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Decentralized sliding mode control of fractional-order large-scale nonlinear systems

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Abstract This paper presents some novel discussions on fully decentralized and semi-decentralized control of fractional-order large-scale nonlinear systems with two distinctive fractional derivative dynamics. First, two decentralized fractional-order sliding mode controllers with different sliding surfaces are designed. Stability of the closed-loop systems is attained under the assumption that the uncertainties and interconnections among the subsystems are bounded, and the upper bound is known. However, determining the interconnections and uncertainties bound in a large-scale system is troublesome. Therefore in the second step, two different fuzzy systems with adaptive tuning structures are utilized to approximate the interconnections and uncertainties. Since the fuzzy system uses the adjacent subsystem variables as its own input, this strategy is known as semi-decentralized fractional-order sliding mode control. For both fully decentralized and semi-decentralized control schemes, the stability of closed-loop systems has been analyzed depend on the sliding surface dynamics by integer-order or fractional-order stability theorems. Eventually, simulation results are presented to illustrate the effectiveness of the suggested robust controllers.

Keywords Fractional-order large-scale nonlinear systems · Fully decentralized and semi-decentralized control · Fractional-order sliding mode control · Fuzzy systems

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1 Introduction

Large-scale nonlinear systems are often composed of multiple interconnected low-dimensional subsystems. Such dynamical systems are widely employed in industry, for example, electric power systems [1], chemical processes [2], robotic manipulators [3], etc. These systems complexities lie in high nonlinearity, large dimensions and interconnections among the subsystems, which make the centralized control strategy computationally burden or hard to implement. Moreover, when the centralized controller fails, the entire system becomes out of control. In contrast, fully decentralized control can be designed for local subsystems from local data with less computational efforts by ignoring the interactions. It is apparent that fully decentralized control may not provide pleasant performance and may not even guarantee system stability for systems with unknown interconnections. On the other side, the advancement of DCS, fieldbus, and communication technologies in industry allows the engineers to introduce semi-decentralized and distributed strategies as new control methodologies [4,5].

One of the dominant challenges in large-scale system control is to develop some robust methods for dealing with the interconnections and consequently global system stability. In [6–15], different decentral-

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ized control methods have been reported. In some of these studies, intelligent methods like adaptive-fuzzy [6,7] and adaptive-neural controllers [8] are used to cope with the interconnections and nonlinearities. Refs [4,9] have suggested a semi-decentralized technique based on direct and indirect adaptive-fuzzy techniques. The semi-decentralized structure means that the i-th subsystem's controller depends not only on the i-th subsystem variables but also on neighbor subsystems variables. This technique is a high-cost control strategy, but in reality the interconnection terms are functions of multiple subsystems states not only one. In recent two decades, the sliding mode control (SMC) has been used in large-scale systems control [10–13] due to its high precision and robust behavior against model uncertainties and interactions. These literatures often assume that the interconnections are bounded by first-order or higher-order polynomials of states. However, some physical systems do not satisfy these conditions, or finding such conditions is challenging. In [3,14,15] some combinations of intelligent techniques and SMC have been reported to manage the mentioned problem. It is worthwhile to notify that whole mentioned discussions on decentralized control of large-scale systems are developed based on integer-order (IO) calculus.

Fractional calculus is an old mathematical branch with a generalization of ordinary differentiation -integration to an arbitrary order. Nearly 300 years, this field was viewed as an only theoretical topic with no practical applications [16]. But in last three decades, it has been used in different branches of engineering and physics such as: reaction–diffusion system [17], electrical circuits [18], rotor-bearing system [19], finance system [20], biological system [21], thermoelectric system [22], and so on. Designing fractional -order (FO) controllers on dynamical systems is another prominent case of mentioned applications. Also fractional-order sliding mode control (FOSMC) is a famous one of these FO controllers.

Recently, various forms of FOSMC have been used to control FO nonlinear systems especially the chaotic systems [23–31]. In [23,24], the FOSMC with a simple linear sliding surface has been designed. Refs [25,26] have developed this method base on terminal sliding surfaces. To remove the chattering of FOSMC a non-chatter sliding manifold proposed in [27], and a second-order structure is suggested in [28]. The sliding mode technique is designed for output tracking of a time-varying reference signal for FO nonlinear systems in [29]. In [30], a passivity-based integral sliding mode controller is considered. Also, authors of [31] have tried to apply a backstepping sliding mode controller for uncertain chaotic systems. Most of the above literatures are common in the following cases:

1 Designing FO sliding mode controllers for small-scale systems: To the author's best knowledge, there are few works on control of FO large-scale systems. Recently, robust decentralized control of FO large-scale linear systems is reported in [32]. Based on our information, there is no prominent work on applying FO sliding mode technique for fully or semi-decentralized control of large-scale nonlinear systems.
2 The FOSMC is employed alone, and lack of adaptive or adaptive-fuzzy structures for uncertainty approximation in proposed controllers is apparent: Based on our knowledge, there are few literatures on the adaptive and adaptive-fuzzy sliding mode control of nonlinear systems. In [33,34], authors have proposed an adaptive-fuzzy sliding mode controller for synchronization of FO nonlinear systems. However, the final result of their work is questionable, because they were careless about some properties of FO calculus [35]. In [36,37], two adaptive sliding mode controllers are constructed to facilitate the stability of systems with unknown uncertainties. However, the presented methods are employed on the small-scale systems.

With the mentioned motivations, we study the fully decentralized and semi-decentralized control of FO large-scale nonlinear systems. Both strategies are designed for two different types of fractional derivatives (Caputo and RL), which can be considered as a comparative research. Also, we found the FO stability theorems presented in [38–40] and properties in [41] really helpful in the closed-loop system haltering and stability analysis.

The rest of this paper is organized as follows: Some fractional calculus preliminaries are presented in Sect. 2. In Sect. 3, FO large-scale nonlinear systems with two distinctive dynamics are introduced. Two decentralized FOSMCs are developed in Sect. 4. Section 5 describes two semi-decentralized FOSMC strategies. Two illustrative examples are provided to confirm the theoretical results in Sect. 6, and finally, conclusions are given in Sect. 7.

2 Fractional calculus preliminaries

In this section, some basic definitions of fractional calculus and two essential FO stability theorems are expressed.

Definition 1 [42] The function $f(t) : R \rightarrow R$ is called C^k-class if the derivatives $f^{(1)}, f^{(2)}, \ldots, f^{(k)}$ exist and be continuous (except for a finite number of points). From the above definition, $f(t) \in C^0, C^1$, and C^∞ are the classes of all continuous, continuously differentiable, and smooth functions, respectively.

Definition 2 [41] The α-th order Riemann–Liouville fractional integration of function $f(t)$ with respect to t is given by

$$I_{0,t}^{\alpha} f(t) = D_{0,t}^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} \mathrm{d}\tau \quad (1)$$

where $\Gamma(\cdot)$ is the Gamma function.

Definition 3 [41] The Grunwald–Letnikov (GL) fractional derivative of function $f(t)$ with fractional-order α is defined as

$$\begin{aligned} {}_{GL}D_{0,t}^{\alpha} f(t) = & \sum_{k=0}^{m-1} \frac{f^{(k)}(0) t^{-\alpha+k}}{\Gamma(-\alpha+k+1)} \\ & + \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{1-m+\alpha}} \mathrm{d}\tau \end{aligned} \quad (2)$$

where $f(t) \in C^m[0, t]$ and $m-1 \leq \alpha < m,\ m \in N$.

Definition 4 [41] The α-th order Caputo fractional derivative of continuous ($f(t) \in C^m[0, t]$) function $f(t)$ is given by follows:

$$\begin{aligned} {}_{C}D_{0,t}^{\alpha} f(t) &= D_{0,t}^{-(m-\alpha)} D^m f(t) \\ &= \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{1-m+\alpha}} \mathrm{d}\tau \end{aligned} \quad (3)$$

where $m-1 < \alpha < m,\ m \in N$.

Definition 5 [41] The Riemann–Liouville (RL) fractional derivative of function $f(t)$ with fractional-order α is defined as

$$\begin{aligned} {}_{RL}D_{0,t}^{\alpha} f(t) &= D^m D_{0,t}^{-(m-\alpha)} f(t) \\ &= \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-m+\alpha}} \mathrm{d}\tau \end{aligned} \quad (4)$$

where $m-1 \leq \alpha < m,\ m \in N$.

Property 1 [41] If $f(t) \in C^0[0, T]$ for $T > 0$ and $\alpha > 0$, then

$$D_{0,t}^{-\alpha} f(t)|_{t=0} = 0 \quad (5)$$

Property 2 [41] If $f(t) \in C^m[0, \infty)$, $m-1 < \alpha < m$, and $m \in N$, then

(a) ${}_{C}D_{0,t}^{\alpha} f(t) = {}_{RL}D_{0,t}^{\alpha} \left(f(t) - \sum_{k=0}^{m-1} \frac{t^k}{k!} f^{(k)}(0) \right)$.
(b) ${}_{RL}D_{0,t}^{\alpha} D_{0,t}^{-\alpha} f(t) = f(t)$.
(c) ${}_{C}D_{0,t}^{\alpha} D_{0,t}^{-\alpha} f(t) = f(t)$ holds for $m = 1$.

Part (c) proof Using parts (a), (b) and Property 1, one can get

$$\begin{aligned} {}_{C}D_{0,t}^{\alpha} \left(D_{0,t}^{-\alpha} f(t) \right) &= {}_{RL}D_{0,t}^{\alpha} \left(D_{0,t}^{-\alpha} f(t) - D_{0,t}^{-\alpha} f(t)|_{t=0} \right) \\ &= {}_{RL}D_{0,t}^{\alpha} \left(D_{0,t}^{-\alpha} f(t) - 0 \right) = f(t) \end{aligned} \quad (6)$$

□

Property 3 [41] If $f(t) \in C^m[0, t]$ then ${}_{RL}D_{0,t}^{\alpha} f(t) = {}_{GL}D_{0,t}^{\alpha} f(t)$.

Property 4 (sequential property) [41]: If $f(t) \in C^1[0, T]$ for some $T > 0$, $\alpha_i \in (0, 1)$ $(i = 1, 2)$ and $\alpha_1 + \alpha_2 \in (0, 1]$, then

$$\begin{aligned} {}_{C}D_{0,t}^{\alpha_1} {}_{C}D_{0,t}^{\alpha_2} f(t) &= {}_{C}D_{0,t}^{\alpha_2} {}_{C}D_{0,t}^{\alpha_1} f(t) \\ &= {}_{C}D_{0,t}^{\alpha_1+\alpha_2} f(t), \quad t \in [0, T] \end{aligned} \quad (7)$$

Note that, the sequential property is very attractive in the sliding mode control, but the continuously differentiability condition restricts it. For more details see Remark 1.

Remark 1 Let consider the sliding surface $f(t) = s(t)$, $\alpha_1 = \alpha$ and $\alpha_2 = 1-\alpha$, then Eq. (7) will be as follows:

$$\dot{s}(t) = {}_{C}D_{0,t}^{\alpha} . {}_{C}D_{0,t}^{1-\alpha} s(t) = {}_{C}D_{0,t}^{1-\alpha} . {}_{C}D_{0,t}^{\alpha} s(t) \quad (8)$$

from mathematical point of view and according to Property 4, Eq. (8) holds for continuous $s(t)$ and $\dot{s}(t)$, while discontinuous $\mathrm{sgn}(s(t))$ function, non-smooth desired values, and sudden changes in disturbances may cause discontinuous $\dot{s}(t)$ which degrade C^1 condition. Using fuzzy approximations, smooth functions ($\tanh(s(t))$, $sat(s(t))$, $D_{0,t}^{-\alpha}(s(t))$) instead of $\mathrm{sgn}(s(t))$ and smooth desired values are approximate practical remedies for the mentioned problem.

Theorem 1 *[38,39] Let $x = 0$ be an equilibrium point for the non-autonomous FO system*

$$ {}_C D_{0,t}^{\alpha} x(t) = f(t, x(t)) \tag{9}$$

where $f(t, x(t))$ satisfies the Lipschitz condition with Lipschitz constant $l > 0$ and $\alpha \in (0, 1)$. Assume that there exists a Lyapunov function $V(t, x(t))$ and class-K functions $\alpha_i (i = 1, 2, 3)$ satisfying

$$\alpha_1 (\|x\|) \le V(t, x(t)) \le \alpha_2 (\|x\|) \tag{10}$$

$$ {}_C D_{0,t}^{\beta} V(t, x(t)) \le -\alpha_3 (\|x\|) \tag{11}$$

where $\beta \in (0\,1)$. Then the system (9) is asymptotically stable.

Remark 2 For Theorem 1, if ${}_C D_{0,t}^{\beta} V(t, x(t)) \le 0$, then the system (9) will be stable [40].

Proof From ${}_C D_{0,t}^{\beta} V(t, x(t)) \le 0$ we can get $V(t, x(t)) \le V(0, x(0))$. Taking into account (10), results $\|x\| \le \alpha_1^{-1}(V(t, x(t))) \le \alpha_1^{-1}(V(0, x(0)))$. Therefore, the equilibrium point $x = 0$ is stable. □

Theorem 2 *[38] Let $x = 0$ be an equilibrium point for the non-autonomous fractional-order system (9). Assume that there exists a Lyapunov function $V(t, x(t))$ satisfying*

$$\alpha_1 \|x\|^a \le V(t, x(t)) \le \alpha_2 \|x\| \tag{12}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} V(t, x(t)) \le -\alpha_3 \|x\| \tag{13}$$

where α_1, α_2, α_3, and a are positive constants. Then the equilibrium point of the system (9) is asymptotic stable.

3 Problem formulation

Consider a class of FO large-scale nonlinear system composed of N interconnected subsystems (S_i). All of the subsystems S_i can be described based on RL derivative as $(i = 1, 2, \ldots, N)$:

$$S_i : \begin{cases} {}_{RL}D^{\alpha} x_{i1}(t) = x_{i2}(t) \\ {}_{RL}D^{\alpha} x_{i2}(t) = x_{i3}(t) \\ \quad\vdots \\ {}_{RL}D^{\alpha} x_{in}(t) = f_i(X_i) + g_i(X_i) u_i(t) \\ \qquad + M_i(X_i, t) + I_i(X_1, \ldots, X_N, t) \end{cases} \tag{14}$$

or based on Caputo derivative

$$S_i : \begin{cases} {}_{C}D^{\alpha} x_{i1}(t) = x_{i2}(t) \\ {}_{C}D^{\alpha} x_{i2}(t) = x_{i3}(t) \\ \quad\vdots \\ {}_{C}D^{\alpha} x_{in}(t) = f_i(X_i) + g_i(X_i) u_i(t) \\ \qquad + M_i(X_i, t) + I_i(X_1, \ldots, X_N, t) \end{cases} \tag{15}$$

where $\alpha \in (0, 1)$ is the order of system, $X_i = [x_{i1}, x_{i2}, \ldots, x_{in}]$ is the state vector of i - th subsystem, $u_i \in R$ is the input, $f_i : R^n \to R$ and $g_i : R^n \to R$ are known functions, $M_i : R^{n+1} \to R$ is model uncertainty and external disturbance term, and $I_i : R^{n \times N+1} \to R$ represents the interconnection between the i-th subsystem and other subsystems. We consider $L_i(X, t) = M_i(X_i, t) + I_i(X_1, \ldots, X_N, t)$ which is called the lumped uncertainty.

Assumption 1 Full state vectors of the system are measurable.

By defining the tracking errors of the i-th subsystem as $e_{i1}(t) = x_{i1}(t) - x_{i1d}(t), \ldots,\ e_{in}(t) = x_{in}(t) - x_{ind}(t)$, the error dynamics of (14) and (15) will be in the following form

$$S_i : \begin{cases} {}_{RL}D^{\alpha} e_{i1}(t) = e_{i2}(t) \\ {}_{RL}D^{\alpha} e_{i2}(t) = e_{i3}(t) \\ \quad\vdots \\ {}_{RL}D^{\alpha} e_{in}(t) = f_i(X_i) + g_i(X_i) u_i(t) - {}_{RL}D^{\alpha} x_{ind}^{(t)} \\ \qquad + L_i(X, t) \end{cases} \tag{16}$$

and

$$S_i : \begin{cases} {}_{C}D^{\alpha} e_{i1}(t) = e_{i2}(t) \\ {}_{C}D^{\alpha} e_{i2}(t) = e_{i3}(t) \\ \quad\vdots \\ {}_{C}D^{\alpha} e_{in}(t) = f_i(X_i) + g_i(X_i) u_i(t) - {}_{C}D^{\alpha} x_{ind}^{(t)} \\ \qquad + L_i(X, t) \end{cases} \tag{17}$$

where sets $X_{id} = [x_{i1d}, x_{i2d}, \ldots, x_{ind}]^T$ and $E_i = [e_{i1}, e_{i2}, \ldots, e_{in}]^T$ are reference vector and tracking error vector of i-th subsystem, respectively, and the large-scale system error vector is given by $E = [E_1, E_2, \ldots, E_N]^T$. The goal is to design robust controllers for the FO systems (14) and (15) such that the state vectors $X_i(t)$, $i = 1, 2, \ldots, N$ track the time-varying reference vectors $X_{id}(t)$ (where $x_{i(j+1)d}(t) = {}_{RL}D^{\alpha} x_{ijd}(t)$ for (14) and $x_{i(j+1)d}(t) = {}_{C}D^{\alpha} x_{ijd}(t)$ for (15), $1 \le j \le n-1$).

4 Fully decentralized FOSMC

In this section, designing the fully decentralized FOSMC for FO large-scale systems (14) and (15) are developed. For this purpose, two types of sliding surfaces are proposed. The suggested controllers are designed based on the Caputo and RL derivatives separately.

Assumption 2 In this section, we assume that the lumped uncertainty $L_i(X, t)$ satisfies the following conditions

$$|L_i(X,t)| \le \psi_{i1} \tag{18}$$

$$\left| {}_{C}D^{1-\alpha} L_i(X,t) \right| \le \psi_{i2} \tag{19}$$

where ψ_{1i}, ψ_{i2} are known positive constants.

4.1 RL-derivative-based dynamics

Consider the following integral FO sliding manifold:

$$\sigma_i(t) = D^{-(1-\alpha)} \left(e_{in}(t) + \sum_{k=1}^{n-1} c_{ik} e_{ik}(t) \right) \tag{20}$$

where $c_{i1}, c_{i2}, \ldots, c_{i(n-1)}$ are selected in such a way that all roots of the polynomial $P(s) = s^{n-1} + c_{i(n-1)} s^{n-2} + \cdots + c_{i2}s + c_{i1}$ are located in the left half of s-plane. By differentiating from both sides of (20), one can obtain

$$\begin{aligned}\dot{\sigma}_i(t) &= D^1 D^{-(1-\alpha)} \left(e_{in}(t) + \sum_{k=1}^{n-1} c_{ik} e_{ik}(t) \right) \\ &= {}_{RL}D^{\alpha} e_{in}(t) + \sum_{k=1}^{n-1} c_{ik\,RL} D^{\alpha} e_{ik}(t)\end{aligned} \tag{21}$$

Putting (16) in (21), leads to

$$\begin{aligned}\dot{\sigma}_i(t) &= f_i(X_i,t) + g_i(X_i,t)u_i(t) - {}_{RL}D^{\alpha} x_{ind}(t) \\ &\quad + \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) + L_i(X,t)\end{aligned} \tag{22}$$

Theorem 3 *Consider the RL-derivative-based error dynamics (16) with the sliding manifold (20) and assumptions 1, 2, the fully decentralized control law*

$$\begin{aligned}u_i(t) = \frac{1}{g_i(X_i,t)} \Big(&-f_i(X_i,t) + {}_{RL}D^{\alpha} x_{ind}(t) \\ &- \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) \\ &- \eta_i \sigma_i(t) - K_{sw-i} sgn(\sigma_i(t)) \Big), \eta_i > 0\end{aligned} \tag{23}$$

guarantees the closed-loop system asymptotic stability, if the switching gain K_{sw-i} be selected as

$$K_{sw-i} \ge \psi_{i1} \ge |L_i(X,t)| \tag{24}$$

then the tracking errors $E_1, E_2, \ldots, E_N$ will converge to zero.

Proof Choose the following continuously differentiable (except in equilibrium point) Lyapunov function candidate

$$V(t,\sigma(t)) = \|\sigma(t)\|_1 = \sum_{i=1}^{N} V_i(t,\sigma_i(t)) = \sum_{i=1}^{N} |\sigma_i(t)| \tag{25}$$

where $V_i(\bullet)$ is the Lyapunov function for each subsystem. Whereas putting (23) in (22) results an IO sliding dynamic (all fractional terms are removed), then applying the IO stability theorems is proper. So by taking time derivative from $V(\bullet)$, one has

$$\dot{V}(t,\sigma(t)) = \sum_{i=1}^{N} \text{sgn}(\sigma_i(t)) \dot{\sigma}_i(t) \tag{26}$$

Substituting the sliding surface dynamics (22), results in

$$\begin{aligned}\dot{V}(t,\sigma(t)) = \sum_{i=1}^{N} \text{sgn}(\sigma_i(t)) \Big(&f_i(X_i,t) + g_i(X_i,t)u_i(t) \\ &- {}_{RL}D^{\alpha} x_{ind}(t) + \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) + L_i(X,t) \Big)\end{aligned} \tag{27}$$

Using the control signal (23), one can obtain

$$\begin{aligned}\dot{V}(t,\sigma(t)) = \sum_{i=1}^{N} \text{sgn}(\sigma_i(t)) \big(&-\eta_i \sigma_i(t) \\ &- K_{sw-i} \text{sgn}(\sigma_i(t)) + L_i(X,t) \big)\end{aligned} \tag{28}$$

Since $\text{sgn}(\sigma_i(t)) \times \text{sgn}(\sigma_i(t)) = 1$ and $\text{sgn}(\sigma_i(t)) \times \sigma_i(t) = |\sigma_i(t)|$, then we have

$$\dot{V}(t, \sigma(t)) = -\sum_{i=1}^{N} \Big(\eta_i \, |\sigma_i(t)| + K_{sw-i} - \text{sgn}(\sigma_i(t)) L_i(X, t)\Big) \tag{29}$$

Choosing $K_{sw-i} \geq \psi_{i1}$, leads to

$$\dot{V}(t, \sigma(t)) \leq -\sum_{i=1}^{N} \eta_i \, |\sigma_i(t)| \leq -\frac{1}{N} \sum_{i=1}^{N} \eta_i \sum_{i=1}^{N} |\sigma_i(t)| = -\Omega \, \|\sigma_i(t)\|_1 , \quad \Omega = \left(\frac{1}{N} \sum_{i=1}^{N} \eta_i\right) > 0 \tag{30}$$

which implies the closed-loop system asymptotic stability based on IO stability theorems. □

4.2 Caputo-derivative-based dynamics

Let the FO sliding surface to be defined as follows:

$$\sigma_i(t) = e_{in}(t) + \sum_{k=1}^{n-1} c_{ik} e_{ik}(t) \tag{31}$$

Taking ${}_C D^{\alpha}$ derivative from both sides of (31), leads to

$${}_C D^{\alpha} \sigma_i(t) = {}_C D^{\alpha} e_{in}(t) + \sum_{k=1}^{n-1} c_{ik} {}_C D^{\alpha} e_{ik}(t) \tag{32}$$

Substituting the error dynamics (17) into (32), results in

$${}_C D^{\alpha} \sigma_i(t) = f_i(X_i, t) + g_i(X_i, t) u_i(t) - {}_C D^{\alpha} x_{ind}(t) + \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) + L_i(X, t) \tag{33}$$

Theorem 4 *For the error dynamics described by Caputo derivative (17) with the sliding manifold (31) and assumptions 1, 2, the decentralized control effort*

$$u_i(t) = \frac{1}{g_i(X_i, t)} \Bigg(-f_i(X_i, t) + {}_C D^{\alpha} x_{ind}(t) - \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) - D^{-(1-\alpha)} \Bigg(\eta_i \sigma_i(t) + K_{sw-i} \tanh\left(\frac{\sigma_i(t)}{\rho_i}\right)\Bigg)\Bigg), 0 < \rho_i < 1 \tag{34}$$

with the sliding gain

$$K_{sw-i} \geq \psi_{i2} \geq \left| {}_C D^{1-\alpha} L_i(X_i, t) \right| \tag{35}$$

guarantee boundedness of the closed-loop system tracking errors $(E_1, E_2, \ldots, E_N)$.

Proof Let the global Lyapunov candidate to be as follows:

$$V(t, \sigma(t)) = \|\sigma(t)\|_1 = \sum_{i=1}^{N} V_i(t, \sigma_i(t)) = \sum_{i=1}^{N} |\sigma_i(t)| \tag{36}$$

By differentiating $V(\bullet)$ with respect to time

$$\dot{V}(t, \sigma(t)) = \sum_{i=1}^{N} \text{sgn}(\sigma_i(t)) \dot{\sigma}_i(t) \tag{37}$$

Since substituting (34) in (33) results a FO sliding dynamic, then using the FO stability theorems will be useful. Hence, using Property 4, the following equation fractionalizes the IO derivative into a fractional type.

$$\dot{\sigma}_i(t) = {}_C D^{1-\alpha} {}_C D^{\alpha} \sigma_i(t) \tag{38}$$

Inserting (38) into (37), results in

$$\dot{V}(t, \sigma(t)) = \sum_{i=1}^{N} \text{sgn}(\sigma_i(t)) \left({}_C D^{1-\alpha} {}_C D^{\alpha} \sigma_i(t)\right) \tag{39}$$

Using the sliding surface dynamics (33), one can obtain

$$\dot{V}(t, \sigma(t)) = \sum_{i=1}^{N} \text{sgn}(\sigma_i(t)) \Bigg({}_C D^{1-\alpha} \Bigg(f_i(X_i, t) + g_i(X_i, t) u_i(t) - {}_C D^{\alpha} x_{ind}(t) + \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) + L_i(X, t)\Bigg)\Bigg) \tag{40}$$

Substituting control signal (34) in (40), we get

$$\dot{V}(t, \sigma(t)) = \sum_{i=1}^{N} \text{sgn}(\sigma_i(t)) \Bigg({}_C D^{1-\alpha} \Bigg(-D^{-(1-\alpha)} \Bigg(\eta_i \sigma_i(t) + K_{sw-i} \tanh\left(\frac{\sigma_i(t)}{\rho_i}\right)\Bigg) + L_i(X, t)\Bigg)\Bigg) \tag{41}$$

Since $\eta \sigma_i(t) + K_{sw-i} \tanh(\sigma_i(t)/\rho_i)$ is a continuously differentiable function, then from Property 2.c we get

$$\dot{V}(t, \sigma(t)) = \sum_{i=1}^{N} (-\eta_i \, |\sigma_i(t)| + \text{sgn}(\sigma_i(t)) \left(-K_{sw-i} \tanh\left(\frac{\sigma_i(t)}{\rho_i}\right) + {}_C D^{1-\alpha} L_i(X, t)\right) \tag{42}$$

By approaching $\rho \to 0$, we have $\tanh(\sigma_i(t)\rho_i) \cong \mathrm{sgn}(\sigma_i(t))$, which leads to

$$\dot{V}(t,\sigma(t)) \cong -\sum_{i=1}^{N}\Big(\eta_i|\sigma_i(t)| + \big(K_{sw-i} - \mathrm{sgn}(\sigma_i(t))_C D^{1-\alpha} L_i(X,t)\big)\Big) \quad (43)$$

Selecting $K_{sw-i} \geq \psi_{i2}$, results in

$$\dot{V}(t,\sigma(t)) \leq -\sum_{i=1}^{N}\eta_i|\sigma_i(t)| \leq -\frac{1}{N}\sum_{i=1}^{N}\eta_i\sum_{i=1}^{N}|\sigma_i(t)| = -\Omega\,\|\sigma_i(t)\|_1 \quad (44)$$

which implies the large-scale system (15) stability based on Theorem 2. □

Remark 3 Inequalities (30) and (44) are derived by the following Chebyshev's sum inequality:

For $a_1 \geq a_2 \geq \cdots \geq a_N$ and $b_1 \geq b_2 \geq \cdots \geq b_N$ then

$$\frac{1}{N}\sum_{i=1}^{N}a_ib_i \geq \left(\frac{1}{N}\sum_{i=1}^{N}a_i\right)\left(\frac{1}{N}\sum_{i=1}^{N}b_i\right) \quad (45)$$

Remark 4 Although the suggested decentralized control strategies, (23) and (34), are developed on large-scale systems with distinctive FO derivatives, there are other dominant differences between them which are listed in below:

1. To guarantee the large-scale system stability, the control law (23) needs a small sliding gain in comparison with the control effort (34) sliding gain.
2. In (34), the sliding surface should be continuously differentiable ($\sigma_i(t) \in C^1$), while this constraint is not necessary for (23) (sequential property).
3. By applying (23), the closed-loop system error trajectories will converge to the origin, while for (34), the error trajectories approach the neighborhood of the origin (due to using $\tanh(\sigma_i(t)/\rho_i)$ function instead of $\mathrm{sgn}(\sigma_i(t))$).

Remark 5 Proposed decentralized SMC techniques, (23) and (34) based on Assumption 2, contain the following limitations:

1. The control laws, (23) and (34), usually need the upper bound of interconnections and model uncertainties in order to assure the stability of closed-loop system. Generally, it is not easy to obtain this knowledge in practice because of the complexities of large-scale systems. Moreover, when an unknown perturbation occurs in one subsystem, it may cause large changes in the interaction bounds, which makes the calculation of the switching gain K_{sw-i} difficult in (23) and (34). Therefore, a plan is needed in order to approximate the interactions bound.
2. Applying functions $\mathrm{sgn}(\sigma_i(t))$ and $\tanh(\sigma_i(t)/\rho_i)$ (with very small ρ_i) value in (23) and (34) can provoke the chattering phenomena, which can degrade C^1 condition (for (34)), and damage both RL- and Caputo-derivative-based physical systems.

5 Semi-decentralized FOSMC

In this section, two adaptive-fuzzy schemes are introduced to approximate the interconnections and uncertainties, so that the objective of stability can be achieved.

5.1 Fuzzy logic system

In this part, the fuzzy logic system is briefly discussed. The basic configuration of the fuzzy system composed of a collection of fuzzy IF-THEN rules, which can be written as follows [43]:

$Rule\ l:\ If\ x_1\ is\ F_1^l\ and ... and\ x_p\ is\ F_p^l\ Then\ y\ is\ A^l$

where the input vector $X = [x_1, \ldots, x_p]^T \in R^p$ and the output variable $y \in R$ denote the linguistic variables of the fuzzy system, $i = 1, 2, \ldots, p$ denotes the number of input for the fuzzy system, and $l = 1, 2, \ldots, M$ denotes the number of the fuzzy rules, F_i^l and A^l are labels of the input and output fuzzy sets, respectively. By using the product inference, singleton fuzzification, and center average defuzzification, the fuzzy system output will be as

$$y(X) = \frac{\sum_{l=1}^{m} y^l\left(\prod_{i=1}^{p}\mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{m}\prod_{i=1}^{p}\mu_{F_i^l}(x_i)} \quad (46)$$

where $\mu_{F_i^l}(x_i)$ and $\mu_{A^l}(y^l) = 1$ are the membership functions of the linguistic variables x_i and y, respectively. By introducing the concept of fuzzy basis function, (46) can be rewritten in the following form

$$y(X) = \theta^T\xi(X) \quad (47)$$

where $\theta = [y^1, \ldots, y^M]^T$ is the parameter vector and $\xi(X) = [\xi^1(X), \ldots, \xi^M(X)]^T$ is a regressive vector which can be defined as

$$\xi^l(X) = \frac{\prod_{i=1}^{p} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{m} \prod_{i=1}^{p} \mu_{F_i^l}(x_i)} \tag{48}$$

5.2 Adaptive-fuzzy interconnection and uncertainty approximation

As mentioned earlier, determining the interconnections, modeling uncertainties, and also external disturbances upper bounds is difficult. But based on the universal approximation property of fuzzy systems it is possible.

5.2.1 RL-derivative-based dynamics

Now, consider the rewritten form of the sliding surface dynamics (22) as follows:

$$\begin{aligned}\dot{\sigma}_i(t) &= f_i(X_i, t) + g_i(X_i, t)u_i(t) - {}_{RL}D^{\alpha} x_{ind}(t) \\ &\quad + \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) + \theta_i^T \xi_i(X)\end{aligned} \tag{49}$$

where $\theta_i = [\theta_{i1}, \ldots, \theta_{iM}]^T$ is the parameter vector, $\xi_i(X)$ is a regressive vector, and $X = [X_1,\ X_2, \cdots,\ X_N]$ is the fuzzy system input vector. Choosing the semi-decentralized control law as

$$\begin{aligned}u_i(t) &= \frac{1}{g_i(X_i, t)}\big(-f_i(X_i, t) + {}_{RL}D^{\alpha} x_{ind}(t) \\ &\quad - \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) - \eta_i \sigma_i(t) - \hat{\theta}_i^T \xi_i(X)\big)\end{aligned} \tag{50}$$

guarantees the large-scale systems (14) stability with the following adaptation mechanism

$$\dot{\tilde{\theta}}_i = -\mu_i \xi_i(X) \sigma_i(t) \tag{51}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter error vector, $\hat{\theta}$ is the estimation vector of the unknown parameter vector θ, and μ_i is a positive constant used for adaptation.

Proof Using (49) and (50) the closed-loop dynamic becomes

$$\begin{aligned}\dot{\sigma}_i(t) &= \theta_i^T \xi_i(X) - \eta_i \sigma_i(t) - \hat{\theta}_i^T \xi_i(X) \\ &= \tilde{\theta}_i^T \xi_i(X) - \eta_i \sigma_i(t)\end{aligned} \tag{52}$$

To study the stability and derive the adaptation law for $\tilde{\theta}$, we consider the following Lyapunov function:

$$\begin{aligned}V(t, Y(t)) &= \alpha(\|Y\|) = \|Y\|_2^2 = \sum_{i=1}^{N} V_i(t, Y_i(t)) \\ &= \sum_{i=1}^{N} \left(\frac{1}{2}\sigma_i^2(t) + \frac{1}{2\mu_i} \tilde{\theta}_i^T \tilde{\theta}_i \right)\end{aligned} \tag{53}$$

where $Y_i = [\sigma_i, \tilde{\theta}_i^T]$. Differentiating (53) along the trajectory (52), one can obtain that

$$\begin{aligned}\dot{V}(t, Y(t)) &= \sum_{i=1}^{N} \left(\sigma_i(t)\dot{\sigma}_i(t) + \frac{1}{\mu_i} \tilde{\theta}_i^T \dot{\tilde{\theta}}_i \right) \\ &= \sum_{i=1}^{N} \left(\sigma_i(t) \left(\tilde{\theta}_i^T \xi_i(X) - \eta_i \sigma_i(t) \right) + \frac{1}{\mu_i} \tilde{\theta}_i^T \dot{\tilde{\theta}}_i \right) \\ &= \sum_{i=1}^{N} \left(-\eta_i \sigma_i^2(t) + \tilde{\theta}_i^T \left(\xi_i(X)\sigma_i(t) + \frac{1}{\mu_i} \dot{\tilde{\theta}}_i \right) \right)\end{aligned} \tag{54}$$

Inserting the adaptation law (51), leads to

$$\dot{V}(t, Y(t)) = \sum_{i=1}^{N} -\eta_i \sigma_i^2(t) \le 0 \tag{55}$$

which assures the large-scale system (14) stability. Therefore, $\sigma_i(t)$ and $\tilde{\theta}_i$ are bounded. Although $\sigma_i(t)$ converges to zero (Barbalat's lemma), the system is not asymptotically stable, because $\tilde{\theta}_i$ is only bounded. □

5.2.2 Caputo-derivative-based dynamics

Consider the rewritten form of the sliding manifold dynamics (33) in the following form:

$$\begin{aligned}{}_{C}D^{\alpha}\sigma_i(t) &= f_i(X_i, t) + g_i(X_i, t)u_i(t) - {}_{C}D^{\alpha} x_{ind}(t) \\ &\quad + \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) + D^{-(1-\alpha)} \theta_i^T \xi_i(X)\end{aligned} \tag{56}$$

Selecting the semi-decentralized control signal

$$\begin{aligned}u_i(t) &= \frac{1}{g_i(X_i, t)} \Bigg(-f_i(X_i, t) + {}_{C}D^{\alpha} x_{ind}(t) \\ &\quad - \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) - D^{-(1-\alpha)} (\eta_i \sigma_i(t) + \hat{\theta}_i^T \xi_i(X)) \Bigg)\end{aligned} \tag{57}$$

with the following adaptation law

$$_C D^{\alpha}\tilde{\theta}_i = -D^{-(1-\alpha)}\mu_i \xi_i(X)\sigma_i(t) \tag{58}$$

guarantees the large-scale system (15) stability.

Proof Substituting (57) in (56), leads to

$$\begin{aligned} _C D^{\alpha}\sigma_i(t) &= D^{-(1-\alpha)}\theta_i^T \xi_i(X) - D^{-(1-\alpha)}\left(\eta_i\sigma_i(t)\right. \\ &\quad \left. + \hat{\theta}_i^T \xi_i(X)\right) \\ &= D^{-(1-\alpha)}\left(\tilde{\theta}_i^T \xi_i(X) - \eta_i\sigma_i(t)\right) \end{aligned} \tag{59}$$

Similar to the previous part, consider the following Lyapunov candidate:

$$\begin{aligned} V(t, Y(t)) &= \alpha(\|Y\|) = \|Y\|_2^2 = \sum_{i=1}^{N} V_i(t, Y_i(t)) \\ &= \sum_{i=1}^{N}\left(\frac{1}{2}\sigma_i^2(t) + \frac{1}{2\mu_i}\tilde{\theta}_i^T\tilde{\theta}_i\right) \end{aligned} \tag{60}$$

Now, by differentiating from (60) along the trajectory (59) and using Properties 2.c and 4, we have

$$\begin{aligned} \dot{V}(t, Y(t)) &= \sum_{i=1}^{N}\left(\sigma_i(t)\dot{\sigma}_i(t) + \frac{1}{\mu_i}\tilde{\theta}_i^T\dot{\tilde{\theta}}_i\right) \\ &= \sum_{i=1}^{N}\left(\sigma_i(t)_C D^{1-\alpha}{}_C D^{\alpha}\sigma_i(t) + \frac{1}{\mu_i}\tilde{\theta}_i^T{}_C D^{1-\alpha}{}_C D^{\alpha}\tilde{\theta}_i\right) \\ &= \sum_{i=1}^{N}\left(\left(\tilde{\theta}_i^T \xi_i(X) - \eta\sigma_i(t)\right)\sigma_i(t) + \frac{1}{\mu_i}\tilde{\theta}_i^T{}_C D^{1-\alpha}{}_C D^{\alpha}\tilde{\theta}_i\right) \end{aligned} \tag{61}$$

Using the adaptation mechanism (58), one can obtain

$$\dot{V}(t, Y(t)) = \sum_{i=1}^{N} -\eta_i\sigma_i^2(t) \le 0 \tag{62}$$

By using Caputo definition (3), Theorem 1 and Remark 2, we get

$$_C D^{1-\alpha}V(t, Y(t)) = D^{-\alpha}\dot{V}(t, Y(t)) \le 0 \tag{63}$$

which results the large-scale system (15) stability, and guarantees $\sigma_i(t)$ and $\tilde{\theta}_i$ boundedness. It is worthwhile to notify that in (63), the Barbalat's lemma is not used for analyzing the sliding manifold $\sigma_i(t)$ convergence to zero, because the Barbalat's lemma is known as an IO stability technique. □

Remark 6 Although, we suggested the adaptation law (58) in FO form in order to have set dynamics, both adaptation mechanisms (51) and (58) in reality are similar. This can be proofed by integrating from both sides of the adaptation laws:

$$\begin{aligned} \dot{\tilde{\theta}}_i &= -\mu_i\xi_i(X)\sigma_i(t) \xrightarrow{D^{-1}} \tilde{\theta}_i(t) - \tilde{\theta}_i(0) \\ &= -\int_0^t \mu_i\xi_i(X)\sigma_i(\tau)d\tau \end{aligned} \tag{64}$$

$$\begin{aligned} _C D^{\alpha}\tilde{\theta}_i &= -D^{-(1-\alpha)}\mu_i\xi_i(X)\sigma_i(t) \xrightarrow{D^{-\alpha}} \tilde{\theta}_i(t) - \tilde{\theta}_i(0) \\ &= -D^{-\alpha}D^{-(1-\alpha)}\mu_i\xi_i(X)\sigma_i(t) \\ &= -\int_0^t \mu_i\xi_i(X)\sigma_i(\tau)d\tau \end{aligned} \tag{65}$$

Remark 7 By substituting the proposed control laws in the corresponding sliding manifold dynamics, we can get the following closed-loop sliding surface dynamics: Fully decentralized strategy:

$$\dot{\sigma}_i(t) = -\eta_i\sigma_i(t) - K_{sw-i}\,\mathrm{sgn}(\sigma_i(t)) + L_i(X, t) \tag{66}$$

$$\begin{aligned} _C D^{\alpha}\sigma_i(t) &= -D^{-(1-\alpha)}\left(\eta_i\sigma_i(t) + K_{sw-i}\tanh\left(\frac{\sigma_i(t)}{\rho_i}\right)\right) \\ &\quad + L_i(X, t) \end{aligned} \tag{67}$$

and semi-decentralized strategy:

$$\begin{aligned} \dot{\sigma}_i(t) &= \tilde{\theta}_i^T \xi_i(X) - \eta_i\sigma_i(t) \\ \dot{\tilde{\theta}}_i &= -\mu_i\xi_i(X)\sigma_i(t) \end{aligned} \tag{68}$$

$$\begin{aligned} _C D^{\alpha}\sigma_i(t) &= D^{-(1-\alpha)}\left(\tilde{\theta}_i^T \xi_i(X) - \eta_i\sigma_i(t)\right) \\ _C D^{\alpha}\tilde{\theta}_i &= -D^{-(1-\alpha)}\mu_i\xi_i(X)\sigma_i(t) \end{aligned} \tag{69}$$

from equations (66) and (68), it is obvious that there is no FO dynamics, while we can find $D^{-(1-\alpha)}$ and $_C D^{\alpha}$ operators in (67) and (69) even in the adaptation law. Therefore, the IO stability analysis can be applied for (66) and (68), and the FO stability analysis should be employed for (67) and (69). Note that, the mentioned concept is not depends on RL or Caputo derivatives, but it is related to how we are defining the sliding manifold.

6 Simulation results

In this section, two illustrative examples are presented to reveal the effectiveness of the proposed control strategies. Simulation results only presented for semi-decentralized controllers (50) and (57). Similar results are achievable for the fully decentralized control laws (23) and (34) by replacing the fuzzy system with a constant sliding gain.

Example 1 Consider the following nonlinear FO large-scale system composed of two subsystems which described by

$$S_1 : \begin{cases} {}_{C,RL}D^{0.8}x_{11} = x_{12} \\ {}_{C,RL}D^{0.8}x_{12} = -x_{11}^3 - x_{12} \\ \quad + \left(1 + e^{-x_{11}+x_{12}^2}\right) u_1 + L_1(X,t) \end{cases}$$

$$S_2 : \begin{cases} {}_{C,RL}D^{0.8}x_{21} = x_{22} \\ {}_{C,RL}D^{0.8}x_{22} = -x_{21} - x_{22}^2 \\ \quad + (2 + \sin(x_{21}))\, u_2 + L_2(X,t) \end{cases}$$

where the lumped uncertainty terms are as follows

$$L_1(X,t) = 0.6\cos(t) + 0.4x_{12}\sin(0.5t) + 0.2x_{21}\sin(3t) + 0.5x_{22}\cos(10t)$$

$$L_2(X,t) = 0.5\sin(5t) + 2x_{11}\sin(x_{21}) + 0.4x_{22}\sin(0.2t)$$

also the reference values and initial conditions are chosen as

$$x_{11d}(t) = \sin((\pi/20)t),$$
$$x_{12d}(t) = {}_{C,RL}D^{\alpha}\sin((\pi/20)t),$$
$$x_{21d}(t) = \sin((\pi/15)t),$$
$$x_{22d}(t) = {}_{C,RL}D^{\alpha}\sin((\pi/15)t)$$
$$(x_{11}(0), x_{12}(0)) = (0,0), \quad (x_{21}(0), x_{22}(0)) = (0,\ 0)$$

Each subsystem states and neighbor subsystem states are considered as the fuzzy system input variables (four variables for each subsystem). Fuzzy sets for input variables are defined according to the membership functions depicted in Fig. 1.

$$\mu_1(x_{ik}) = \exp(-10x_{ik}^2)$$
$$\mu_2(x_{ik}) = 1 - \mu_1(x_{ik})$$
$$i,k = 1,2$$

Two fuzzy sets for each input variable have been found sufficient. Therefore, the number of fuzzy rules will be $2 \times 2 \times 2 \times 2 = 16$.

Based on (50) and (57), the controller parameters are picked as follows:

$$c_{11} = c_{21} = 1, \quad \eta_1 = \eta_2 = 20, \quad \mu_1 = \mu_2 = 100$$

For Caputo-derivative-based dynamics (15) and control effort (57), the simulations are performed using MATLAB toolbox called Ninteger [44]. The ${}_CD^{\alpha}$ operator is approximated via Crone method in frequency range [0.01 100] rad/s and $n = 10$. For RL-derivative-based dynamics (14) and control law (50), the ${}_{RL}D^{\alpha}$ operator has been approximated (from Property 3) by GL-derivative discrete-time algorithm with the sampling interval $h = 0.005$ [45]. This algorithm is executed by S-function blocks in MATLAB.

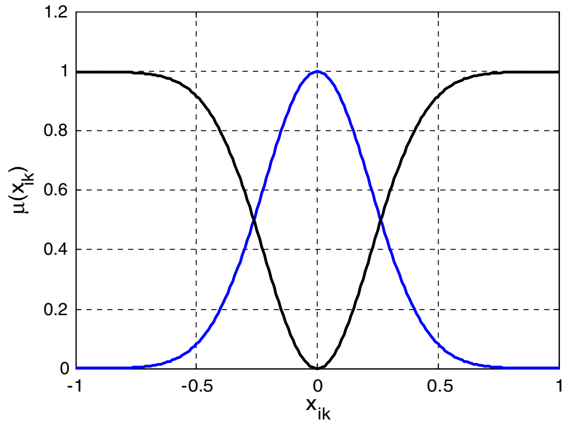

Fig. 1 Fuzzy sets assigned to input variables (x_{11}, x_{12}, x_{21} and x_{22})

The simulation results of semi-decentralized FO-SMC based on RL and Caputo-derivatives are given in Figs. 2 and 3, respectively. As can be seen from Figs. 2 and 3, the responses of semi-decentralized FOSMC appear to be very satisfactory, since they track desired trajectories with low deviation and small control signals. Moreover, there are some differences in the transient behaviors between RL- and Caputo-based dynamics. For instant: the control signal amplitude for initial times in Fig. 2 is higher than Fig. 3, also the states x_{12} and x_{22} oscillation in Fig. 3 is a little high.

Example 2 To show more results of the suggested controllers, we consider the double inverted pendulum [9] by replacing IO derivatives with FO ones. The dynamic model of the system can be described as

$$S_1 : \begin{cases} {}_CD^{0.85}x_{11} = x_{12} \\ {}_CD^{0.85}x_{12} = \left(\frac{m_1 gr}{J_1} - \frac{kr^2}{4J_1}\right)\sin(x_{11}) \\ \quad + \frac{kr}{2J_1}(l-b) + \frac{1}{J_1}u_1 + L_1(X) \end{cases}$$

$$S_2 : \begin{cases} {}_CD^{0.85}x_{21} = x_{22} \\ {}_CD^{0.85}x_{22} = \left(\frac{m_2 gr}{J_2} - \frac{kr^2}{4J_2}\right)\sin(x_{21}) \\ \quad - \frac{kr}{2J_2}(l-b) + \frac{1}{J_2}u_2 + L_2(X) \end{cases}$$

where $\theta_1 = x_{11}$ and $\theta_2 = x_{21}$ are the angular displacements of the vertical reference value for each pendulum, $L_1(X) = (kr^2/kr^2 4J_1)\sin(x_{21})$ and

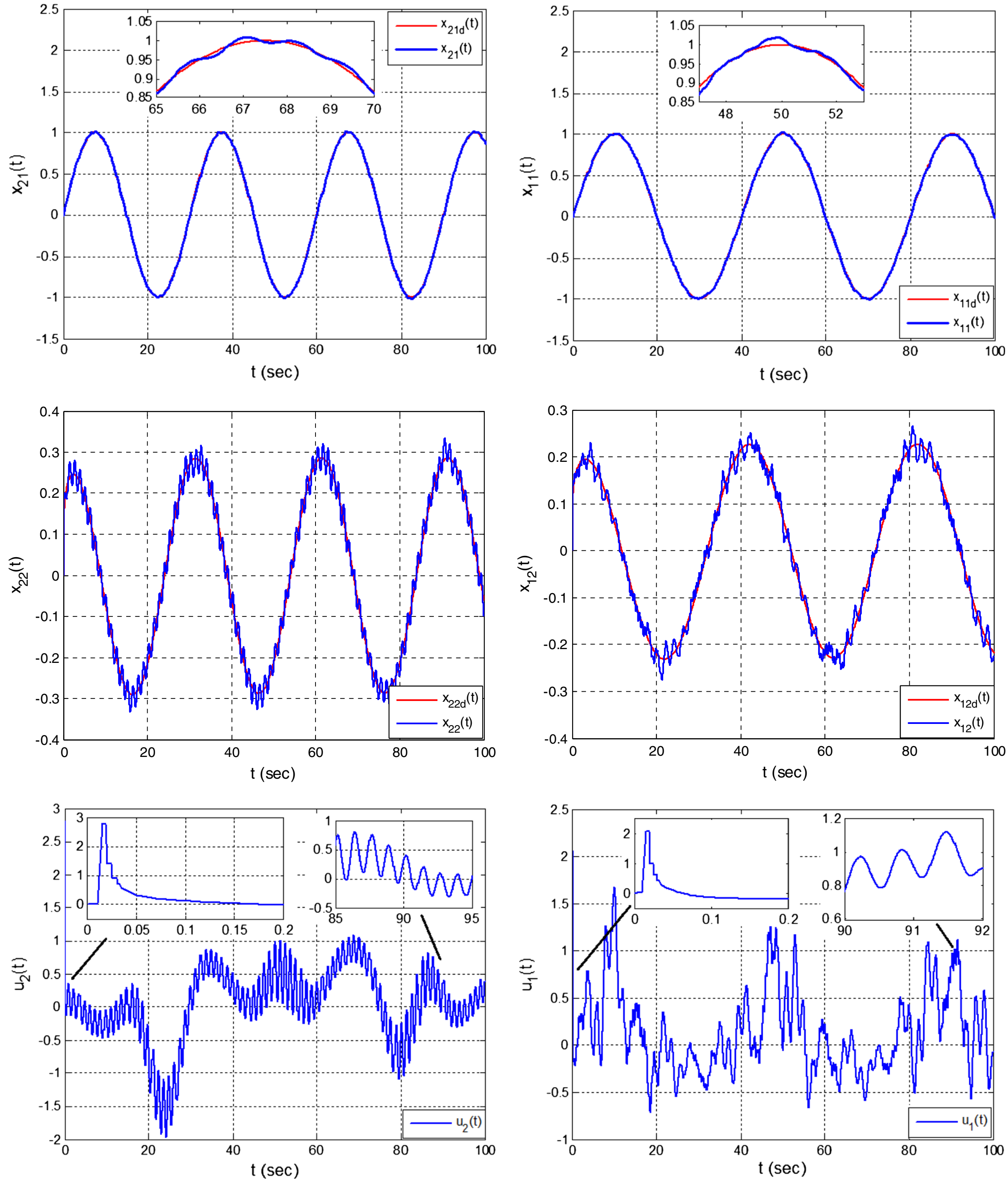

Fig. 2 The responses of (RL-derivative-based) system under the semi-decentralized FOSMC; *Right* states x_{11}, x_{12} and control effort u_1 of subsystem 1; *Left* states $x_{21},\ x_{22}$ and control effort u_2 of subsystem 2

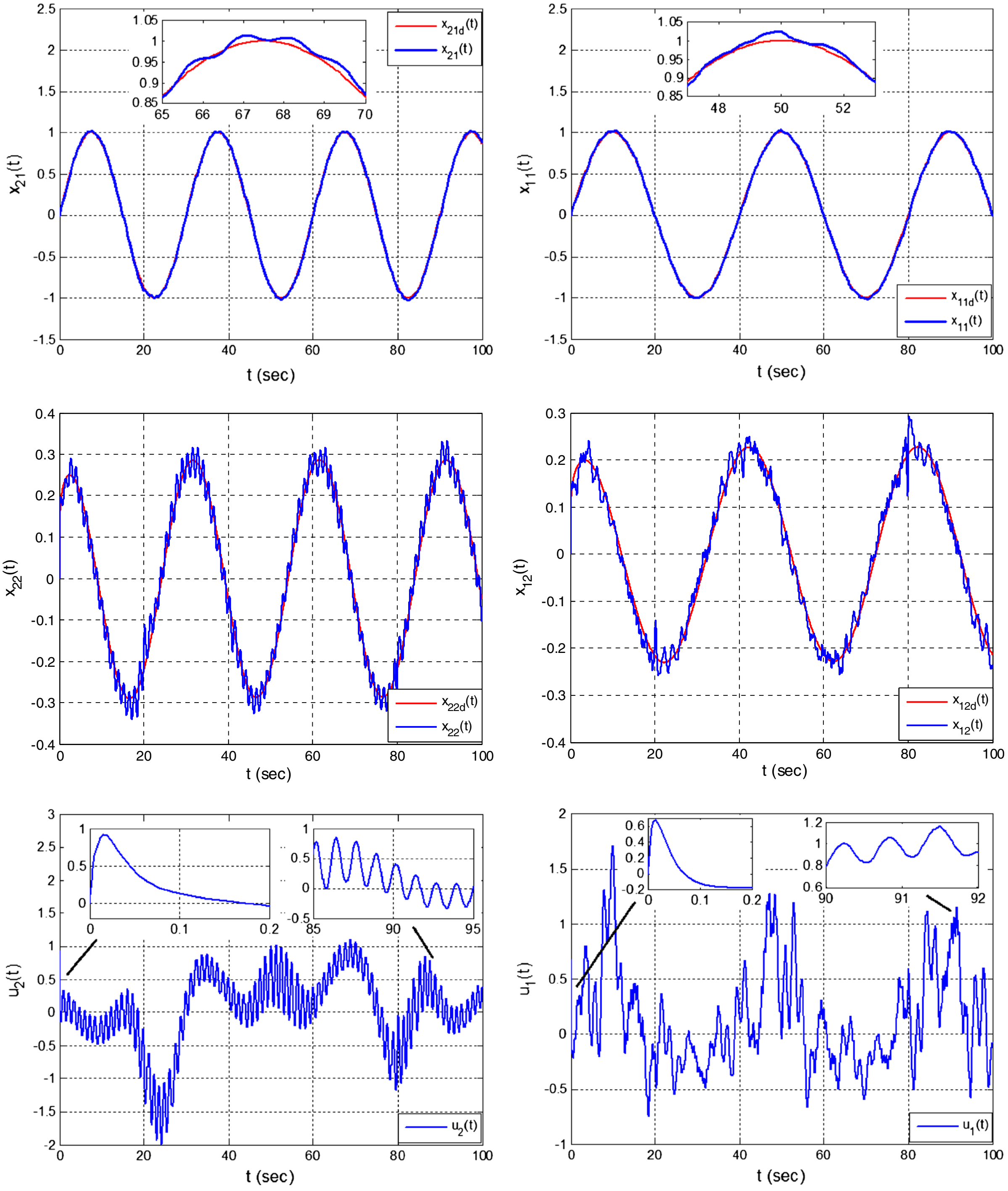

Fig. 3 The responses of (Caputo-derivative-based) system under the semi-decentralized FOSMC; *Right* states x_{11}, x_{12} and control effort u_1 of subsystem 1; *Left* states x_{21}, x_{22} and control effort u_2 of subsystem 2

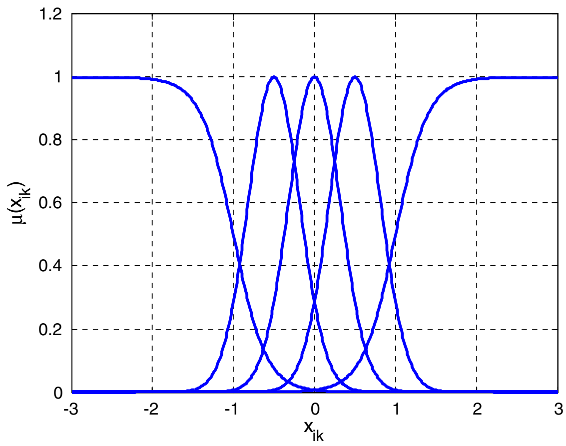

Fig. 4 Fuzzy sets assigned to input variables (x_{12} and x_{21})

$L_2(X) = (kr^2/kr^2 4J_2)\sin(x_{12})$ are the interconnections terms, m_1 and m_2 are the end masses, J_1 and J_2 are the moments of inertia, r is the pendulum height, k is the spring constant of the linker spring, l is the spring natural length, b is the distance between the pendulum hinges, and finally g is the gravitational acceleration. These parameters values are considered as follows:

$m_1 = 2$ kg $m_2 = 2.5$ kg $J_1 = 0.5$ kg

$J_2 = 0.625$ kg $r = 0.5$ m $k = 100$ N/m $l = 0.5$ m

$b = 0.4$ m $g = 9.81$ m/sec^2

and the initial conditions are selected as

$(x_{11}(0), x_{12}(0)) = (\pi/3, 0)$,

$(x_{21}(0),\ x_{22}(0)) = (-\pi/3, 0)$

Note that, the pendulum dynamics are demonstrated only by Caputo-derivative in order to avoid the ini-

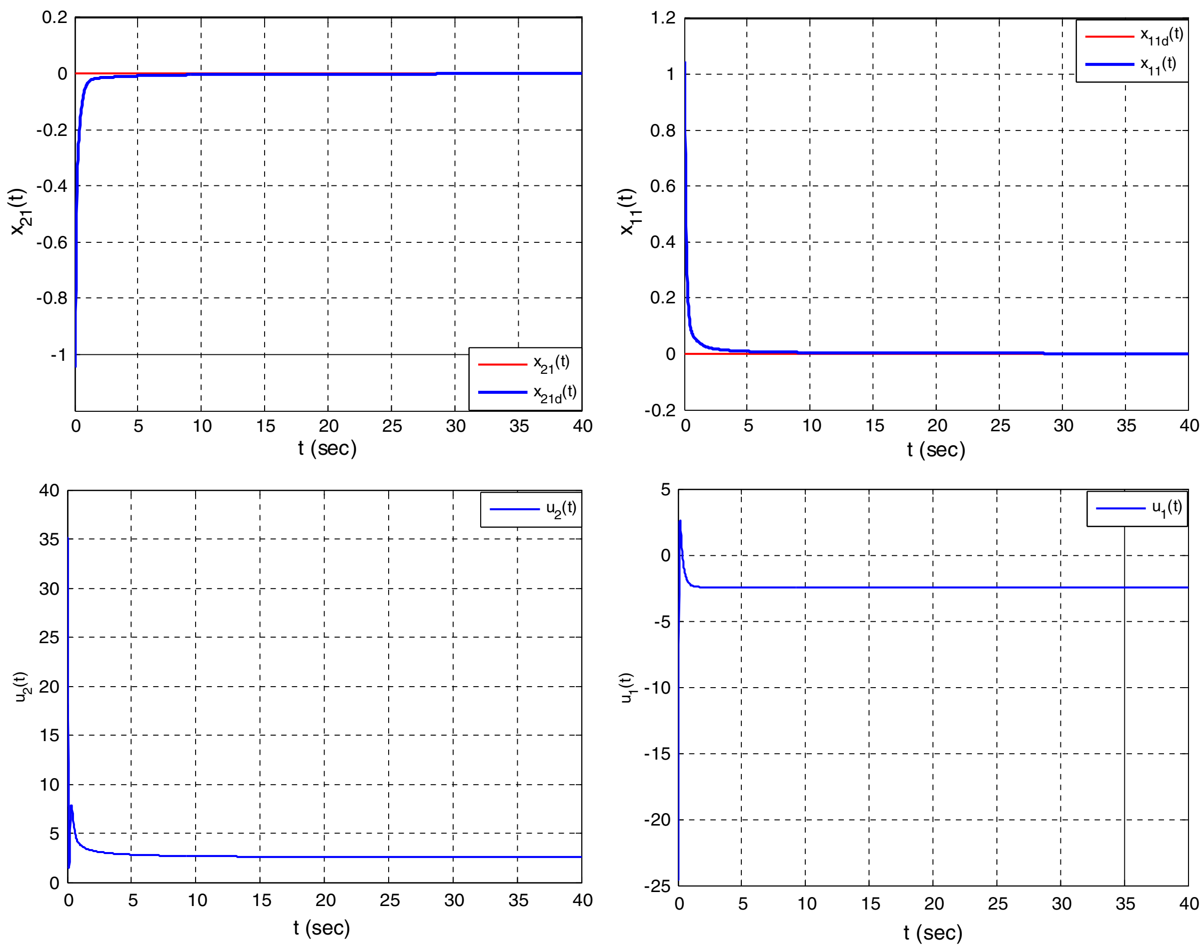

Fig. 5 The responses of (Caputo-derivative-based) system under the semi-decentralized FOSMC; *Right* state x_{11} and control effort $u_1$of subsystem 1; *Left* state x_{21} and control effort u_2 of subsystem 2

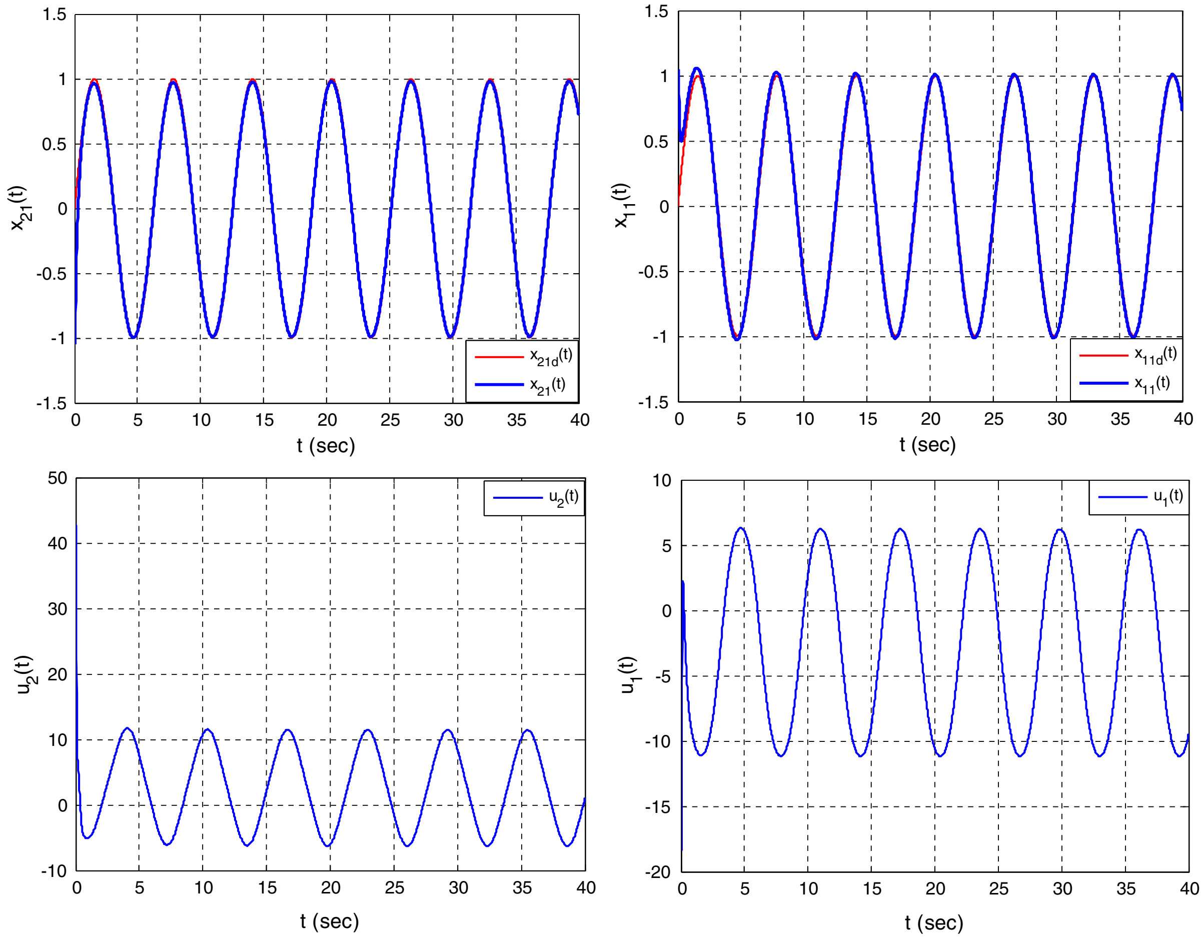

Fig. 6 The responses of (Caputo-derivative-based) system under the semi-decentralized FOSMC; *Right* state x_{11} and control effort $u_1$of subsystem 1; *Left* state x_{21} and control effort u_2 of subsystem 2

tial condition interpretation problem of RL derivatives. We consider one input variable for each fuzzy system, whereas the interconnection terms are only depended on one of the variables of neighbor subsystem. Moreover, the fuzzy sets of input variable are defined according to the membership functions which are illustrated in Fig. 4. From this figure, it is obvious that the number of fuzzy rules will be five.

$$\mu_{1i}(x_{ik}) = 1/(1 + \exp(-5 \times (x_{ik} - 1)))$$
$$\mu_{2i}(x_{ik}) = \exp(-5 \times (x_{ik} - 0.5)^2)$$
$$\mu_{3i}(x_{ik}) = \exp(-5 \times (x_{ik})^2)$$
$$\mu_{4i}(x_{ik}) = \exp(-5 \times (x_{ik} + 0.5)^2)$$
$$\mu_{5i}(x_{ik}) = 1/(1 + \exp(5 \times (x_{ik} + 1)))$$

According to (57), the following parameters can be used for this example simulation

$$c_{11} = c_{21} = 5, \quad \eta_1 = \eta_2 = 20, \quad \mu_1 = \mu_2 = 100$$

The system states and control efforts are presented in Figs. 5 and 6 for different reference values in order to check the performance of semi-decentralized FOSMC. In Fig. 5, the reference values for each subsystem are selected as $x_{11d}(t) = 0$, $x_{12d}(t) = 0$, $x_{21d}(t) = 0$, and $x_{22d}(t) = 0$. Moreover, the reference values for Fig. 6 are chosen as $x_{11d}(t) = x_{21d}(t) = \sin(t)$, and $x_{12d}(t) = x_{22d}(t) = {}_C D^{\alpha} \sin(t)$. By taking a glance at these figures, the closed-loop system behavior seems satisfactory.

7 Conclusion

In this paper, the fully decentralized and semi-decentralized FOSMC schemes have been developed for two different classes of FO large-scale nonlinear systems for the first time. First, two fully decentralized controllers are designed for RL- and Caputo-based systems with the known interconnections and uncertainties. In the second step, to approximate the interconnections and uncertainties we proposed two semi-decentralized controllers using fuzzy systems. The closed-loop systems stability is proved using IO and FO stability theorems depend on existing IO and FO operators in the sliding surface dynamic. For system with RL dynamics: an integral sliding surface is used which removes fractional dynamics of the closed-loop system. As a result, we employed IO stability theorems to stability analyzing in both fully and semi-decentralized strategies. For the Caputo-based system: we applied a simple linear sliding surface which a closed-loop system with FO dynamics is its outcome. Hence the FO stability theorems are applied to guarantee system stability with fully decentralized and semi-decentralized controllers. Finally, computer simulations revealed the good efficiency of the suggested control techniques in trajectory tracking of two FO large-scale case studies.

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