Continuous Extended Kalman Filter for the Localization of Nonlinear Stochastic State-Space System

Hamed Keshmiri Neghab
dept. of control engineering
Ferdowsi University of Mashhad
Mashhad, Iran
Keshmirineghab.hamed@mail.um.ac.ir

Naser Pariz
dept. of control engineering
Ferdowsi University of Mashhad
Mashhad, Iran
n-pariz@um.ac.ir

***Abstract*- This paper derives a nonlinear observer design problem for noisy Continuous Nonlinear Stochastic State-Space Systems using the Continuous Extended Kalman Filter for Accurate Trajectory Tracking, which uses a system's Jacobian linearization along the current best estimate of its trajectory. The efficiency of Continuous Extended Kalman Filter is shown on a Trajectory Tracking Problem with sufficiently long sampling periods. Here, we showed that changing principal diagonal of the initial covariance matrix, improve the trajectory tracking.**

Keywords—continuous exended kalman filter, localization, stochastic, jacobian

I. INTRODUCTION

Extended Kalman Filter (EKF) is used to reduce the noise in corrupted measurements. In estimation theory, the EKF is the nonlinear version of the Kalman Filter which linearizes about an estimate of the current mean and covariance[1]. In the real world, most systems are nonlinear, so some attempt was immediately made to apply this filtering method to nonlinear systems. Most of this work was done at NASA. EKF is used in several systems such as Trajectory Tracking problem and are generally preferred, because due to its advantages like accurate target tracking and cheaper in cost. In this paper, we simulated a noisy continuous nonlinear plant model using Continuous EKF(CEKF) to track a specific position of autonomous car landmark even if measurement is corrupted by noise. Also, we use CEKF to evaluate and minimize Covariance matrix P(t). Moreover, we calculated the Kalman gain and associated estimation error covariance matrix and the estimated states and estimated outputs of a continuous nonlinear stochastic state-space system using CEKF [2].

II. METHODS

First, we simulate a nonlinear stochastic state-space plant with noise on the states and outputs. Then, we use the CEKF to estimate the position of landmark in distance and angle from the tip of autonomous car. The autonomous car has the reference at the front and, as it moves, the fixed landmark changes position related to the car. Noise corrupts the measurement of the sensors, which in this case affects localization. We use the CEKF to track the landmark position. Since CEKF can give a more accurate measurement of the landmark location [3].

A. Stochastic System Models

Stochastic system models represent the effects of noise on the plant, actuators, and/or sensors. In addition to the state-space matrices *A*, *B*, *C*, and *D*, stochastic models contain the following variables: Vectors *w* and *v* represent process noise and measurement noise, respectively. Matrices *G* and *H* relate *w* to the states and outputs, respectively. The following equations define continuous stochastic state-space models:

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \qquad (1)$$

$$y(t) = Cx(t) + Du(t) + Hw(t) + v(t)$$

Moreover we specify the mean vectors $E\{w\}$ and $E\{v\}$, the auto-covariance matrices $Q = Cov\{w,w\}$, $R = Cov\{v,v\}$, and the cross-variance matrix $N = Cov\{w,v\}$.

$$Q = E\{w.w^T\} - E\{w\}.E^T\{w\} \ (2)$$

$$R = E\{v.v^T\} - E\{v\}.E^T\{v\}$$

$$N = E\{w.v^T\} - E\{w\}.E^T\{v\}$$

B. Simulating Nonlinear Stochastic Models:

Before we deploy a controller to a Real Time target, we can test that the controller performs as expected in the presence of noise. To perform this test, we use the Second-Order Statistics Noise Model to generate values of w(k) and v(k) to simulate the behavior of a Nonlinear stochastic state-space model. In this situation, we can use the Correlated Gaussian Random Noise to generate Gaussian-distributed values of w(k) and v(k).

C. Using Continuous EKF to Estimate Model States

In the real world, controllers typically receive measurements that are corrupted by noise. Also, we typically cannot measure state values ($\hat{x}$). If we want to calculate all state values, we can use an EKF. EKF linearize the nonlinear system around the current estimate by computing a matrix of partial derivatives called a Jacobian matrix. The EKF evaluates the Jacobian matrix with the current estimated states at each time step [4]. The function $f(\hat{x}, u, t)$ can be used to compute the predicted state from the previous estimate and similarly the function $h(\hat{x}, u, t)$ can be used to compute the predicted measurement from the predicted state that defined by the following equations:

$$\dot{x}(t) = f(x, u, t) + w(t) \quad (3)$$

$$y(t) = h(x, u, t) + v(t)$$

The following equations illustrate the computations that the CEKF performs to produce the estimated states $\hat{x}(t)$ of the plant.

$$\dot{\hat{x}}(t) = f(\hat{x}, u, t) + L(t)[y(t) - \hat{y}(t)] \quad (4)$$

$$\hat{y}(t) = h(\hat{x}, u, t)$$

$$\dot{P}(t) = F_x(\hat{x}, u, t)P(t) + P(t)F_x^T(\hat{x}, u, t) + Q(t)$$

$$L(t) = [P(t)H_x^T(\hat{x}, u, t) + N(t)]R^{-1}(t)$$

$$[P(t)H_x^T(\hat{x}, u, t) + N(t)]R^{-1}(t)[P(t)H_x^T(\hat{x}, u, t) + N(t)]^T$$

$$F_x(\hat{x}, u, t) = \left.\frac{\partial f(x, u, t)}{\partial x(t)}\right|_{x(t)=\hat{x}(t)}$$

$$H_x(\hat{x}, u, t) = \left.\frac{\partial h(x, u, t)}{\partial x(t)}\right|_{x(t)=\hat{x}(t)}$$

Where

- $\hat{x}(t)$ is the estimated state
- $\hat{y}(t)$ is the estimate output
- $P(t)$ is the estimated error covariance
- $L(t)$ is the Kalman Filter gain
- $F_x(\hat{x}, u, t)$ is the Jacobian matrix of $f(x, u, t)$ with respect to x evaluated at $\hat{x}(t)$
- $H_x(\hat{x}, u, t)$ is the Jacobian matrix of $h(x, u, t)$ with respect to x evaluated at $\hat{x}(t)$

To use the CEKF, first we define the Continuous Nonlinear Stochastic Model whose states we want to estimate. For instance, consider a plant model of a vehicle moving with constant velocity and whose acceleration is slightly perturbed. Also, assume a radar measures the range r and the bearing angle θ of the vehicle, both corrupted by additive noise. The following equations describe this plant model.

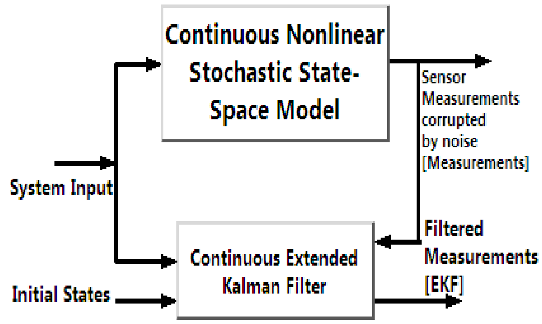


Fig. 1. The Process of using an Extended Kalman Filter.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & .5 \\ 0 & 0 & .5 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w_3 \\ w_4 \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \sqrt{x_1^2 + x_2^2} \\ \tan^{-1}\left[\frac{x_2}{x_1}\right] \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Where x_1 and x_2 are the x and y position in the Cartesian frame system, x_3 and x_4 are the x and y velocities, and y_1 and y_2 are the range r and bearing angle θ, respectively. We define the system model by calculating the vector-valued functions $f(x, u, t)$ and $h(x, u, t)$ for the continuous plant model, where:

$$f(x, u, t) = \begin{bmatrix} 0 & 0 & 1 & .5 \\ 0 & 0 & .5 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} (6)$$

$$h(x, u, t) = \begin{bmatrix} \sqrt{x_1^2 + x_2^2} \\ \tan^{-1}\left[\frac{x_2}{x_1}\right] \end{bmatrix}$$

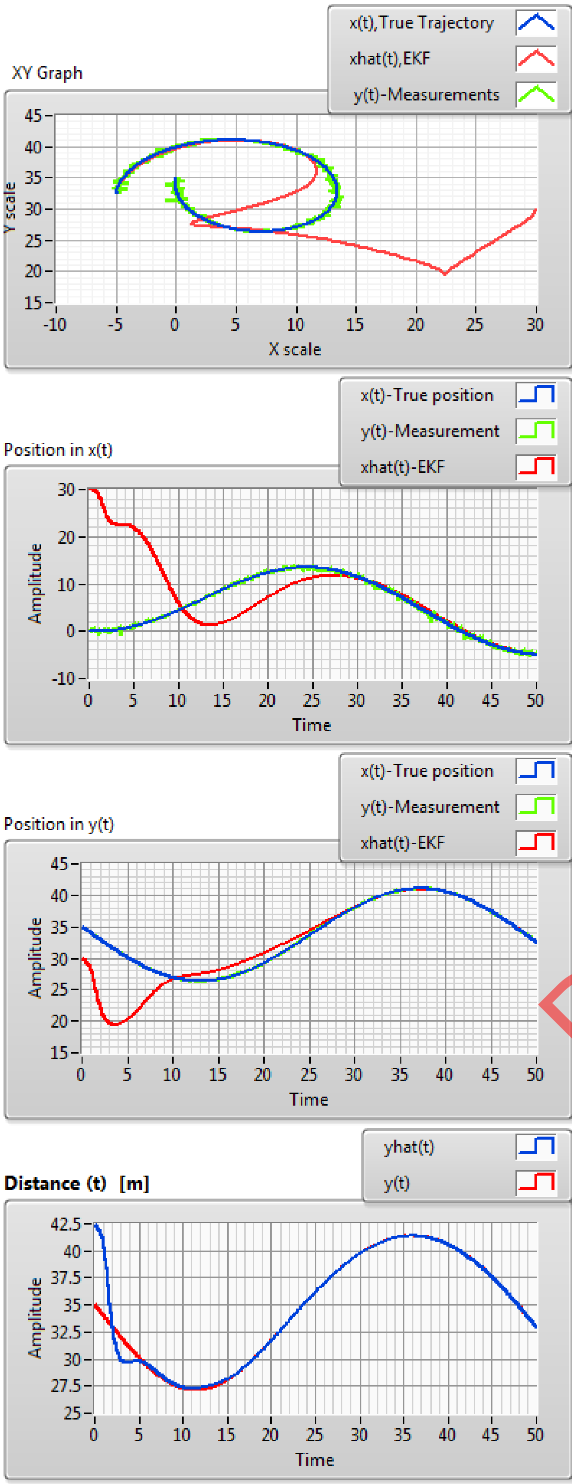
III. RESULT AND DISSCUTION

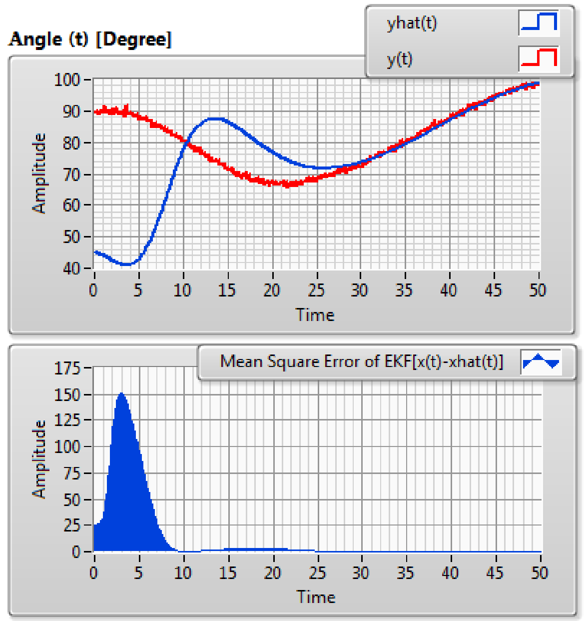
The following figures illustrate the resulting graph of the estimated states and estimated outputs. The *x* and *y* positions of the estimated states closely match the true system states.

Here, we change the Initial Covariance Matrix to see how it affects the CEKF. Further, changing the initial covariance matrix, its impact on the position of landmark in distance and angle from tip of autonomous car were investigated.

The results show that changing principal diagonal of the initial covariance matrix, improve the trajectory tracking.

As can be observable in fig. 2, with P(0)= diag [0.01,0.01,0.01,0.01], trajectory tracking did not have required accuracy and MSE was highly increased.



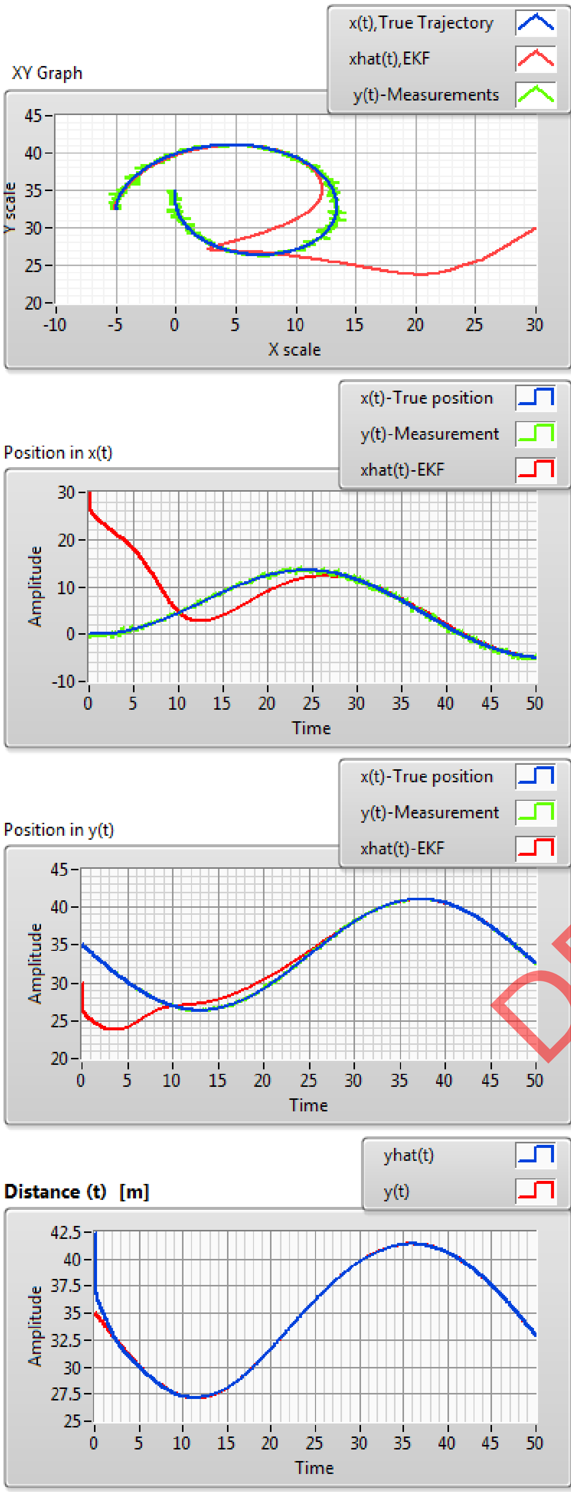


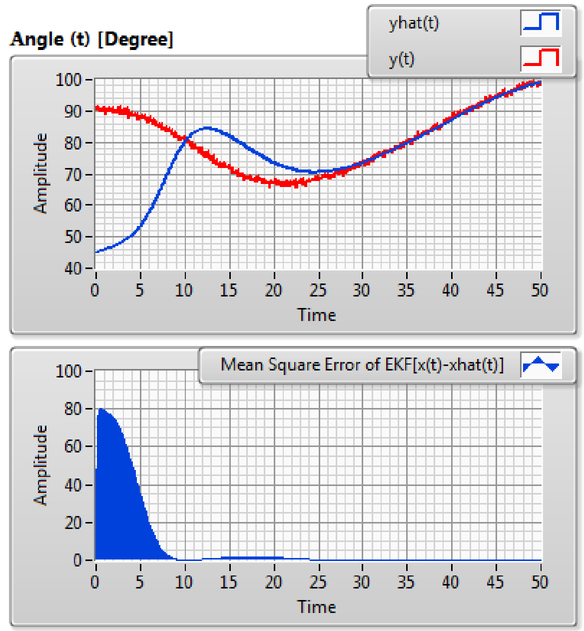
Initial Covariance matrix P(0)

0.01	0	0	0
0	0.01	0	0
0	0	0.01	0
0	0	0	0.01

Fig. 2. Localization with Extended Kalman Filter with Initial Covariance matrix P(0)=diag[0.01,0.01,0.01,0.01]

As shown in fig. 3, with P(0)= diag [1,1,0.01,0.01], trajectory tracking gained more accuracy and MSE was significantly decreased as equal as 80 amp. Notably, tracking in angle and position x(t) showed higher improvement.



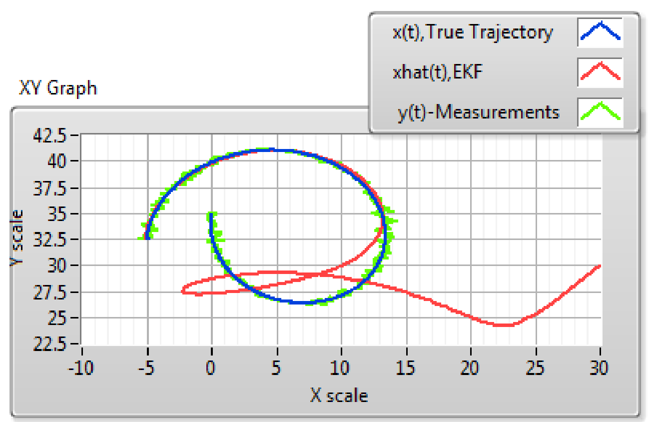


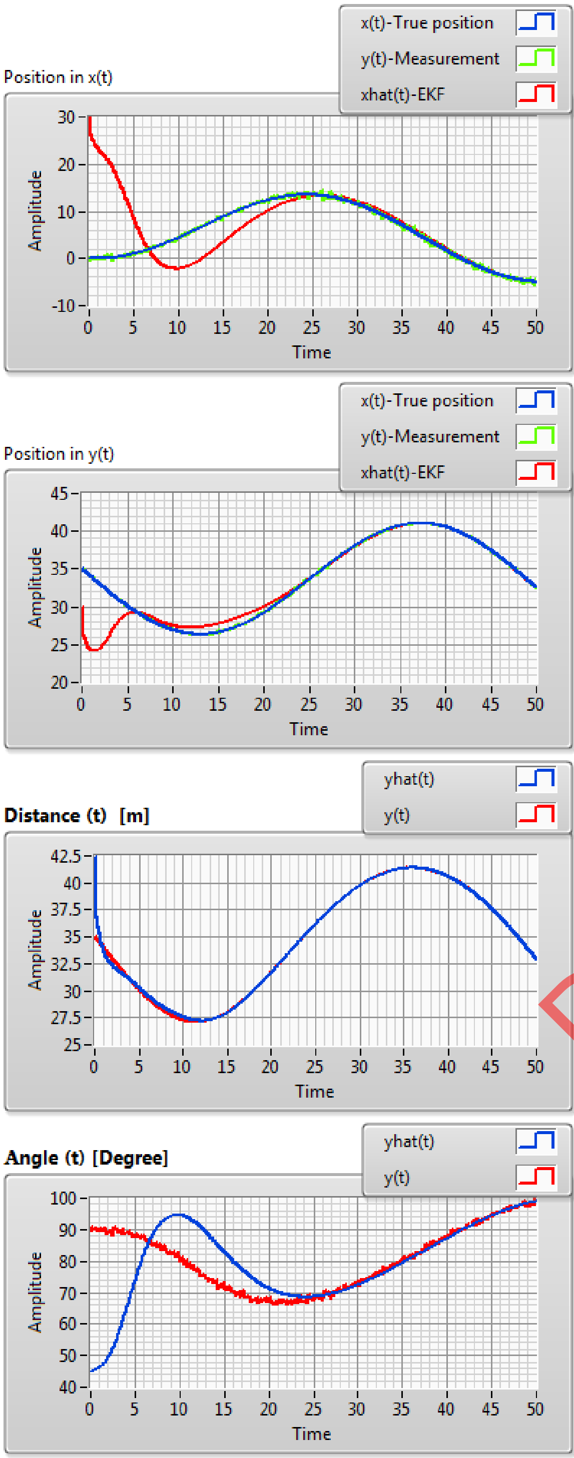
Initial Covariance matrix P(0)

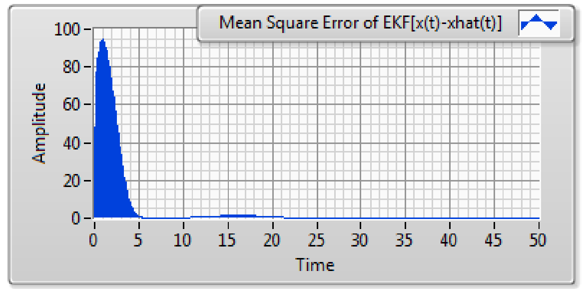
1	0	0	0
0	1	0	0
0	0	0.01	0
0	0	0	0.01

Fig. 3. Localization with Extended Kalman Filter with Initial Covariance matrix P(0)=diag[1,1,0.01,0.01]

As demonstrated in fig. 4, with P(0)= diag [1,1,1,1], the accuracy of trajectory tracking reduced in comparison with P(0)= diag [1,1,0.01,0.01] but improved comparing with P(0)= diag[0.01,0.01,0.01,0.01].







Initial Covariance matrix P(0)

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Fig. 4. Localization with Extended Kalman Filter with Initial Covariance matrix P(0)=diag[1,1,1,1]

Results demonstrated that trajectory tracking with CEKF by with Initial Covariance matrix P(0)=diag[1,1,0.01,0.01] is more accurate than P(0)=diag[0.01,0.01,0.01,0.01] and P(0)=diag[1,1,1,1]. That is due to the fat that MSE with P(0)=diag[1,1,0.01,0.01] is close to 80 that is less than two other matrixes. This can be observed in XY graph.

IV. CONCLUSION

This paper presented a performance of CEKF for localization of Trajectory Tracking Problem, using measurements that corrupted by noise. Through simulations, it was observed that, the CEKF technique significantly more preferable and gives more precise result than the Kalman filter and other Trajectory Tracking optimization techniques. Results show that the best method that can be used for Continuous Nonlinear Stochastic State-Space Systems with Nonlinear or Non differentiable observations, is the CEKF that more flexible and robust and accurate state estimation.

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