Lyapunov-Based Input-Output Feedback Linearization Control of Induction Motor drives Considering Online MTPA Strategy and Iron Loss

Mohammad-Ali Salahmanesh
Electrical Engineering Department, Engineering Faculty, Ferdowsi University of Mashhad, Mashhad, Iran
m.salahmanesh@mail.um.ac.ir

Hossein Abootorabi Zarchi
Electrical Engineering Department, Engineering Faculty, Ferdowsi University of Mashhad, Mashhad, Iran
abootorabi@um.ac.ir

Hamidreza Mosaddegh Hesar
Electrical Engineering Department, Engineering Faculty, Ferdowsi University of Mashhad, Mashhad, Iran
h.mosaddegh@ferdowsi.um.ac.ir

Abstract—**In this paper a non-linear field oriented control (FOC) of induction motor (IM) drives is presented which is based on input-output feedback linearization (IOFL) technique incorporating the iron loss. The total stability of the proposed controller is proven using the Lyapunov control theory. In addition, the model-based maximum torque per ampere (MTPA) control is achieved considering the iron loss as an important factor of performance degradation in the MTPA strategy. Based on this strategy, the optimal relation between d and q axis stator current is determined using the gradient approach. The Lyapunov-based IOFL control regulates both the IM torque and the MTPA realization criterion by selecting these as outputs. Simulation results are finally presented to show the capability of the proposed control method.**

Keywords- Induction motor (IM); input-output feddback linearization (IOFL); lyapunov control theory; maximum torque per ampere (MTPA); iron loss.

NOMENCLATURE

$\vec{v}$, $\vec{i}$, $\vec{\lambda}$	Voltage, current, flux vectors
T_e	Electromagnetic torque
R	Resistance
R_i	Iron loss resistance
L	Self-inductance
L_l	Leakage inductance
L_m	Coupling inductance between stator and rotor
p	Pole pair number of stator
ω_r	Electrical rotor speed
ω	Angular synchronous speed
σ	Leakage factor $(1-(L_m^2 / L_s.L_r))$
Subscripts	
s,r	Stator and rotor
d,q	Rotating d-q frame axis
D,Q	Stationary reference frame

I. INTRODUCTION

Due to the well-known benefits such as low cost, high reliability, and simple construction, the induction motor (IM) has been attracted much attention in the last decades. The control of the IM drives constitutes, however, a challenging problem because of its non-linear dynamics. To achieve the better performance of IM drives, the field-oriented control (FOC) scheme has been introduced by Blaschke in the early 1970s [1]. The FOC, also known as vector control is developed by decomposing the stator current into flux and torque producing components controlled separately. The proportional-integral (PI)-based FOC approach partially linearizes the IM drive by controlling it in the rotor flux frame [2]. To entirely linearize the IM non-linear model, this control scheme can be upgraded by implementing a non-linear controller such as IOFL. To linearize the non-linear systems, the conventional approaches can be used in which by neglecting high-order dynamics, the first-order approximation of system dynamics is derived at a desired operating point [3]. This linearization method is, therefore, proper for the IM drives operated at constant rotor speed. However, IM behavior is inherently non-linear and other approaches must be used. The IOFL is a technique applied to the non-linear plants such as the IM. Accordingly, to use the linear control approaches, a non-linear system is transformed into a linear one [4]. After applying the input-output linearization, it is possible that a number of the system dynamics known as internal dynamics eliminate. Since, the internal dynamics are non-linear and non-autonomous, determining the stability of internal dynamics is generally difficult under the steady-state and transient conditions [5,6]. This means that by adjusting the controller gains, properly, the control system is only locally stable [7]. In this paper, an IOFL technique is proposed in which the stability of non-linear controller is proven using the Lyapunov control theory. To determine the optimal relation between d and q-axis stator currents, an on-line MTPA strategy is also introduced considering the iron loss as an important factor of performance degradation in the MTPA strategy. Detailed description of the proposed control strategy will be presented in the following Sections. The rest of this paper is structured as

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follows: Section II describes the IM model taking into account the iron loss, as well as the proposed MTPA strategy and the Lyapunov-based IOFL control for IM. Section III renders the simulation results and the conclusions are given in Section IV.

II. IM IOFL-BASED FOC CONTEROLLER

A. IM model including iron loss

An equivalent circuit for the IM model including iron loss is illustrated in Fig. 1, in the stationary reference frame. The space vector motor model is as follows [8]:

$$\frac{R_i}{R_i+R_s}\vec{v}_s=(R_s\parallel R_i)\vec{i}_s+\frac{d\vec{\lambda}_s}{dt} \tag{1}$$

$$0=(R_s\parallel R_i)\vec{i}_r+\frac{d\vec{\lambda}_r}{dt}-j\omega_r\vec{\lambda}_r \tag{2}$$

$$\vec{\lambda}_s=L_{ls}\vec{i}_s+L_m\vec{i}_m \tag{3}$$

$$\vec{\lambda}_r=L_{lr}\vec{i}_r+L_m\vec{i}_m \tag{4}$$

The electromagnetic torque of IM can be determined as:

$$T_e=\frac{3p}{2}\frac{L_m}{L_r}(\lambda_{rd}i_{sq}-\lambda_{rq}i_{sd}) \tag{5}$$

The rotor flux oriented control is obtained by the following conditions:

$$\lambda_{rd}=\left|\vec{\lambda}_r\right| \quad , \quad \lambda_{rq}=0 \tag{6}$$

In this way, the equations of the IM are reduced to fourth order. In the non-linear control, (1)-(4) are rewritten in the form of affine as follows:

$$\dot{X}=f(x)+g_1(x)v'_{ds}+g_2(x)v'_{qs} \tag{7}$$

where

$$f(x)=\begin{bmatrix} \frac{L_m}{\sigma L_s L_r}\left(\frac{1}{T_r}(x_3-L_m x_1)\right)-\frac{1}{\sigma L_s}\left(R_s\parallel R_i\right)x_1+\omega x_2 \\ \frac{L_m}{\sigma L_s L_r}\left(\frac{1}{T_r}(-L_m x_2)-x_4 x_3\right)-\frac{1}{\sigma L_s}\left(R_s\parallel R_i\right)x_2+\omega x_1 \\ -\frac{1}{T_r}(x_3-L_m x_1) \\ \frac{1}{j}\left(T_e-T_L-Bx_4\right) \end{bmatrix}$$

$$g_1(x)=\begin{bmatrix} \frac{1}{\sigma L_s} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad , \quad g_2(x)=\begin{bmatrix} 0 \\ \frac{1}{\sigma L_s} \\ 0 \\ 0 \end{bmatrix}$$

with

$$x=[i_{sd},i_{sq},\lambda_{rd},\omega_r]\,,\; v'_{sd}=\frac{R_i}{R_i+R_s}v_{sd}\,,\; v'_{sq}=\frac{R_i}{R_i+R_s}v_{sq}$$

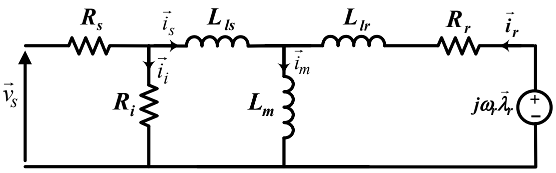

Fig. 1. IM equivalent circuit in the stationary reference frame including iron loss

B. Proposed MTPA strategy

To perform the proposed MTPA strategy, the torque expression must be modified so that this equation to be a function of orthogonal stator current components. So, (5) can be changed as follows [9]:

$$T_e=\left(\frac{\beta^2-1}{\beta^2+1}\right)\cdot\left(\frac{R_i}{\beta\omega}\right)\left(i_{sq}i_{sd}-\frac{i_{sq}^2}{\beta}\right) \tag{8}$$

where $\beta=\frac{R_i}{\omega L_{lr}}+\frac{R_i}{\omega L_m}$.

In MTPA, the minimization of stator current magnitude is selected as the objective function. Applying the gradient approach, the criterion of MTPA realization is achieved as follows [10]:

$$y_1=\frac{\partial T_e}{\partial i_{sd}}\times\frac{\partial i_s^2}{\partial i_{sq}}-\frac{\partial T_e}{\partial i_{sq}}\times\frac{\partial i_s^2}{\partial i_{sd}}=0\Rightarrow y_1=i_{sq}^2+\frac{2}{\beta}i_{sq}i_{sd}-i_{sd}^2=0 \tag{9}$$

By doing some calculations on (9), we have:

$$i_{sq}=\pm i_{sd}\,\xi\Rightarrow\delta=\tan^{-1}(\xi) \tag{10}$$

where $\xi=\left(\frac{\sqrt{\beta^2+1}-1}{\beta}\right)$ is MTPA factor.

Although in ideal condition, the optimal current angle is constant ($\delta=\tan^{-1}i_{sq}/i_{sd}=\pm\pi/4$), but in the non-ideal condition, this angle is smaller than $\pi/4$ dependent on β value. The variation of ξ versus frequency has been plotted in Fig. 2.

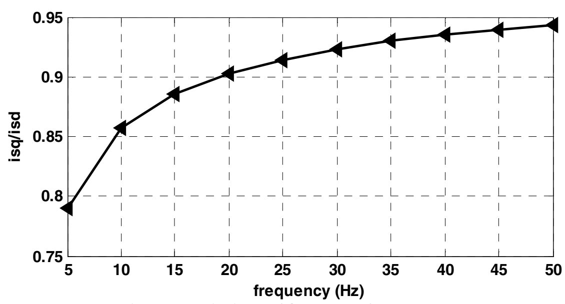

Fig. 2. Variation of ξ versus frequency

C. Lyapunov-based IOFL control for IM

In the non-linear controller, the control objective is the tracking of the electromagnetic torque and another providing MTPA strategy. The error signals are defined as follows:

$$e_1=y_1-y_{1ref} \tag{11}$$

$$e_2=y_2-y_{2ref} \tag{12}$$

where $y_1 = x_2 - \xi x_1$ and $y_2 = T_e$ are related to the MTPA strategy and the IM torque, respectively. According to the criterion of MTPA realization $y_{1,ref} = 0$. The error dynamics are then given by:

$$\dot{e}_1 = \dot{x}_2 - \xi \dot{x}_1 \tag{13}$$

$$\dot{e}_2 = \dot{T}_e - \dot{T}_{eref} \tag{14}$$

Therefore:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} L_f e_1 \\ L_f e_2 \end{bmatrix} + \begin{bmatrix} Lg_1 e_1 & Lg_2 e_1 \\ Lg_1 e_2 & Lg_2 e_2 \end{bmatrix} \begin{bmatrix} v'_{ds} \\ v'_{qs} \end{bmatrix} - \begin{bmatrix} 0 \\ \dot{T}_{eref} \end{bmatrix} \tag{15}$$

where $L_f y_i, (i = 1{,}2)$, are the directional (or Lie) derivative of state function $y(x): R^n \to R$ along a vector field $f(x) = [f_1(x), \dots, f_n(x)]$:

$$L_f y(x) = \sum_{i=1}^{n} \frac{\partial y(x)}{\partial x} f_i(x) \tag{16}$$

According to defined dynamics, the lie derivative functions are:

$$\begin{aligned} L_f e_1 = &-\xi\left(\frac{L_m}{\sigma L_s L_r}\left(\frac{1}{T_r}(x_3 - L_m x_1) \right) - \frac{1}{\sigma L_s}(R_s \| R_i) x_1 + \omega x_2 \right) + \\ &\left(\frac{L_m}{\sigma L_s L_r}\left(\frac{1}{T_r}(-L_m x_2) - x_4 x_3 \right) - \frac{1}{\sigma L_s}(R_s \| R_i) x_2 - \omega x_1 \right) \end{aligned} \tag{17}$$

$$L_{g1} e_1 = -\xi\left(\frac{1}{\sigma L_s} \right) \quad , \quad L_{g2} e_1 = \left(\frac{1}{\sigma L_s} \right) \tag{18}$$

In the rotor flux oriented control, the torque equation is described as:

$$T_e = \frac{3P}{2} \frac{L_m}{L_r}(x_3 x_2) \tag{19}$$

Due to (19), the lie derivative functions of the second dynamic are defined as:

$$L_f e_2 = \frac{3P}{2} \frac{L_m}{L_r} \left(\begin{aligned} &x_3\left(\frac{L_m}{\sigma L_s L_r}\left(\frac{1}{T_r}(-L_m x_2) - x_4 x_3 \right) - \frac{1}{\sigma L_s}(R_s \| R_i) x_2 - \omega x_1 \right) \\ &+ x_2\left(-\frac{1}{T_r}(x_3 - L_m x_1) \right) \end{aligned} \right) \tag{20}$$

$$L_{g1} e_2 = 0 \quad , \quad L_{g2} e_2 = \frac{3p}{2} \frac{L_m}{\sigma L_s L_r} x_3 \tag{21}$$

In this step, to provide the overall stability of control system, the Lyapunov function is given as:

$$V = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \tag{22}$$

The derivative function is therefore obtained as:

$$\begin{aligned} \dot{V} &= e_1 \dot{e}_1 + e_2 \dot{e}_2 = \\ &e_1\left(L_f e_1 + L_{g1} e_1 v'_{ds} + L_{g2} e_1 v'_{qs} \pm \alpha e_1 \right) + \\ &e_2\left(L_f e_2 + L_{g2} e_2 v'_{qs} - \dot{T}_{eref} \pm \beta e_2 \right) \end{aligned} \tag{23}$$

where $\alpha, \beta > 0$ are controller gains.

According to (23), $\dot{V}$ is negative if the control inputs are considered as:

$$L_{g1} e_1 v'_{ds} + L_{g2} e_1 v'_{qs} = -\alpha e_1 - L_f e_1 \tag{24}$$

$$L_{g2} e_2 v'_{qs} = -L_f e_2 + \dot{T}_{eref} - \beta e_2 \tag{25}$$

$$v'_{qs} = \frac{1}{L_{g2} e_2}\left(-L_f e_2 + \dot{T}_{eref} - \beta e_2 \right) \tag{26}$$

$$v'_{ds} = \frac{1}{L_{g1} e_1}\left(-\alpha e_1 - L_f e_1 - L_{g2} e_1 v'_{qs} \right) \tag{27}$$

III. Simulation Results

The proposed control scheme is simulated in MATLAB/SIMULINK environment on the IM with the parameters shown in Table I. Based on the described control approach, the overall block diagram of the IM drive is shown in Fig. 4. The simulation results are illustrated in Figs 5-13. In this paper, according to Fig. 3, experimentally identified equivalent iron loss resistance values of a prototype IM acquired in [9] is used for MTPA strategy.

The electromagnetic torque and its reference are presented in Fig 5. As shown, the torque tracks its step command by fast dynamic response. Fig. 6 shows that the MTPA realization criterion follows its reference value, which is zero, and the MTPA strategy is, therefore, realized. The rotor and synchronous speeds are illustrated in Fig. 7. As depicted, the speed increases and decreases rather linearly as the desired torque is stepped up and stepped down. According to Fig. 2, the variation of frequency changes the ξ value. Hence, as shown in Fig. 8, by varying the torque command and therefore speed variations, the ξ value depended to frequency is determined by (10).

TABLE I: THE IM PARAMETERS [9]

Parameter	Value	Parameter	Value
Pole-Pair	2	R'_r (Ω)	1.1237
Rated torque (N.m)	10	L_{ls} (H)	0.047
Rated voltage (v)	180 (L-L)	L'_{lr} (H)	0.0206
R_s (Ω)	1.3012	L_m (H)	0.1863

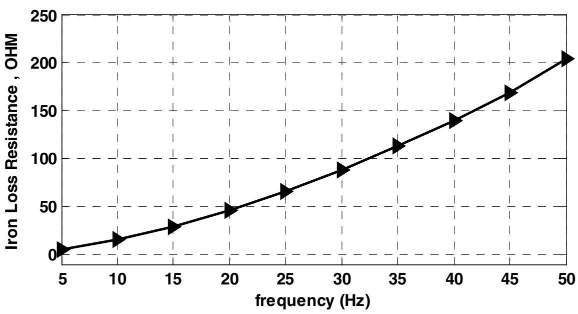

Fig. 3. Identified values of equivalent iron loss resistance for the IM [9]

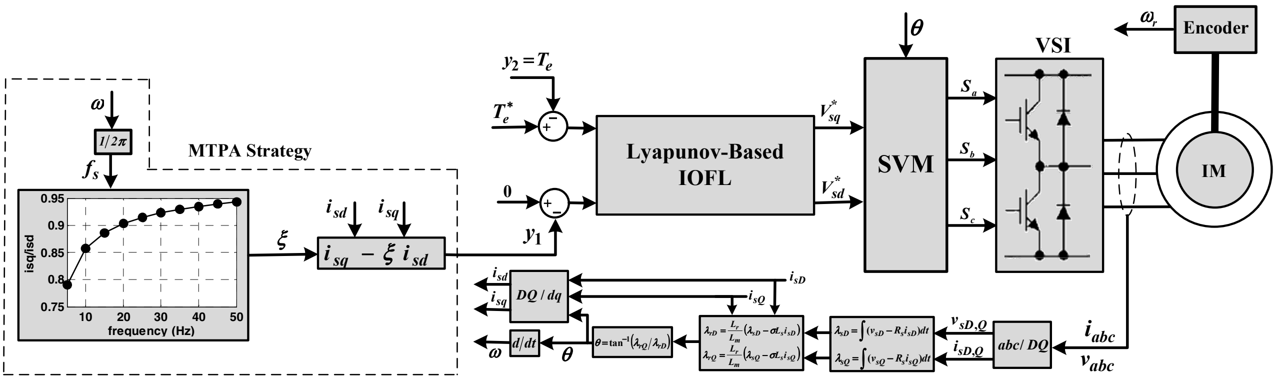

Fig. 4. The proposed MTPA strategy for IOFL control based IM drives

Figs. 9 and 10 illustrate the d-q axis stator currents and the stator phase current waveform, respectively. Considering Fig. 10, the magnitude and frequency of stator current are changed by varying the torque command. The d-q axis flux components, the stator current trajectory, and the flux linkage trajectory are respectively shown in Figs 11-13. Since the balanced power supply has been applied to the IM drive, the trajectories of stator current and flux linkage are circle-shaped, in the stationary reference frame (DQ reference frame).

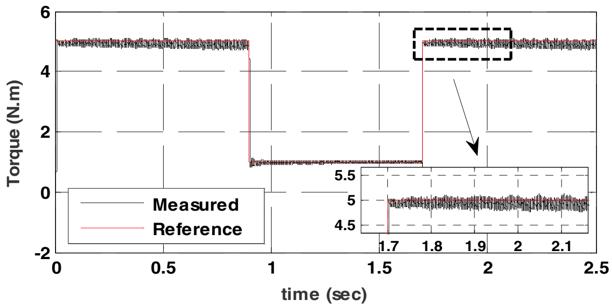

Fig. 5. Electromagnetic torque

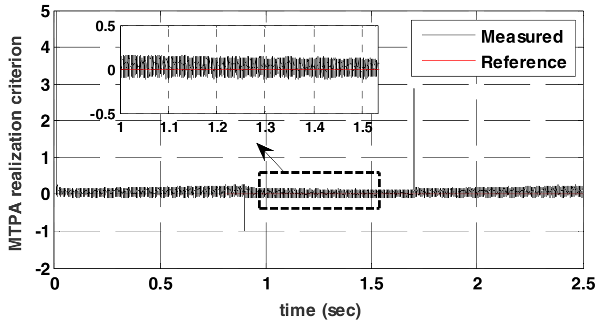

Fig. 6. MTPA realization criterion

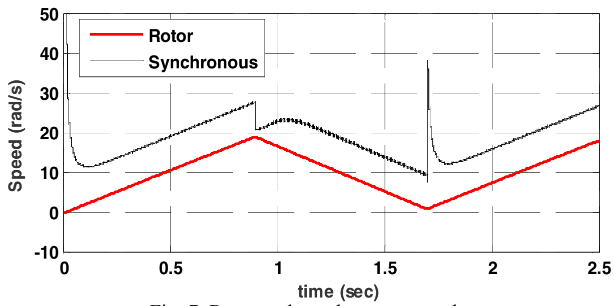

Fig. 7. Rotor and synchronous speeds

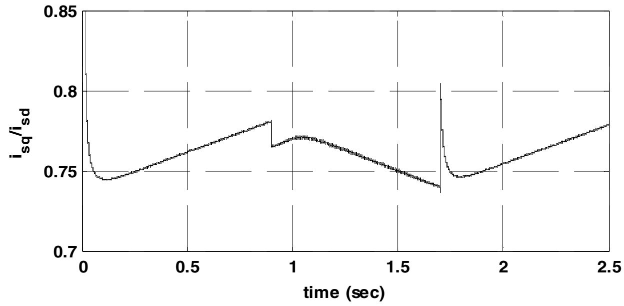

Fig. 8. Variation of $\xi = i_{sq}/i_{sd}$ by changing the frequency

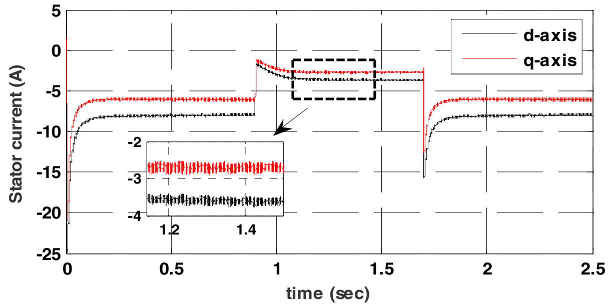

Fig. 9. d-q axis stator currents

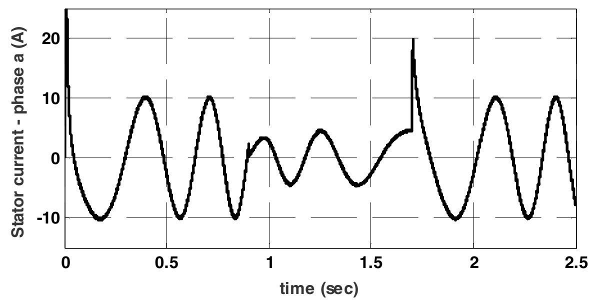

Fig. 10. Stator current with MTPA

IV. CONCLUSION

In this paper, an IOFL controller has been designed for FOC-based three-phase IM drive. To prove the stability of internal dynamics eliminated by applying the input-output linearization, the Lyapunov control theory was used. The features of this control approach are fast transient response, and high accuracy. In addition, an online improved MTPA approach was suggested

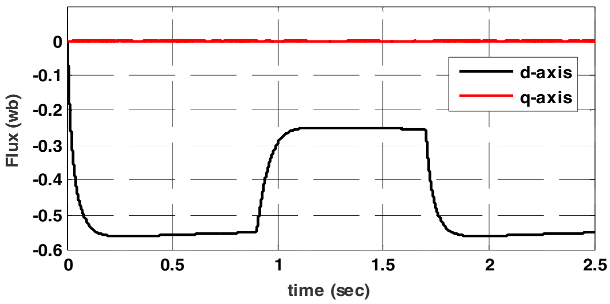

Fig. 11. d-q axis flux components

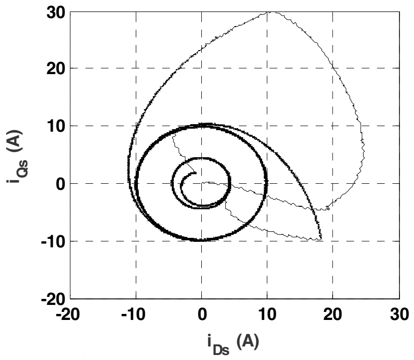

Fig. 12. Stator current trajectory

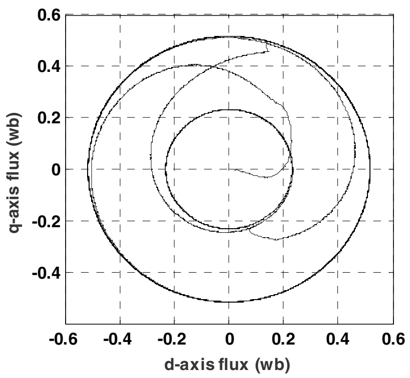

Fig. 13. Flux linkage trajectory

which is simple in structure and includes the effect of iron loss. In this regard, the MTPA strategy is satisfied when the gradient vectors of the stator current magnitude and torque are parallel. The simulation results have clearly verified the undoubted potential and effectiveness of the presented control scheme.

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