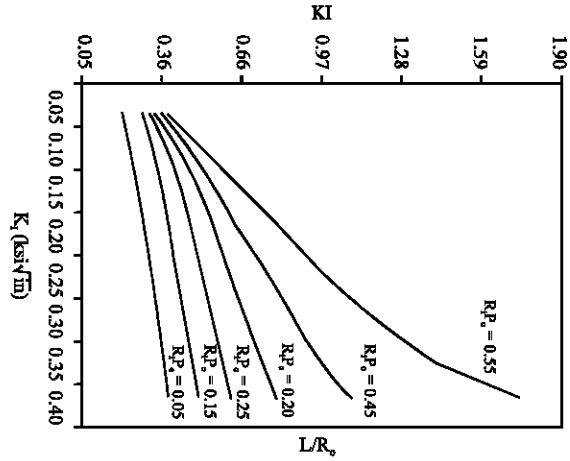

Fig. 6: Variations of stress intensity factor in terms of variations of crack length for different radial ratios and ν = 0.33

Table 4: Coefficients relate to Eq. 5 for ν = 0.33

e	2.239
f	1.494
g	0.756
h	0.557
i	-0.945
j	-12.698
m	3.460
n	-1.409

Table 5: Comparison between results of emulator with results of finite element analyzing (FRANC2D software)

L/R_o	R_i/R_o	Emulator (K_I)	Finite element (K_I)
0.1	0.25	0.3411	0.3816
0.15	0.15	0.3436	0.3609
0.2	0.35	0.5624	0.5620
0.25	0.55	1.0235	0.9927
0.3	0.15	0.4452	0.4401
0.35	0.05	0.3707	0.3610
0.4	0.25	0.5920	0.6252
0.3	0.35	0.6920	0.6713

The stress intensity factor variations in due to Poisson's ratio simulator compare with emulator accuracy is very small, therefore we ignore of the Poisson's ratio variations in calculations. So, Eq. 5 is a suitable emulator to predict stress intensity factors in the tip of crack for geometrical variations of cracked rotating disks. In this case study we can calculate stress intensity factors quickly without using finite element software by replacing geometrical parameters of crack and length of rotating disk in Eq. 5.

CONCLUSION

In this research, geometrical parameters variations and Poisson's ratio variable are studied and we approached to an algebraic equation to predict stress intensity factor for crack growth in rotary disks. At first, an algebraic equation came to hand for ν = 0.33, which results showed stress intensity factor variations in simulator as compared to Emulator accuracy is very small. So, a good Emulator has been achieved to predict stress intensity factors in the tip of crack and finally results of simulator (finite element software FRANC2D) and results of emulator (algebraic equation) have been compared. The results obtained from two methods are too near to others, thus the emulator acts very accurate.

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