# Robust Maximum Torque per Ampere Strategy for Permanent Magnet Synchronous Motor Based on PI-Sliding Mode Controller

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Abstract - Since the applications of Permanent Magnet Synchronous Machines (PMSMs) are developing in various industries, stator loss minimization in these machines is significantly important. In this regard, the d and q axis currents should be controlled where the load torque is provided with minimum stator current. This paper proposes a novel Maximum Torque per Ampere (MTPA) strategy for PMSMs based on a PI-Sliding Mode Controller (SMC) considering machine parameter variations. In the proposed drive system, the speed is controlled by the q axis current. Moreover, the d axis current is controlled via a two layer controller. In the first layer of the proposed controller, the minimum stator current is determined based on the machine nominal parameters through a PI-SMC. With due attention to the machine parameters are varied, the minimum stator current is not precise. Hence, the second layer controller is applied in order to shift the operation point to the correct minimum current point. Consequently, the load torque is precisely provided with minimum stator current followed by the absence of any steady-state torque and speed ripples. The proposed control scheme is simulated in MATLAB/Simulink environment. Simulation results validate the performance of the proposed control system.

Index Terms - Permanent Magnet Synchronous Machine (PMSM), Maximum Torque per Ampere (MTPA), PI-Sliding Mode Controller (SMC).

### I. INTRODUCTION

Currently, the PMSMs are broadly utilized in various applications as wind turbines, Electrical Vehicles (EVs), pump, fans and etc. Generally, PMSMs are categorized in two main groups: Surface Permanent Magnet Synchronous Machines (SPMSMs), which only produce electromagnetic torque, and Interior Permanent Magnet Synchronous Machines (IPMSMs), which produce both electromagnetic and reluctance torques. The machine torque is dependent on both the d and q axis currents ( $I_d$  and  $I_q$ ). Numerous  $I_d$  and  $I_q$ control algorithms are proposed in literature. Most of which are based on efficiency improvement (maximum efficiency, minimum copper loss and minimum core loss). Some studies focused on copper loss minimization. In this way,  $I_d$  and  $I_q$  are determined in such a way that, the load torque is supplied with minimum stator current. These control approach are called Maximum Torque per Ampere (MTPA) strategies [1-3].

MTPA strategies for SPMSMs are merely fulfilled by constraining the  $I_d$  at zero [4]. However, for MTPA realization in IPMSMs, the machine parameters such as d and q axis inductances ( $L_d$  and  $L_q$ ), back EMF, air gap flux and etc. should be specified.

In the proposed MTPA procedure in [4], the MTPA operation condition is realized according to the nominal machine parameters. However, as the machine parameters vary, the MTPA strategy is not satisfied accurately. Since the machine parameters are dependent on the flux saturation level, stator current amplitude and temperature variations, their precise values is required for accurate MTPA realization [5-6]. To solve this problem, some references utilize parameter identification schemes [7-9]. Nevertheless, applying these methods significantly increase the system computational burden and complication. Further, they result in torque ripples.

Other MTPA strategies which are based on search algorithms, are parameter independent [10]. However, they have slow response and inapplicable for drive systems with periodic load variations. To overcome the problems associated with search based algorithms, some developed models are proposed in [11-13]. Although, these models are complicated and their inductances are considerably current dependent. In the MTPA scheme proposed in [14] a look-up table is considered. But all parameters variations are not taken into account. Signal injection for MTPA detection is proposed in [15]. These strategies suffer from torque ripples and involve long convergence time.

In this paper a novel two layer control strategy for MTPA realization in a PMSM is proposed which accounts for parameters variations. In the first layer the MTPA strategy is implemented based on the nominal parameters. The second layer controller, by applying small step changes in the input and inspecting the system output after each step, ascertains the stator current which leads to supply the load with minimum stator current.

The rest of the paper is organized as follows: the overall system, its restrictions and its equations are described in section II. In section III the two-layer proposed controller is described in details. The simulation results are presented in section IV and section V concludes the paper.



Fig. 1 The overall control system block-diagram

## **II. SYSTEM STRUCTURE**

The overall system block-diagram is illustrated in Fig. 1. As shown, the reference torque is designated based on the rotor speed and its reference value. As the produced torque is more dependent on  $I_q$  than  $I_d$ , the speed is controlled with  $I_q$ . Hence the copper loss minimization strategy is performed by controlling  $I_d$ .

The IPMSM stator voltages in the synchronous reference frame can be written as:

$$V_d = R_s I_d + L_d \frac{dI_d}{dt} - L_q \omega I_q \tag{1}$$

$$V_q = R_s I_q + L_q \frac{dI_q}{dt} + L_d \omega \, Id + \omega \, \varphi \tag{2}$$

where,  $V_d$  and  $V_q$  are d and q axis voltages, respectively,  $R_s$  is the stator resistance,  $\varphi$  is the linkage flux and  $\omega$  is the rotor speed. The produced torque is obtained as (3):

$$T_{e} = 1.5P(\varphi I_{g} + (L_{d} - L_{g})I_{d}I_{g})$$
(3)

where, P stands for the number of pole pairs.

As  $I_q$  and  $I_d$  are independent of each other, various current vectors can be selected to produce the desired torque. The constant torque curves in the  $I_q - I_d$  plane is the locus of all current vectors, which can produce the same output torque. (Fig. 2). However, as there are some restrictions in IPMSM such as thermal and demagnetization current limitations, all of the current vectors in Fig. 2 aren't feasible. Figure 3 represents the range of the current vector variations in the  $I_q - I_d$  plane, considering thermal and demagnetizing current limitations [16].

Various produced torque in the  $I_q - I_d$  plane are shown in Fig. 4. As demonstrated T<sub>1</sub> could be produced by various current vector. Nevertheless, there exist just one current vector which has the minimum amplitude. Similarly, if the desired torque be T2, various current vectors are feasible, only one of them leads to minimum stator current. By specifying all of the MTPA points in the  $I_q - I_d$  plane, MTPA curve is achieved.

MTPA curve equation (5) is obtained from (3) and (4).

$$I_s = \sqrt{I_d^2 + I_q^2} < I_{S\max}$$
<sup>(4)</sup>

$$I_d(I_q) = \frac{-\varphi \pm \sqrt{\varphi^2 + 4(L_d - L_q)^2 I_q^2}}{2(L_d - L_q)}$$
(5)



Fig. 3 Thermal and demagnetizing current limitations in  $I_q$  -  $I_d$  plane



Fig. 4 MTPA trajectory for different torque in  $I_q$  -  $I_d$  plane

By manipulating (5), the MTPA trajectory is attained as:

$$H = I_d^2 (L_d - L_a) + I_d \varphi - I_a^2 (L_d - L_a) = 0$$
(6)

In (6), the absolute value of H indicates the distance of the operation point from the MTPA curve. Its sign ascertains the operation point location relative to the MTPA curve. If H has a positive sign, the operation point is located in the right side of the MTPA curve else it is on the left side.

It should be noted that, if the machine is of SPMSM type,  $L_d$  and  $L_q$  are equal and the MTPA curve is located on the q axis. The more the difference between  $L_d$  and  $L_q$ , the more interval has the MTPA curve from the q axis. If only the q axis inductance be in its saturation area, the MTPA curve is shifted to the right side. In this condition by decreasing  $I_d$ , the desired torque could be produced with reduced stator current amplitude. If the d axis inductance be in its saturation area and the deviation of the other parameters from their nominal values be negligible, the MTPA trajectory will be moved to the left side. Then by increasing  $I_d$ , the desired torque with minimum stator current amplitude will be obtained.

#### III. PROPOSED CONTROL STRATEGY

According to the above discussions, for stator current amplitude  $(I_S)$  minimization, the operation point should be on the MTPA curve. MTPA realization is performed through  $I_q$  and  $I_d$  fine-tuning. Since,  $I_q$  is adjusted by the speed controller, the MTPA is fulfilled by  $I_d$  regulation. In the proposed control



Fig. 5 Proposed two layer controller

strategy  $I_d$  is controlled by means of a two layer controller which is shown in Fig.5.

#### A. First Layer controller

The d axis reference voltage  $(V_d^*)$  is determined by a PI-SMC (Fig. 5). From (6) it is clear that when the operation point is on the MTPA curve, H=0. Accordingly, the sliding surface is defines as (7).

$$S = H = I_d^2 (L_d - L_q) + I_d \varphi - I_q^2 (L_d - L_q) = 0$$
<sup>(7)</sup>

According to (7), if S > 0, the  $I_d$  should be reduced to shift the operation point on the MTPA curve. Therefore:

$$S\dot{I}_d < 0$$
 (8)

Figure 6 illustrates the sliding surface and the procedure of the proposed SMC. If the operation point is at the point A and the rotor speed be constant,  $I_d$  is increased and  $I_q$  is decreased. Hence the operation point is transmitted to the point C. If the operation point be at the point B, by reducing  $I_d$ and increasing  $I_q$  the MTPA condition is realized.

Generally, the SMC is robust and has low sensitivity to the parameter variations. Despite its advantages, there exists control parameter chattering in SMCs. In order to reduce parameter chattering and improve its dynamic response in the proposed control system a PI controller is employed in addition to the SMC. Consequently, the d axis reference current  $(I_d^*)$  is determined as (9).

$$I_{d}^{*} = (k_{p} + \frac{k_{l}}{s})(-H - Ksign(H))$$
(9)

It should be noted that by adjusting the speed to its command value and forcing H to be zero,  $I_q$  and  $I_d$  are determined and  $I_s$  is minimized based on the nominal parameters values. However, the machine parameters such as inductances, flux, resistances and etc. change. As a result, the operation point does not moved to the real MTPA trajectory. To overcome this problem and achieve the true MTPA point, a second layer controller is applied.

#### B. Second Layer controller

In the second layer controller, one of the machine inputs is varied in small steps and the stator current variations are analyzed. Indeed, d axis voltage  $(V_d)$  is increased and then



decreased in small steps. If the operation point of the system is on the MTPA curve,  $I_S$  will increase in both cases. The stator current reduction shows the parameters changes.

Since the sensitivity of the produced torque on  $I_d$  is low, the speed deviations due to d axis voltage step changes are slight. Speed deviation causes the q axis current controller to adjust the  $I_q$  such that the rotor speed follow its command value. This results in very low ripples in the torque and rotor speed.

The flow chart of the proposed two-layer control algorithm is shown in Fig. 7. As illustrated, the value of H is determined according to d and q axis current values, first. Then,  $I_q$  and  $I_d$  are adjusted such that the desired torque is produced and H equals to zero. Afterward,  $V_d$  is altered by a small step and  $I_s$  is evaluated. The d axis voltage change is repeated until the maximum power point is specified. In the following, it is checked if the operation point is changed or not. If there exist a variation in the operation point, the minimum stator current determination process is repeated, otherwise, the operation point remains unchanged. Consequently, in the steady-state, the motor speed has no ripples.



Fig. 7 The flow chart of the proposed two-layer control algorithm

#### IV. SIMULATION RESULTS

In order to evaluate the performance of the proposed control strategy, the proposed system, is simulated in the MALAB/Simulink environment. The simulation parameters are tabulated in table I [17]. As the d and q axis inductances are current dependent, their values are considered variable (Fig. 8) [18]. The simulation results are illustrated in figures 9 and 10.

SVSTEM	DAD	AMETED	VA S	LIES	

Parameter	Value		
Nominal power	2 kW		
Phase number	3		
Pair pole	1		
Reference Speed	$150 \frac{rad}{sec}$		
Motor Inertia	$0.003 Kg.m^2$		
Friction coefficient	$0.0034 \frac{Nm.s}{rad}$		
Stator impedance	3.25 Ω		
Nominal D axis Inductance	0.018 H		
Nominal Q axis Inductance	0.034 H		
Flux	0.341Wb.N		
$V_d$	[0.5 0.35 0.25 0.1 0.1 0.1] Volt		
Nominal stator current	10 A		
DC bus voltage	208 Volt		

Figure 9 (a) represents the load and electromagnetic torque variations. As shown, the speed controller is fast and compensates for the load and electromagnetic torque deviations in few milliseconds. It should be noted that the difference between the load and the electromagnetic torques is due to the machine rotational losses.

The rotor speed is demonstrated in Fig. 9(b). As it is clear, the rotor speed appropriately follows its reference value. When the second layer controller changes the d axis voltage, the torque varies. Subsequently, the rotor speed has ripples. In steady state  $V_d$  remain unchanged, thence, in this condition rotor speed has not any ripple.

Voltages of d and q axis are shown in figures 9 (c) and (d), respectively. As it is clear, at t=4s, when the torque alters, the first layer controller, specifies the values of  $V_d$  and  $V_q$ . Then, in the second layer controller  $V_d$  is increased by a small step. The stator current value increases as  $V_d$  does. Next,  $V_d$  is decreased. It is observed that the stator current amplitude is reduced. Therefore,  $V_d$  reduction is repeated until t=8s, when the stator current increases. Thus, at this time, the voltage of d axis is fixed to its latter value, which results in the minimum stator current amplitude. Now, the d and q axis voltages are remain unchanged until the operation point changes.

In the new operation condition at t=13.5s, the load torque is reduced. As a result,  $I_q$  and  $I_d$  vary and speed controller and SMC specify new reference values for d and q axis currents. After the transient operation, the second layer controller starts. This controller augments  $V_d$ , first. This results in stator current increasing. After that, this voltage is decreased which causes stator current augmentation too. As a result, the proposed algorithm concludes that the operation point is on the real MTPA curve.



Fig. 9: Proposed controller dynamic and steady-state response to load torque step change: (a) electromagnetic and load torque, (b) rotor speed, (c) d axis voltage and (d) q axis voltage



Fig. 10 proposed controller dynamic and steady-state response to load torque step change (a): d axis current (b): q axis current (c): stator current amplitude

Since the load torque is low, the machine inductances are in their linear area. Hence, the machine inductance deviations from their nominal values are negligible. Therefore, there is no difference between the real and the nominal MTPA curves.

The d and q axis and stator current waveforms are represented in Fig. 10 (a), (b) and (c), respectively. As it is expected, the q axis current variations are proportional to the torque variations. Alternatively, the d axis current changes such that MTPA based on nominal parameters is obtained ( $t=5sec \ or \ t=14sec$ ). Afterward, the values of  $I_d$  and  $I_q$  change such that the load is supplied with minimum stator current ( $t=8.5 \ sec$ ).

#### V. CONCLUSION

A novel MTPA control strategy based on PI-SMC, which accounted for machine parameters variations, proposed. At first, the overall control system was presented. Then, the machine restrictions such as thermal and demagnetizing current limitations were discussed. The load torque could be supplied by various current vectors. However, in order to have minimum copper losses, the current vector should be on the MTPA realization curve which was parameter dependent. In the following, the stator current vector was controlled by a PI-SMC controller with a well-defined sliding surface, to be on the MTPA trajectory in various operating conditions. Since this curve was defined based on the nominal machine parameter values, a second layer controller was added to the PI-SMC controller which accounted for parameters variations. The proposed system implementation in MATLAB/Simulink environment, verified that it could satisfy the true MTPA realization condition, despite the variations existed in machine parameters. Furthermore, the speed ripples was very low in the transient operation condition and was canceled in the steadystate.

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