

Backstepping time-scale separation in control of a class of nonlinear systems with uncertainty

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Abstract— In this paper an adaptive control approach is investigated for completely non-affine pure-feedback systems with linear parameterization. In this approach, the idea of positive function of linearly connected parameters is coupled effectively with combination of backstepping and time scale separation. Subsequently, a fast dynamical equation is derived from the original subsystem, where the solution is used to approximate the corresponding ideal virtual/actual control inputs. Furthermore, the adaptation law of unknown parameters can be derived based on Lyapunov theory in the backstepping technique. The closed loop stability and the state regulation of these systems are all proved. Finally the simulation results for an electromechanical system are provided to demonstrate the effectiveness of the proposed approach.

Keywords— Pure-feedback systems, linear parameterization, time scale separation, electromechanical system.

I. INTRODUCTION

A more general class of triangular systems is represented by pure-feedback systems. Pure-feedback systems have no affine aspect of the variables which can be used as virtual controls. Mechanical systems, aircraft flight control systems, biochemical process, Duffing oscillator are the examples of pure-feedback systems. By using the conventional backstepping technique, finding the explicit virtual controls to stabilize the pure-feedback system is quite restrictive and difficult [1]. In addition, another difficulty with the backstepping method for the pure-feedback systems is the problem of "explosion of complexity" because of the repeated differentiations of the virtual control inputs. Adaptive neural networks (NNs) control schemes in [2] and [3] are used for a class of pure-feedback nonlinear systems, where the last equation of the controlled system is in affine form to circumvent the algebraic loop problem. Adaptive fuzzy or NNs control approaches in [4]–[8] are studied for some classes of uncertain pure-feedback nonlinear systems without or with time delays, where an implicit theorem is utilized to declare the existence of continuous desired feedback controllers, and NNs or fuzzy logic systems [9]–[12] are applied to approximate these desired feedback controllers. In [13, 14], by employing singular perturbation theory in the backstepping procedure, solutions of fast dynamic equations are considered as the time derivatives of the virtual/actual control inputs.

In this paper, an adaptive control scheme for completely non-affine pure-feedback systems in the presence of parametric uncertainties is investigated. Therefore, the class of systems considered here is much more general than the previous works.

This paper is organized as follows. Preliminary results as well as problem formulation are presented in section 2. In section 3, we develop the controller structure. Finally the simulation results for an electromechanical system and some conclusion remarks are given in sections 4 and 5.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Preliminaries on singular perturbation theory

Consider the problem of solving the state equation [15]

$$\begin{aligned} \dot{x}(t) &= f(t, x(t), z(t), \varepsilon) & x(0) &= \xi(\varepsilon) \\ \varepsilon \dot{z}(t) &= g(t, x(t), z(t), \varepsilon) & z(0) &= \eta(\varepsilon) \end{aligned} \quad (1)$$

Where $\xi(\varepsilon)$ and $\eta(\varepsilon)$ are smooth. It is assumed that the functions f and g are continuously differentiable in their arguments for $(t, x, z, \varepsilon) \in [0, \infty) \times D_x \times D_z \times [0, \varepsilon_0]$ where $D_x \subset R^n$ and $D_z \subset R^m$ are open connected sets, $\varepsilon_0 \gg 0$. If $g(t, x, z, 0) = 0$ has $l \geq 1$ isolated real roots $z = h_a(t, x)$, $a = 1, 2, \dots, l$, for each $(t, x) \in [0, \infty) \times D_x$ when $\varepsilon = 0$, the model (1) is in 'standard form'.

Let $v = z - h(t, x)$. From singular perturbation theory, the 'reduced system' is defined as

$$\dot{x}(t) = f(t, x(t), h(t, x(t)), 0) \quad x(0) = \xi(0) \quad (2)$$

and the 'boundary layer system' with the new time scale $\tau = t/\varepsilon$ is represented by

$$\frac{dv}{d\tau} = g(t, x, v + h(t, x(t)), 0) \quad v(0) = \eta_0 - h(0, \xi_0) \quad (3)$$

Where $\eta_0 = \eta(0)$ and $\xi_0 = \xi(0)$, $(t, x) \in [0, \infty) \times D_x$ are treated as fixed parameters

Theorem 1: Consider the singular perturbation system (1) and Assume that the following assumptions are satisfied for all $(t, x, \varepsilon) \in [0, \infty) \times B_r \times [0, \varepsilon_0]$,

$$(A1) f(t, 0, 0, \varepsilon) = 0 \text{ and } g(t, 0, 0, \varepsilon) = 0 .$$

(A2) The equation $\theta = g(t, x, z, 0)$ has an isolated root $z = h(t, x)$ such that $h(t, 0) = 0$.

(A3) The functions f, g, h and their partial derivatives up to order 2 are bounded for $z - h(t, x) \in B_\rho$.

(A4) The origin of the boundary layer system $\frac{dv}{d\tau} = g(t, x, v + h(t, x), 0)$ is exponentially stable, uniformly in (t, x) .

(A5) The origin of the reduced system $\dot{x} = f(t, x, h(t, x), 0)$ is exponentially stable. There is a Lyapunov function $V(t, x)$ for the reduced system which satisfies

$$c_1 \|x\|^2 \leq V(t, x) \leq c_2 \|x\|^2$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, h(t, x), 0) \leq -c_3 \|x\|^2,$$

For some positive constants $c_i, i = 1, \dots, 3$.

Then, there exists a positive constant ε^* such that for all $\varepsilon < \varepsilon^*$, the origin of (1) is exponentially stable.

Remark 1: According to the proof procedure of theorem 1 (theorem 11.4 in [15]), if the assumption (A5) is replaced by (A5)'

(A5)' The origin of the reduced system $\dot{x} = f(t, x, h(t, x), 0)$ is asymptotically stable. There is a Lyapunov function $V(t, x)$ for the reduced system which satisfies

$$V(t, x) > 0$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, h(t, x), 0) \leq -c_3 \|x\|^2,$$

Then, in theorem 1 we can conclude that There exists a positive constant ε^* such that for all $\varepsilon < \varepsilon^*$, the origin of (1) is asymptotically stable.

B. Problem statement

Consider the following pure-feedback system with linear parameterization

$$\dot{x}_i(t) = f_{i1}(\bar{x}_i(t), x_{i+1}(t)) + f_{i2}^T(\bar{x}_i(t), x_{i+1}(t))\theta,$$

$$i = 1, \dots, n - 1$$

$$\dot{x}_n(t) = f_{n1}(\bar{x}_n(t), u(t)) + f_{n2}^T(\bar{x}_n(t), u(t))\theta \quad (4)$$

where $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i$ and $u \in R$ are the system states and control input respectively. $\theta \in R^s$ denotes an unknown constant vector, $(\bar{x}_i, x_{i+1}) \in D_{\bar{x}_{i+1}}, (\bar{x}_n, u) \in D_{\bar{x}_n} \times D_u$, $D_{\bar{x}_i} \in R^i$, $D_{\bar{x}_{i+1}} \in R^{i+1}$, $D_{\bar{x}_n} \in R^n$, $D_\theta \in R^s$ and $D_u \in R$ are domains containing their respective origins. $f_{i1}: D_{\bar{x}_{i+1}} \rightarrow R$, $f_{i2}: D_{\bar{x}_{i+1}} \rightarrow R^s$, $f_{n1}: D_{\bar{x}_n} \times D_u \rightarrow R$, $f_{n2}: D_{\bar{x}_n} \rightarrow R^s$ are continuously differentiable non-linear functions in their arguments.

Remark 2: as illustrated above, f_{i2} and f_{n2} can be developed as the function of x_{i+1} and u respectively which finally leads to more general expanded class of systems.

The control objective is to design a control law $u(t)$ for system (4) such that the origin of the system is asymptotically stable.

Assumption 1: $(\partial f_{i1}/\partial x_{i+1}), (\partial f_{i2}/\partial x_{i+1}), (\partial f_{n1}/\partial u)$ and $(\partial f_{n2}/\partial u)$ are bounded away from zero for $\bar{x}_{i+1} \in \Omega_{\bar{x}_{i+1}} \subset D_{\bar{x}_{i+1}}$ and $(\bar{x}_n, u) \in \Omega_{\bar{x}_n, u} \subset D_{\bar{x}_n} \times D_u$, where $\Omega_{\bar{x}_{i+1}}$ and $\Omega_{\bar{x}_n, u}$ are compact sets; that is $(\partial f_{i1}/\partial x_{i+1}), (\partial f_{i2}/\partial x_{i+1}), (\partial f_{n1}/\partial u)$ and $(\partial f_{n2}/\partial u)$ are either positive or negative. Without losing the generality, we assume $(\partial f_{i1}/\partial x_{i+1}) > 0$, $(\partial f_{i2}/\partial x_{i+1}) > 0$, $(\partial f_{n1}/\partial u) > 0$ and $(\partial f_{n2}/\partial u) > 0$.

Remark 3: Assumption (A4) in Theorem 1 can be verified locally via linearization [15]. If there exists $\varphi_0 > 0$ such that the Jacobian matrix $(\partial g/\partial v)$ satisfies the eigenvalue condition $\text{Re}[\lambda\{\partial g(t, x, v + h(t, x), 0)/\partial v\}] \leq -\varphi_0 < 0$ for all $x \in D_x$, then Assumption (A4) is satisfied

III. CONTROLLER DESIGN

The adaptive control is developed by combination of backstepping, singular perturbation theory and the idea of positive function of linearly connected parameters term. Similar to the backstepping method, this design procedure contains n steps. Applying time-scale separation concept, the virtual control laws $\alpha_i, i = 1, \dots, n - 1$ are obtained. At each step, the idea of positive function of linearly connected parameters is applied. Finally, the actual control law u is designed at step n and in addition the update law of unknown parameters is derived. The design procedure is presented in the following. Introduce the change of coordinates $z_1 = x_1$ and $z_i = x_i - \alpha_{i-1}$ where $i = 2, \dots, n$.

Step 1. We start with the first equation of (4) by considering x_2 as the control variable. The derivative of z_1 is given as

$$\dot{z}_1(t) = f_{11}(x_1, z_2 + \alpha_1) + f_{12}^T(x_1, z_2 + \alpha_1)\theta, \quad (5)$$

Then, α_1 as the first virtual controller can be specified as the solution of

$$f_{11}(x_1, z_2 + \alpha_1) + f_{12}^T(x_1, z_2 + \alpha_1)\theta = -k_1 z_1, \quad (6)$$

resulting in the asymptotically stable closed-loop dynamics $\dot{z}_1 = -k_1 z_1$ for the first subsystem. $k_1 > 0$ is the first control gain. However, in (6), because of the non-affine property of the non-linear functions, α_1 cannot be explicitly computed. According to the following fast dynamics based on time-scale separation concept, an approximate virtual controller is designed

$$\varepsilon_1 \dot{\alpha}_1 = -\text{sign}\left(\frac{\partial Q_1}{\partial \alpha_1}\right) Q_1(\bar{z}_2, \alpha_1, \hat{\theta}), \quad (7)$$

with the initial condition $\alpha_1(0) = \alpha_{1,0}$, $\varepsilon_1 \ll 1$, $\bar{z}_2 = [z_1, z_2]^T$,

$$Q_1(\bar{z}_2, \alpha_1, \hat{\theta}) = k_1 z_1 + f_{11}(x_1, z_2 + \alpha_1) + f_{12}^T(x_1, z_2 + \alpha_1) \mathcal{E}(\hat{\theta}), \quad (8)$$

where $\hat{\theta}$ is an estimate of θ and $\mathcal{E}(\hat{\theta}) \in R^s$ is a positive function of $\hat{\theta}$ whose derivative is bounded away from zero; i.e. $(\partial \mathcal{E}(\hat{\theta}) / \partial \hat{\theta})$ is either positive or negative.

Let $\alpha_1 = h_1(\bar{z}_2, \hat{\theta})$ be an isolated root of $Q_1(\bar{z}_2, \alpha_1, \hat{\theta}) = 0$. Then the reduced system is defined as

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 + f_{12}^T(x_1, z_2) (\theta - \mathcal{E}(\hat{\theta})), \\ z_1(0) &= z_{1,0}, \end{aligned} \quad (9)$$

and the boundary layer system can be represented by

$$\frac{dy_1}{d\tau_1} = -\text{sign}\left(\frac{\partial Q_1}{\partial \alpha_1}\right) Q_1(\bar{z}_2, y_1 + h_1(\bar{z}_2, \hat{\theta}), \hat{\theta}), \quad (10)$$

where $y_1 = \alpha_1 - h_1(\bar{z}_2, \hat{\theta})$ and $\tau_1 = t/\varepsilon_1$.

Considering the control Lyapunov function $V_1 = \frac{1}{2}z_1^2$ and using the reduced system (9), it is deduced that

$$\begin{aligned} \dot{V}_1 &\leq -k_1 z_1^2 + z_1 f_{12}^T(x_1, z_2) (\theta - \mathcal{E}(\hat{\theta})) \\ &\leq -k_1 z_1^2 + z_1 f_{12}^T(x_1, z_2) \tilde{\theta}, \end{aligned} \quad (11)$$

which $\tilde{\theta} = \theta - \mathcal{E}(\hat{\theta})$.

Step i ($i = 2, \dots, n - 1$). The derivative of z_i is expressed as

$$\dot{z}_i = f_{i1}(\bar{x}_i(t), x_{i+1}(t)) + f_{i2}^T(\bar{x}_i(t), x_{i+1}(t))\theta - \dot{\alpha}_{i-1} \quad (12)$$

Similar to step 1, we should find α_i such that

$$f_{i1}(\bar{x}_i, z_{i+1} + \alpha_i) + f_{i2}^T(\bar{x}_i, z_{i+1} + \alpha_i)\theta - \dot{\alpha}_{i-1} = -k_i z_i \quad (13)$$

where $k_i > 0$ is the i th positive control gain. In this step, the time derivative of the virtual control input $\dot{\alpha}_{i-1}$ is appeared which has been designed in the previous step $\dot{\alpha}_{i-1} = -\text{sign}\left(\frac{\partial Q_{i-1}}{\partial \alpha_{i-1}}\right) Q_{i-1}(\bar{z}_i, \alpha_{i-1}, \hat{\theta})/\varepsilon_{i-1}$. Therefore, the “explosion of complexity” arising from the calculation of this term is avoided.

To overcome the non-affine property, the i th approximate virtual controller can be designed as the following i th fast dynamics

$$\varepsilon_i \dot{\alpha}_i = -\text{sign}\left(\frac{\partial Q_i}{\partial \alpha_i}\right) Q_i(\bar{z}_{i+1}, \alpha_i, \hat{\theta}) \quad (14)$$

where $\alpha_i(0) = \alpha_{i,0}$, $\varepsilon_i \ll 1$, $\bar{z}_{i+1} = [z_1, \dots, z_{i+1}]^T$ and

$$\begin{aligned} Q_i(\bar{z}_{i+1}, \alpha_i, \hat{\theta}) &= k_i z_i + f_{i1}(\bar{x}_i, z_{i+1} + \alpha_i) + \\ &f_{i2}^T(\bar{x}_i, z_{i+1} + \alpha_i) \mathcal{E}(\hat{\theta}) - \dot{\alpha}_{i-1}, \end{aligned} \quad (15)$$

Let $\alpha_i = h_i(\bar{z}_{i+1}, \hat{\theta})$ be an isolated root of $Q_i(\bar{z}_{i+1}, \alpha_i, \hat{\theta}) = 0$. Then the reduced system is defined as

$$\begin{aligned} \dot{z}_i &= -k_i z_i + f_{i2}^T(\bar{x}_i, z_{i+1}) (\theta - \mathcal{E}(\hat{\theta})), \\ z_i(0) &= z_{i,0} \end{aligned} \quad (16)$$

and the boundary layer system can be represented by

$$\frac{dy_i}{d\tau_i} = -\text{sign}\left(\frac{\partial Q_i}{\partial \alpha_i}\right) Q_i(\bar{z}_{i+1}, y_i + h_i(\bar{z}_{i+1}, \hat{\theta}), \hat{\theta}), \quad (17)$$

where $y_i = \alpha_i - h_i(\bar{z}_{i+1}, \hat{\theta})$ and $\tau_i = t/\varepsilon_i$. Considering the control Lyapunov function $V_i = V_{i-1} + \frac{1}{2}z_i^2$ and using the reduced system (16), it is deduced that

$$\begin{aligned} \dot{V}_i &\leq (\sum_{j=1}^{i-1} -k_j z_j^2 + z_j f_{j2}^T(\bar{x}_j, x_{j+1}) \tilde{\theta}) - k_i z_i^2 \\ &\quad + z_i f_{i2}^T(\bar{x}_i, x_{i+1}) (\theta - \mathcal{E}(\hat{\theta})) \\ &\leq \sum_{j=1}^i -k_j z_j^2 + z_j f_{j2}^T(\bar{x}_j, x_{j+1}) \tilde{\theta}, \end{aligned} \quad (18)$$

Step n. In the last step, the actual control input u appears and is at our disposal. We derive the z_n dynamics

$$\dot{z}_n = f_{n1}(\bar{x}_n, u) + f_{n2}^T(\bar{x}_n, u) \theta - \dot{\alpha}_{n-1} \quad (19)$$

we now obtain an approximate actual control input via time-scale separation to satisfy

$$f_{n1}(\bar{x}_n, u) + f_{n2}^T(\bar{x}_n, u) \theta - \dot{\alpha}_{n-1} = -k_n z_n \quad (20)$$

as

$$\varepsilon_n \dot{u} = -\text{sign}\left(\frac{\partial Q_n}{\partial u}\right) Q_n(\bar{z}_n, u, \hat{\theta}) \quad (21)$$

with the initial condition $u(0) = u_0$, $\varepsilon_n \ll 1$ and

$$\begin{aligned} Q_n(\bar{z}_n, u, \hat{\theta}) &= k_n z_n + f_{n1}(\bar{x}_n, u) + f_{n2}^T(\bar{x}_n, u) \mathcal{E}(\hat{\theta}) - \\ &\dot{\alpha}_{n-1} \end{aligned} \quad (22)$$

$\bar{z}_n = [z_1, \dots, z_n]^T$. k_n is the n th control gain.

Let $u = h_n(\bar{z}_n, \hat{\theta})$ be an isolated root of $Q_n(\bar{z}_n, u, \hat{\theta}) = 0$. Then the reduced system is defined as

$$\begin{aligned}\dot{z}_n &= -k_n z_n + f_{n2}^T(\bar{x}_n, u)(\theta - \Xi(\hat{\theta})), \\ z_n(0) &= z_{n,0},\end{aligned}\tag{23}$$

and the boundary layer system can be represented by

$$\frac{dy_n}{d\tau_n} = -\text{sign}\left(\frac{\partial Q_n}{\partial u}\right) Q_n(\bar{z}_n, y_n + h_n(\bar{z}_n, \hat{\theta}), \hat{\theta}), \tag{24}$$

where $y_n = u - h_n(\bar{z}_n, \hat{\theta})$ and $\tau_n = t/\varepsilon_n$. We choose the Lyapunov function $V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$ where Γ is a positive definite matrix. The resulting derivatives of V_n is given as

$$\begin{aligned}\dot{V}_n &\leq \left(\sum_{j=1}^{n-1} -k_j z_j^2 + z_j f_{j2}^T(\bar{x}_j, x_{j+1}) \tilde{\theta} \right) - k_n z_n^2 \\ &\quad + z_n f_{n2}^T(\bar{x}_n, u) \tilde{\theta} - \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} * \frac{\partial \Xi(\hat{\theta})}{\partial \tilde{\theta}} \\ &\leq (\sum_{j=1}^n -k_j z_j^2) - \tilde{\theta}^T [\Gamma^{-1} \dot{\tilde{\theta}} * \frac{\partial \Xi(\hat{\theta})}{\partial \tilde{\theta}}] - \\ &\quad \sum_{j=1}^{n-1} f_{j2}(\bar{x}_j, x_{j+1}) z_j - f_{n2}(\bar{x}_n, u) z_n\end{aligned}\tag{25}$$

where $\dot{\tilde{\theta}} * \frac{\partial \Xi(\hat{\theta})}{\partial \tilde{\theta}}$ is defined as the element-by-element multiplication of array $\dot{\tilde{\theta}}$ by $\frac{\partial \Xi(\hat{\theta})}{\partial \tilde{\theta}}$.

Finally, we can eliminate the $\tilde{\theta}^T$ term from (24) by designing the adaptation law as

$$\dot{\tilde{\theta}} = \Gamma [\sum_{j=1}^{n-1} f_{j2}(\bar{x}_j, x_{j+1}) z_j + f_{n2}(\bar{x}_n, u) z_n] / \frac{\partial \Xi(\hat{\theta})}{\partial \tilde{\theta}}, \tag{26}$$

Where $\Gamma [\sum_{j=1}^{n-1} f_{j2}(\bar{x}_j, x_{j+1}) z_j + f_{n2}(\bar{x}_n, u) z_n] / \frac{\partial \Xi(\hat{\theta})}{\partial \tilde{\theta}}$ is defined as the element-by-element division of $\Gamma [\sum_{j=1}^{n-1} f_{j2}(\bar{x}_j, x_{j+1}) z_j + f_{n2}(\bar{x}_n, u) z_n]$ by $\frac{\partial \Xi(\hat{\theta})}{\partial \tilde{\theta}}$.

Therefore, the derivative of V_n is

$$\dot{V}_n \leq \sum_{j=1}^n -k_j z_j^2 \tag{27}$$

By using the Lasalle's Theorem, this Lyapunov function guarantees the asymptotic stability of the origin of reduced system (9), (16) and (23).

Theorem 2: Consider the singular perturbation problem of the pure-feedback system (4) and the controllers (7), (14), (21). Assume that the following conditions are satisfied for all $(\bar{z}_{i+1}, \alpha_i - h_i(\bar{z}_{i+1}, \hat{\theta})) \in D_{\bar{z}_{i+1}} \times D_{y_i}$ for some domains $D_{\bar{z}_{i+1}} \subset R^{i+1}$ and $D_{y_i} \subset R$, which contain their respective origins, where $i = 1, \dots, n$, $\bar{z}_{n+1} = \bar{z}_n$, $D_{\bar{z}_{n+1}} = D_{\bar{z}_n}$ and $a_n = u$.

$$\text{B1)} f_{i1}(0,0) = 0, f_{i2}(0,0) = 0, Q_i(0,0,\hat{\theta}) = 0$$

B2) On any compact subset of $D_{\bar{z}_{i+1}} \times D_{y_i}$, the equation $0 = Q_i(\bar{z}_{i+1}, \alpha_i, \hat{\theta})$ has an isolated root $\alpha_i = h_i(\bar{z}_{i+1}, \hat{\theta})$ such that $h_i(0, \hat{\theta}) = 0$.

B3) The functions Q_i, h_i and their first partial derivatives respect to their arguments are bounded.

B4) $(\bar{z}_{i+1}, y_i) \mapsto (\partial Q_i / \partial \alpha_i)(\bar{z}_{i+1}, y_i + h_i(\bar{z}_{i+1}, \hat{\theta}), \hat{\theta})$ is bounded below by some positive constant for all $\bar{z}_{i+1} \in D_{\bar{z}_{i+1}}$.

Then, the origins of (10), (17) and (24) are exponentially stable. Moreover, there exists a positive constant ε_i^* such that for all $\varepsilon_i < \varepsilon_i^*$, the origin of (4) is asymptotically stable.

Proof: We show that the conditions in theorem 2 satisfy assumptions (A1)-(A5). Assumptions (B1) - (B3) directly denote that Assumptions (A1) and (A3) hold respectively. The exponential stability of the boundary layer system (10), (17) and (24) can be obtained locally by linearization with respect to y_i . Using Assumption 1 , remark 3 and (B4) yields

$$\text{sign}\left(\frac{\partial Q_i}{\partial \alpha_i}\right) = \text{sign}\left(\frac{\partial f_{i1}}{\partial \alpha_i} + \frac{\partial (f_{i2}^T)}{\partial \alpha_i} \Xi(\hat{\theta})\right) > 0 \tag{28}$$

This confirms that the boundary layer system has a locally exponentially stable origin.

Finally, since in previous section we showed the asymptotic stability of the origin of reduced system (9), (16) and (23), by considering remark 1, assumption (A5)' is satisfied.

According to theorem 1 and remark 1, there exists a constant $\varepsilon_i^* > 0$ such that for $0 < \varepsilon_i < \varepsilon_i^*$, the origins of the systems (5), (7), (12), (14), (19) and (21) are asymptotically stable. It follows that $z_i \rightarrow 0$ and $\alpha_i \rightarrow 0$ as $t \rightarrow \infty$. Since $x_1 = z_1$ and $x_i = z_i + \alpha_{i-1}$, it can be concluded that the origin of the pure-feedback system (4) is asymptotically stable.

Remark 4: The idea of using positive function $\Xi(\hat{\theta})$ instead of $\hat{\theta}$ is just for satisfying (28).

V. SIMULATION RESULTS

To further show the effectiveness of the proposed adaptive controller, we consider the electromechanical system shown in Fig. 1.

The dynamics of the electromechanical system is described by the following equation [16]

$$\begin{aligned}D\ddot{q} + B\dot{q} + N\sin(q) &= \tau \\ M\dot{q} + H\tau &= V - K_m\dot{q} \\ \text{where } D &= \frac{J}{K_r} + \frac{mL_0^2}{3K_r} + \frac{M_0L_0^2}{K_r} + \frac{2M_0L_0^2}{5K_r}, \quad N = \frac{mL_0G}{2K_r} + \frac{M_0L_0G}{K_r} \\ \text{and } B &= \frac{B_0}{K_r}.\end{aligned}\tag{29}$$

J is the rotor inertia, m is the link mass, M_0 is the load mass, L_0 is the link length, R_0 is the radius of the

load, G is the gravity coefficient, B_0 is the coefficient of viscous friction at the joint, $q(t)$ is the angular motor position (and hence the position of the load), τ is the motor armature current, and K_τ is the coefficient which characterizes the electromechanical conversion of armature current to torque. M is the armature inductance, H is the armature resistance, K_m is the back-emf coefficient, and V is the input control voltage. The values of the parameters are chosen as $J = 1.625 \times 10^{-3} \text{ Kg.m}^2/\text{s} = 0.506 \text{ Kg}\cdot\text{m} \cdot R_0 = 0.023 \text{ m}\cdot M_0 = 0.434 \text{ Kg}\cdot L_0 = 0.305 \text{ m}\cdot B_0 = 16.25 \times 10^{-3} \text{ N.m.s/rad} \cdot M = 25.0 \times 10^{-3} \text{ H} \cdot H = 5.0\Omega$, and $K_\tau = K_m = 0.90 \text{ N.m/A}$.

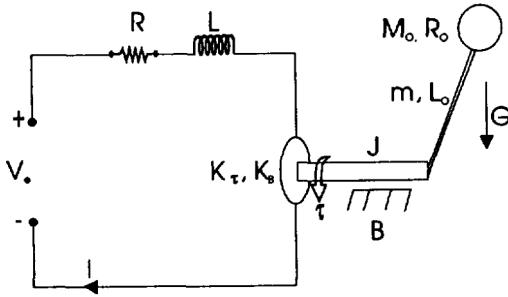


Fig. 1. Schematic of electromechanical system.

Introduce the variable change $x_1 = q$, $x_2 = \dot{q}$, $x_3 = \tau$ and $u = V$. The dynamics given by (29) can be written in the following form:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \theta_1 \sin(x_1) + \theta_2 x_2 + \theta_3 x_3 \\ \dot{x}_3 &= \theta_4 x_2 + \theta_5 x_5 + \theta_6 u\end{aligned}\quad (30)$$

Where $\theta = [-\frac{N}{D} \ -\frac{B}{D} \ \frac{1}{D} \ -\frac{K_m}{M} \ -\frac{H}{M} \ \frac{1}{M}]^T$ is the vector of unknown parameters. The control object is to design an adaptive control law $u(t)$ such that the origin of the system is asymptotically stable.

The initial conditions set to $x_1(0) = -1$, $x_2(0) = 1.5$, $x_3(0) = 2$, $u(0) = 0$, $\hat{\theta}(0) = [1 \ 1 \ 2 \ 3 \ 0 \ 2]^T$. The design parameters for the proposed control system are adopted as follows: Γ is a 6×6 identity matrix, $k_1 = k_2 = 3$, $k_3 = 5$, $\varepsilon_1 = \varepsilon_2 = 0.1$, $\varepsilon_3 = 0.2$. Furthermore, $\mathcal{E}(\hat{\theta}) = \exp(\hat{\theta})$ is chosen.

The fast dynamic for determining the solution of control signal is designed as

$$0.1\dot{\alpha}_1 = -\text{sign}\left(\frac{\partial Q_1}{\partial \alpha_1}\right) Q_1(z_1, z_2, \alpha_1, \hat{\theta}) \quad (31)$$

$$0.1\dot{\alpha}_2 = -\text{sign}\left(\frac{\partial Q_2}{\partial \alpha_2}\right) Q_2(z_1, z_2, z_3, \alpha_1, \alpha_2, \hat{\theta}) \quad (32)$$

$$0.2\dot{u} = -\text{sign}\left(\frac{\partial Q_3}{\partial u}\right) Q_3(z_1, z_2, z_3, \alpha_1, \alpha_2, u, \hat{\theta}) \quad (33)$$

$$\dot{\hat{\theta}} = \left[\frac{z_2 \sin(x_1)}{\exp(\hat{\theta}_1)} \ \frac{z_2 x_2}{\exp(\hat{\theta}_2)} \ \frac{z_2 x_3}{\exp(\hat{\theta}_3)} \ \frac{z_3 x_2}{\exp(\hat{\theta}_4)} \ \frac{z_3 x_3}{\exp(\hat{\theta}_5)} \ \frac{z_3 u}{\exp(\hat{\theta}_6)} \right]^T \quad (34)$$

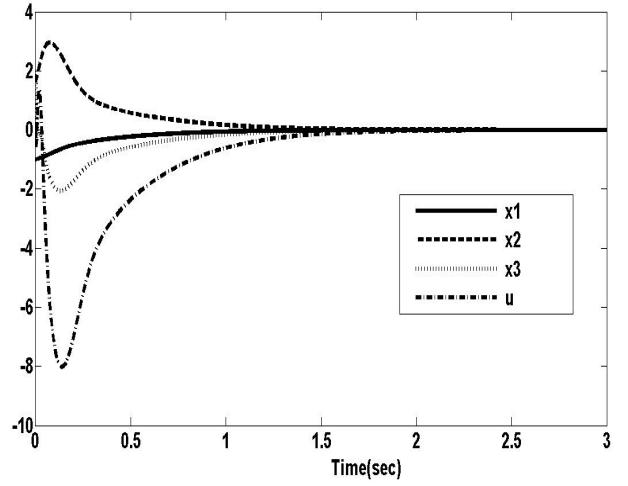


Fig.2. Transient response of the closed loop system.

where Q_1 , Q_2 and Q_3 are defined in (8), (15) and (22) respectively.

The simulation result in Fig. 2 shows dynamic performance and actual control effort $u(t)$ of the closed-loop system. This figure indicates that, despite of the parameterization of this system, the stability of the closed-loop system as well as asymptotic state regulation, with a satisfactory dynamic performance of the system states and control input can be achieved by the proposed approach.

VI. CONCLUSION

In this paper, an adaptive control method has been developed for completely non-affine pure-feedback systems in the presence of parametric uncertainties. By combination of back-stepping and singular perturbation concept and coupling it effectively with the idea of positive function of linear connected parameters, virtual/actual control inputs as well as adaptation law of unknown parameters have been derived. The proposed control approach can succeed in dealing with the linear parameterization and non-affine property. The stability proof is carried out by presenting the theorem in singular perturbation theory and the asymptotic stability of the origin of these systems is achieved. The

simulation results for electromechanical system show that the proposed approach works reasonably well.

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