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NLO radiative correction to the Casimir energy in Lorentz-violating scalar field theory

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Abstract

Violation of the Lorentz symmetry has important effects on physical quantities including field propagators. Therefore, in addition to the leading order, the sub-leading order of quantities may be modified. In this paper, we calculate the next to leading (NLO) radiative corrections to the Casimir energy in the presence of two perfectly conducting parallel plates for ϕ^4 theory with a Lorentz-breaking extension. We do the renormalization and investigate these NLO corrections for three distinct directions of the Lorentz violation; temporal direction, parallel and perpendicular to the plates.

1. Introduction

The Casimir effect which is a physical manifestation of changes in the quantum vacuum fluctuations for different configurations, was discovered by H.B.G. Casimir in 1948 [1]. He showed the existence of this effect as an attractive force between two infinite parallel uncharged perfectly conducting plates in vacuum (for a general review on the Casimir effect, see Refs. [2,3]). Sparnaay [4] and Arnold et al. [5] experimentally observed the Casimir force for such a con-

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figuration. Also, the other measurements, with greatly improved precisions, have been done for various geometries [6–8].

In addition to the leading Casimir energy, the next to leading order (NLO) radiative corrections to this effect is an exciting subject of discussion. The first endeavors to calculate the leading radiative corrections to the Casimir energy were reported in [9]. Also, many works on the radiative corrections to the Casimir energy for various cases exist in the literature (see for instance [10,11]). In the case of a real massive scalar field, NLO correction to the Casimir energy has been computed in [2,12]. We have also calculated one loop radiative corrections to the Casimir energy in [13].

In original quantum field theory (QFT), the Lorentz symmetry is preserved. However, there are some theories which present models with Lorentz symmetry violation (for example [14, 15]). Naturally, Lorentz symmetry violation arises from, for example, existence of space-time anisotropy [16,17] or non-commutativity [18,19] or a spacetime varying coupling constant [20, 21]. Investigations of Casimir effect with Lorentz-breaking symmetry for QED theory have been done (see please [22–24]). It has also been studied recently for a real massive scalar field in [25].

In this paper we calculate the NLO correction to the Casimir energy in an interacting scalar field theory, $\lambda\phi^4$, with a Lorentz violating term. Our configuration is two perfectly conducting parallel plates. We work within the renormalized perturbation theory, therefore we need to reconsider the renormalization for this theory. Naturally, the counterterms needed for renormalization, are modified due to the existence of new Lorentz violating terms in the Lagrangian.

To take the physical result and resolve infinities problem, we use a well-known approach called *Boyer method* [26]; also is known as *Box Renormalization Scheme* (BRS). This method uses a completely physical approach by enclosing the whole system in a box of volume $V = L^3$ which finally may tend to infinity in such a way that difference between the zero point energies of two different configurations is calculated. It removes all ambiguities associated with appearance of the infinities without resorting to any other schemes such as analytic continuation approach. It is notable that, in BRS the subtraction procedure in calculation of Casimir energy takes place in two physical configurations with similar nature, which is another advantage of BRS.

We organized our paper as follows:

We introduce our model for Lorentz-breaking symmetry of the theory in section 2. We shall see that energy-momentum tensor and Klein-Gordon (KG) equation is modified. In section 3, we survey renormalization of the related theory within a Lorentz-breaking case. In section 4 we calculate the NLO radiative correction to the Casimir energy for ϕ^4 theory with Lorentz-breaking symmetry. We note that at this stage we consider the existence of Lorentz-symmetry parameter in two cases: 1. time-like (TL), and 2. space-like (SL). Finally, in last section we state our conclusions.

2. The Lorentz-breaking ϕ^4 theory

2.1. The model

In this section, we present the Lorentz symmetry breaking for a scalar field theory due to an anisotropy of space-time. We do this by insertion an additional term in the KG Lagrangian density

$$\mathcal{L}(x) = \frac{1}{2}\left(\partial_\mu\phi\right)^2 + \frac{1}{2}c\,(u\cdot\partial\phi)^2 - \frac{1}{2}m_0^2\phi^2, \tag{1}$$

where m_0 is the bare mass and the dimensionless parameter c, which is much smaller than one, manifests the Lorentz symmetry breaking of the system by a coupling between the derivative of the scalar field ϕ and a constant four-vector u^μ. Adding a self-interaction term to Eq. (1) we get

$$\mathcal{L}(x) = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{1}{2}c\,(u\cdot\partial\phi)^2 - \frac{1}{2}m_0^2\phi^2 - \frac{\lambda_0}{4!}\phi^4, \tag{2}$$

where λ_0 is our bare coupling. The equation of motion for Lagrangian (1) reads as

$$\left[\Box + c(u\cdot\partial)^2 + m_0^2\right]\phi(x) = 0. \tag{3}$$

It is obvious that this modified KG equation, for $c = 0$ reverts to the original KG equation of motion with the following dispersion relation:

$$\omega_n^2 = \left|\mathbf{k}^\perp\right|^2 + k_n^2 + m_0^2. \tag{4}$$

The violation of Lorentz symmetry has vital consequences such as modification of dispersion relation which directly affects the propagator of the field. We consider this effect in three different cases. In the first case we assume that the Lorentz violation is in the time direction. The second and third are the SL Lorentz violations in the directions parallel (pl-SL) and perpendicular (pr-SL) to the plates.

2.2. *Propagator in bounded space*

To calculate radiative corrections to any physical quantity, including Casimir energy, we need to know the exact form of propagator. In this subsection we first derive the propagator, suitable for Casimir effect problem, in the context of standard quantum field theory (without any Lorentz-violating term). Our configuration is two parallel plates located at $z = \pm a/2$ perpendicular to z-axis with a separation a. We suppose the fields satisfy Dirichlet boundary conditions (DBCs) on the plates,

$$\phi(x)\Big|_{z=\pm a/2} = 0. \tag{5}$$

Being d the dimension of space-time, the field ϕ is defined with quantized modes as

$$\begin{aligned}\phi(x) = &\int \frac{d^{d-2}\mathbf{k}^\perp}{(2\pi)^{d-1}} \sum_{n=1}^{\infty} \left(\frac{1}{a\omega_n}\right)^{1/2} \\ &\times \left\{ e^{-i(\omega_n t - \mathbf{k}^\perp.\mathbf{x}^\perp)} \sin\left[k_n(z+\frac{a}{2})\right]\mathbf{a}_n + e^{i(\omega_n t - \mathbf{k}^\perp.\mathbf{x}^\perp)} \sin\left[k_n(z+\frac{a}{2})\right]\mathbf{a}_n^\dagger \right\},\end{aligned} \tag{6}$$

where $\mathbf{k}^\perp$ and $k_n = \frac{n\pi}{a}$ denote the momenta parallel and perpendicular to the plates, respectively.

Here, $\mathbf{a}_n^\dagger$ and $\mathbf{a}_n$ are creation and annihilation operators, respectively, with the following commutation relations:

$$[\mathbf{a}_n, \mathbf{a}_{n'}^\dagger] = \delta_{n,n'}, \quad [\mathbf{a}_n, \mathbf{a}_{n'}] = [\mathbf{a}_n^\dagger, \mathbf{a}_{n'}^\dagger] = 0,$$

and $\mathbf{a}|0\rangle = 0$ defines the vacuum state in the presence of boundary conditions. One may easily find Feynman Green's function of the KG equation as

$$G_F(x,x') = i\frac{2}{a}\int\frac{d\omega}{2\pi}\int\frac{d^{d-2}\mathbf{k}^\perp}{(2\pi)^{d-2}} \times\sum_n \frac{e^{-i\omega(t-t')}e^{-i\mathbf{k}^\perp.(\mathbf{x}^\perp-\mathbf{x}'^\perp)}\sin\left[k_n(z+\frac{a}{2})\right]\sin\left[k_n(z'+\frac{a}{2})\right]}{\omega^2-k^{\perp 2}-k_n^2-m_0^2+i\epsilon}. \tag{7}$$

We then find Euclidean Green's function by the following definitions:

$$\omega_E = -i\omega \quad ; \quad \mathbf{k}_\mathbf{E}^\perp = \mathbf{k}^\perp,$$

which finally leads to (we need only $G_F(x,x)$ in our calculations)

$$G_F(x,x) = \frac{2}{a}\int\frac{d\omega_E}{2\pi}\int\frac{d^{d-2}\mathbf{k}_E^\perp}{(2\pi)^{d-2}}\sum_n\frac{\sin^2\left[k_n(z+\frac{a}{2})\right]}{\omega_E^2+k_E^{\perp 2}+k_n^2+m_0^2+i\epsilon}. \tag{8}$$

2.2.1. TL vector case

Choosing the four-vector to be TL, $u^\mu=(1,0,0,0)$, the second term in Eq. (3) becomes $c\partial_0^2$. Hence, the dispersion relation (4) takes the form

$$(1+c)\omega_n^2 = k^{\perp 2}+k_n^2+m_0^2. \tag{9}$$

Therefore, we can find the propagator for this case by replacing $\omega^2\to(1+c)\omega^2$ in Eq. (8)

$$G_F(x,x) = \frac{2}{a}\int\frac{d\omega_E}{2\pi}\int\frac{d^{d-2}\mathbf{k}_E^\perp}{(2\pi)^{d-2}}\sum_n\frac{\sin^2\left[k_n(z+\frac{a}{2})\right]}{(1+c)\omega_E^2+k_E^{\perp 2}+k_n^2+m_0^2+i\epsilon} \tag{10}$$

Changing variable $\omega'=\sqrt{1+c}\;\omega_E$, we obtain

$$G_F(x,x) = \frac{2}{a(1+c)^{1/2}}\int\frac{d^{d-1}k}{(2\pi)^{d-1}}\sum_n\frac{\sin^2\left[k_n(z+\frac{a}{2})\right]}{k^2+k_n^2+m_0^2+i\epsilon}, \tag{11}$$

where $\mathbf{k}=(\omega',\mathbf{k}_E^\perp)$. Performing the angular integration, finally we have

$$\begin{aligned} G_F(x,x) &= \frac{2}{a(1+c)^{1/2}}\Omega_{d-1}\int\frac{dkk^{d-2}}{(2\pi)^{d-1}}\sum_n\frac{\sin^2\left[k_n(z+\frac{a}{2})\right]}{k^2+k_n^2+m_0^2+i\epsilon} \\ &= \frac{4}{a(1+c)^{1/2}(4\pi)^{\frac{d-1}{2}}\Gamma(\frac{d-1}{2})}\int dkk^{d-2}\sum_n\frac{\sin^2\left[k_n(z+\frac{a}{2})\right]}{k^2+k_n^2+m_0^2+i\epsilon}, \end{aligned} \tag{12}$$

where the solid angle $\Omega_d=\frac{2\pi^{d/2}}{\Gamma(d/2)}$, with $\Gamma(x)$ being the Gamma function, corresponds to the area of a unit sphere in d dimensions.

2.2.2. SL vector case

In SL case we choose three distinct directions for four-vector u^μ; $u^\mu=(0,1,0,0)$, $u^\mu=(0,0,1,0)$ and $u^\mu=(0,0,0,1)$. In this case the Lorentz-breaking term in (3) is $-c\partial_i^2$ with $i=x,y$ or z. There is no difference between the physics of the first two vectors (pl-SL case), which are parallel to the plates, and the dispersion relations for both cases are also the same. Choosing $u^\mu=(0,0,1,0)$ for instance, Eq. (4) becomes

$$\omega_n^2 = k_x^2+(1-c)k_y^2+k_n^2+m_0^2. \tag{13}$$

Changing the variables $\mathbf{k} = (\omega, \mathbf{k}'^{\perp}_{E})$ with $k'_y = \sqrt{1-c}\, k_y$, in a similar manner to the TL case, the Green's function is derived as:

$$G_F(x,x) = \frac{4}{(1-c)^{1/2} a (4\pi)^{\frac{d-1}{2}} \Gamma(\frac{d-1}{2})} \int dk k^{d-2} \sum_n \frac{\sin^2\left[k_n(z+\frac{a}{2})\right]}{k^2 + k_n^2 + m_0^2 + i\epsilon}. \tag{14}$$

Now, for the last case (pl-SL), $u^\mu = (0, 0, 0, 1)$ is normal to the plates and Eq. (4) becomes

$$\omega_n^2 = k^{\perp 2} + (1-c)k_n^2 + m_0^2. \tag{15}$$

In this case, the Euclidean Feynman propagator is derived as

$$\begin{aligned} G_F(x,x) &= \frac{4}{a(4\pi)^{\frac{d-1}{2}} \Gamma(\frac{d-1}{2})} \int dk k^{d-2} \sum_n \frac{\sin^2\left[k_n(z+\frac{a}{2})\right]}{k^2 + (1-c)k_n^2 + m_0^2 + i\epsilon} \\ &= \frac{4}{a(4\pi)^{\frac{d-1}{2}} \Gamma(\frac{d-1}{2})} \int dk k^{d-2} \sum_n \frac{\sin^2\left[k_n(z+\frac{a}{2})\right]}{k^2 + k'^2_n + m_0^2 + i\epsilon}, \end{aligned} \tag{16}$$

where $k'_n = n\pi/a'$ with $a' = a/\sqrt{1-c}$.

For the future use, in the case of free space without plates, we note that the propagator for a Lorentz symmetry breaking theory becomes

$$\begin{aligned} G_F(x,x) &= \frac{1}{(1\pm c)^{\frac{1}{2}}} \int \frac{d^d k}{(2\pi)^d} \frac{i}{k^2 - m_0^2} \\ &= \frac{1}{(1\pm c)^{\frac{1}{2}} (4\pi)^{\frac{d}{2}}} \frac{\Gamma(1-\frac{d}{2})}{(m_0^2)^{1-\frac{d}{2}}}, \end{aligned} \tag{17}$$

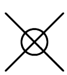
where $+$ $(-)$ is used for TL (SL) vector case.

3. Renormalization up to order λ

At the level of quantum corrections, all unphysical quantities such as m_0 and λ_0 need to be renormalized. Therefore, we need to do a renormalization procedure to extract the physical m and λ from the bare parameters m_0 and λ_0 (see [27]). Here, we work within the standard renormalized perturbation theory. In the Lagrangian (2), after rescaling the fields by a field strength renormalization Z, namely $\phi = Z^{\frac{1}{2}}\phi_r$ we have

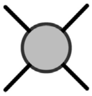
$$\begin{aligned} \mathcal{L} = \frac{1}{2}(\partial_\mu \phi_r)^2 + \frac{1}{2}c(u\cdot\partial\phi_r)^2 - \frac{1}{2}m^2\phi_r^2 - \frac{\lambda}{4!}\phi_r^4 \\ + \frac{1}{2}\delta_Z(\partial_\mu\phi_r)^2 - \frac{1}{2}\delta_m\phi_r^2 - \frac{\delta_\lambda}{4!}\phi_r^4, \end{aligned} \tag{18}$$

where $\delta_m = m_0^2 Z - m^2$, $\delta_\lambda = \lambda_0 Z^2 - \lambda$ and $\delta_Z = Z - 1$ are the counterterms. Then, we have two new Feynman rules from the above Lagrangian

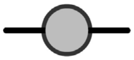
 $= -i\delta_\lambda$

 $= i[(c \pm 1)\, p^\mu p_\mu \delta_Z - \delta_m],$ (19)

where $+$ $(-)$ along with $\mu = 0$ $(\mu = i)$ are used for TL (SL) vector case (for more details see [28]). The counterterms are totally fixed by two renormalization conditions:



$$= -i\lambda \qquad (s = 4m^2, t = u = 0) \tag{20}$$



$$= \frac{i}{p^2 - m^2} + (\text{terms regular at } p^2 = m^2).$$

From the first renormalization condition it is obvious that $\delta_\lambda = O(\lambda^2)$. The second renormalization condition which gives the physical mass m, up to order λ, can be written as

$0 =$  $+$ 

$$= -\frac{1}{2}\frac{i\lambda}{(1 \pm c)^{1/2}(4\pi)^{\frac{d}{2}}}\frac{\Gamma(1 - \frac{d}{2})}{(m^2)^{1-\frac{d}{2}}} + i[(c \pm 1)p^\mu p_\mu \delta_Z - \delta_m] \tag{21}$$

where we have used Eq. (17). Therefore, δ_Z up to order $\mathcal{O}(\lambda)$ is zero, and

$$\delta_m = -\frac{\lambda}{2(1 \pm c)^{1/2}(4\pi)^{\frac{d}{2}}}\frac{\Gamma(1 - \frac{d}{2})}{(m^2)^{1-\frac{d}{2}}}. \tag{22}$$

4. Radiative correction to the Casimir energy

The Casimir energy is defined as follows:

$$E_{\text{Cas.}} = E_{\text{B}} - E_{\text{F}}, \tag{23}$$

where E_{B} and E_{F} are the vacuum expectation values of the energies with and without plates, respectively. As we stated in the Introduction, the tree-level study of the Casimir energy with Lorentz violation have been performed in [25]. In this reference, to remove the infinities and derive the finite physical result, authors have used Abel-Plana summation (APS) formula which converts an integral into a sum. Their results for TL $(+)$ and pl-SL $(-)$ cases are

$$E^{(0)}_{\text{Cas.}} = -\frac{L^2(1 \pm c)^{\frac{1}{2}} a m^4}{6\pi^2}\int_1^\infty \frac{(v^2 - 1)^{\frac{3}{2}} dv}{e^{2amv} - 1}, \tag{24}$$

and for pr-SL is

$$E^{(0)}_{\text{Cas.}} = -\frac{L^2 a' m^4}{6\pi^2}\int_1^\infty \frac{(v^2 - 1)^{\frac{3}{2}} dv}{e^{2a'mv} - 1}. \tag{25}$$

Here, in order to calculate the radiative corrections, along with APS formula we use BRS [26]. In this approach, we first compare the energies in two various configurations: when the plates are at $\pm a/2$ as compared to $\pm b/2$. We confine each configuration in a box with edges are located

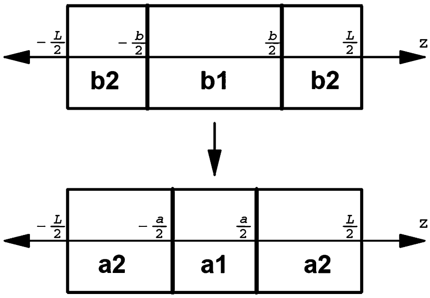


Fig. 1. The labels a1, *etc.* denote the appropriate sections in each configuration separated by the plates.

at $\pm L/2$ in all directions (see Fig. 1). Now, the Casimir energy is defined as

$$E_{\text{Cas.}} = \lim_{b/a\to\infty}\left[\lim_{L/b\to\infty}(E_a - E_b)\right], \tag{26}$$

where,

$$E_a = E_{a1} + 2E_{a2}, \quad E_b = E_{b1} + 2E_{b2}. \tag{27}$$

The radiative corrections to the zero point energy in the (for example) a1 part, i.e. $z \in [\frac{-a}{2}, \frac{a}{2}]$, are

$$\Delta E_{a1} = E^{(1)}_{a1} + E^{(2)}_{a1} + \dots = \int_V d^3\mathbf{x}\langle\Omega|\mathcal{H}_I|\Omega\rangle$$
$$= i\int_V d^3\mathbf{x}\left(\frac{1}{2} + \frac{1}{8} + \frac{1}{8} + \dots\right), \tag{28}$$

where $|\Omega\rangle$ is the vacuum state in the presence of interaction. Up to order λ we have

$$E^{(1)}_{a_1} = i\int_V d^3\mathbf{x}\left(\frac{1}{2} + \frac{1}{8}\right)$$
$$= i\int_V d^3\mathbf{x}\left[-\frac{i}{2}\delta_m G_{a1}(x,x) - \frac{i\lambda}{8}G^2_{a1}(x,x)\right], \tag{29}$$

where $G_{a_1}(x,x)$ is the propagator of the real scalar field in region a1 (we drop the subscript 'F' for simplicity).

4.1. TL & pl-SL vector cases

To calculate the first term in Eq. (29), $E^{(1),F}_{a_1}$, using Eqs. (12) and (22) and carrying out the spatial integration, one obtains the correction to the vacuum energy in region a1, up to $\mathcal{O}(\lambda)$, as:

$$E_{a_1}^{(1),F} = \frac{1}{2}\int_V \delta_m G_{a1}(x,x)d^3\mathbf{x}$$
$$= -\frac{\lambda\sqrt{\pi}\Gamma(1-\frac{d}{2})L^{d-2}}{2(1\pm c)(4\pi)^d(m^2)^{1-\frac{d}{2}}\Gamma(\frac{d-1}{2})}\int_0^\infty dk k^{d-2}\sum_n \frac{1}{k^2+k_{a1,n}^2+m^2}, \tag{30}$$

where $k_{a1,n} = \frac{n\pi}{a}$. Integrating over momentum k yields

$$\frac{1}{2}\int_V \delta_m G_{a_1}(x,x)d^3\mathbf{x} = \frac{\lambda\pi^{\frac{3}{2}}\Gamma(1-\frac{d}{2})\sec(\frac{d\pi}{2})L^{d-2}}{4(1\pm c)(4\pi)^d(m^2)^{1-\frac{d}{2}}\Gamma(\frac{d-1}{2})}\sum_n \omega_{a_1,n}^{d-3}, \tag{31}$$

where $\omega_{a_1,n} = (m^2+k_{a_1,n}^2)^{1/2}$. This is one of the four terms (related to the $a1$ region) that contribute to the NLO radiative correction for Casimir energy Eq. (26). To derive the Casimir energy from Eq. (31), we apply APS formula [29],

$$\sum_{n=1}^{\infty} g(n) = -\frac{g(0)}{2} + \int_0^\infty g(x)dx + i\int_0^\infty \frac{g(it)-g(-it)}{e^{2\pi t}-1}dt, \tag{32}$$

with,

$$g(n) = \omega_{a1,n}^{d-3} + \omega_{a2,n}^{d-3} - \omega_{b1,n}^{d-3} - \omega_{b2,n}^{d-3}. \tag{33}$$

We note that the $g(0)$ term vanishes. Also the second term on the right hand side of Eq. (32), with respect to suitable changing of variables in the four integrals below, vanishes:

$$\frac{a}{\pi}\int_0^\infty dk'\left(m^2+k'^2\right)^{(d-3)/2} + 2\frac{L-a}{2\pi}\int_0^\infty dk'\left(m^2+k'^2\right)^{(d-3)/2}$$
$$-\frac{b}{\pi}\int_0^\infty dk'\left(m^2+k'^2\right)^{(d-3)/2} - 2\frac{L-b}{2\pi}\int_0^\infty dk'\left(m^2+k'^2\right)^{(d-3)/2} = 0. \tag{34}$$

Finally, we calculate branch-cut terms in Eq. (32). Assuming $f(x) = [x^2-(\frac{am}{\pi})^2]^{(d-3)/2}$ we have

$$B(a) = i\int_0^\infty \frac{f(it)-f(-it)}{e^{2\pi t}-1}dt = -2\left(\frac{\pi}{a}\right)^{d-3}\int_{\frac{am}{\pi}}^\infty \frac{[t^2-(\frac{am}{\pi})^2]^{(d-3)/2}}{e^{2\pi t}-1}$$
$$= -\sum_{j=1}^{\infty}\frac{2K_{\frac{d-2}{2}}(2amj)\Gamma(\frac{d-1}{2})}{a^{d-3}\pi^{3/2}(\frac{am}{j})^{\frac{2-d}{2}}}, \tag{35}$$

where $K_n(x)$ is the modified Bessel function of order n. To calculate the integral, we have used the identity

$$\frac{1}{e^{2\pi t}-1} = \sum_{j=1}^{\infty} e^{-2\pi jt}. \tag{36}$$

Therefore, we obtain

$$E^{(1),F}_{a_1} = -\frac{\lambda\pi^{\frac{3}{2}}\Gamma(1-\frac{d}{2})\sec(\frac{d\pi}{2})L^2}{4(1\pm c)a^3(4\pi)^d(\tilde{m}^2)^{1-\frac{d}{2}}\Gamma(\frac{d-1}{2})}\sum_{j=1}^{\infty}\frac{2K_{\frac{d-2}{2}}(2\tilde{m}j)\Gamma(\frac{d-1}{2})}{\pi^{3/2}(\frac{\tilde{m}}{j})^{\frac{2-d}{2}}}, \tag{37}$$

where $\tilde{m} = ma$ is a dimensionless parameter. Then, according to Eq. (26) the contribution of the Eq. (31) to Casimir energy is

$$E^{(1),F}_{\text{Cas.}} = \lim_{b/a\to\infty}\left[\lim_{L/b\to\infty}\left(E^{(1),F}_{a_1} - E^{(1),F}_{b_1} + E^{(1),F}_{a_2} - E^{(1),F}_{b_2}\right)\right]. \tag{38}$$

Taking the limits, only the first term survives. Finally, we take the limit $d \to 4$,

$$E^{(1),F}_{\text{Cas.}} = -\sum_{j=1}^{\infty}\frac{\lambda\tilde{m}^3L^2}{512(1\pm c)a^3\pi^4}\frac{1}{j}\left[K_1(2\tilde{m}j)\left(\ln(\frac{\tilde{m}^3}{16\pi^2 j}) + \gamma - 1\right) + K_1'(2\tilde{m}j)\right], \tag{39}$$

where $K_q'(x) = \frac{\partial}{\partial q}K_q(x)$ and γ is the Euler-Mascheroni number.

The contribution of the second term in Eq. (29) to the Casimir energy, $E^{(1),S}_{a_1}$, without Lorentz violating terms, have been calculated in Ref. [11] using BRS:

$$\begin{aligned}
E^{(1),S}_{a_1} &= \frac{\lambda}{8}\int_V G^2_{a1}(x,x)d^3\mathbf{x} \\
\to E^{(1),S}_{\text{Cas.}} &= -\lambda L^2\frac{B(a)}{128\pi^2} \\
&\times\left[\frac{B(a)}{a} - \frac{m}{a} + \frac{m^2}{\pi}(\ln 2 + 1/2)\right] \quad \text{(no Lorentz violation)} \qquad (40) \\
&= -\lambda L^2\sum_{j=1}^{\infty}\frac{m}{128\pi^3}\frac{K_1(2amj)}{j} \\
&\times\left[\frac{m}{\pi a}\sum_{j'=1}^{\infty}\frac{K_1(2amj')}{j'} + \frac{m}{a} - \frac{m^2}{\pi}(\ln 2 + 1/2)\right]. \qquad (41)
\end{aligned}$$

But, when we have a TL (pl-SL) Lorentz breaking term, an extra factor $\frac{1}{\sqrt{1+c}}$ ($\frac{1}{\sqrt{1-c}}$), as we see in Eq. (12) (Eq. (14)), is multiplied to the propagator. Therefore to derive the Casimir energy contribution we only need to multiply the factor $\frac{1}{1+c}$ ($\frac{1}{1-c}$) to Eq. (41). Accordingly, using Eqs. (41) and (39), we can write NLO radiative correction to the Casimir energy as

$$\begin{aligned}
E^{(1)}_{\text{Cas.}} &= E^{(1),F}_{\text{Cas.}} + E^{(1),S}_{\text{Cas.}} \qquad (42) \\
&= \frac{-\lambda L^2}{(1\pm c)}\sum_{j=1}^{\infty}\left\{\frac{m}{128\pi^3}\frac{K_1(2amj)}{j}\left[\frac{m}{\pi a}\sum_{j'=1}^{\infty}\frac{K_1(2amj')}{j'} + \frac{m}{a} - \frac{m^2}{\pi}(\ln 2 + 1/2)\right]\right. \\
&\left. - \frac{m^3}{512\pi^4}\frac{1}{j}\left[K_1(2amj)\left(\ln(\frac{a^3m^3}{16\pi^2 j}) + \gamma - 1\right) + K_1'(2amj)\right]\right\}.
\end{aligned}$$

From this result it is obvious that the influence of the Lorentz-symmetry breaking parameter appears only in a factor.

Two special limits are interesting to calculate; the large mass $ma \gg 1$, and small mass $m \to 0$ limits:

$$\begin{cases} E^{(1)}_{\text{Cas.}} \overset{am\gg 1}{\longrightarrow} \dfrac{3\,L^2}{1024\pi^{7/2}}\dfrac{\lambda}{(1\pm c)a^3}(am)^{5/2}\,\ln(am)\,e^{-2am}, \\ E^{(1)}_{\text{Cas.}} \overset{m\to 0}{\longrightarrow} -\dfrac{L^2\lambda}{512\pi^4(1\pm c)a^3}\left(\displaystyle\sum_{j=1}^{\infty}\frac{1}{j^2}\right)^2 = -\dfrac{L^2\lambda}{18432(1\pm c)a^3}, \end{cases} \tag{43}$$

with $+$ $(-)$ for TL (pl-SL) case.

4.2. pr-SL vector case

For the pr-SL vector case, $u^\mu = (0, 0, 0, 1)$, we do not need to do new calculation. In this case, applying Eq. (16) leads us to the following expression for Eq. (31):

$$\frac{1}{2}\int_V \delta_m G_{a_1}(x,x)d^3\mathbf{x} = \frac{\lambda\pi^{\frac{3}{2}}\Gamma(1-\frac{d}{2})\sec(\frac{d\pi}{2})L^{d-2}}{4(1-c)^{1/2}(4\pi)^d(m^2)^{1-\frac{d}{2}}\Gamma(\frac{d-1}{2})}\sum_n {\omega'}^{d-3}_{a_1,n}, \tag{44}$$

where ${\omega'}_{a_1,n} = (m^2 + {k'}^2_{a_1,n})^{1/2}$. Therefore, the Eq. (35) becomes

$$B(a') = -\sum_{j=1}^{\infty}\frac{2K_{\frac{d-2}{2}}(2a'mj)\Gamma(\frac{d-1}{2})}{{a'}^{d-3}\pi^{3/2}(\frac{a'm}{j})^{\frac{2-d}{2}}} = -\sum_{j=1}^{\infty}\frac{2K_{\frac{d-2}{2}}\left(\frac{2amj}{\sqrt{1-c}}\right)\Gamma(\frac{d-1}{2})}{(1-c)^{\frac{3-d}{2}}a^{d-3}\pi^{3/2}\left(\frac{am}{j\sqrt{1-c}}\right)^{\frac{2-d}{2}}}, \tag{45}$$

and hence, we get

$$E^{(1),F}_{a_1} = -\frac{\lambda\pi^{\frac{3}{2}}\Gamma(1-\frac{d}{2})\sec(\frac{d\pi}{2})L^2}{4(1-c)^{\frac{6-d}{4}}a^3(4\pi)^d(\tilde{m}^2)^{1-\frac{d}{2}}\Gamma(\frac{d-1}{2})}\sum_{j=1}^{\infty}\frac{2K_{\frac{d-2}{2}}\left(\frac{2\tilde{m}j}{\sqrt{1-c}}\right)\Gamma(\frac{d-1}{2})}{\pi^{3/2}\left(\frac{\tilde{m}}{j}\right)^{\frac{2-d}{2}}}. \tag{46}$$

Now, we use the above equation to compute Eq. (38), and take the limit $d \to 4$, to get

$$\begin{aligned} E^{(1),F}_{\text{Cas.}} = \frac{\lambda L^2 m^3}{(1-c)^{1/2}512\pi^4}\sum_{j=1}^{\infty}\frac{1}{j}\Bigg[&K_1(\frac{2amj}{\sqrt{1-c}})\left(\ln(\frac{a^3m^3}{16\pi^2 j}) + \ln(1-c) + \gamma - 1\right) \\ &+K_1'(\frac{2amj}{\sqrt{1-c}})\Bigg]. \end{aligned} \tag{47}$$

Similarly, for the second term in Eq. (29), now the Eq. (40) becomes

$$\begin{aligned} E^{(1),S}_{\text{Cas.}} &= -\frac{\lambda L^2}{(1-c)^{1/2}}\frac{B(a')}{128\pi^2}\left[\frac{B(a')}{a} - \frac{m}{a} + \frac{m^2}{\pi}(\ln 2 + 1/2)\right] \\ &\overset{d\to 4}{=} -\frac{\lambda L^2}{(1-c)^{1/2}}\sum_{j=1}^{\infty}\frac{m}{128\pi^3}\frac{1}{j}K_1(\frac{2amj}{\sqrt{1-c}})\Bigg[\frac{m}{\pi a}\sum_{j'=1}^{\infty}\frac{1}{j'}K_1(\frac{2amj'}{\sqrt{1-c}}) \\ &\quad +\frac{m}{a} - \frac{m^2}{\pi}(\ln 2 + 1/2)\Bigg]. \end{aligned} \tag{48}$$

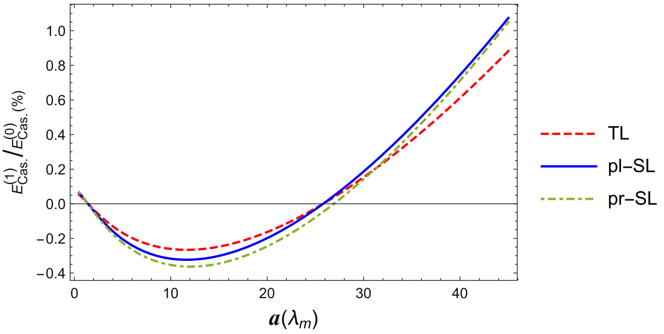


Fig. 2. The ratio between the first order radiative corrections and leading terms, $E^{(1)}_{\text{Cas.}}/E^{(0)}_{\text{Cas.}}$, in terms of plates separation a, for $c = 0.1$, $\lambda = 0.1$ and $m = 1$; λ_m is the Compton wavelength of the scalar field.

Therefore, the result for the radiative correction of Casimir energy for the pr-SL vector case can be written as

$$\begin{aligned}
E^{(1)}_{\text{Cas.}} &= E^{(1),F}_{\text{Cas.}} + E^{(1),S}_{\text{Cas.}} \\
&= -\frac{\lambda L^2}{(1-c)^{1/2}} \sum_{j=1}^{\infty} \left\{ \frac{m^3}{512\pi^4} \frac{1}{j} \left[K_1(\frac{2amj}{\sqrt{1-c}}) \left(\ln(\frac{a^3 m^3}{16\pi^2 j}) + \ln(1-c) + \gamma - 1 \right) \right. \right. \\
&\quad \left. + K_1'(\frac{2amj}{\sqrt{1-c}}) \right] \\
&\quad \left. + \frac{m}{128\pi^3} \frac{1}{j} K_1(\frac{2amj}{\sqrt{1-c}}) \left[\frac{m}{\pi a} \sum_{j'=1}^{\infty} \frac{1}{j'} K_1(\frac{2amj'}{\sqrt{1-c}}) + \frac{m}{a} - \frac{m^2}{\pi}(\ln 2 + 1/2) \right] \right\}.
\end{aligned} \tag{49}$$

We can also compute the large mass and massless limits:

$$\begin{cases}
E^{(1)}_{\text{Cas.}} \xrightarrow{am \gg 1} \dfrac{3\,L^2}{1024\pi^{7/2}} \dfrac{\lambda}{(1-c)^{1/2} a^3} (am)^{5/2} \ln(am)\, e^{\frac{-2am}{\sqrt{1-c}}}, \\
E^{(1)}_{\text{Cas.}} \xrightarrow{m \to 0} -\dfrac{L^2 \lambda (1-c)^{1/2}}{512\pi^4 a^3} \left(\displaystyle\sum_{j=1}^{\infty} \frac{1}{j^2} \right)^2 = -\dfrac{L^2 \lambda (1-c)^{1/2}}{18432 a^3}.
\end{cases} \tag{50}$$

In Fig. 2, we have illustrated the variation of the ratio between the first order radiative corrections and leading terms, $E^{(1)}_{\text{Cas.}}/E^{(0)}_{\text{Cas.}}$, in terms of plates separation, for three distinct cases TL, pl-SL and pr-SL. We have also plotted this ratio in terms of Lorentz violating parameter c in Fig. 3.

There exist many measured and derived values of coefficients for Lorentz violation in the Standard Model Extensions (see [30]) which have been tabulated in Ref. [31]. If we can compare our scalar model with the only existing scalar sector in the Standard Model, the Higgs sector, we find $|c| < 10^{-6}$ from nuclear β decay [32] and for the space-like case $|c| < 10^{-19}$ from laser interferometry [20]. From Fig. 3, we see that the ratio between the correction and leading term of the Casimir energy for these limits is some finite value (about -0.3%).

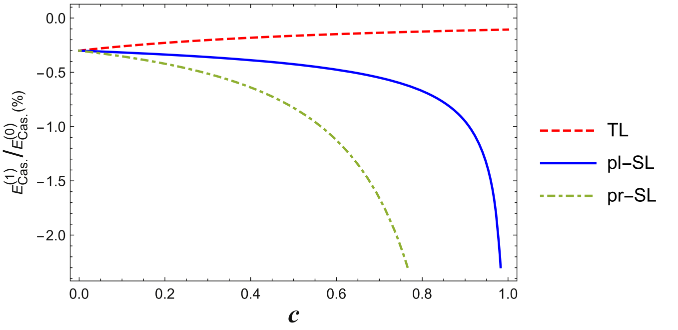


Fig. 3. The variation of the ratio between first order radiative correction and leading term, $E^{(1)}_{\text{Cas.}}/E^{(0)}_{\text{Cas.}}$, in terms of the Lorentz violating parameter c, with $\lambda = 0.1$, $m = 1$ and $a = 10(\lambda_m)$.

5. Conclusions

In this paper we have calculated the next to leading order radiative correction to the Casimir energy for ϕ^4 theory with Lorentz-breaking symmetry in the context of renormalized perturbation theory. Our approach to calculate this energy is box renormalization method introduced firstly by Boyer [26] and used for example in [11,33–35]. The violation of symmetry breaking can be appeared in the Lagrangian by insertion of a term which couples the derivative of a field to a constant vector u^μ. This additional term in the Lagrangian modifies the dispersion relation and accordingly propagators of the fields. Therefore, in addition to the leading terms of physical quantities, all their sub-leading corrections are also affected. In three separate cases of the Lorentz violation, violation in the time direction (TL), in the directions parallel (pl-SL) and perpendicular (pr-SL) to the plates, the leading terms of Casimir energy for ϕ^4 theory have been recently calculated in [25]. Here, we have investigated NLO corrections. We have plotted our results in Figs. 2 and 3.

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