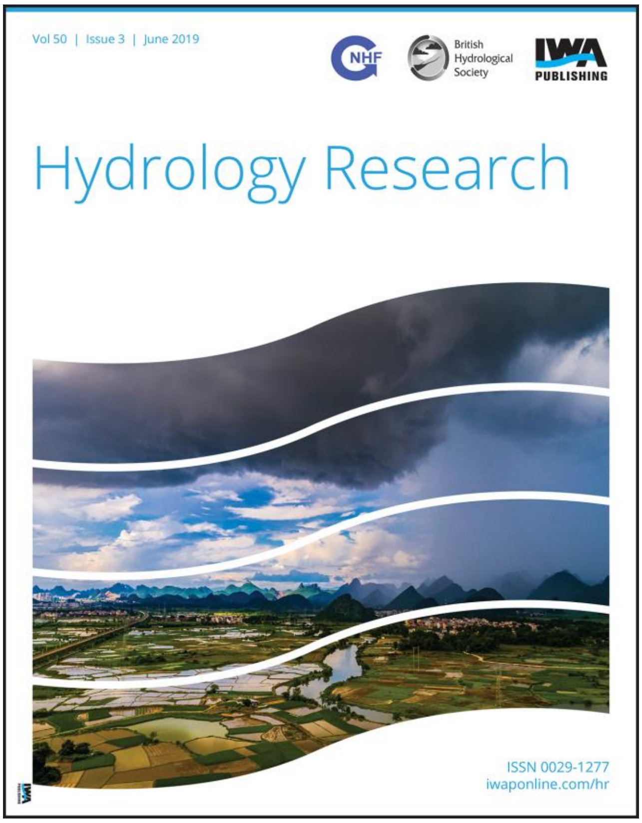
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Stage-discharge estimation in compound open channels with composite roughness

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ABSTRACT

Determination of stage-discharge relationships in natural rivers is extremely important in flood control projects. The importance of rating curves in rivers and the difficulty of its establishment show the need for simpler and more precise methods. Determination of rating curves in compound channels, especially with composite roughness, has proved to be difficult because of significant variations in hydraulic parameters from the main channel to the floodplains. In the current paper, a new approach that is based on the concept of the cross-sectional isovel contours is introduced for estimation of the stage-discharge curves in compound channels. The multivariate Newton's method is applied to the difference between the observed and estimated data to optimize the exponent values of the governing parameters. The proposed method is verified by comparing predictions obtained by using it with some experimental datasets. For compound channels with composite roughness, the stage-discharge curves obtained by the proposed method are more accurate than those obtained by using available conventional methods.

Key words | composite roughness, compound channel, isovel contours, parameter minimization, stage-discharge curve

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INTRODUCTION

Determination of the stage-discharge curves in compound open channels is essential from the point of view of river engineering. Most of the time water is present in the main channel, but the floodplains have an important role to play during flood events because they increase the conveyance capacity of channels. Several researchers have proposed different methods for predicting the stage-discharge relation in compound channels. In conventional methods, discharge in compound channels can be calculated based on the single and divided channel methods (SCM and DCM). If the channel is taken as a single unit, the flow rate will be underestimated due to the reduced main channel velocity. When the sub-sections of a compound channel are taken independently, the total discharge is generally overestimated due to neglecting the momentum transfer between the main channel and the floodplains (Stephenson & Kolovopoulos 1990). The exact solution should be found somewhere between the SCM and DCM methods (Sahu *et al.* 2011). The cross-section of a compound channel may be subdivided into the main channel and the floodplain as shown in Figure 1. Typically, these divisions are made using straight vertical lines at the edges of the main channel, which are assumed to be shear-free and as a result are not included in the wetted perimeter. So far, various methods have been introduced by researchers in order to improve the performances of the SCM and DCM.

Lambert & Myers (1998) have proposed the weighted divided channel method (WDCM) by investigating the experimental works. The WDCM is based on the fact that the real discharge should be located between an

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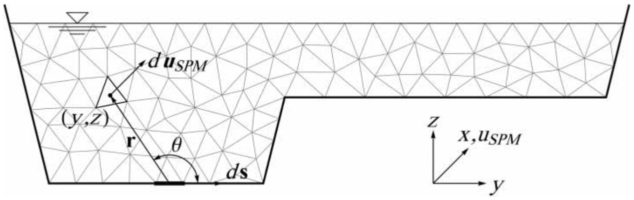

Figure 1 | Illustrative geometry for the effect of boundary on the velocity of an arbitrary point with coordinates (*y*,*z*) in a compound channel cross-section.

underestimated value of SCM and overestimated value of DCM. So, a weighted multiplier is introduced to modify the velocities of the main channel and floodplain. The minimum weighting factor is zero, and the maximum is one. It is applied to both the in-bank and over-bank(s) to give the improved mean velocity estimations for these areas.

Ackers (1993) used experimental data and proposed the concept of coherence (COH), which works based on the ratio of the discharges that are calculated by the SCM and DCM methods. Khatua *et al.* (2012) proposed the modified divided channel method (MDCM) to calculate the discharge in compound channels especially for the straight smooth compound channels with wide floodplains. The results obtained by MDCM give a rough indication of the efficacy of the method for practical applications both in small-scale and large-scale experimental data.

The exchange discharge method (EDM) was proposed by Bousmar & Zech (1999). The momentum transfer is estimated as the product of the velocity difference between the main channel and the floodplain at the interface by the mass discharge exchanged through this interface due to turbulence. Accurate estimation of exchanged discharge with errors generally less than 5% are obtained, while the momentum correction enables the use of actual roughness coefficients, corresponding to the actual river bed material (Bousmar 2002). For non-prismatic flows, the EDM is extended by taking into account, in the momentum transfer, a mass transfer corresponding to the geometrical change. Its accuracy is similar to the WDCM, but the model has the advantage of being a physically based model without numerous parameter fittings (Bousmar & Zech 1999). As a secondary goal, for estimation of water surface profile the velocity gradient was conceptualized as an additional head loss in gradually varied flow equation. When roughness distribution along the wetted perimeter is not uniform in compound channels, more complexities in estimation of discharge will be encountered. In compound channels with large floodplains and considerable roughness, a kind of discontinuity at the level of the main channel will happen due to the floodplain drag (Smart 1992). Determination of equivalent roughness has been investigated by the results obtained from 17 different methods using DCM (Yen 2002). Major differences have been observed in the results obtained by different techniques. It is not actually an easy task to discern the best discharge estimation method among a number of techniques.

The main difficulty associated with the DCM method in natural rivers is distinguishing the border of the main channel and the floodplain. Sellin & Van Beesten (2004) have analyzed the effects of flow resistance in compound channels on the results obtained within five years of observations on the floodplain of the Blackwater River in southern England. It has been found that at higher overbank flow n values increase drastically with depth. However, they remain constant after cutting the berm vegetation. Huai *et al.* (2013) have estimated the apparent shear stress acting on the vertical interface between the main channel and floodplain (with and without vegetation) in prismatic compound open channels using artificial neural networks (ANNs).

Cao *et al.* (2006) presented a formula for flow resistance and momentum flux in compound open channels. The proposed formula was implemented in the St. Venant equations for evaluating conveyance, roughness and stage-discharge relationship in compound open channels. Liao & Knight (2007) analytically proposed a relationship to estimate the discharge in straight trapezoidal open channels. The results indicate good agreement between the analytical and experimental rating curves. Sun & Shiono (2009) used an experimental model in straight compound channels with and without one-line emergent vegetation along the floodplain edge to predict the rating curves. They proposed a new formula for friction factors for with and without vegetation cases using flow parameters and vegetation density. Rezaei & Knight (2010) presented experimental results of overbank flow in compound channels with non-prismatic floodplains and different convergence angles. The results show that the discharge evolution seems linear in the lower water depths, whereas it is non-linear for higher water depths, and the mass transfer in the second half of the converging reach is higher than that in the first half. Yang *et al.* (2012) developed a new model to estimate the discharge in symmetric, compound open channels based on the energy concept. They have utilized the application of the momentum transfer coefficient to predict the discharge distributions of the floodplains and main channel. Mohanty & Khatua (2014) proposed a new relationship, which is derived partly based on the percentages of shear stress carried by the floodplains and partly based on the percentage of area occupied by the floodplains to estimate the rating curves in compound channels especially with wide floodplain(s).

Maghrebi *et al.* (2016) were the first to present a method for estimating stage-discharge curves using the least amount of hydraulic data that is comprised of the geometrical information of the cross-section and discharge at any desired water level. In continuation of their studies, Maghrebi *et al.* (2017) proposed a new relationship that was mainly aimed at increasing the accuracy of the results in compound cross-sections. As a matter of fact, the introduced new approach on the estimation of rating curves in different types of conduits is founded on the basis of the multiplication of inter- and extrapolation of multi-variables, which affects the discharge at a certain water stage.

The main objective of the present paper is to look for the most general form of an expression which can be used to predict the rating curves in any kind of open channels with any distribution of roughness including composite compound as well as hydraulic parameters. Then the best relationship is selected based on the minimum error statistics such as normalized root mean square error (NRMSE) and mean absolute percentage error (MAPE). Although the number of the selected cross-sectional shapes and hydraulic parameters which are engaged in the generated relationships are not limited to those implemented in the current study, it is believed that any other combinations of the shapes and parameters will not lead to a simpler, and more accurate relationship.

FUNDAMENTALS OF THE ISOVEL DERIVATION METHOD

Maghrebi (2006) proposed the concept of isovel contours to predict the discharge in any cross-sectional shape of a conduit which is known as SPM (Single Point in Maghrebi's approach). An extension of the isovel contours is used for the estimation of the rating curve relationship. Figure 1 shows a typical asymmetric compound channel cross-section which is covered with triangular meshes. The centroid of each triangular element is where the boundary effects are calculated. In order to increase the accuracy of the method, the number of meshes has been increased along the boundary and water surface.

All of the conventional velocity distributions such as logarithmic, velocity defect and power laws are functions of y which is measured from the boundary in a vertical direction. For open channels with large aspect ratios, the power law velocity profile for the longitudinal velocity component u can be defined in the following form:

$$\frac{u}{u_*} = c\left(\frac{y}{k_s}\right)^{\frac{1}{m}} \tag{1}$$

where u_* is the shear velocity, k_s is the Nikuradse sand equivalent roughness and c is a coefficient. The exponent $1/m$ is almost independent of the Reynolds number, and the velocity profiles are therefore almost similar. Over a wide range of the Reynolds numbers $m = 7$ agreed well with a large number of experimental measurements. However, most of the natural and artificial channels do not take the form of a large aspect ratio cross-section. In order to deal with these types of open channels, the idea of using r instead of y in velocity distribution functions was proposed by Maghrebi (2006). It was inspired by the Biot–Savart law (Halliday & Resnick 1981) which can be used to calculate the intensity of a magnetic field generated at a point in space by a piece of wire carrying a stationary current. In the field of hydraulics, velocity at an arbitrary point with the coordinates (y, z) at the flow section was quantified by the integration along the whole wetted perimeter as follows (Figure 1):

$$u_{SPM}\,(y, z) = \int_{boundary} c_3(r^{\frac{1}{7}}) sin\,\theta\, ds \tag{2}$$

Since the above equation is valid for any triangular mesh inside the flow area, the total mean cross-sectional velocity generated by the wetted perimeter is given by:

$$U_{SPM} = \frac{\int_A u_{SPM}\ (y, z) dA}{A} \tag{3}$$

In Figure 1 a schematic sketch of the implemented mesh in the flow section is shown. As a matter of fact, the actual mesh size is finer than that shown in the figure. When the number of grids is increased to more than 500, the calculated value of U_{SPM} will remain nearly unchanged. As can be determined from Figure 2, when the number of grids is increased from say 450 to 850, U_{SPM} will be decreased by less than about 0.2%. For all sections without considering the water level, a total number of 850 grids has been implemented in calculation of U_{SPM}, which leads to finer mesh for lower water levels.

DETERMINATION OF STAGE-DISCHARGE RELATIONSHIP

All of the parameters which are involved in discharge can also be involved in the stage-discharge relationship. The main contributing parameters are listed as cross-sectional area A, wetted perimeter P, width of the free surface T, total perimeter P_t $(= P + T)$, equivalent Manning roughness n, bed slope S_0 and U_{SPM} which has the concept of mean cross-sectional velocity since the isovel contours can be obtained by the use of this parameter. Accordingly,

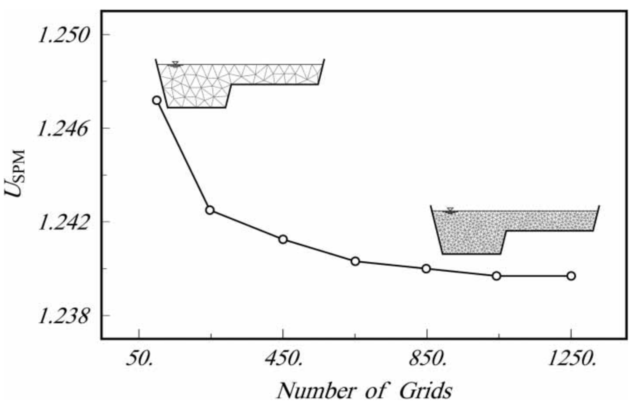

Figure 2 | Influence of the grid numbers in calculating USPM on FCF-S06 at the highest water level.

discharge Q can be expressed by the following relationship:

$$Q = f(A, P, T, P_t, U_{SPM}, n, S_0) \tag{4}$$

The effect of the bed slope of the channel S_0 which stays fixed at all water levels can be omitted from Equation (4). The general form of the proposed relationship to estimate the stage-discharge curve is presented as follows:

$$\frac{Q_e}{Q_r} = \left(\frac{A_e}{A_r}\right)^{a1}\left(\frac{P_e}{P_r}\right)^{a2}\left(\frac{T_e}{T_r}\right)^{a3}\left(\frac{(P_t)_e}{(P_t)_r}\right)^{a4}\left(\frac{(U_{SPM})_e}{(U_{SPM})_r}\right)^{a5}\left(\frac{n_e}{n_r}\right)^{a6} \tag{5}$$

where the subscripts e and r are referred to as the estimated and referenced values, respectively. The most challenging issue in Equation (5) is the problem of determining the exponent values a_1, a_2, a_3, a_4, a_5 and a_6. Several steps must be taken in order to calculate the exponents. The first step is collection of some of the data taken from the theoretical and observational rating curves for different hydraulic cross-sections. The exact stage-discharge relations for the triangular, rectangular and circular sections at different water levels can be calculated by the use of the Manning formula. Additionally, due to the lack of a unique analytical solution for the stage-discharge relation in compound channels, the experimentally measured discharges in compound channels of FCF-Series01 (Knight 1992) are considered as the most accurate available data. The specifications of the compound channels, which is shown in Figure 3, are given in Table 1.

The parameters n_c, n_f, s_c, s_f, bw_c, bw_{f1}, bw_{f2}, h and H_f are the Manning roughness in the main channel and the floodplain, the side slope of the main channel, the side slope of the floodplain, the width of the main channel, the width of the floodplain on the left and right sides, the depth of the main channel and the depth of the floodplain, respectively (Figure 3).

The multivariate Newton's method is applied to the difference between the exact and estimated values of (Q,H) to optimize the exponent values of the governing equation (Equation (5)). Before computing the exponents, it should be mentioned that they can be extracted from different conditions because the proposed relationship can be formed from three to six different parameters. They are shown on the bottom row of Figure 4. U_{SPM} and n are

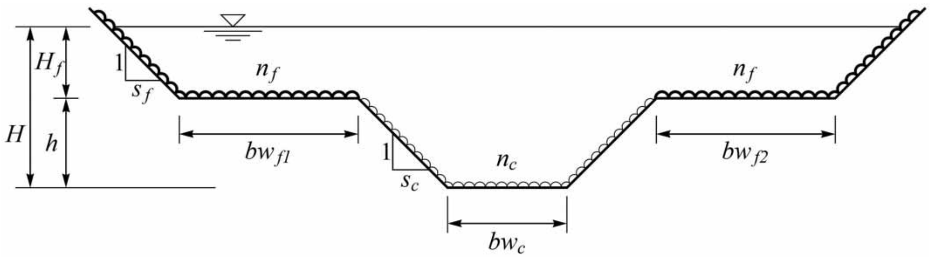

Figure 3 | General characteristics of compound channels.

Table 1 | Geometric and hydraulic specifications of the compound channel cross-sections

Test	S_0 ($\times 10^{-3}$)	n_c	n_f	S_c	S_f	Bw_c (m)	bw_{f1} (m)	bw_{f2} (m)	h (m)	H_f (m)
FCF-S01	1.027	0.01	0.01	1	0	1.5	4.1	4.1	0.15	0.15
FCF-S02	1.027	0.01	0.01	1	1	1.5	2.25	2.25	0.15	0.15
FCF-S03	1.027	0.01	0.01	1	1	1.5	0.75	0.75	0.15	0.15
FCF-S06	1.027	0.01	0.01	1	1	1.5	2.25	0	0.15	0.15
FCF-S10	1.027	0.01	0.01	2	1	1.5	2.25	2.25	0.15	0.15
P & T-S1-S	0.3	0.011	0.011	0.5	0	0.203	0.381	0.381	0.102	0.1
P & T-S2-S	0.3	0.011	0.011	0.5	0	0.305	0.381	0.381	0.102	0.1
P & T-S1-R1	0.3	0.011	0.014	0.5	0	0.203	0.381	0.381	0.102	0.1
P & T-S2-R2	0.3	0.011	0.014	0.5	0	0.305	0.381	0.381	0.102	0.1
P & T-S1-R2	0.3	0.011	0.018	0.5	0	0.203	0.381	0.381	0.102	0.1
P & T-S2-R2	0.3	0.011	0.018	0.5	0	0.305	0.381	0.381	0.102	0.1
P & T-S1-R3	0.3	0.011	0.022	0.5	0	0.203	0.381	0.381	0.102	0.1
P & T-S2-R3	0.3	0.011	0.022	0.5	0	0.305	0.381	0.381	0.102	0.1

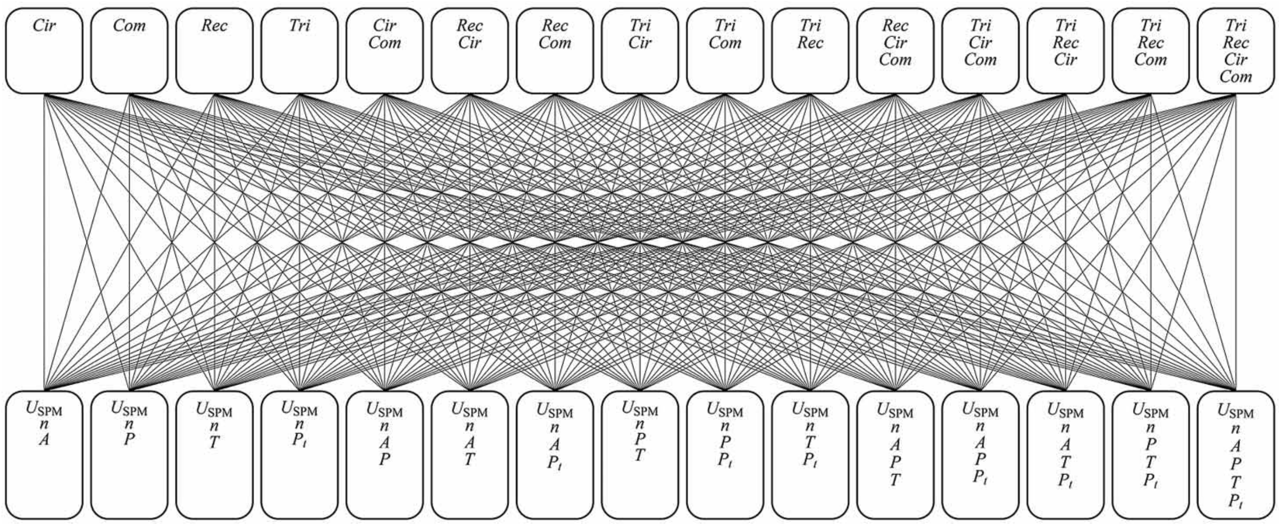

Figure 4 | Schematic diagram shows all of the possible combinations with the selected variables.

included in all of the relationships. On the other hand, all combinations of the four reference cross-sections including triangular, rectangular, circular and compound sections can be used in the minimization process. They are shown on the top row of Figure 4. In total, 225 different relationships are extracted as shown in Figure 4. In Figure 4, the abbreviations *Tri*, *Rec*, *Cir* and *Com* represent the triangular, rectangular, circular and compound cross-sections in FCF-Series01, respectively.

In the determination of the exponent values involved in Equation (5), the geometric and hydraulic parameters of $A_{\max}$, $P_,$ (P_t), U_{SPM} and n in four selected sections at all water levels should be evaluated. Figure 5(a)–5(d) indicates the variations of these parameters for the selected sections. Their maximum values are given as $A_{\max}$, $P_{\max}$, $(P_t)_{\max}$, U_{SPM} and $n_{\max}$, which are mainly associated with the highest water level $H_{\max}$, as presented in Table 2. The Manning roughness coefficient n is actually the equivalent roughness which can be calculated by a number of different methods (Yen 2002). For a uniform boundary roughness, it takes the corresponding roughness coefficient. However, when the roughness distribution along the wetted perimeter is non-uniform, an equivalent roughness at each water level should be implemented. In the proposed method, the rating curve can be estimated from any arbitrary water level which is considered as a reference level.

In the continuity equation $Q = AU$, velocity has a power of 1. Moreover, since the role of U_{SPM} is the same as velocity, it is concluded that $a_5 = 1$. In addition, we have $a_6 = -1$ due to the inverse relation between discharge and roughness according to the Manning formula. Therefore, the remaining task is the evaluation of a_1, a_2, a_3 and a_4. This can be achieved by the minimization of the statistical measure $\overline{NRMSE}$, which is defined in the following form:

$$\overline{NRMSE} = \frac{\frac{1}{N}\sum_{j=1}^{N}\sqrt{\frac{1}{N}\sum_{j=1}^{N}(Q_r)_i\left(\frac{A_j}{A_i}\right)^{a1}\left(\frac{P_j}{P_i}\right)^{a2}\left(\frac{T_j}{T_i}\right)^{a3}\left(\frac{(P_t)_j}{(P_t)_i}\right)^{a4}\left(\frac{(U_{SPM})_j}{(U_{SPM})_i}\right)^{1}\left(\frac{n_j}{n_i}\right)^{-1} - (Q_e)_j}}{(Q_r)_{\max} - (Q_r)_{\min}} = \frac{\overline{RMSE}}{(Q_r)_{\max} - (Q_r)_{\min}} \quad (6)$$

The subscripts i and j in Equation (6) are referred to as the reference and estimated values, respectively and N is the total number of investigated points for each stage-discharge curve. In other words, $(Q_r)_i\left(\frac{A_j}{A_i}\right)^{a1}\left(\frac{P_j}{P_i}\right)^{a2}\left(\frac{T_j}{T_i}\right)^{a3}\left(\frac{(P_t)_j}{(P_t)_i}\right)^{a4}\left(\frac{(U_{SPM})_j}{(U_{SPM})_i}\right)^{1}\left(\frac{n_j}{n_i}\right)^{-1}$ is equal to Q_e which is the estimated discharge at the jth level.

According to Equation (6), considering a single observed discharge as the reference, a unique stage-discharge curve can be estimated. Therefore, the *RMSE* parameter can be calculated based on each single curve. Then, the average value of this parameter, which is shown by $\overline{RMSE}$, can be computed from all of the calculated values of *RMSEs*. The denominator of Equation (6) is composed of the difference between the maximum and minimum reference discharges. An objective function like *F*, which can be defined generally in the following form, is introduced since it is desirable to establish a rating curve which is derived from all of the selected sections. Now the minimization process should be implemented on *F*:

$$F = \overline{NRMSE_{Tri}} + \overline{NRMSE_{Rec}} + \overline{NRMSE_{Cri}} + \overline{NRMSE_{Com}} \quad (7)$$

where the subscripts *Tri, Rec*, *Cir* and *Com* are defined previously. The number of parameters on the right-hand side of Equation (7) depends on the number of the implemented sections. In order to examine the performance of each relationship, classification should be carried out. The statistical measure of the normalized root mean square error, $\overline{NRMSE}$, is used for this purpose. All of the relationships extracted from the four rectangular, circular, triangular and compound sections are collected in a group named Group A with the corresponding value of $\overline{NRMSE_A}$. The variations of $\overline{NRMSE_A}$ for all 225 relationships, is shown in Figure 6. It can be seen that the least amount of $\overline{NRMSE_A}$ is related to relationship number 225 which is generated by using six parameters and four different sections. Meanwhile, it can also be observed that the

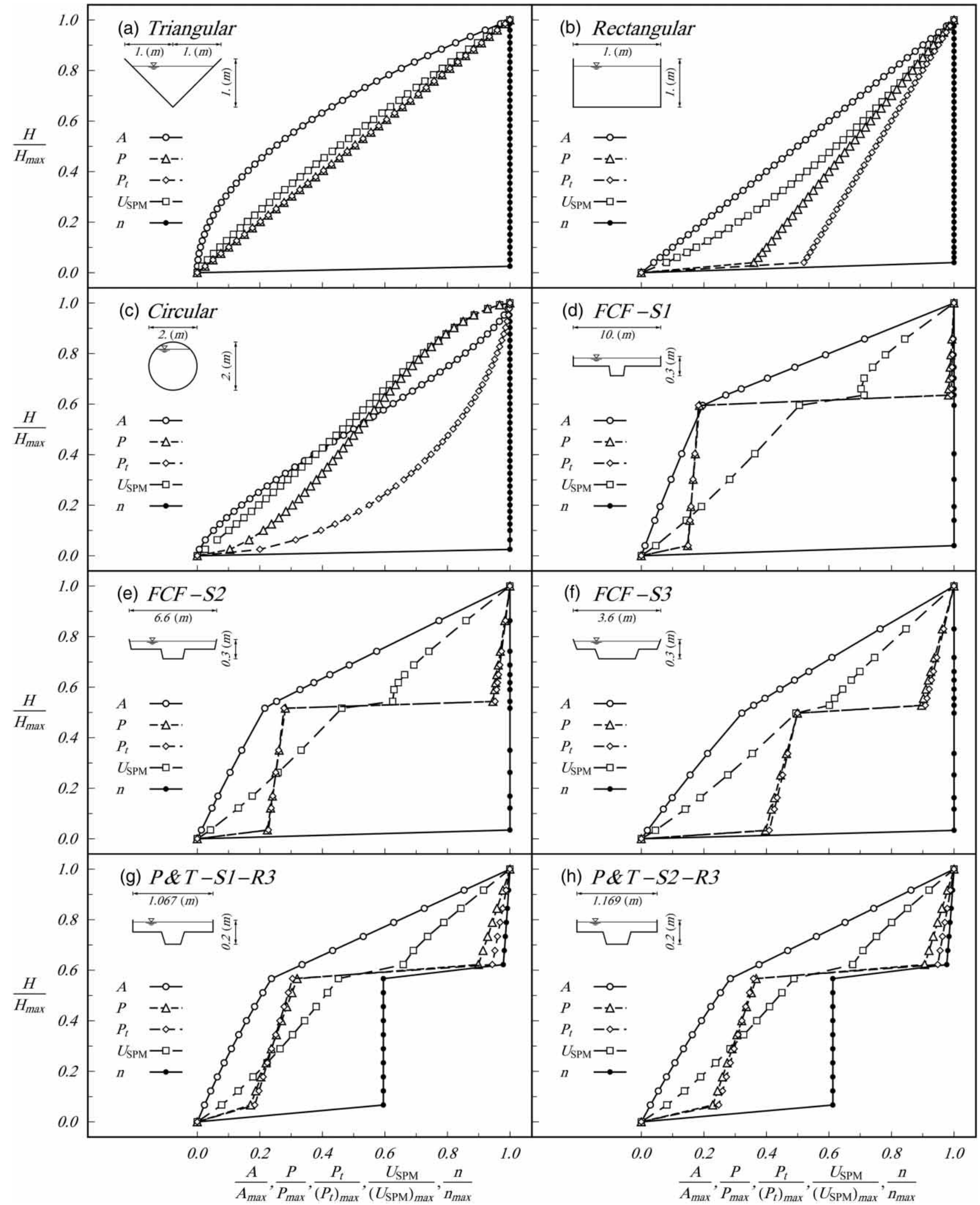

Figure 5 | Variation of the relative values of A, P, P_t, U_{SPM} and n for different cross-sections; (a)–(d) are used for the relationship development and (e) to (h) for observing its performance.

corresponding values for five-parameter relationships in Group B do not differ significantly from the six-parameter relationships in Group A. For example, we have $\overline{NRMSE}_{205} = 0.033$ and $\overline{NRMSE}_{225} = 0.032$. Now the question is how the trivial difference of $\overline{NRMSE}_s$ in relationships 205 and 225 (the best relationships of Groups B and A,

Table 2 | Maximum values for the parameters engaged in Figures 6 and 8

Test	H_{max} (m)	A_{max} (m^2)	P_{max} (m)	$(P_t)_{max}$ (m)	$(U_{SPM})_{max}$	n_{max}
FCF-S01	0.250	1.248	10.324	20.324	1.147	0.01
FCF-S02	0.287	1.135	6.814	13.390	1.254	0.01
FCF-S03	0.299	0.762	3.846	7.444	1.172	0.01
FCF-S06	0.301	0.885	4.603	8.957	1.208	0.01
FCF-S10	0.279	1.142	7.037	13.897	1.211	0.01
P & T-S1-S	0.18	0.109	1.349	2.416	0.484	0.011
P & T-S2-S	0.18	0.127	1.451	2.620	0.518	0.011
P & T-S1-R1	0.18	0.109	1.349	2.416	0.484	0.013
P & T-S2-R2	0.18	0.127	1.451	2.620	0.518	0.0128
P & T-S1-R2	0.18	0.109	1.349	2.416	0.484	0.0157
P & T-S2-R2	0.18	0.127	1.451	2.620	0.518	0.0151
P & T-S1-R3	0.18	0.109	1.349	2.416	0.484	0.0184
P & T-S2-R3	0.18	0.127	1.451	2.620	0.518	0.0179

respectively) affects the performance of stage-discharge curves when they are applied to compound channels with composite roughness.

In order to answer this question, rating curves are estimated from each equation in two laboratory compound channels FCF-S2 and FCF-S3 (Knight 1992) and two rough compound channels P & T-S1-R3 and P & T-S2-R3 (Prinos & Townsend 1984) and then $NRMSE$ values are calculated for each of them. Figure 5(e)–5(h) indicate the variations of H, A, P, P_t, U_{SPM} and n where their maximum values are shown in Table 2.

The total value of $\overline{NRMSE}$ among the second group of the compound cross-sections is named Group B. According to Figure 6, $\overline{NRMSE}_B$ value is negligible for zones with five and six parameters. The summation of $\overline{NRMSE}_A$ and $\overline{NRMSE}_B$ is shown as:

$$\overline{NRMSE}_T = \overline{NRMSE}_A + \overline{NRMSE}_B \tag{8}$$

In fact, the lowest value of $\overline{NRMSE}_T$ is not the only criterion for selection of the best relationship among all relationships. By considering other criteria such as the number of parameters and cross-sections used in the proposed relationship, it can be concluded that the best relationship is the one with the lowest $NRMSE_T$ as well as the minimum number of parameters and cross-sections involved in the formulation of the problem.

In order to propose a universal relationship, the statistical performance of the most accurate extracted relationships based on three, four, five and six parameters corresponding with the relationship numbers 48, 133, 205 and 225, respectively are compared with each other in Table 3. Although the configuration of relationship number 205 is completely different from that of 225, the percentage of the difference in absolute mean error $MAPE$ is only limited to 0.004%. One of the most attractive reasons to choose the relation 205 as the best one is its simplicity and high accuracy.

Relationship number 205 includes five parameters, namely A, P, P_t, U_{SPM} and n as well as two geometric cross-sections (rectangular and compound cross-section) in the determination of exponents of a_1, a_2, a_3 and a_4. Finally, Equation (5) can be displayed in the following form for the selected relationship:

$$Q_e = Q_r\left(\frac{A_e}{A_r}\right)^{0.97}\left(\frac{P_e}{P_r}\right)^{-1.27}\left(\frac{(P_t)_e}{(P_t)_e}\right)^{0.83}\left(\frac{(U_{SPM})_e}{(U_{SPM})_r}\right)^{1}\left(\frac{n_e}{n_r}\right)^{-1} \tag{9}$$

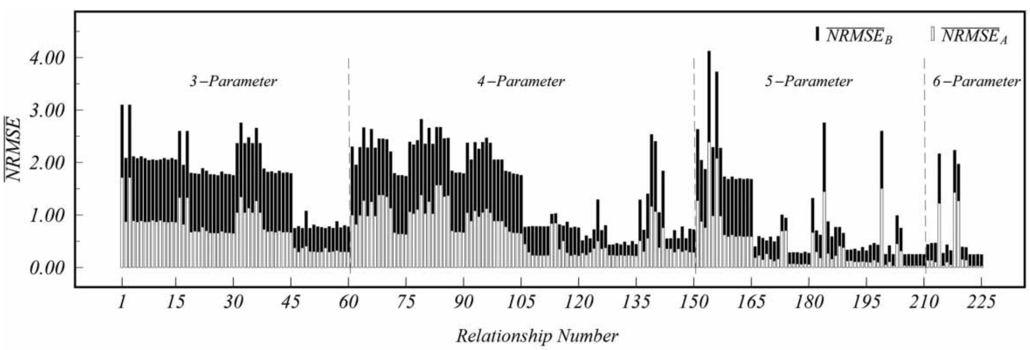

Figure 6 | Variation of $\overline{NRMSE}$ for the cross-sections including in Groups A and B.

Table 3 | Performance evaluation of the best relationships selected from each class of three, four, five and six parameters

	Exponents value							
Equation number	***A***	***P***	***T***	***P_t***	***U_{SPM}***	***n***	***NRMSE***	***MAPE (%)***
48	0.761	0	0	0	1	−1	0.094	12.78
133	1.006	−0.488	0	0	1	−1	0.056	7.80
205	0.972	−1.268	0	0.832	1	−1	0.033	4.479
225	0.979	−1.183	0.047	0.701	1	−1	0.032	4.475

A comparison between the estimated and observed rating curves based on Equation (9) for different cross-sections is shown in Figure 7. As can be seen, the rating curve is established for each section using only three different levels. The results indicate that the performance of the model is highly accurate.

ERROR ANALYSIS

In order to examine the performance of the proposed model, some of the statistical measures including NRMSE and MAPE are calculated based on the observed discharge Q_r and the estimated data Q_e. They are given as follows:

$$MAPE\ (\%) = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{(Q_r)_i - (Q_e)_i}{(Q_r)_i}\right| \tag{10}$$

$$NRMSE = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}((Q_r)_i - (Q_e)_i)^2}}{(Q_r)_{max} - (Q_r)_{\min}} \tag{11}$$

The results of the statistical measures calculated based on different models using Equations (10) and (11), are shown in Figure 8. For triangular, rectangular and circular cross-sections, the results show that except for the first observed values at the lowest water level, the estimated discharge values based on other points are within 3.8% and 0.02 for the *MAPE* and *NRMSE*, respectively. From Figure 8(d)–8(f), it can be seen that the mean values of *MAPE* and *NRMSE* for *FCF* sections are within 5.3% and 0.03, respectively. Figure 8(g) and 8(h) illustrate that the maximum value of *MAPE* and *NRMSE* for compound channels with rough floodplains are within 8.2% and 0.12, respectively.

VALIDATION OF THE STAGE-DISCHARGE RELATIONSHIP

For further validation of the proposed relationship, it is applied to eight compound cross-sections which are selected from the experimental works of Knight (1992) and Prinos & Townsend (1984). After calculating the geometrical parameters and U_{SPM}, one can easily depict the dimensionless variations of the cross-sectional area A, wetted perimeter P, total perimeter P_t and velocity parameter U_{SPM}. In Table 2, $H_{\max}$, $A_{\max}$, $P_{\max}$, $(P_t)_{\max}$, $(U_{SPM})_{\max}$ and $n_{\max}$ for the experimental works of compound channels are given. From Figure 9, the stage-discharge curves based on a couple of observed data about Q and H at any arbitrary level can be plotted.

Figure 10(a)–10(h) show the stage-discharge curves obtained from the EDM, SCM and WDCM methods. They are compared with the results of the proposed method for the corresponding compound channels.

Considering the results of EDM with the assumed value of $\Psi^t = 0.16$ at the lower levels above the floodplain, the estimated discharge is lower than that of the experimental ones. However, by increasing the water level, the difference in discharges becomes smaller. Due to the integrity of the whole cross-section in the SCM, where its hydraulic behavior is completely different from the compound sections, the obtained results are not in good agreement with the observed data. The results of WDCM are in good agreement with the observed stage-discharge data taken from the experimental results of *FCF* channels. It should be noted that the calibration of this method is made by the use of *FCF* observational data. The best capability of the proposed model is that it does not need any calibration while other methods do.

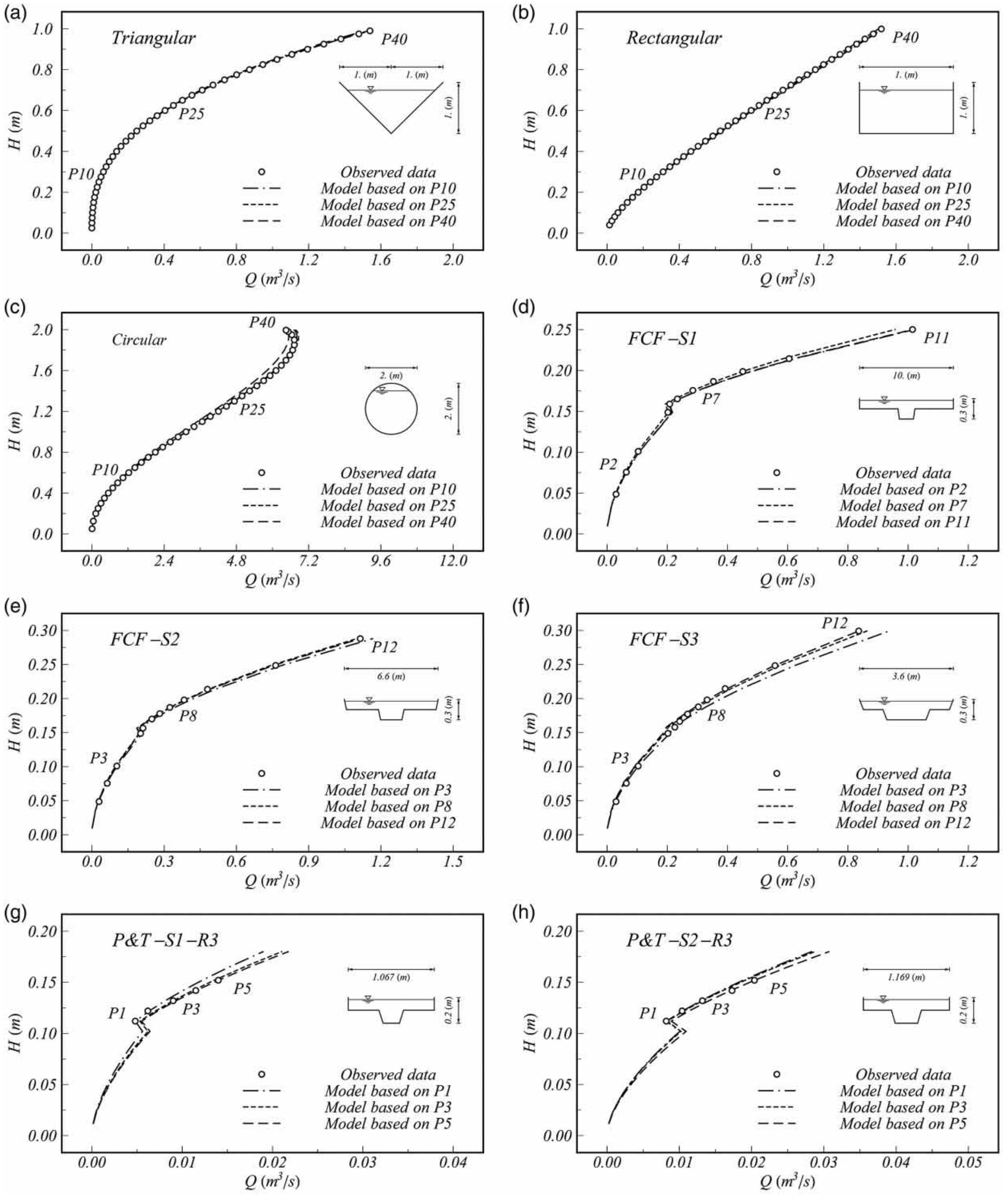

Figure 7 | The results of stage-discharge curves based on the proposed model using different reference points and cross-sections corresponding to Figure 5.

In the calculation of errors, it is only possible to compare the results when the water level is higher than the floodplain level.

In comparison with other cross-sections, the performances of WDCM in *FCF* cross-sections are much better because of experimental data such as the *FCF* data which are used to develop the WDCM method. The average values of the statistical measures, i.e. $\overline{MAPE}$ and $\overline{NRMSE}$, are equal to 6.4% and 0.06, respectively. The accuracy of EDM is highly related to the selection of Ψ^t. If a good value is chosen for this parameter, the corresponding error associated with this method will be reduced. With the

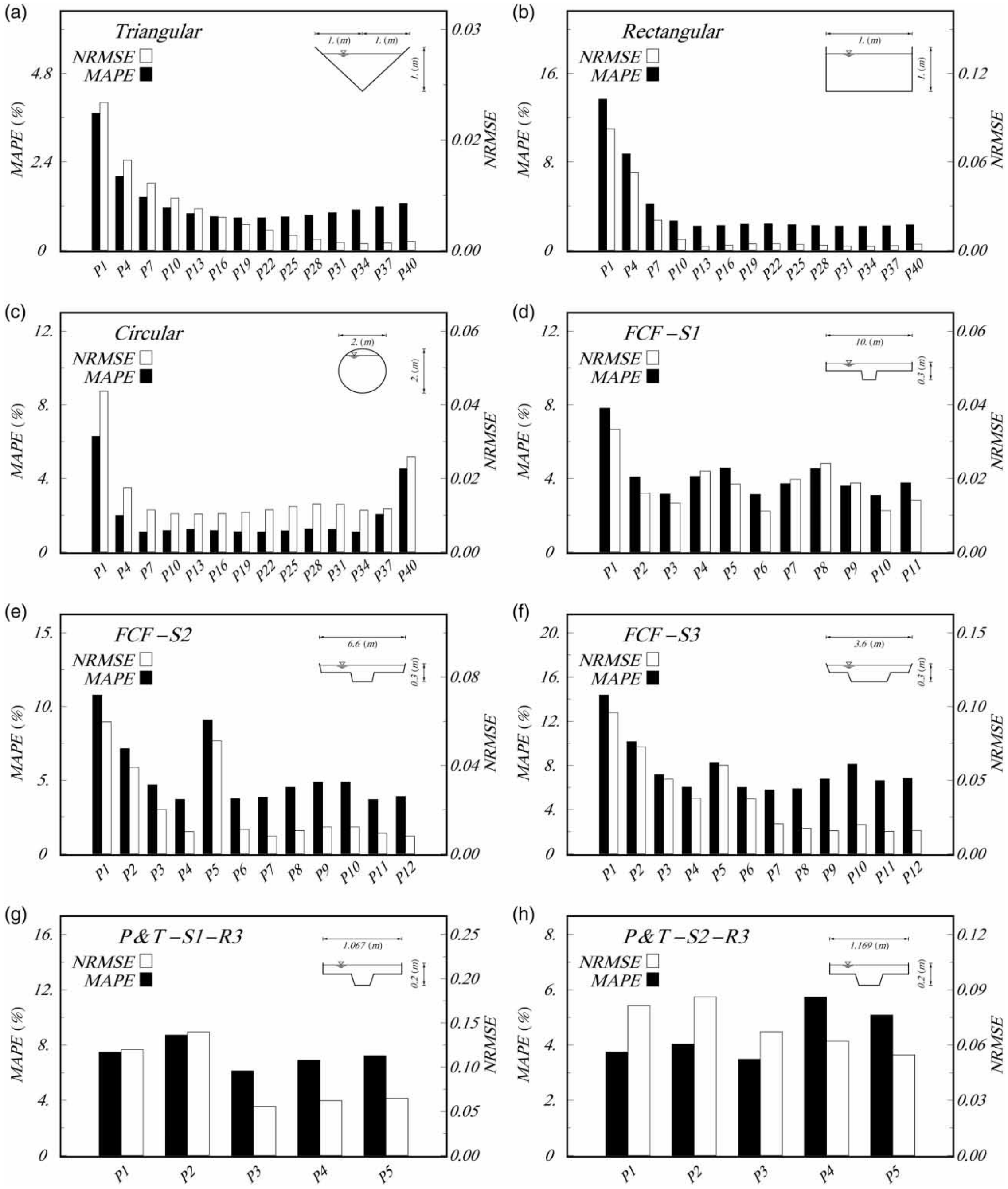

Figure 8 | A comparison of *MAPE* and *NRMSE* obtained by the proposed model and conventional models at different reference levels.

recommended value of 0.16 which is proposed by Bousmar (2002), $\overline{MAPE}$ and $\overline{NRMSE}$ are within 4.7% and 0.05, respectively. The average value of $\overline{MAPE}$ obtained from the SCM method is roughly 13%.

From Figure 11, it can be seen that for the different reference points the obtained values of the statistical parameters are changed. It should be noted that the accuracy of the results is very much related to the accuracy of the observed data. The calculated values of $\overline{MAPE}$ and $\overline{NRMSE}$ based on the proposed model are quite acceptable within 4.9% and 0.053, respectively, when compared with other methods. Even in some cases, these values are lower than other

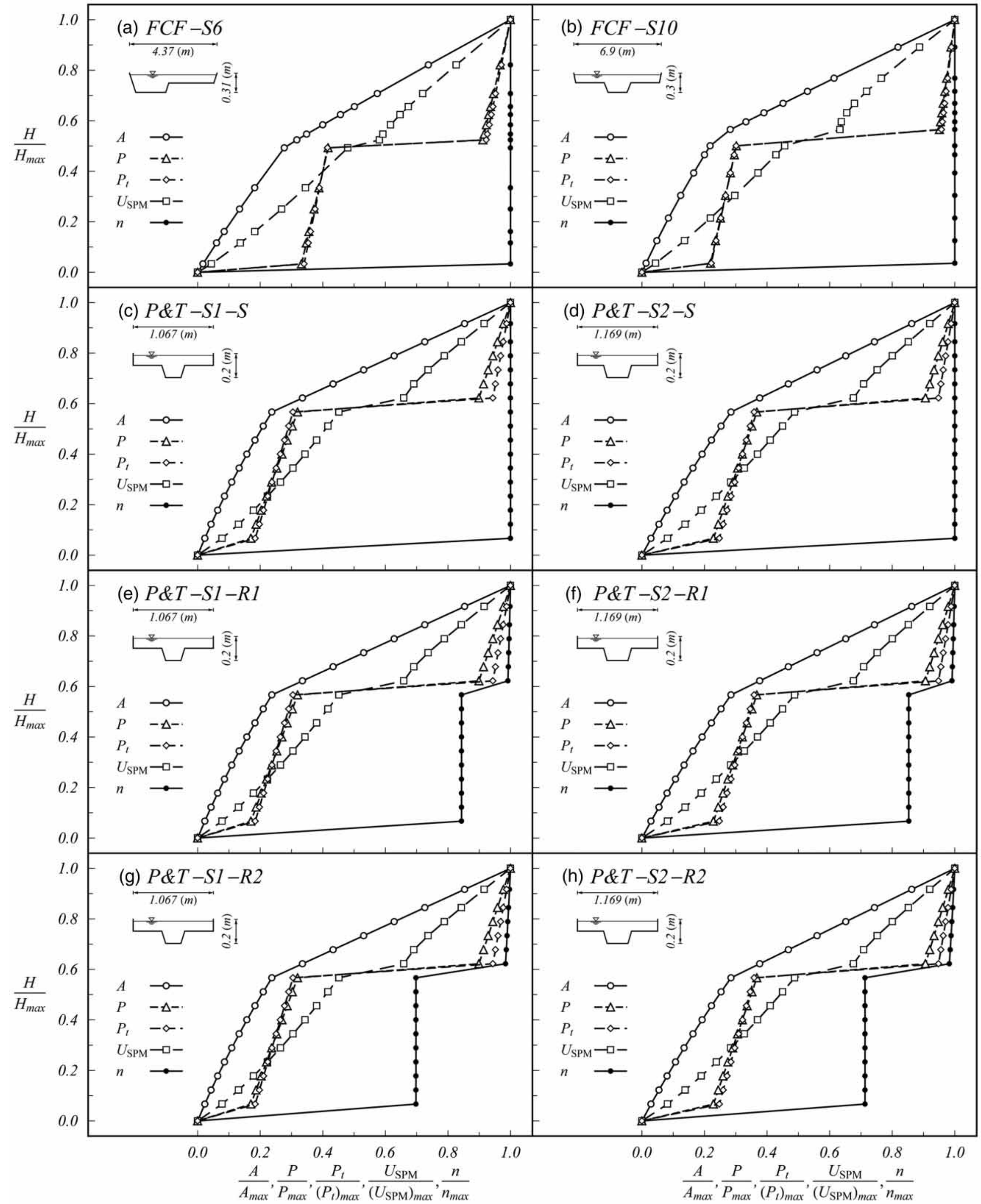

Figure 9 | Variation of the relative values of A, P, P_t and U_{SPM} for different cross-sections.

methods. The average values of the above-mentioned statistical measures based on the whole water levels are within these values. The stage-discharge curves obtained by the proposed method are more accurate than the other two methods as seen in Figure 11(f)–11(h) for compound channels with composite roughness.

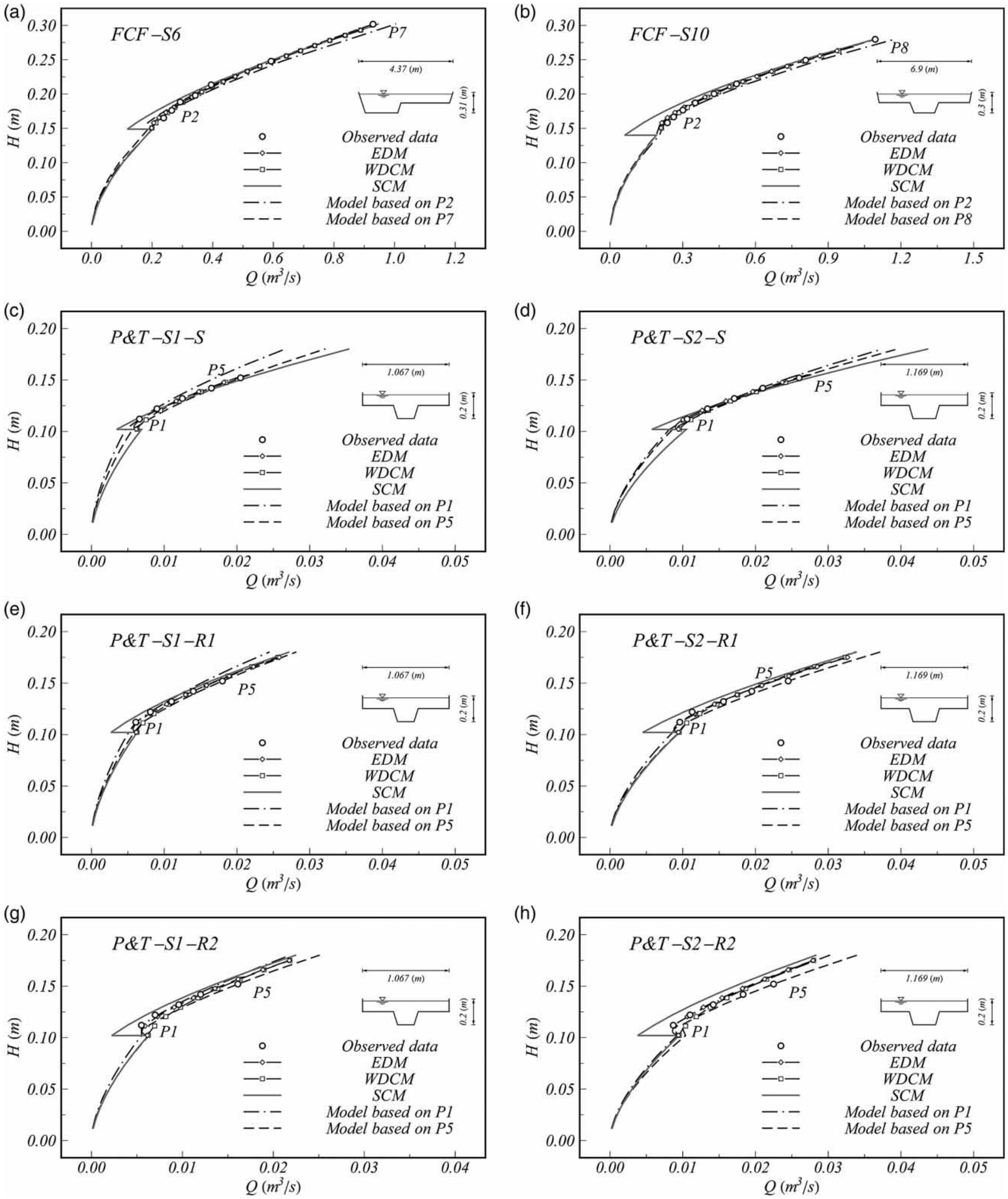

Figure 10 | The results of stage-discharge curves based on the proposed model and conventional methods in comparison with the experimental works at different cross-sections.

APPLICATION TO A NATURAL RIVER

The Batu River is located in Malaysia. During the monsoon season in recent years, it has experienced several flood occurrences. The cross-sectional geometry of the Batu River is extracted as shown in Figure 12(a), which is largely asymmetrical (Hin *et al.* 2008). Therefore, implementation of the 1D approach to predict the stage-discharge

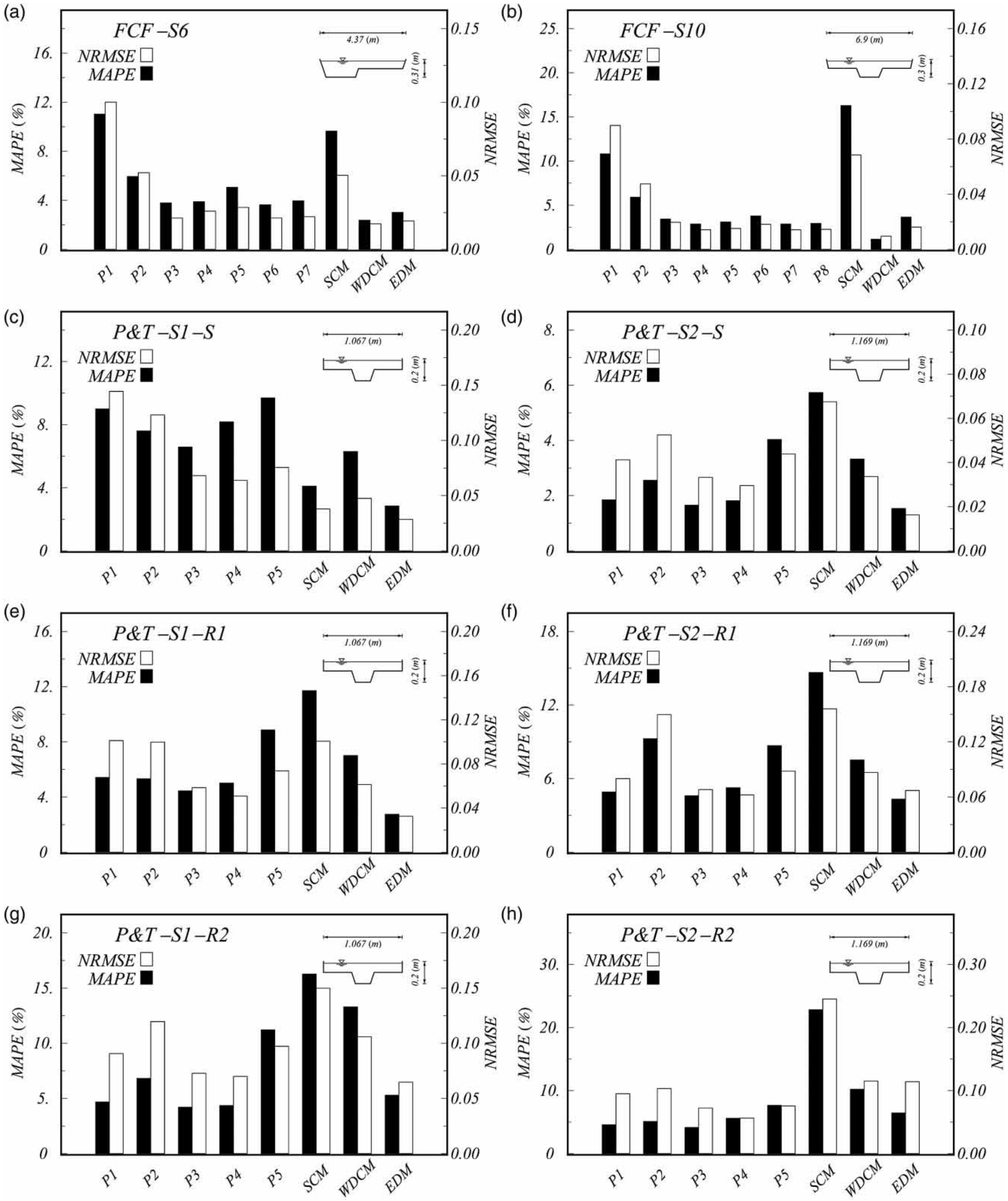

Figure 11 | A comparison of *MAPE* and *NRMSE* obtained by the proposed model and conventional models at different reference levels.

relationship without consideration of the momentum transfer will lead to unrealistic results (Khatua *et al.* 2012).

Now the model is applied to this river. Based on Equation (9), the values of A, P, P_t, n and U_{SPM} should be calculated at each stage. For this purpose, variation of each value for the whole range of stages varying between zero and H_{max} in a dimensionless form for the Batu River is plotted in Figure 12(b). The values of A_{max}, P_{max}, $(P_t)_{\max}$, n_{max} and

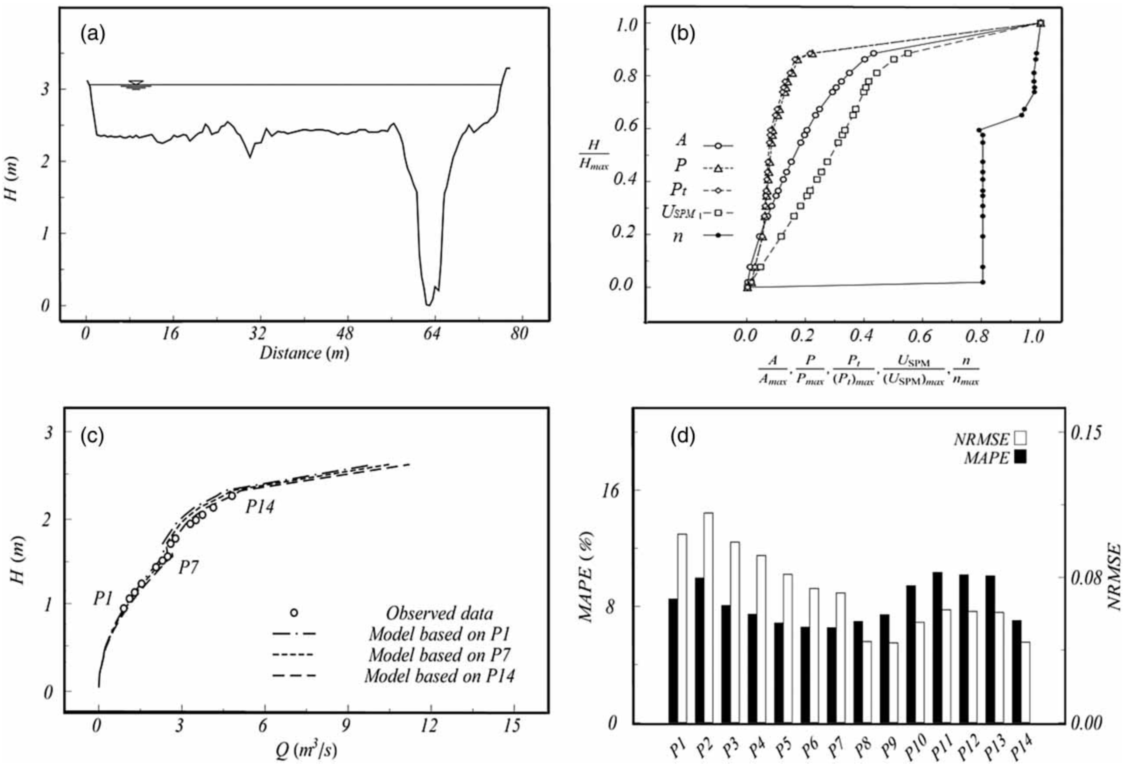

Figure 12 | The Batu River (a) cross-sectional geometry, (b) variation of the parameters affecting the discharge, (c) estimated stage-discharge curves as well as the measured values and (d) statistical values of *MAPE* and *NRMSE*.

U_{max} are the maximum values at H_{max}. In the Batu River the maximum value of stage is $H_{max} = 2.6$ m. The values of A_{max}, P_{max}, $(P_t)_{max}$, n_{max} and $(U_{SPM})_{max}$ are 28.19 m^2, 74.67 m, 147.46 m, 0.103 and 13.98, respectively. Figure 12(c) shows the estimated rating curves in the Batu River which is extracted for the reference levels P1, P7 and P14. It can be observed that the estimated rating curves obtained by the present method are very close to the observed data.

The bar charts in Figure 12(d) show the statistical measures of *NRMSE* and *MAPE* in the Batu River. With respect to the value of *MAPE* for the Batu River, it can be seen that the lowest and highest values are 7 and 10%, respectively, with an average value of nearly 8.2%. It should be noted that the accuracy of estimated results highly depends on the accuracy of referenced data.

CONCLUSIONS

The stage-discharge curve is one of the key parameters for decision-making in the management of water resources and river engineering. In order to estimate the discharge in compound channels using traditional methods, the flow cross-sections should be divided into a number of subsections. However, in the proposed model, the whole flow cross-section is considered as a unified and integrated section. As a result, the most general form of an expression which can be used for extrapolation of the stage-discharge curve in any kind of open channel with any type of roughness distribution including composite compound channels is introduced. In order to set up the new expression, a combination of the selected geometries of the flow sections and the corresponding calculated parameters, namely A, P, P_t, U_{SPM} and n, are implemented. From each combination a unique relationship is extracted. At the end of this stage 225 different formulae are proposed, then the best formula is selected based on the performance of each formula using the minimum error statistics such as *NRMSE* and *MAPE*. All of the variables involved, including the geometrical and hydraulic parameters such as A, P, P_t, U_{SPM} and n, are assumed to be not only known at the referenced level but also at the level where we are actually looking for its discharge. Presentation of the performance of the new relationship in comparison to others shows that the proposed expression has a very good capability in the estimation of stage-discharge curves. The most significant advantage of the proposed method in the estimation of rating curves is its inherent simplicity, which does not need any calibration.

REFERENCES

Ackers, P. 1993 Flow formulae for straight two-stage channels. *J. Hydr. Res.* **31** (4), 509–531.

Bousmar, D. 2002 *Flow Modelling in Compound Channels: Momentum Transfer Between Main Channel and Prismatic or Non-Prismatic Floodplains*. UCL, Louvain-la-Neuve, Belgium.

Bousmar, D. & Zech, Y. 1999 Momentum transfer for practical flow computation in compound channels. *J. Hydr. Eng.* **125** (7), 696–706.

Cao, Z., Meng, J., Pender, G. & Wallis, S. 2006 Flow resistance and momentum flux in compound open channels. *J. Hydr. Eng.* **132** (12), 1272–1282.

Halliday, D. & Resnick, R. 1981 *Fundamentals of Physics*. John Wiley & Sons, New York.

Hin, L. S., Bessaih, N., Ling, L. P., Ghani, A. A., Zakaria, N. A. & Seng, M. Y. 2008 A study of hydraulic characteristics for flow in equatorial rivers. *Int. J. River Basin Manage.* **6** (3), 213–223.

Huai, W., Chen, G. & Zeng, Y. 2013 Predicting apparent shear stress in prismatic compound open channels using artificial neural networks. *J. Hydroinform.* **15** (1), 138–146.

Khatua, K., Patra, K. & Mohanty, P. 2012 Stage-discharge prediction for straight and smooth compound channels with wide floodplains. *J. Hydr. Eng.* **138** (1), 93–99.

Knight, D. 1992 *SERC Flood Channel Facility Experimental Data – Phase A. Report SR314*. Hydraulic Research, Wallingford, UK, pp. 1–14.

Lambert, M. & Myers, W. 1998 Estimating the discharge capacity in straight compound channels. *Proc. Inst. Civil Eng. Water Maritime Energy* **130** (2), 84–94.

Liao, H. & Knight, D. 2007 Analytic stage-discharge formulas for flow in straight prismatic channels. *J. Hydr. Eng.* **133** (10), 1111–1122.

Maghrebi, M. F. 2006 Application of the single point measurement in discharge estimation. *Adv. Water Resour.* **29** (10), 1504–1514.

Maghrebi, M. F., Arash Ahmadi, A., Attari, M. & Faghfour Maghrebi, R. 2016 New method for estimation of stage-discharge curves in natural rivers. *Flow Meas. Instrum.* **52**, 67–76.

Maghrebi, M. F., Kavousizadeh, A., Maghrebi, R. F. & Ahmadi, A. 2017 Stage-discharge estimation in straight compound channels using isovel contours. *Hydrol. Process.* **31** (22), 3859–3870.

Mohanty, P. K. & Khatua, K. K. 2014 Estimation of discharge and its distribution in compound channels. *J. Hydrodynam.* **26** (1), 144–154.

Prinos, P. & Townsend, R. 1984 Comparison of methods for predicting discharge in compound open channels. *Adv. Water Resour.* **7** (4), 180–187.

Rezaei, B. & Knight, D. W. 2010 Overbank flow in compound channels with nonprismatic floodplains. *J. Hydr. Eng.* **137** (8), 815–824.

Sahu, M., Khatua, K. & Mahapatra, S. 2011 A neural network approach for prediction of discharge in straight compound open channel flow. *Flow Meas. Instrum.* **22** (5), 438–446.

Sellin, R. & Van Beesten, D. 2004 Conveyance of a managed vegetated two-stage river channel. In: *Proceedings of the Institution of Civil Engineers – Water Management*. Thomas Telford Ltd, London, pp. 21–33.

Smart, G. 1992 Stage-discharge discontinuity in composite flood channels. *J. Hydr. Res.* **30** (6), 817–833.

Stephenson, D. & Kolovopoulos, P. 1990 Effects of momentum transfer in compound channels. *J. Hydr. Eng.* **116** (12), 1512–1522.

Sun, X. & Shiono, K. 2009 Flow resistance of one-line emergent vegetation along the floodplain edge of a compound open channel. *Adv. Water Resour.* **32** (3), 430–438.

Yang, Z., Gao, W. & Huai, W. 2012 Estimation of discharge in compound channels based on energy concept. *J. Hydr. Res.* **50** (1), 105–113.

Yen, B. C. 2002 Open channel flow resistance. *J. Hydr. Eng.* **128** (1), 20–39.

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