

## How Does Large-scale Wind Power Generation Affect Energy and Reserve Prices?

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**Abstract-** Intermittent nature of wind power faced ISO and power producers with new challenges. Wind power uncertainty has increased the required reserve capacity and deployment reserve. Consequently, large-scale wind power generation increases ISO costs and consequently reserve prices. On the other hand, since wind power producers are price taker, large-scale wind power generation decreases residual demand and consequently decreases energy and reserve prices. In this paper, impacts of large-scale wind power generation on energy and reserve markets are studied. To this end, we need to know bids of power producers. But, bids of power producers are unknown and changes if wind power penetration is varied. To overcome this problem, first equilibrium of day-ahead energy market is computed at the presence of large-scale wind power generation considering hour-ahead deployment reserve market scenarios. Then, equilibrium of hour-ahead reserve market is computed considering results of day-ahead market. Finally, impacts of large-scale wind power generation on energy and reserve markets are studied at the markets equilibria. The presented model is applied to an 18-unit power system and the results are analyzed.

**Keyword:** Supply Function Equilibrium, Energy Market, Reserve Market, Wind Power Penetration.

### Abbreviations

EPEC	Equilibrium Problem with Equilibrium Constraints
ISO	Independent System Operators
KKT	Karush Kuhn Tucker optimality conditions
MCP	Market Clearing Price
SFE	Supply Function Equilibrium
WPPs	Wind Power Producers

### Nomenclature

#### A. Sets

$F$	Set of set of generating firms
$S$	Set of forecast error or reserve scenarios
$S_n$	Set of negative reserve scenarios
$S_p$	Set of positive reserve scenarios

#### B. Indices

$i$	Index for generating firm
$s, s_0$	Index for scenario

#### C. Constants

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$a_i$	Intercept of cost function of generating firm $i$ (\$/MWh)
$b_i$	Slope of supply function of generating firm $i$ (\$/MW <sup>2</sup> h)
$Q_{gi}^{\max}$	Maximum generation capacity of firm $i$ (MW)
$Q_s^r$	Total required positive or negative reserve at scenario $s$ (MW)
$Q_0$	Total day-ahead load (MW)
$Q_D$	Residual load at day-ahead market (MW)
$Q_w$	Wind power forecast (MW)
$Q_w^{\max}$	Max. power output of wind power in system (MW)
$N_f$	Numbers of firms
$N_s$	Numbers of reserve scenarios
$\rho_s$	Probability of occurring reserve scenario $s$
$\sigma_w$	Standard deviation of wind power forecast error
$\sigma_D$	Standard deviation of load forecast error
$\sigma$	Standard deviation of wind and load forecast error

#### C. Variables

$r_{is}/r_{is_0}$	Reserve of firm $i$ at scenario $s/s_0$ (MW)
$r_{isn}$	Negative reserve of firm $i$ at scenario $s$ (MW)

$r_{isp}$	Positive reserve of firm $i$ at scenario $s$ (MW)
$Q_{gi}$	Generation power of firm $i$ in day-ahead energy market (MW)
$\alpha_{ei}$	Intercept of bid of firm $i$ at day-ahead energy market (\$/MW)
$\alpha_{ris}$	Intercept of bid of firm $i$ at scenario $s$ of hour-ahead reserve market (\$/MW)
$\lambda_e$	MCP of day-ahead energy price (\$/MW)
$\lambda_{rs}, \lambda_{rs_0}$	MCP of reserve market at scenario $s/s_0$ (\$/MW)

## 1. INTRODUCTION

Penetration of renewable energy resources is growing very fast around the world. Wind power, by supplying 10.4% of the total electricity demand of EU's electricity [1], [2], is one of the most important renewable resources. Despite the environmental benefits of wind power, increasing the installed capacity of Wind Power Producers (WPPs) affects the long-term and short-term operation strategies of the Independent System Operators (ISO) and generating firms, and provides some new challenges. The output power of WPPs is uncertain and is not fully controllable. Although the forecasting tools of WPPs' output power have improved significantly in the recent years, their forecast error is still considerable. Hence, the most important challenge in the electricity markets with large-scale penetration of renewable resources is keeping the balance of power production and consumption at each moment of the electricity market.

Impacts of large-scale penetration of WPPs on electricity markets have been studied in the literature. Studies can be classified into two main viewpoints: market players' viewpoint including WPPs and non-WPPs and ISO's viewpoint. References [3-7] study the problem from the viewpoint of market players. Reference [3] proposes an optimization model for maximizing the profit of a generating firm that participates in both day-ahead and reserve market. Mutual impacts of electricity and reserve markets are ignored and energy and reserve market prices are assumed to be known. Reference [4] proposes a bidding strategy for WPPs in both energy and reserve market. Energy and reserve market prices are assumed to be known and wind uncertainty is extracted from wind speed PDF using Monte-Carlo simulation. In [5], it is shown that WPPs can increase their profit by bidding in both energy and reserve markets. In this situation, part of wind power variations is diverted into the reserve market, reducing the need for additional reserve required to balance short-term variations of wind power. Reference [6] proposes an offering strategy for

WPPs in energy and reserve markets considering imbalance penalties for wind power forecast error. Impacts of wind power generation on energy and reserve market prices are ignored. Impacts of forming coalitions between renewable power producers on uncertainty reduction, market power, and strategic bidding of renewable power producers in day-ahead electricity markets are studied in [7]. Reserve electricity market is not considered in this study. References [8-15] study the problem from the viewpoint of ISO. Reference [8], proposes Equilibrium Problem with Equilibrium Constraints (EPEC) approach for modeling an electricity market with large-scale intermittent resources. Reserve electricity market and forecast error of the intermittent resources are not considered in the study. Reference [9] proposes a scenario-based stochastic programming, look-ahead dispatch, risk-limiting dispatch, and robust optimization for handling the uncertainty caused by large-scale WPPs. The goal is providing economic benefits for the system while keeping the system as much as reliable. Reserve market is not considered in the proposed model. In reference [10] impacts of large-scale integration of WPPs on the electricity market prices is studied. Perfect competition is assumed for electricity market. Time series-based prediction models are considered in this study to investigate the impact of wind power forecast accuracy on electricity market prices. Reserve electricity market is not considered in [10]. Simulation results show spikes in market prices due to lack of a suitable power balancing mechanism. In reference [11] a two-stage stochastic model for determining the required reserve level in the electricity market with large-scale wind power penetration is introduced. Perfect competition is assumed for electricity market in [11]. In Reference [12], to study interactions among energy storage systems, wind farms and conventional generators, a bi-level equilibrium model is introduced. Reserve market is not considered in this study and cost functions of firms are considered linear. References [13] and [14] propose an EPEC approach to find the equilibrium of an oligopolistic electricity market in which WPP joins with a thermal power plant and the aggregated firm participates in the energy electricity market as a single firm. Reserve electricity market is not considered in these studies. Supply function model is used to model competition in [8, 13-14]. In reference [15] an EPEC approach is presented to find the equilibrium in an oligopolistic market with large-scale strategic WPPs. Energy and real-time markets are considered in [15]. The marginal costs of units are assumed to be constant and each market player submits a price bid for participation in the electricity market.

In this paper, impacts of large-scale integration of WPPs in electricity markets on energy and reserve market prices is studied. To this end, assuming large-scale wind power integration, an SFE model for day-ahead energy market considering uncertainty in hour-ahead reserve market, and an SFE model for hour-ahead reserve market considering day-ahead market results are presented. The proposed model consists of coupled optimization problems. Each optimization problem is a bi-level problem with two inner optimization problem. The proposed approach is applied to a case study and the impacts of large-scale wind power integration on energy and reserve prices are studied and simulation results are discussed.

Compared to references [8], [13-14], this paper considers both energy and reserve markets while reserve electricity market is not considered in references [8, 13-14]. Compared to reference [15], this paper considers supply function model as competition model of the market which is a more realistic model than Bertrand model for studying the electricity markets. References [10] and [11] assume perfect competition model for the electricity markets while this paper mode oligopolistic electricity market which is a more realistic structure for electricity markets.

The rest of the paper is organized as follows, in section 2, the optimization problem is defined and assumptions are presented. Problem is formulated in sections 3. Simulation results are discussed in section 4 and finally, conclusions are presented in section 5.

## 2. PROBLEM DEFINITION

In this paper, an electricity market that consists of a day-ahead energy market and an hour-ahead reserve market is considered. Electricity market includes a large-scale WPP. Suppose market regulator would like to know how large-scale wind power generation affects energy and reserve prices.

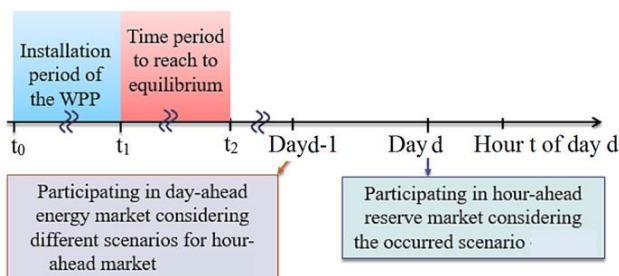


Fig. 1. Time horizon of the model

To answer the regulator's inquiry, it is assumed that WPP has been installed and market players have adapted

to the new condition and have reached to their optimal strategies. Hence, it is assumed that the energy and reserve markets are in their Nash equilibrium. In this research, we study one hour of the energy and reserve markets after passing a mid-term from installing of large-scale WPP, when the markets approached their equilibrium. In fact, it is a short-term study in the medium-term future. According to the above-mentioned assumptions, short-run operational constraints such as minimum up/down time constraints and ramp rate constraints are ignored

### 2.1. Modeling assumptions

In this study, it is assumed that the understudy electricity market is an oligopoly. It is assumed that required capacity for reserve is provided in a long-term market or through capacity payment. The amount of deployment reserve is determined in the hour-ahead market, when there is no uncertainty in demand and wind power generation. The aim of this paper is to study the impacts of large-scale wind power generation on energy and deployment reserve prices. Hence, hereafter the word "reserve" represents "deployment reserve" in this paper. Pool structure and supply function model are chosen for energy and reserve markets. Market players compete with each other for increasing their profits. It is assumed the market players are rational and try to maximize their profit by offering optimal bids. Each generation firm offers a supply function for the energy market, a supply function for positive reserve, and a supply function for negative reserve market. It is assumed that The slopes of supply functions of each generating firm are equal to the slopes of its marginal cost of the firm and the intercepts of its supply functions are chosen such that the total profit of the firm in both energy and reserve markets are maximized.

It is assumed that slopes of energy and reserve supply functions of each generating firm are equal to slope of marginal cost function of the firm and intercepts of its supply functions are chosen such that total profit of the firm in both energy and reserve markets are maximized. Suppose a large-scale WPP is added to the electricity market.

Large-scale WPP is a single wind farm or a set of wind farms that its output power is high enough to affect the electricity market price. Large-scale penetration of WPPs in electricity markets is not out of reach even in today's electricity markets. For instance, in Denmark 60% of the total demand was supplied by WPPs in January 2016 [16]. Germany produced 36% of its consumption by renewable resources in 2017. Germany could also produce about 100% of its consumption from renewable

resources on 1st January 2018 [17]. WPP is a price-taker firm and all of its generation power is purchased by ISO. In order to decrease wind power forecast error of WPP, WPP is penalized for forecast error more than a specified value. Although forecast error penalty increases the forecast accuracy and consequently affects the strategic behavior of other players, it does not appear in the modeling since WPP is not a strategic player. The demand is assumed inelastic. Uncertainty in both demand and wind power forecast is considered by some discrete scenarios. The hour-ahead market must cover forecast errors in demand and wind power in addition to hour-ahead load. Transmission constraints are ignored. Uniform pricing is considered in energy and reserve markets. In order to study the impact of large-scale wind power penetration on energy and reserve electricity markets, bids of different market players should be known. Market players are not willing to expose their bids. So bids of market players are unknown and vary in different conditions. Moreover, installing a new WPP changes the bids of market players. To overcome this problem, it is assumed that the market has reached to its Nash equilibrium in presence of WPP. So bids of market players at the Nash equilibrium of the joint energy and reserve markets are used to study the impacts of large-scale wind power generation on the electricity market.

## 2.2. Time horizon

Time horizon of the problem is shown in Fig.1. In time axis of Fig.1,  $t_0$  is the present time. The market regulator is willing to study the impacts of adding a large-scale wind power plant on the electricity market. Based on Fig.1, it is assumed that installation of wind power plant starts at  $t_0$  and finishes at  $t_1$ . During the period between  $t_1$  and  $t_2$  energy and reserve electricity markets run frequently and it is assumed that the markets reach their Nash equilibrium by  $t_2$ . This means that the market players optimize their bids by changing their offers and watching the behavior of other players, frequently. At this point, none of the market players is willing to change its bid unilaterally since his or her profit would not increase [18].

In order to run the market for hour  $t$  of the day  $d$ , ISO predicts the output power of WPP on day  $d-1$  and send the result to market players. Market players participate in the electricity market at day  $d-1$  by offering a supply function for selling energy at hour  $t$  of day  $d$ . Market players consider the profits from both energy and reserve electricity markets when submitting their bids to the ISO. Electric energy market at hour  $t$  of day  $d$  is considered as a deterministic market because it runs based on forecasted values of load and output power of WPP.

Reserve market, which runs one hour before the hour  $t$  of day  $d$  is also a deterministic market because the values of load and output power of WPP can be forecasted precisely one hour before the real time. Since these two markets do not run simultaneously, market players take into account the reserve market for hour  $t$  of day  $d$  when they bid in the energy market for hour  $t$  of day  $d$ , on day  $d-1$ . Considering reserve market of hour  $t$  of day  $d$ , on day  $d-1$ , is confronted with uncertainty. Generating firms take into account the uncertainty in reserve market by considering different scenarios for demand and wind power forecast error when they would like to determine and submit their bids on day  $d-1$ .

## 3. PROBLEM FORMULATION

In order to study the impact of large-scale penetration of WPP on electric energy and reserve markets, the bids of market players in energy and reserve markets in presence of WPP should be determined. To this end, bids of market players at Nash equilibrium are used. Hence, we need to find the Nash equilibrium of the joint energy and reserve markets in presence of a large-scale WPP.

On day  $d-1$ , the energy market is a deterministic market and it is scheduled based on forecasting value of wind power and demand. Considering reserve market on day  $d-1$  is confronted with uncertainties caused by forecast errors of the wind power and demand. Since WPP is assumed to be price taker, it can be modeled as a negative load. Hence, forecast error scenarios of demand and WPP's output can be described by a single set of discrete scenarios called forecast error scenarios. These scenarios and their probabilities can change based on the volume of demand and forecasted power of WPP. Demand and wind data are assumed to be non-correlated. The scenario in which real-time demand minus real-time wind power is smaller/greater than the forecasted demand minus the forecasted wind power is called a positive/negative reserve scenario.

Positive and negative reserves of firm  $i$  at scenario  $s$  are shown by  $r_{isp}$  and  $r_{isn}$ , respectively. Since in each scenario one of  $r_{isp}$  or  $r_{isn}$  is zero,  $r_{is}$  is used for indicating positive or negative reserve of firm  $i$  at scenario  $s$ .  $S_p$  and  $S_n$  represent the set of positive and negative reserve scenarios, respectively, and  $S = S_p \cup S_n$  is the set of all reserve scenarios.

Assume that  $Q_{gi}$  and  $r_{is}$  are power and reserve of firm  $i$  in energy market and reserve market at scenario  $s$ , respectively. The marginal cost function of firm  $i$  at scenario  $s$  is  $MC_i = a_i + b_i(Q_{gi} + r_{is})$ . As mentioned before, each firm submits a supply function for selling

electric energy, a supply function for selling positive reserve, and a supply function for selling negative reserve to ISO. The slopes of these functions are equal to the slope of the marginal cost function of firm  $i$  and their intercepts are chosen such that the submitted functions maximize the profit of the firm. Hence, the bid of firm  $i$  for selling energy is  $bid_{ei}(Q_{gi}) = \alpha_{ei} + b_i Q_{gi}$  and for selling positive reserve at scenario  $s \in S_p$  is equal to  $bid_{ris}(r_{isp}) = \alpha_{risp} + b_i(Q_{gi} + r_{isp})$  and for selling negative reserve at scenario  $s \in S_n$  is  $bid_{ris}(r_{isn}) = \alpha_{risn} + b_i(Q_{gi} + r_{isn})$ . Since only positive or negative reserve is required at scenario  $s$ , bids for selling positive and negative reserves can be shown by  $bid_{ris}(r_{ris}) = \alpha_{ris} + b_i(Q_{gi} + r_{is})$

On day  $d-1$ , each market player submits his or her proposed supply function to ISO. The ISO determines Market Clearing Price (MCP) and generation power of each firm by maximizing social welfare of the energy market. The social welfare maximization problem of ISO in energy market can be formulated as follows.

$$Max (J_{ISO\_E}) = -\sum_{i=1}^{N_f} \left[ \alpha_{ei} Q_{gi} + \frac{1}{2} b_i Q_{gi}^2 \right] \quad (1)$$

$$s.t. \quad \sum_{i=1}^{N_f} Q_{gi} = Q_D \quad (\lambda_e) \quad (2)$$

Where,  $Q_D$  represent residual demand, which is equal to demand minus forecasted wind power,  $N_f$  represent numbers of firms, and  $\lambda_e$  is Lagrangian multiplier of equality constraint of Eq. (2) and is equal to MCP in energy market.

In practice usually, there is no day-ahead reserve market and it is an hour-ahead reserve market. However, when a power producer would like to bid in day-ahead energy market he or she considers hour-ahead reserve market. Hence, the power producer determines his or her bids so that his or her total profit in day-ahead energy market and hour-ahead reserve market is maximized. Since day-ahead energy and hour-ahead reserve markets have been run many times, power producers have enough experiences to determine their bids so that their total profits in day-ahead energy and hour-ahead reserve markets are maximized. Since on day  $d-1$ , the value of required reserve is not known and depends to forecast error of demand and wind power, different forecast error scenarios are considered for demand and wind on day-ahead reserve market.

In order to consider hour-ahead reserve market on day  $d-1$ , social welfare at each forecast error scenario is maximized. Since ISO optimization for each scenario is

independent of other scenarios, instead of maximizing social welfare for each forecast error scenario, the sum of social welfares of the ISO at different scenarios is maximized as below:

$$Max (J_{ISO\_R}) = Max \sum_{s=1}^{N_s} (J_{ISO\_R}^s) \quad (3)$$

$$= -\sum_s \sum_{f=1}^{N_f} \left[ (\alpha_{fjs} + b_f Q_{gf}) r_{fs} + \frac{1}{2} b_f r_{fs}^2 \right]$$

$$s.t. \quad \sum_{f=1}^{N_f} r_{fs} = Q_s^r, \quad \forall s \in S, (\lambda_{rs}) \quad (4)$$

Where  $Q_s^r$  represents the total required positive or negative reserve at scenario  $s$  and  $\lambda_{rs}$  is Lagrangian multiplier of equality constraint of Eq. (4) and is equal to MCP for reserve market at scenario  $s$ .

Each market player tries to maximize its profit in the energy and reserve electricity markets. From the viewpoint of market players on day  $d-1$ , energy market for day  $d$  is a deterministic market while reserve market is a probabilistic market due to different forecast errors and consequently reserve scenarios. So, profit optimization problem of market player  $i$  is formulated as follows:

$$Max E (\pi_{is}) \quad (5)$$

$$= \sum_s \rho_s \left[ \lambda_e Q_{gi} + \lambda_{rs} r_{is} - a_i (Q_{gi} + r_{is}) - \frac{b_i}{2} (Q_{gi} + r_{is})^2 \right]$$

$$s.t. \quad Q_{gi} \geq 0 \quad (6)$$

$$Q_{gi} \leq Q_{gi}^{max} \quad (7)$$

$$Q_{gi} + r_{is} \geq 0 \quad \forall s \in S_n \quad (8)$$

$$r_{is} \geq 0 \quad \forall s \in S_p \quad (9)$$

$$r_{is} \leq 0 \quad \forall s \in S_n \quad (10)$$

$$Q_{gi} + r_{is} \leq Q_{gi}^{max} \quad \forall s \in S_p \quad (11)$$

Where  $\rho_s$  is the probability of occurring the reserve scenario  $s$ . The first term of the objective function of Eq. (5) represents revenue from energy market, the second term represents expected revenue of firm  $i$  from reserve market over different scenarios. The third term represents the total cost of firm  $i$  for producing energy and reserve.

In order to find Supply Function Equilibrium (SFE) of the energy market, optimization problems of all firms must be solved together. SFE problem is a coupled bi-level problem. In the outer level of each problem profit of a firm is maximized. The outer optimization problem has two inner optimization problem. In the first inner problem, social welfare in energy market is solved and in the second inner problem, social welfare in reserve market is solved.

SFE problem for day-ahead market considering hour-ahead market scenario is depicted in Fig. 2. A method for solving these coupled bi-level optimizations is to find KKT optimally conditions of inner problems and adding them to the optimization problems of the firms (outer problems). These optimization problems are called revised optimization problems. To find the equilibrium of the market, KKT optimally conditions of revised profit optimization problems of all firms are solved together.

As mentioned before, reserve market which runs one hour before hour t of day d is a deterministic market because output power of each market player in the energy market has been determined already and values of demand and output power of wind power plant can be forecasted precisely.

On day d, each of the forecasted error scenarios may occur. For each scenario, equilibrium point for reserve electricity market is calculated. ISO optimization for reserve market at scenario  $s_0$  is formulated as below:

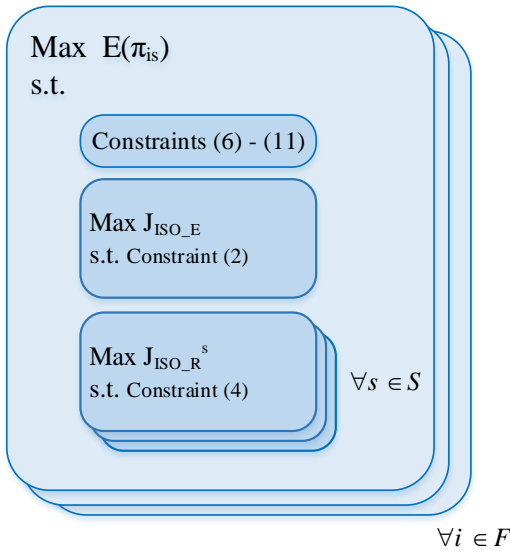


Fig. 2. SFE problem for day-ahead market considering hour-ahead market scenarios

$$Max \left( J_{ISO\_R}^{s_0} \right) = - \sum_{i=1}^{N_f} \left[ \left( \alpha_{ris_0} + b_i Q_{gi} \right) r_{is_0} + \frac{1}{2} b_i r_{is_0}^2 \right] \tag{12}$$

$$s.t. \sum_{i=1}^{N_f} r_{is_0} = Q_{s_0}^r \quad \left( \lambda_{rs_0} \right) \tag{13}$$

Where  $Q_{gi}$  is assumed to be known and fixed. The profit optimization problem of firm  $i$  for a specific reserve scenario is as below:

$$Max \left( \pi_{is_0} \right) = \left[ \lambda_e Q_{gi} + \lambda_{rs_0} r_{is_0} - a_i \left( Q_{gi} + r_{is_0} \right) - \frac{b_i}{2} \left( Q_{gi} + r_{is_0} \right)^2 \right] \tag{14}$$

s.t.

$$r_{is_0} \geq 0 \quad \text{if } s_0 \in S_p \tag{15}$$

$$Q_{gi} + r_{is_0} \leq Q_{gi}^{max} \quad \text{if } s_0 \in S_p \tag{16}$$

$$r_{is_0} \leq 0 \quad \text{if } s_0 \in S_n \tag{17}$$

$$Q_{gi} + r_{is_0} \geq 0 \quad \text{if } s_0 \in S_n \tag{18}$$

Similar to the day-ahead electricity market, SFE can be found for each forecast error scenario that may happen in the hour-ahead reserve market. Fig. 3 shows SFE problem for hour-ahead market. As Fig. 3 shows, in order to find the SFE of the reserve market for a specific forecast error scenario, KKT optimally conditions of the inner problem, i.e., ISO optimization Eqs. (12) - (13), is added to the profit optimization of each firm. Then the SFE is found by writing KKT optimally conditions of revised optimization problems for all firms and solving them together.

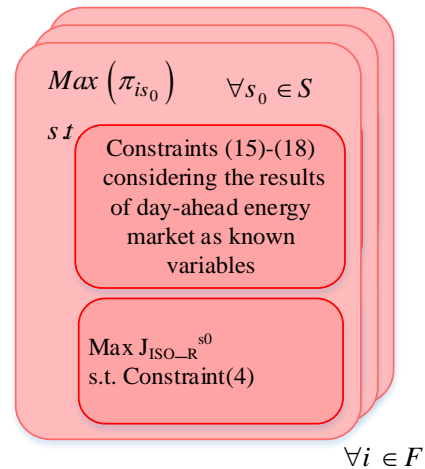
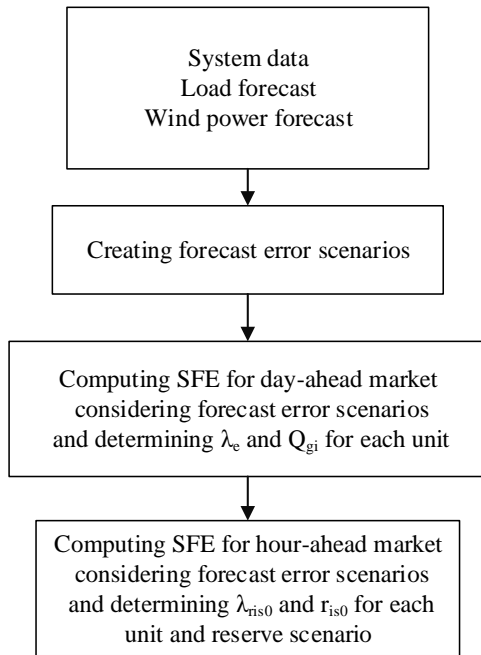


Fig. 3. SFE problem for hour-ahead market



**Fig. 4.** Overview of modelling and solution

An overview of problem solution procedure is presented in Fig. 4.

#### 4. CASE STUDY

In this section, the proposed model is applied to an 18-generator test system and results are discussed. Test system consists of three sets of generators. Each set has 6 generators. The first set consists of generators in IEEE 30-bus test system which are introduced in Table 1. Marginal cost functions of generators in the second (third) set are generated by multiplying the slope of marginal cost functions of the first set in 1.1 (0.9) and multiplying the intercepts of marginal cost functions of the first set in 0.9 (1.1). The aim of adding generators of the second and third generator sets is to increase the number and diversity of generators in the understudy electricity market.

Characteristics of load and WPP are given in Table 3. As Table 3 shows, installed capacity of WPP is equal to 50% of total demand and expected output power of WPP is 29.75% total demand which is similar to situation in some countries like Denmark, Portugal and Spain [19]. Weibull PDF is used to model wind speed uncertainty. In order to consider all different scenarios that may happen for wind power generations, 11 wind power generation scenarios are generated by the sampling method presented in Fig. 5 [20-21]. These scenarios are referred to as forecasted wind generation scenarios and are given in Table 2. In order to model wind power forecast error,

12 scenarios are defined around each forecasted wind power scenario. These scenarios are referred to as wind power forecast error scenarios. It is assumed that the distribution of forecast error of WPP's output power is  $N(0, \sigma_w)$  [22-25], where  $\sigma_w$  is a linear function of forecasted power  $\sigma_w = 0.001 + 0.1 \times Q_w / Q_w\text{-cap}$  [26]. Demand uncertainty is also modelled by a normal PDF  $N(0, \sigma_D)$  where  $\sigma_D$  is 2% of demand [27]. Since it is assumed that WPP's output power and demand are not correlated, PDF of wind and load uncertainty can be expressed by a normal PDF  $N(0, \sigma)$  where  $\sigma = \sqrt{\sigma_w^2 + \sigma_D^2}$  [28-30]. A set of forecast error scenarios are defined for each forecasted demand and wind power scenario using aggregated normal PDF. Forecast error scenarios for two different forecasted wind power scenarios i.e. 20% and 60% of installed capacity and 20 GW demand are depicted in Fig. 6. Reserve scenarios are generated by adding hour-ahead demand to each forecast error scenario. Reserve scenarios (s1... s12) for each forecasted wind generation and 20 GW demand scenario (w0... w10) are depicted in Fig 7.

**Table 1.** Parameters of marginal cost of the firms

	$a_i$ (\$/MWh)	$b_i$ (\$/MWh <sup>2</sup> )	$Q_i^{\max}$ (MW)
<b>Firm 1</b>	20	0.0020	1920
<b>Firm 2</b>	17.5	0.0175	1920
<b>Firm 3</b>	10	0.0625	1200
<b>Firm 4</b>	32.5	0.0083	1320
<b>Firm 5</b>	30	0.0250	720
<b>Firm 6</b>	30	0.025	960

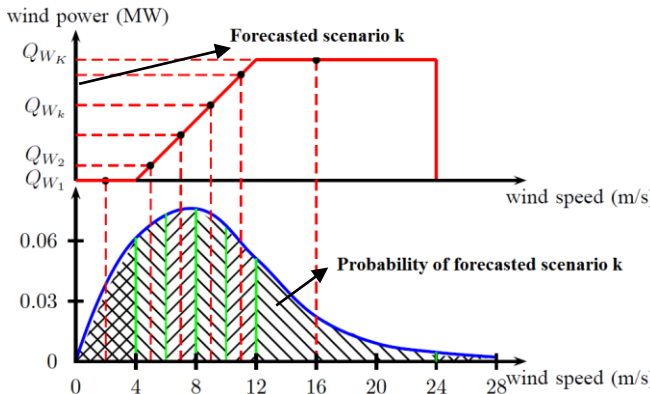
**Table 2.** Forecasted wind generation scenarios

Probability (%)	Wind generation		Scenario number
	(GW)	% of wind capacity	
8.47	0	0	w0
8	1	10	w1
9.47	2	20	w2
10.17	3	30	w3
10.20	4	40	w4
9.71	5	50	w5
8.84	6	60	w6
7.75	7	70	w7
6.56	8	80	w8
5.38	9	90	w9
15.45	10	100	w10

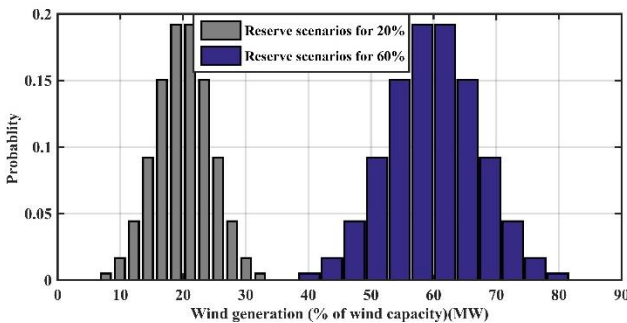


**Table 3.** Test system data

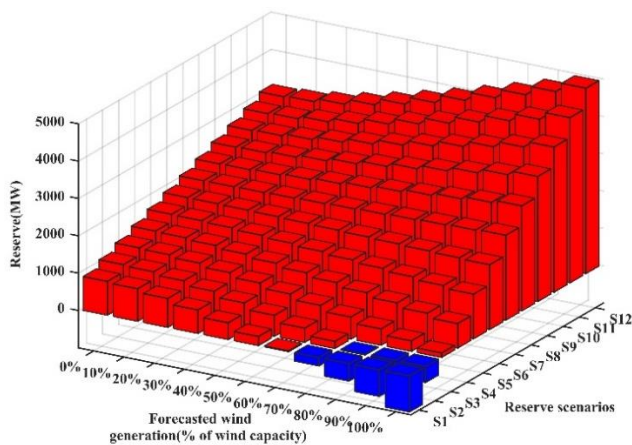
<b>Load</b>	Forecasted day-ahead load (GW)	20
	Forecasted hour-ahead load (GW)	2
<b>WPP</b>	WPP capacity (GW)	10
	Expected output power (GW)	5.95
	Scale parameter of Weibull PDF of wind speed (m/s)	10
	Shape parameter of Weibull PDF of wind speed	1.8
	Cut in speed of wind turbines (m/s)	2.5
	Rated output speed of wind turbines (m/s)	14
Cut out speed of wind turbines (m/s)	25	



**Fig. 5.** Proposed sampling method for creating wind power generation scenarios



**Fig. 6.** Forecast error scenarios around two forecasted wind generation scenarios (20% and 60%) and 20 GW demand



**Fig. 7.** Reserve scenarios ( $s_1 \dots s_{12}$ ) for each forecasted wind generation scenario (0-100%) red color used for positive and blue used for negative scenarios

In the next subsections, first impacts hour-ahead reserve market on the energy and reserve market prices are studied and then simulation results are discussed in different cases.

**4.1 Energy and reserve prices for a specific forecasted power of WPP**

In order to study the energy and reserve market prices for a certain wind generation forecast, assumed that at a certain hour of day  $d$  forecasted output power of WPP is equal to 60% of WPP installed capacity. Forecast error scenarios and reserve scenarios for forecasted output power of WPP are depicted in Figs. 6 and 7. Energy market price, reserve market prices for different reserve scenarios and the expected value of reserve prices are presented in Table 4. Energy market price is equal to 41.67 \$/MWh which is greater than reserve price in first three reserve scenarios and lower than reserve market prices in next eight reserve scenarios.

This happens because reserve prices depend directly on required reserve in reserve market. In the first few scenarios, required reserve and consequently competition level is not high enough that the reserve market prices get greater than energy market. When required reserve gets greater than 738.1 MW, competition in the reserve market reaches to a level that leads to the reserve market prices greater than energy market price.

**4.2. Energy and reserve prices**

Energy price and generation powers of different units are computed by computing equilibrium of day-ahead electricity market considering different hour-ahead reserve market scenarios, i.e. Wind forecast error scenarios, is computed for each wind generation scenario. Then equilibrium of hour-ahead reserve market is computed for each forecast error scenario.

**Table 4.** Energy and reserve prices for different reserve scenarios for 6GW forecasted wind generation scenarios.

Reserve prices (\$/MWh)	Reserve (MW)	Scenario number
40.30	16.900	S1
41.10	377.50	S2
41.53	738.10	S3
41.93	1098.6	S4
42.33	1459.2	S5
42.73	1819.7	S6
43.13	2180.3	S7
43.53	2540.8	S8
43.95	2901.4	S9
44.39	3261.9	S10
44.84	3622.5	S11
45.29	3983.1	S12
<b>Expected reserve price (\$/MWh)</b>		42.94
<b>Energy price (\$/MWh)</b>		41.67



Energy price, maximum and expected value of positive and, minimum and expected value of negative reserve prices over different forecast error scenarios are depicted in Fig. 8 for different wind generation scenarios. Since WPP is assumed to be a zero-bid price-taker producer, as wind generation increases residual demand for non-wind power producers decreases from 20 GW to 10 GW and consequently energy price decreases from 54.6 to 38.8 \$/MWh as shown in Fig. 8. As wind power generation increases, two factors affect energy and positive and negative reserve prices. The first factor is an increase in max wind power forecast error. As wind power generation increases standard deviation of aggregate demand and wind power forecast error increases from 400 to 1010.2 MW [16]. Increase in standard deviation of wind power forecast increases the max forecast error and consequently, increases the required reserve. Increase in required reserve increases the price of positive reserve and decreases the price of the negative reserve. The second factor is decrease in residual demand and consequently total non-wind generation. The decrease in total non-wind power generation decreases the positive and negative reserve prices. The second factor is dominant in this test system and consequently, positive and negative reserve prices decrease as wind power generation increases, as it is shown in Fig. 8.

**4.3. Impacts of different parameters on Energy and reserve prices**

The behaviour of reserve market price is affected by different parameters like amount of wind power generation, the standard deviation of wind power forecast error scenarios and the amount of hour-ahead load. In order to determine impacts of the above-mentioned parameters on energy and reserve prices, it is assumed that two of these parameters are constant and remained one changes step by step.

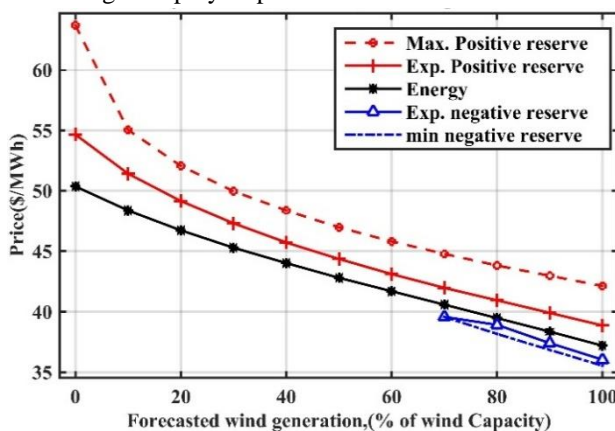


Fig. 8. Energy and reserve market prices at different forecasted wind power generation scenarios

In each step day-ahead market equilibrium considering hour-ahead market and hour-ahead market equilibrium for each wind power forecast error are computed. Fig. 9 shows energy and reserve prices versus wind generation power assuming hour-ahead load and standard deviation of wind power forecast are constant at 1GW and 5%, respectively. As it was discussed in the last subsection, standard deviation of wind power generation and residual demand affect energy and positive and negative reserves prices. In this case, the standard deviation of wind power generation is constant and max wind power forecast error decreases due to increase in wind power generation and consequently decrease in residual demand. In Fig. 9, compared with Fig. 8 by increasing the forecasted power of WPP, the difference between positive reserve price and energy price decreases because the constant assumed standard deviation of wind forecast error in Fig. 9 gets lower than variable standard deviation in Fig. 8 which leads to lower uncertainty in the system and consequently lower available power in reserve market.

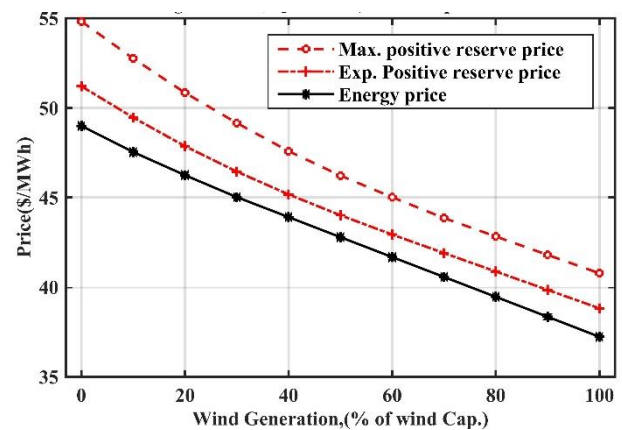


Fig. 9. Energy and reserve prices versus forecasted wind power generation power at hour-ahead load 1GW and standard deviation of wind power forecast are constant 5%

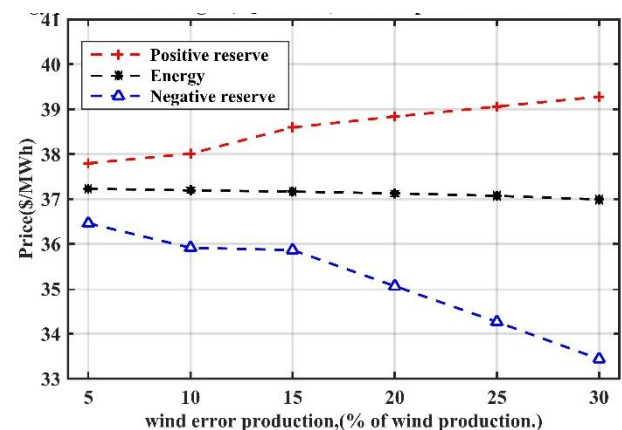


Fig. 10. Energy and reserve prices versus standard deviation of forecast error

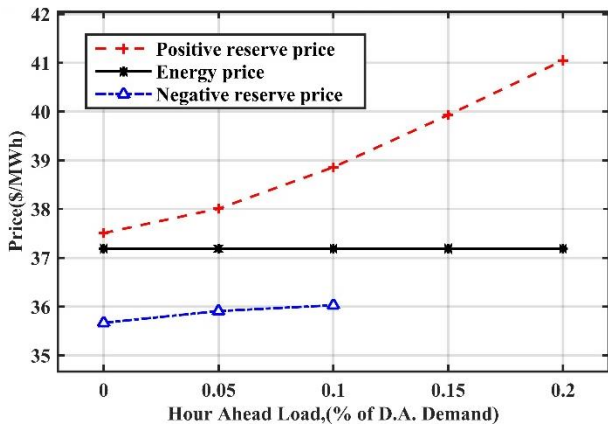


Fig. 11. Energy and reserve prices versus hour-ahead load

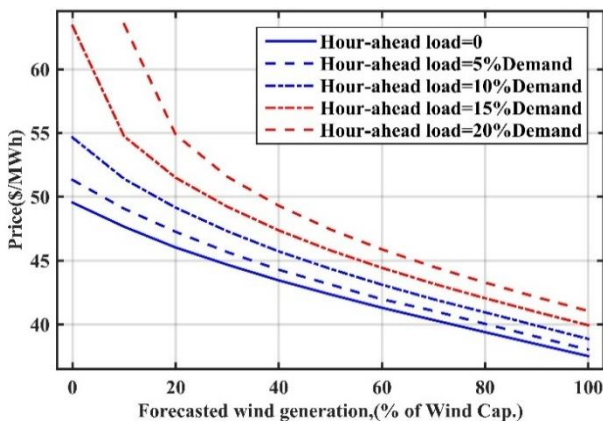


Fig. 12. Expected positive market prices for different values for different forecasted wind power generations

Figure 10 shows energy and reserve prices versus standard deviation of wind power forecast assuming wind power generation and hour-ahead load are constant at 10GW and 2GW respectively. As standard deviation of wind power forecast error increases from 5% to 30% of wind power generation the required positive and negative reserves increase. Increase in required positive and negative reserves increases the price of positive reserve up to 39.2 \$/MWh and decrease the price of the negative reserve up to 33.4 \$/MWh as it is shown in Fig. 10. Fig. 11 shows variations of energy and expected reserve prices versus hour-ahead load assuming wind power generation and standard deviation of wind power for hour-ahead load are constant at 10GW and 1GW respectively. Fig. 12 shows energy and expected positive reserve prices versus different wind power forecast scenarios for different hour-ahead load levels. Both Fig. 11 and Fig. 12 shows that as hour-ahead load increases non-wind power generation increases and consequently positive and negative reserve pieces increases.

Figure 12 also shows when wind power forecast is low, the sensitivity of reserve prices on the hour-ahead load is higher than the case that wind power forecast is high. This happens because when wind power forecast is low,

wind power forecast error is also low and hour-ahead load takes a larger partition of demand in the reserve market compared to the case that wind power forecast is high.

#### 4.4 Effects of large-scale integration of wind power on the profit of the market players

Large-scale integration of wind power in the electricity market affects the profit of market player. In this subsection, profits of two market players are calculated for different forecasted wind generation scenarios and results are discussed. The first firm is the fourth firm of the third set of generators (F4\_3) that is a low-cost firm. The second firm is the fifth firm of the first set of generators (F5\_1) that is a high-cost firm. Profits of the firms in energy market for different forecasted wind power scenarios are shown in Fig. 13. Fig. 13 shows that profits of firm F4\_3 and firm F5\_1 in energy market decreases by increasing forecasted power of WPP up to 83% and 86%, respectively. This happens due to reduction in energy market price and scheduled power of each firm Profits of the firms in reserve market in different forecasted wind generation and reserve scenarios are compared in Fig.14 and Fig. 15.

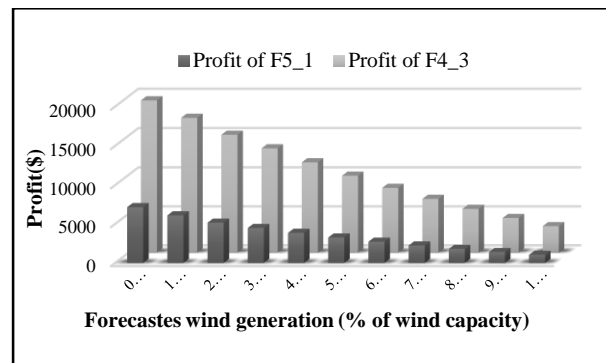


Fig. 13. Comparing profit of F4\_3 and F5\_1 firms in energy market.

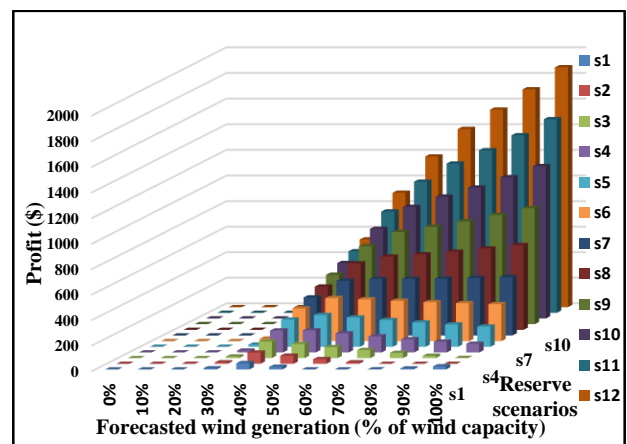


Fig. 14. Profit of the firm F4\_3 in reserve market for different forecasted wind power and different scenarios.

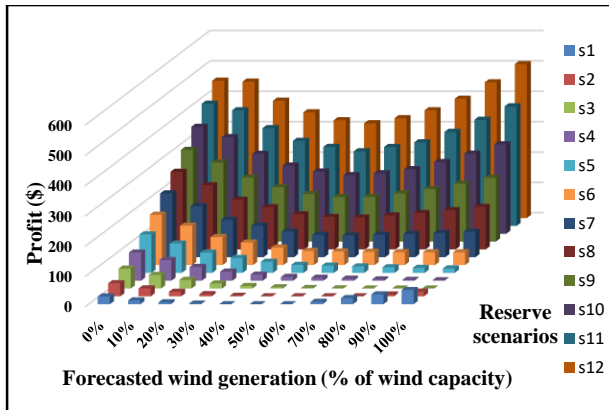


Fig. 15. Profit of the firm F5\_1 in reserve market in different forecasted wind generation and reserve scenarios.

Figure 14 and Figure 15 show that expected profits of the firm F4\_3 and firm F5\_1 in reserve market increases by increasing forecasted power of WPP up to 35.64% and 32% of total profit, respectively. When forecasted power of WPP is low residual demand is high and low-cost firms like firm F4\_3, generate their maximum capacity and do not participate in reserve market. In this situation, higher cost firms like Firm F5\_1 can increase their profit by participating in reserve market. By increasing the forecasted wind power, residual demand decreases and firm F4\_3 cannot win all of its capacity in energy market and participates in reserve market to increase its profit. In this case, the profit of high-cost firm F5\_1 decreases due to increasing the number of low-cost market players in the reserve market. When the forecasted power of WPP gets higher than 50% of its capacity all firms participate in the reserve market. In this case, dominant factor for the profit of each firm is the occurred forecasted error scenario. In the scenarios that forecasted error or residual demand in reserve market is low, the profits decrease by increasing the forecasted power of WPP, but when residual demand in reserve market is high the profits increase by increasing the forecasted power of WPP. In the case that reserve power is negative, i.e. when forecasted wind power is about 70%-100% of wind capacity and if a negative reserve scenario happens, both firms gain profit by reducing their output power in reserve market.

**4.5 Evaluation of impacts of different factors on the energy and reserve market prices**

In order to study the impacts of different factors on the energy and market prices, four following cases are defined, simulated and discussed. Proposed cases are as follows:

**Case 1:** In this case, it is assumed that the understudy electricity market consists of 18 non-wind generators. The only uncertainty in the electricity market is demand

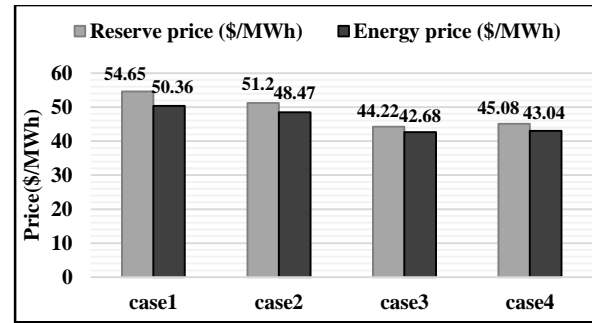


Fig. 16. Energy and reserve market prices for different cases

uncertainty.

**Case 2:** Considering the base case assumptions and adding a strategic market player with marginal cost function parameters equal to 90% of the most expensive firm and generating capacity equal to the average generation power of WPP, i.e., 5.95 GW.

**Case 3:** Considering the base case assumptions and adding a price taker market player without uncertainty and with generating capacity equal to the average generation power of WPP, i.e., 5.95 GW.

**Case 4:** Considering the base case assumptions and adding a WPP with a 10 GW installed capacity.

Energy and expected positive reserve market prices are presented in Fig. 17. Since negative power imbalance is zero in most of the scenarios, negative expected reserve price variations are ignored in this subsection.

Energy and reserve market prices in case 1 are more than other cases due to the no integration of wind production in this case. Adding a strategic player in case 2, increases the installed capacity and consequently the competition and hence decreases energy price from 51.20\$/MWh to 48.47\$/MWh and positive reserve price from 54.65\$/MWh to 51.20\$/MWh. Converting the strategic player of case 2 to a zero bid price taker player in case 3 reduces the residual demand for non-wind generators and consequently decreases energy price from 48.48\$/MWh to 42.68\$/MWh and positive reserve price from 51.20\$/MWh to 44.22\$/MWh. Adding uncertainty to the added unit in case 3 and converting it to a wind generator in case 4 increases energy price from 42.68\$/MWh to 43.04\$/MWh and positive reserve price from 44.22\$/MWh to 45.08\$/MWh.

In conclusion, by adding WPP energy price decreases 1.89\$/MWh due to increase in total generation capacity and consequently increase in competition. It decreases 6.98\$/MWh due to zero bid price taker nature of the added generator. It increases 0.86\$/MWh due to wind uncertainty. Similarly, by adding WPP positive price decreases 1.89\$/MWh due to increase in total generation capacity and consequently increase in competition. It

decreases 5.79\$/MWh due to zero bid price taker nature of the added generator. It increases 0.36\$/MWh due to wind uncertainty and consequently increases in required positive reserve.

#### 4.6 Comparing the proposed method with perfect competition method

In perfect competition method it is assumed that each firm proposes its marginal cost function as its bid function to the ISO, i.e.  $\alpha_{ei}=a_i$ . So, since the bids of market players are known, it is not necessary to solve firms' optimization problem and equilibrium problem turns into solving only the ISO optimization problem. Hence, day-ahead energy market operation considering different reserve scenarios of hour-ahead market, in perfect competition method, can be formulated as below:

$$Max (J_{ISO\_E}) = - \sum_{f=1}^{N_f} \left[ a_{ef} Q_{gf} + \frac{1}{2} b_f Q_{gf}^2 \right] \quad (19)$$

$$- \sum_s \sum_{f=1}^{N_f} \left[ (a_{fs} + b_f Q_{gf}) r_{fs} + \frac{1}{2} b_f r_{fs}^2 \right]$$

$$s.t. \sum_{f=1}^{N_f} Q_{gf} = Q_D \quad (\lambda_e) \quad (20)$$

$$\sum_{f=1}^{N_f} r_{fs} = Q_s^r \quad (\lambda_{rs}), \forall s \in S \quad (21)$$

Same reasoning leads to below optimization for hour-ahead operation for each reserve scenario  $s_0$ :

$$Max (J_{ISO\_R}^{s_0}) = - \sum_{f=1}^{N_f} \left[ (a_{fs_0} + b_f Q_{gf}) r_{fs_0} + \frac{1}{2} b_f r_{fs_0}^2 \right] \quad (22)$$

$$s.t. \sum_{f=1}^{N_f} r_{fs_0} = Q_{s_0}^r \quad (\lambda_{rs_0}), \forall s_0 \in S \quad (22)$$

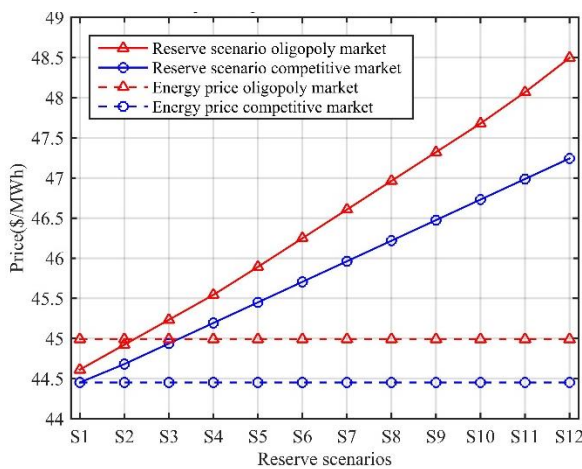


Fig. 17 . Comparing energy and reserve prices in oligopoly and competitive markets

Simulation results for forecasted wind generation equal to 30% of installed capacity are presented in Fig. 17 for different forecast error scenarios. As it is expected, since the bids in competitive markets are lower than bids in oligopoly market the market prices in the competitive markets gains lower than market prices in energy market. As forecast error and consequently the required reserve increases, market players bid higher prices in oligopoly than competitive reserve market. Hence, difference between reserve prices in oligopoly and competitive markets increases.

### 5. CONCLUSION

In this paper, impacts of large-scale integration of WPPs in electricity markets on energy and reserve market prices is studied. To this end, assuming large-scale wind power integration, an SFE model for day-ahead energy market considering uncertainty in hour-ahead reserve market, and an SFE model for hour-ahead reserve market considering day-ahead market results are presented. The proposed approach is applied to a case study. Simulation results are summarized as below:

- Large-scale integration of wind power in energy market decreases the energy market price due to reduction in residual demand for market players.
- Reserve market price in different forecast error scenarios may be lower or greater than energy market price based on forecasted wind power and amount of required reserve. However, expected positive/negative reserve prices are greater/lower than energy market price, respectively.
- Two factors affect the reserve market prices. Increase in forecasted wind generation, increases required positive and negative reserves and consequently increases/decreases positive/negative reserve prices. On the other hand, increase in forecasted wind generation, decreases output power of the firms in energy market, reduces market players' marginal cost in reserve market and consequently decreases reserve market prices. Results show that second factor is dominant and positive and negative reserve prices decrease as wind power generation increases.
- As standard deviation of wind power generation increases positive reserve price increase and negative reserve price decreases since the required positive and negative reserve increase.
- As hour-ahead load increases, positive and negative reserve prices increase since required positive reserve increases and required negative reserve decreases.



- When forecasted wind power is low, low-cost firms prefer to participate in energy market only and higher cost firms participate in both markets to gain more profit. As the forecasted wind power increases, low-cost firms also participate in reserve market and reduce the profits of high-cost firms from reserve market.
- Results also show that although large-scale integration of wind power reduces energy and reserve prices, increase in uncertainty increases the market prices slightly.

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