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Abstract:

We address the logistics and planning problem of delivering Ready Mixed Concrete (RMC) to a set of demand customers from multiple depots. The RMC Dispatching Problem (RMCDP) is closely related to the Vehicle Routing Problem (VRP) with the difference that demand nodes in the RMCDP may be visited more than once by a truck. This class of routing problems can be represented using Mixed-Integer Programming (MIP) and is known to be NP-hard. Solving RMC delivery problems is often achieved through heuristics and meta-heuristic based methods as exact solution approaches are often unable to find optimal solutions efficiently, in particular when multiple depots are represented in the model. Although a variety of methods are available to solve MIP models, in this paper we attempt to solve the RMCDP using a Lagrangian relaxation technique. Namely, we derive a solution algorithm based on Lagrangian relaxation to reduce the complexity of the initial MIP model and show that the proposed relaxation is able to provide promising computational results on a realistic dataset representative of an active RMCDP in the region of Adelaide, Australia.

Keywords: Lagrangian Relaxation, Duality, MIP, Ready Mixed Concrete, Dispatching.

1. Introduction

In this paper, we address the problem of delivering Ready Mixed Concrete (RMC) to a set of demand customers. This logistics and planning problem arise in many real-world applications where a large amount of RMC needs to be delivered to several construction sites while respecting some scheduling and haul time constraints. The underlying routing problem within RMC delivery is closely related to the Vehicle Routing Problem (VRP), with the difference that in the RMC Dispatching Problem (RMCDP) a customer may be visited more than once by the same truck to be entirely serviced. The RMCDP can be represented mathematically using Mixed Integer Programming (MIP) and therefore this class of routing problems requires dedicated models and solution methods. In this contribution, we introduce a novel Lagrangian relaxation *(1)* approach to solve the RMCDP.

In RMCDP it is desirable to find the best allocation of delivery trucks to depots and customers so that transportation costs are minimized. In this paper, we attempt to solve this NP-hard problem using Lagrangian Relaxation. Lagrangian relaxation has been widely used to solve hard Integer Programming (IP) models and attempts to relax the original problem by representing a set of constraints as penalties within the objective function through the use of Lagrangian multipliers. Though there are many modeling strategies and approaches that have been developed by researchers in the past *(1)*, in this paper, we focus on a simple implementation of Lagrangian Relaxation where we dualize the flow constraints and implement a basic sub-gradient algorithm (non-differentiable optimization method) to obtain the values for the Lagrangian multipliers. We test this solution algorithm using a realistic dataset representative of an active multi-depot RMCDP in the region of Adelaide, Australia and report promising results.

The paper is organized as follows: Section 2 summarizes the existing literature on the RMC delivery problems and related works; Section 3 presents the mathematical formulation of the MIP proposed to represent the RMCDP; Section 4 introduces a novel Lagrangian relaxation approach for the RMCDP; Section 5 details the implementation of the proposed solution algorithm and the results obtained and Section 6 concludes this research.

2. Literature Review

In this section, we briefly summarize the existing literature on RMC delivery problems and the related VRP formulations.

Most of the work on RMC delivery has been published within the last decade and several formulations have been proposed for single-depot and for multi-depots dispatching problems. Single-depot RMCDPs aim to represent small to medium sized delivery problems which only have an active batch plant and an assumed homogeneous fleet. The multi-depot variant seeks to represent the case where multiple batch plants (depots) are available to load RMC into delivery trucks and a wide range of trucks is typically available within the fleet. Feng and Wu *(2)* introduced a single-depot model which focuses on minimizing idle times. Due to the complexity of the RMCDP, the authors solve this model heuristically and recently introduced a more advanced model *(3)* to refine their approach. Naso *et al. (4)* introduced a multi-objective model for multi-depot RMCDP with a homogeneous fleet of trucks. Their model is also able to take into account hired trucks as well as out-sourced deliveries. However, the proposed formulation requires a large number of decision variables as well as side constraints, hence the authors use a Genetic Algorithm (GA) to tackle the problem. Yan *et al. (5)* introduced a decomposition based formulation for the single-depot RMCDP with a homogeneous fleet. In this formulation, the authors decompose a customer according to the number of required deliveries. Subsequently, a several variants of this formulation were proposed which incorporate additional features of the RMC delivery problem, such as overtime consideration *(6)*, incident management and stochastic travel times *(7)*. Lin *et al. (8)* presented a formulation that introduces uncertainty in the demand for RMC and minimizes the total waiting time. The authors represent the RMC dispatching problem as a job shop problem where the construction site represents a job and trucks represent workstations. This model can be used to address single-depot dispatching problem with a heterogeneous fleet. Another model using a similar approach was presented by Schmid *et al. (9)* for a single-depot RMCDP with a heterogeneous fleet. The authors present a MIP model that seeks to avoid unsupplied customers by penalizing the unsatisfied customers in the objective function. A more advanced version of this model was subsequently introduced *(10)*. Recently, a single-depot formulation for the homogeneous truck case was proposed and shown to optimally reduce the number of decision variables when the scheduling considerations can be omitted *(11)*.

In an effort to address the case of multi-depot RMCDP, Asbach *et al. (12)* introduced a node-decomposition oriented formulation which proved to be a promising approach to tackle this variant. In this formulation, a depot is divided into a set of sub-depots based on the number of possible loading slots at that depot. Similarly, a customer is divided into a set of sub-customers according to the number of required deliveries. While this approach is shown to reduce the number of side constraints in the obtained MIP, the number of decision variables may increase significantly. This decomposition approach was subsequently used along with different solution methods for the multi-depot RMCDP *(13- 17)*. The present paper builds on this research and introduces a novel solution method based on Lagrangian relaxation. The RMCDP can also be perceived as a VRP with capacity, split deliveries *(18)* with the addition of scheduling constraints that can be introduced in the formulation with dedicated time window side constraints *(19)*. However, if no node decomposition is conducted, a careful attention must be given to the arising flow constraints within VRP-based formulation to represent the possibility that depot and customer nodes may be visited more than once during the course of the operations period.

In this paper, we present formulation based on the aforementioned depot and customer node decomposition and introduce a novel a Lagrangian relaxation based solution algorithm to improve the computational tractability of the mutli-depot RMDCP.

3. Mathematical Formulation

In this section, we introduce the mathematical formulation of the RMCDP. In a RMC batch plant, the specifications of concrete mix are designed and raw materials are mixed together based on orders. Then fresh concrete is loaded into a truck. The loaded truck hauls the concrete and pours it at the destination and then returns to the batch plant. In practice, the mixing part is performed automatically while the rest of the process is handled by human experts. Dispatchers are responsible for deciding to send a truck from a batch plant at a specific time to a project. This job becomes more complicated when a dispatcher need to make calculated decisions for supplying concrete for a certain project that is located between two or more batch plants. This decision-making task involves the management of the distance from batch plants (depots) to customers' facilities, the total amount of concrete required (demand), the temporal spacing time between deliveries, the truck RMC capacities as well as the time of the first unload. Based on this information the dispatch manager needs to manage the supply to each customer and each of their projects while trying to keep all customers pleased. The dispatcher makes decisions about the location(s) of supplier batch plant(s), time of delivery and the size of trucks. Having several active batch plants and several projects increase the complexity of this process and the role of the dispatch manager becomes more critical, as the entire RMC system works according to the schedule that is developed by the dispatch manager.

This concrete delivery problem can be represented as a logistics and transportation planning problem where delivery trucks are to be routed from a set of start nodes to a RMC depot, supply RMC to a set of demand customers and finish their journey at a final node. We can decompose the journey of a vehicle in four types of trips, as depicted by Fig. 1:

1. **Start–Depot:** trips from the start nodes to the first assigned depot nodes in the route.
2. **Depot–Customer:** trips from the depot nodes to the customer nodes (the vehicle carry RMC).
3. **Customer–Depot:** trips from the customer nodes to the depot nodes (the vehicle is empty and will be loaded at the next depot).
4. **Customer–Final:** trips from the customer nodes to the final nodes.

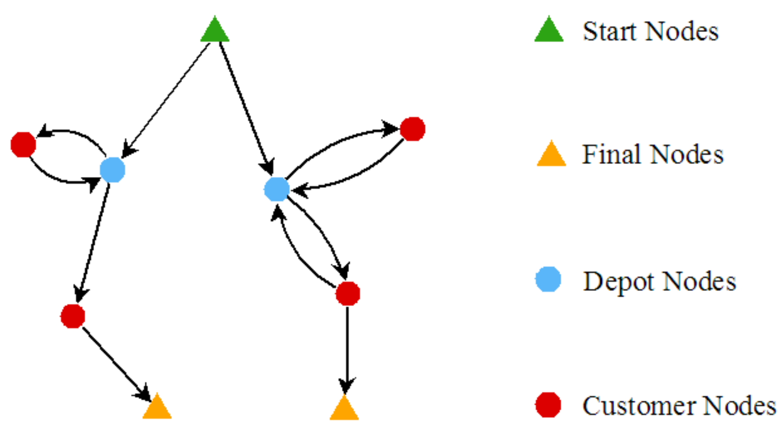

Fig. 1 – An example of a solution for the RMCDP with four types of trips.

Since a depot and/or a customer may be visited more than once by the same truck within the delivery period, the traditional network flow formulations for VRP cannot be used to directly to represent this dispatching problem. We use a depot/customer node decomposition to transform an original RMCDP instance into a network flow instance where each sub-depot and sub-customer can only be visited at most once during the operation period. We use the following notation for the sets and parameters used throughout the paper:

N	set of nodes: $N = S \cup D \cup C \cup F$
A	set of arcs
$\bar{C}$	set of customer nodes
$\bar{D}$	Set of depot nodes
C	set of sub-customer nodes
D	set of sub-depot nodes
S	set of start nodes
F	set of final nodes
K	set of vehicles
Q_u	demand of customer $u \in \bar{C}$
c_k	capacity of vehicle $k \in K$
s_u	service time at depot $u \in \bar{D}$
β_u	penalty for not servicing customer $u \in \bar{C}$
$[L_u, U_u]$	feasible time-window for node $u \in \bar{C} \cup \bar{D}$
z_{uvk}	travel cost on arc $(u, v) \in A$ for vehicle $k \in K$
t_{uvk}	travel time on arc $(u, v) \in A$ for vehicle $k \in K$
γ	maximum concrete haul time
T	operations period

To ensure that the capacity of the depot is respected, we compute the maximum number of loading slots at a depot $u \in \bar{D}$ according to its service time s_u. Hence the number of sub-depot nodes over the operations period T is given by $\left\lfloor \frac{T}{s_u} \right\rfloor$. Similarly, the number of deliveries required to service a customer $u \in \bar{C}$ must respect the capacity of the trucks used to deliver RMC. Two cases can be identified:

- Homogeneous fleet: in this case, all trucks have the same capacity and therefore the number of sub-customer nodes is given by $SC_u = \left\lceil \frac{Q_u}{c} \right\rceil$ where $c = c_k, \forall k \in K$.
- Heterogeneous fleet: in this case, trucks may have a different capacity and therefore we can determine an upper bound on the number of sub-customer nodes which is given by $SC_u = \left\lceil \frac{Q_u}{min_{k \in K\{c_k\}}} \right\rceil$.

Hence each customer node $u \in \bar{C}$ can be decomposed into SC_u sub-customer nodes and we can then define the cluster sets $C_u, \forall u \in \bar{C}$ composed of all the sub-customer nodes $i_1 \dots i_{SC_u}$ that represents the real customer node u:

$$C_u \equiv \{i_1 \dots i_{SC_u}\}, \qquad \forall u \in \bar{C}$$

The deliveries of RMC to customer nodes can then be planned using the cluster sets in order to ensure that the model does not plan more deliveries than requested to satisfy the demand.

We denote $N = S \cup D \cup C \cup F$ the set of nodes used in the routing process and we denote A the set of valid trips, that is, $A \subset N \times N$ is the set of arcs that can be used to construct a solution for the RMCDP. Precisely, only trips of the four types listed above can be used within the journey of a truck, hence the set of valid trips is given by:

$$A \equiv \{\{(u,v): u \in S, v \in D\} \cup \{(u,v): u \in D, v \in C\} \cup \{(u,v): u \in C, v \in D\} \cup \{(u,v): u \in C, v \in F\}\}$$

Note that set A excludes all trips between two depots or two customers. Fig. 2 illustrates this depot/customer node decomposition: each sub-depot and sub-customer node is visited at most once in the solution.

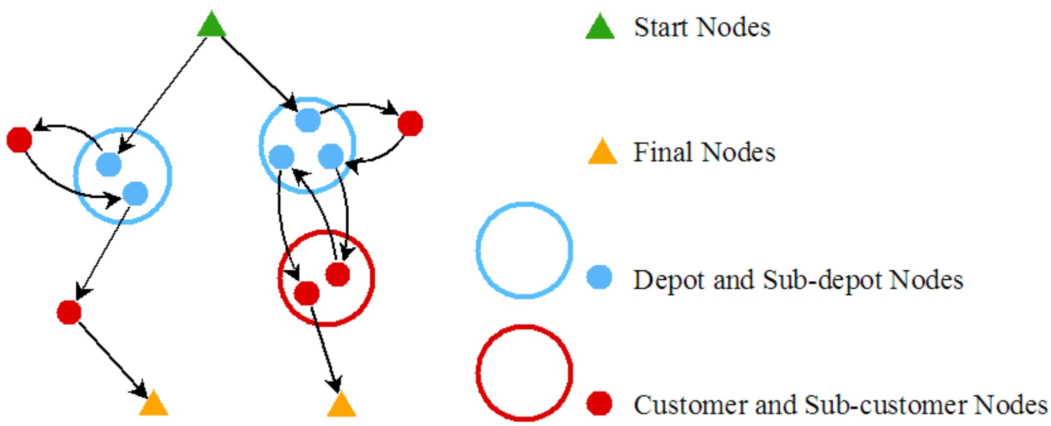

Fig. 2 – Decomposition of depot and customer nodes into sub-depot and sub-costumer.

The decision variables of the proposed RMCDP model can be divided into three categories: namely, the routing variables x_{uvk} are defined as

$$\forall\, (u,v) \in A, \forall\, k \in K, \quad x_{uvk} \equiv \begin{cases} 1 & if\ vehicle\ k\ uses\ arc\ (u,v) \\ 0 & otherwise \end{cases} \tag{1}$$

the assignment variables y_u are defined as

$$\forall\, u \in C, \quad y_u \equiv \begin{cases} 1 & if\ customer\ j\ is\ serviced; \\ 0 & otherwise. \end{cases} \tag{2}$$

and the timing variables w_u are defined as

$$\forall\, u \in C \cup D, \quad w_u \in [L_u, U_u]. \tag{3}$$

where L_u and U_u are lower and upper bounds on the feasible arrival time at node u. The proposed model for the RMCDP is represented by Equations (4)-(15).

$$\min \sum_{(u,v)\in A} \sum_{k\in K} z_{uvk} x_{uvk} + \sum_{u\in \bar{C}} (1 - y_u)\beta_u \tag{4}$$

Subject to:

$$\sum_{u\in S} \sum_{v\in D} x_{uvk} = 1 \qquad \forall\, k \in K \tag{5}$$

$$\sum_{u\in C} \sum_{v\in F} x_{uvk} = 1 \qquad \forall\, k \in K \tag{6}$$

$$\sum_{u\in N} x_{uvk} - \sum_{u\in N} x_{vuk} = 0 \qquad \forall\, k \in K\ , \forall\, v \in C \cup D \tag{7}$$

$$\sum_{k\in K} \sum_{u\in N} x_{uvk} \leq 1 \qquad \forall\, v \in C \cup D \tag{8}$$

$$x_{uvk} + x_{vuk} \leq 1 \qquad \forall\, u \in D, \forall\, v \in C, \forall k \in \mathrm{K} \tag{9}$$

$$\sum_{v\in D} \sum_{i\in C_u} \sum_{k\in K} x_{vik} c_k \geq Q_u y_u \qquad \forall\, u \in \bar{C} \tag{10}$$

$$-M(1 - x_{uvk}) + s_u + t_{uvk} \leq w_v - w_u \qquad \forall\, u \in D, \forall\, v \in C, \forall k \in \mathrm{K} \tag{11}$$

$$M(1 - x_{uvk}) + \Upsilon + s_u \geq w_v - w_u \qquad \forall\, u \in D, \forall\, v \in C, \forall k \in \mathrm{K} \tag{12}$$

$$x_{uvk} \in \{0,1\} \qquad \forall\, (u,v) \in A, \forall k \in \mathrm{K} \tag{13}$$

$$y_u \in \{0,1\} \qquad \forall u \in C \tag{14}$$

$$w_u \in [L_u, U_u] \qquad \forall u \in C \cup D \tag{15}$$

The objective function (4) seeks to minimize the transportation costs while supplying a maximum number of customers. Constraints (5) and (6) enforce that each vehicle commences its journey from a start node and terminates it at a final node. Constraint (7) is the flow conservation constraint and ensures that no truck is left behind at a sub-depot or a sub-customer node. Constraint (8) states that no sub-depot or sub-customer node may be visited more than once. Constraint (9) is a subtour elimination constraint, namely it guarantees that no pair of sub-customer and sub-depot nodes forms a subtour. Constraint (10) links the routing variables to the assignment variables and allows variable y_u to be equal to one at the condition that the demand of the sub-customer node u is supplied. Constraints (11) and (12) are time windows constraints that into consideration the load and unload time at depot and customer nodes, the travel time between these nodes as well as the perishable goods consideration (maximum concrete haul time). They ensure that a trip can be planned only if both sub-depot and sub-customer nodes can be visited during the specified time windows. Finally, Constraints (13), (14) and (15) define the domain of the decision variables.

The model represented by Equations (4)-(15) is a Mixed-Integer Linear Program (MILP) which can be solved by enumerative algorithms such as Branch-and-Bound and/or Branch-and-Cut which are widely implemented in off-the-shelf optimization software. However, the potentially large number of variables induced by the node decomposition may significantly affect the computational tractability of the proposed formulation. In the following section, we present a novel Lagrangian relaxation approach to solve the RMCDP represented by Equations (4)-(15).

4. Solution Method

In this section, we present the solution method developed to address the above multi-depot RMCDP. Our approach is based on the Lagrangian relaxation theory *(1)*. One of the difficulties in designing efficient Lagrangian relaxations is that it can be difficult to choose the set of constraint to pass in the objective function so as to yield the best possible approximation of the initial model while improving the computational tractability of the model. We first present the basic theory of Lagrangian relaxation before presenting the scheme adopted in this study.

Let Z_{IP} be the optimal value of an IP problem defined as:

$$Z_{IP} = \{\min C^T x : Ax = b, x \in X, x\ integer\}$$

where $Ax = b$ is a set of linear constraints, C is a cost vector and X represents the feasible region of the variables. Let Z_{LP} be the optimal value of the Linear Programming (LP) relaxation of the IP model obtained by dropping the integrality conditions. The Lagrangian relaxation of the IP relative to $Ax = b$ with a vector λ unrestricted in sign is:

$$Z_{L(\lambda)} = \{\min C^T x + \lambda^T (b - Ax) : x \in X, x\ integer\}$$

λ is known as the Lagrange multipliers (20) and by passing some of the constraints in the objective function, the Lagrangian relaxation relative to $Ax = b$ seeks to penalize this relaxed problem by iteratively adjusting the values of the Lagrange multipliers. Let $Z_L \equiv \max_\lambda\{Z_{L(\lambda)}\}$, it is well known from the Lagrangian relaxation theory that $Z_{LP} \leq Z_L \leq Z_{IP}$. Finally, if problem the IP is feasible and its Lagrangian relaxation possesses an integral optimal solution, then $Z_{LP} = Z_L$.

In the course of this study, we have tried to dualize multiple set of constraints within the set of Equations (5)-(15) and it has been observed that dualizing the first flow constraint (5) provided the best outcome. For this reason, we present the Lagrangian relaxation relative to this set of constraint. Namely, we can pass Equation (5) into the initial objective function (4) using the Lagrange multipliers λ_k for each $k \in K$; the Lagrangian relaxation model is represented by Equations (16)-(26).

$$\min \sum_{(u,v)\in A} \sum_{k\in K} z_{uvk} x_{uvk} + \sum_{u\in \bar{C}} (1 - y_u)\beta_u + \sum_{k\in K} \lambda_k \left(1 - \sum_{u\in S} \sum_{v\in D} x_{vuk} c_k\right) \tag{16}$$

Subject to:

$$\sum_{u\in C} \sum_{v\in F} x_{uvk} = 1 \qquad \forall\, k \in K \tag{17}$$

$$\sum_{u\in N} x_{uvk} - \sum_{u\in N} x_{vuk} = 0 \qquad \forall\, k \in K\ , \forall\, v \in C \cup D \tag{18}$$

$$\sum_{k\in K} \sum_{u\in N} x_{uvk} \leq 1 \qquad \forall\, v \in C \cup D \tag{19}$$

$$x_{uvk} + x_{vuk} \leq 1 \qquad \forall\, u \in D, \forall\, v \in C, \forall k \in \mathrm{K} \tag{20}$$

$$\sum_{v\in D} \sum_{i\in C_u} \sum_{k\in K} x_{vik} c_k \geq Q_u y_u \qquad \forall\, u \in \bar{C} \tag{21}$$

$$-M(1 - x_{uvk}) + s_u + t_{uvk} \leq w_v - w_u \qquad \forall\, u \in D, \forall\, v \in C, \forall k \in \mathrm{K} \tag{22}$$

$$M(1 - x_{uvk}) + \varUpsilon + s_u \geq w_v - w_u \qquad \forall\, u \in D, \forall\, v \in C, \forall k \in K \qquad (23)$$
$$x_{uvk} \in \{0,1\} \qquad \forall\,(u,v) \in A, \forall k \in K \qquad (24)$$
$$y_u \in \{0,1\} \qquad \forall u \in C \qquad (25)$$
$$w_u \in [L_u, U_u] \qquad \forall u \in C \cup D \qquad (26)$$

The problem represented by Equations (16)-(26) is a MIP that necessitates the Lagrange multipliers λ_k to be computed beforehand. In order to determine the values of the Lagrange multipliers, we initialize them with the marginal values (dual) of the LP relaxation of the initial MIP defined by Equations (4)-(15). We then use a generic sub-gradient optimization algorithm to iteratively update the values of the Lagrange multipliers *(17)*. Convergence of the sub-gradient algorithm is achieved when the maximal relative gap between two consecutive values of the Lagrange multipliers is lower than a predefined value. The pseudo-code of the solution algorithm is given in Fig. 3.

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1. Solve the LP relaxation of the model represented by Equations (4)-(15) and let LB be its optimal value. Initialize the variables λ_k^0 with the dual of Constraint (5).
2. Let $\theta = 2$, $best = -\infty$ and $nImp = 0$
3. While $convergence$ is $FALSE$ do:
   a. Solve the Lagrangian relaxation model represented by Equations (16)-(26). Let LB_i be the value of the objective function at iteration i and let x^i be the current optimal solution.
   b. If $(LB_i > best)$ then $best = LB_i$ and $nImp = 0$; else $nImp = nImp + 1$ and if $(nImp > 1)$then $\theta = \theta + 2$ and $nImp = 0$.
   c. Let $\gamma_k^i = 1 - \sum_{u \in S} \sum_{v \in D} x_{uvk}^i$, for each $k \in K$
   d. Let $norm = \sum_{k \in K} \sqrt{\gamma_k^i}$ and let $step = \theta \frac{LB - LB_i}{norm}$
   e. Let $\lambda_k^{i-1} = \lambda_k^i$ and $\lambda_k^i = \max(0, \lambda_k^i + step * \gamma_k^i)$, for each $k \in K$
   f. Let $\Delta = \max_k |\lambda_k^{i-1} - \lambda_k^i|$
   g. If $(\Delta < gap)$ then $convergence = TRUE$
4. End.

Fig. 3 – Lagrangian Relaxation Algorithm for the RMCDP

In the next section, we implement the above Lagrangian relaxation algorithm and report our results on a realistic case study.

5. Case Studies and Computational Results

In this section, we present case studies synthesized from realistic RMC dispatch operations and compare the performance of the proposed Lagrangian relaxation algorithm with regards to the initial MIP represented by Equations (4)-(15). Our approach is tested by field data which belong to an active RMCDP in the region of Adelaide, Australia. Four instances of different sizes (i.e. number of customers, demand volumes) are tested. Given that we use a node-decomposition approach based on the demand of each customer, the complexity of the instances is strongly related to the number of sub-customers and the number of delivery trucks available.

The Lagrangian relaxation algorithm was developed in GAMS, a algebraic modeling system on a RedHat ® CentOS ® 5.9 Linux server with 8 3.60 GHz Intel ® Xeon ® CPUs with a 188 GB physical memory; this algorithm uses IBM CPLEX version 12.4.0.1 to solve the LP relaxations and the MIPs iteratively *(21)*. We use an optimality gap of 0.001%, that is $gap = 0.001\%$ in the Lagrangian relaxation algorithm. The penalty β_u is set to a large enough number for all customers. The computational results are summarized in Table 1 which also contains a summary of the characteristics of each instance tested (number of depots, sub-depots, customers, sub-customers and delivery trucks available). Table 1 describes the computational performance of the Lagrangian relaxation algorithm presented in Fig. 3 as well as the one obtained when the initial MIP is solved directly.

Table. 1 – Summary of the computational results

Problem Instance						**Algorithm run time (seconds)**		**Objective value (km)**	
ID	**nb Depots**	**nb sub-depots**	**nb customers**	**nb sub-customers**	**nb trucks**	**MIP**	**Lagrangian**	**MIP**	**Lagrangian**
1	2	106	26	40	17	5.57	4.54	27.137327	27.137327
2	4	230	33	63	29	37.15	36.89	37.156975	37.156975
3	4	320	39	112	31	246.43	239.98	62.829725	62.829725
4	4	320	55	153	33	7086.8	3428.87	90.636275	90.636275

For all the instances tested, the Lagrangian relaxation algorithm found the same solutions as the initial MIP, hence all the solutions obtained are feasible and optimal. In each instance, all the customers are fully serviced. The Lagrangian approach is found to be always faster than solving the initial MIP directly with an improvement of the run time varying from 1% (Instance 2) to almost 51% (Instance 4). The results obtained suggest that the improvement rate tends to increase with the size of the instance, although the improvement for Instance 1 is inferior to the one for Instance 2. Instance 4 is reported to be more difficult to solve than other instances as run time increases significantly for both the initial MIP and the Lagrangian relaxation algorithm.

To further illustrate the model behavior, we give the location and demand data for Instance 3 which is representative of a regular RMCDP, in Tables 2 and 3; and plot the optimal solution obtained by the Lagrangian relaxation algorithm for this instance in Fig. 4 as well as the optimal schedule of the trucks in Fig. 5. In this instance among the 31 trucks available 25 have a capacity of 7.6 m3 and 6 have a capacity of 6.2 m3. 9 start nodes and 9 final nodes are used to organize the journey of the RMC delivery trucks and sub-depots are available to load RMC into trucks. The service time is 15 minutes and the maximum haul time is 90 minutes. The operations period is from 4am to 6pm but most of the depots are only available later in the day. Some of the trucks' start and final nodes are existing depots, hence the schedule for these trucks is shown to start and end at depots nodes. Due to the scheduling conflicts for loading RMC at depot nodes and unloading at customer nodes, a variable amount of idle time is observed.

Table 2 – Location of depots for Instance 3

Depot	x coordinate	y coordinate
1	138.502	-35.1183
2	138.7034	-34.927
3	138.7189	-34.6505
4	138.5225	-34.8394

Table 3 – Location and demand of customers for Instance 3

Customer	Demand (in m3)	x coordinate	y coordinate
1	3.6	138.618973	-35.030293
2	7.2	138.649551	-34.855896
3	1	138.71077	-34.842735
4	41.8	138.502533	-35.140484
5	7.1	138.596481	-34.92733
6	3.2	138.755905	-34.60482
7	11	138.668945	-34.937206
8	5.4	138.521423	-35.095875
9	24.6	138.665482	-34.933838
10	5.7	138.504166	-35.156281
11	5	138.682953	-34.863651
12	4	138.520737	-35.03788
13	4.6	138.548233	-34.938145
14	105.4	138.701111	-34.873985
15	2.4	138.534256	-34.927444
16	3	138.517944	-34.869549
17	5.1	138.601013	-34.720016
18	15.8	138.539276	-34.935966
19	9.9	138.653519	-34.967152
20	5.2	138.58194	-34.892113
21	4.6	138.542847	-35.132805
22	2.7	138.602371	-34.982914
23	1.0	138.702927	-34.836113
24	2.5	138.59906	-34.907127
25	2.4	138.601318	-34.83226
26	6.0	138.552429	-34.875774
27	1.0	138.66188	-34.687962
28	1.2	138.615326	-34.941086
29	2.0	138.661392	-34.688042
30	24.2	138.577606	-34.892906
31	4.8	138.491425	-35.102406
32	4.8	138.537745	-34.976188
33	1.1	138.577911	-34.920647
34	2.4	138.546844	-34.836704
35	14.0	138.646454	-34.791466
36	83.0	138.598944	-34.696476
37	16.4	138.539932	-34.873688
38	1.5	138.592926	-34.995338
39	134.5	138.627487	-34.726559

6. conclusion

The application of a Lagrangian relaxation algorithm to solve the RMCDP has been examined in this paper. The RMCDP is a logistics and planning problem arising in many real-world applications where readymade concrete must be delivered from a set of loading depots to a set of demand customers. The problem can be represented using MIP and is closely related to the VRP with the difference that depot and customer nodes may be visited more than once during the operations period. We have implemented a novel Lagrangian relaxation algorithm on realistic instances representative of an active RMCDP in the region of Adelaide, Australia and report promising results. Namely the computational tractability of the model has been improved due to the dualization of a set of flow constraints and the dualized MIP was able to find the global optimum. Further, there is scope to fine tune the proposed solution approach by implementing different methods for solving the Lagrangian dual (piecewise linear function) other than a generic sub-gradient optimization method. Besides, there is always scope to further refine by experimenting with multiple constraints and also by using a nested Lagrangian relaxation approach.

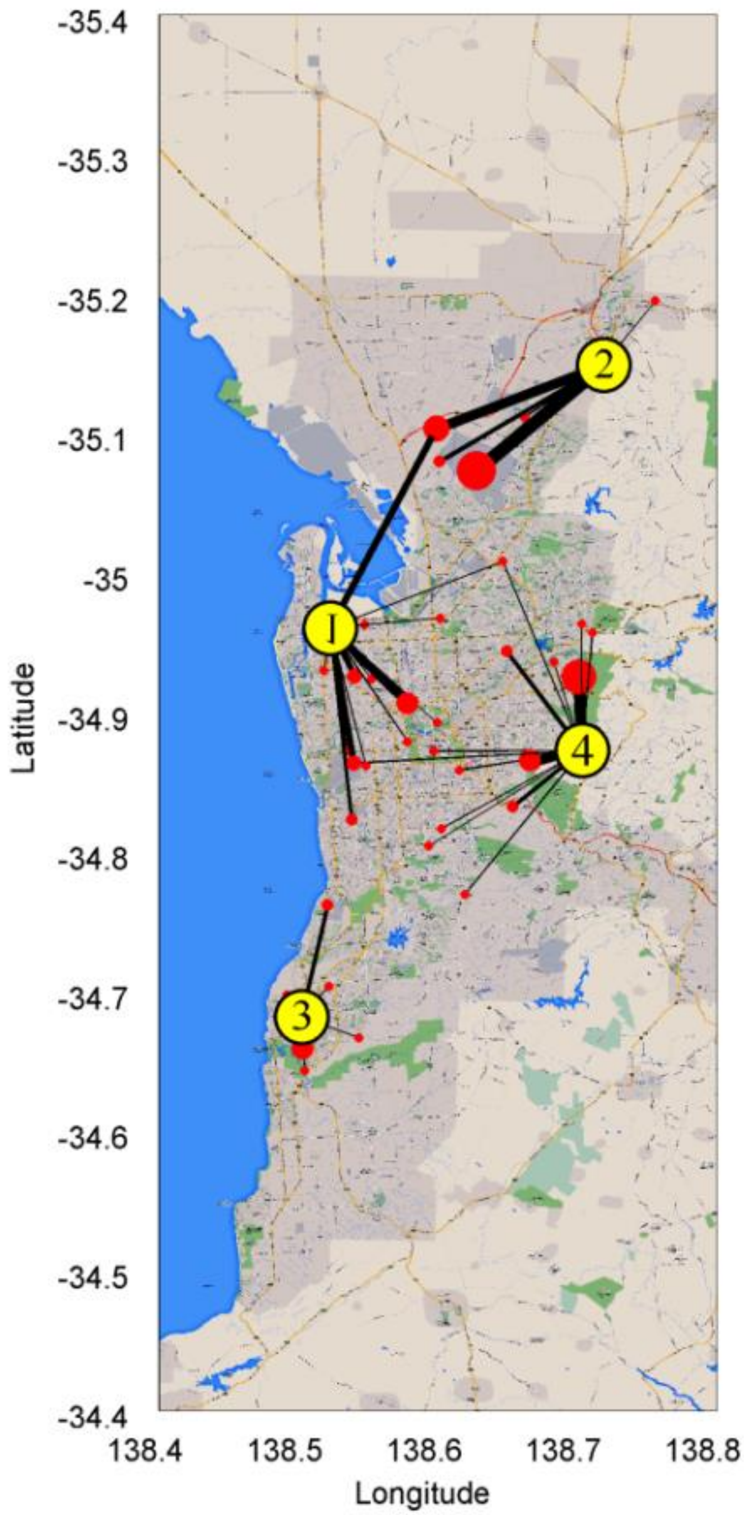

Fig. 4. – Optimal solution obtained with the Lagrangian Relaxation Algorithm for the RMCDP in the region of Adelaide, Australia.

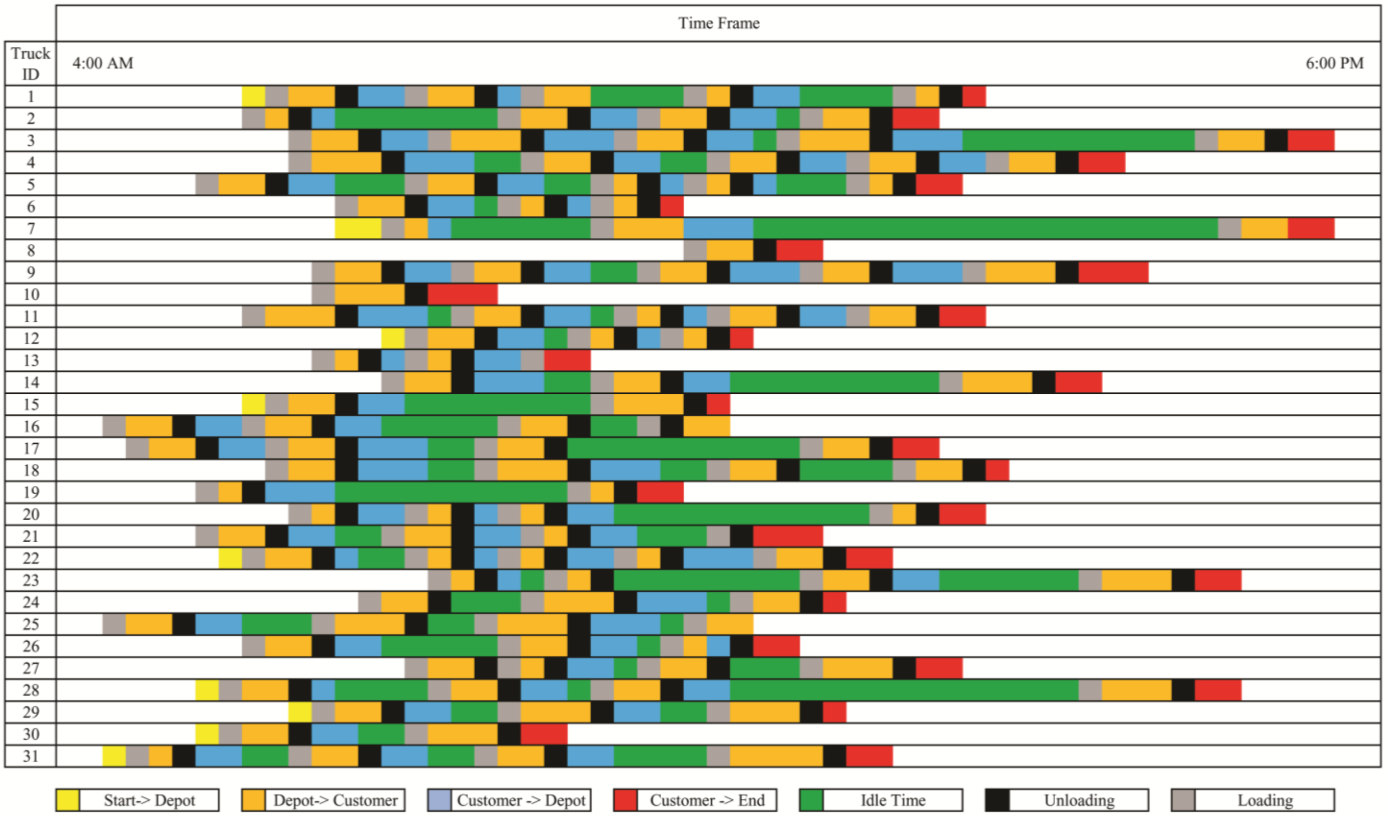

Fig. 5 – Optimal schedule of the RMC delivery trucks

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