This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRD.2019.2897402, IEEE Transactions on Power Delivery

Robust Wide Area Fault Location Considering Network Parameters Error

Seyed Hossein Mortazavi [1], M. Hossein Javidi[1], Ebadollah Kamyab[2], and Fatemeh Daburi Farimani[1]

[1]Dept. of Electrical Eng., Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
[2]Department of Operation, Khorasan Regional Electric Company (KREC), Mashhad, Iran

***Abstract*—Network parameters error may seriously affect the accuracy of fault location results, even making them unacceptable in extreme cases. This paper proposes a new method which incorporates these errors in wide area fault location problem using phasor measurement units(PMU) to estimate inaccurate network parameters employing robust state estimation. Our proposed method is composed of two stages. The first stage or the primary stage provides an initial point for the second stage which is the main contribution of this paper. Additionally, the set of measurements contaminated with bad data could be obtained from the first stage and its intersection with leverage measurements are then eliminated from the measurement set before feeding it into the second stage. Finally, fault point and faulted line parameters are estimated by the main(second) stage of the method. The proposed method has been applied to two test systems, namely Institute of Electrical and Electronics Engineers (IEEE) 14-Bus and IEEE 57-Bus systems to evaluate its accuracy. Simulation results indicate high accuracy and consistency of the proposed method.**

***Index Terms*—Fault impedance , fault location, leverage measurement, phasor measurement unit, robust state estimation**

I. INTRODUCTION

FAULT diagnostic accuracy in fault location (FL) problem is of high importance in power system protection. Much effort has been performed to increase the FL solution accuracy. In this respect, the errors associated with the measurements of PMUs is usually considered in FL estimation problem. Actually, value of faulted line parameters which are used in real time, may be incorrect due to various reasons such as outdated data during network changes resulted from structure, temperature and environmental conditions and so on. Therefore, it is necessary to incorporate them in the FL problems. This paper considers the probability of faulted line parameters being inaccurate in FL procedure.

Synchronized phasor measurement units are essential in monitoring, data acquisition, control, operation and protection of power system through wide-area measurement system (WAMS) technology [1]. Two main groups of fault location methods include one terminal methods [2, 3, 4], and multi-terminal methods [5, 6, 7, 8]. Multi-terminal methods are more acceptable and more accurate due to gathering the information of different parts of the network.

Corresponding Author: mortazaviravari.se@mail.um.ac.ir
h-javidi@um.ac.ir
ebadkamyab@yahoo.com
fateme.daburifarimani@mail.um.ac.ir

Multi-terminal methods are divided into synchronized [5], and unsynchronized [6, 7, 8] types. Unsynchronized multi-terminal methods are less accurate than the synchronized ones, since they estimate the fault location regardless of the time of gathering the data related to different points of the network. PMUs play a fundamental role in synchronized FL methods. Financial issues force utilities to minimize the number of installed PMUs, and many of researches focus on this matter in wide area fault location methods [9, 10, 11]. Depending on the number of PMUs in the network, it may be observable or non-observable. Therefore, the methods used in FL problems with minimum number of PMUs, especially when the network is not observable, may be of high importance.

The method proposed in [9], presents a PMU placement strategy to make the network observable when fault occurs. The faulted line is assumed to be known in that paper. Observability criteria for the FL method increases the minimum number of required PMUs in large networks. The FL method proposed in [10], uses bus impedance matrix. The number of installed PMUs in this method is not sufficient to make the network observable.

An efficient fault location algorithm for large transmission networks in addition to an optimal PMU placement scheme, is presented in [11]. In this method, the accurate FL is estimated based on numerous different probable faulted points candidates in various lines. In the first stage, the faulted region is identified using the concept of matching degree index to restrict the search area. The second stage determines the accurate location of the fault.

By linearizing the equations of [11], a new FL method is proposed in [12]. The new method calculates fault type, location and impedance based on voltage data using minimum number of installed PMUs. Moreover, a PMU placement strategy based on a set of fundamental rules is introduced [12].

A new solution for FL which uses a linear weighted least square algorithm to provide a closed form solution is proposed in [13]. The suggested method incorporates only the error associated with voltage measurements.

In [14], Dobakhshari and Ranjbar used Newton-Raphson method to solve equations in [13]. One of the advantages of the method presented in [14], is the ability to estimate the fault point in presence of unsynchronized measurement data. Also, a bad data detection and identification scheme has been included in [13] and [14].

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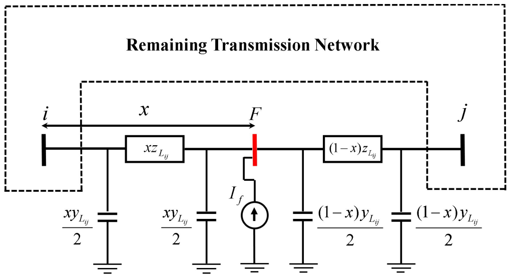

Fig. 1. A faulted line in the network

A robust state estimation method for fault location, considering measurement errors, with the aim of accuracy enhancement is presented in [15]. In this paper, FL problem is modeled as an optimization problem using voltage and current measurement data. Bad data detection and elimination are carried out by using least absolute value (LAV) estimator.

As mentioned before, while the error of measuring units have been considered in some wide area FL estimation methods, the error associated with network parameters, have not been considered so far.

This paper, estimates the fault point and the parameters of faulted line, even when the number of PMUs is not enough to make the network observable. Considering measurement errors, these parameters are modified during the proposed FL algorithm. Hence, faulted line parameters are estimated more accurately, improving the accuracy of fault location results.

An important feature of the proposed FL method is the elimination of leverage measurements which have been identified as bad data in the first or primary step. This is because of the fact that leverage measurements containing error may highly affect the estimated parameters and may lead to incorrect answers. Furthermore, our proposed FL algorithm has the advantage of detecting and eliminating the leverage measurements which may be hard to be recognized in conventional FL methods.

The paper is organized as follows: Section II presents the developed equations of the proposed method for a robust FL estimation problem. In section III, the proposed algorithm consisting of primary and main FL problems utilizing LAV estimation method is presented. Evaluating simulation results related to different types of faults and the validity of the algorithm is investigated in section IV. Finally, Section VI states the concluding remarks.

II. Derivation of the Equations

As illustrated in Fig.1, it is assumed that a fault has occurred at distance x from bus i along line L_{ij}. Fault point is considered as a fictitious bus denoted by F, in which the fault current of $-I_f$ is injected. To increase the accuracy of the results, transmission lines are modeled by their equivalent π in equations (1) and (2).

$$z_{ij}^{+}(x) = Z_{C_{ij}}^{+} \sinh(\gamma_{ij} l_{ij} x) \tag{1}$$

$$\frac{y_{ij}^{+}(x)}{2} = \frac{\tanh(\gamma_{ij} l_{ij} x)}{Z_{C_{ij}}^{+}} \tag{2}$$

where, z_{ij}^{+}, y_{ij}^{+} and $Z_{C_{ij}}^{+}$ stand for positive-sequence impedance, admittance and characteristic impedance of the line L_{ij}, respectively. γ_{ij} represents the propagation constant of the faulted line L_{ij} with the length of l_{ij}. According to the superposition principle, voltage change in bus k may be calculated by equation (3) [15].

$$\Delta V_k^{+} = V_k^{+,p} - V_k^{+,f} = Z_{kf}^{+,M}(x) I_f^{+} \tag{3}$$

where, ΔV_k^{+} is the voltage change which is the subtraction of pre-fault ($V_k^{+,p}$) and post-fault positive-sequence voltages ($V_k^{+,f}$). Equation (3) can be used for buses equipped with PMUs. $Z_{kf}^{+,M}(x)$ represents the element located at k^{th} row and f^{th} column of the modified positive-sequence bus impedance matrix. I_f^{+} denotes the positive-sequence of the fault current. $Z_{kf}^{+,M}(x)$ can be written as equation (4), where Z_{ki}^{+} and Z_{kj}^{+} are elements in k^{th} row located at i^{th} and j^{th} columns of the positive-sequence of the pre-fault bus impedance matrix, respectively [16].

$$Z_{kf}^{+,M}(x) = Z_{ki}^{+} f_1(x) + Z_{kj}^{+} f_2(x) \tag{4}$$

$f_1(x)$ and $f_2(x)$ are calculated by equations (6) and (7), respectively. Equations (6) and (7) can be approximated by x and $(1-x)$ according to [15].

By replacement of (4) in (3), the voltage changes is computed from (5).

$$\Delta V_k^{+} = \left[Z_{ki}^{+} f_1(x) + Z_{kj}^{+} f_2(x) \right] I_f^{+} \tag{5}$$

$$f_1(x) = \frac{\sinh(\gamma_{ij} l_{ij} (1-x))}{\sinh(\gamma_{ij} l_{ij})} \approx 1 - x \tag{6}$$

$$f_2(x) = \frac{\sinh(\gamma_{ij} l_{ij} x)}{\sinh(\gamma_{ij} l_{ij})} \approx x \tag{7}$$

Assuming $I_f^{+} = x_1 + jx_2$, $F_1(x, x_1, x_2) = Z_{ki}^{+} f_1(x) I_f^{+}$ and $F_2(x, x_1, x_2) = Z_{kj}^{+} f_2(x) I_f^{+}$ and applying these assumptions to equation (5), the real and imaginary parts of the voltage change can be expressed as (8) and (9).

$$\Delta V_k^{+,re} = Z_{ki}^{+,re} F_1^{re} - Z_{ki}^{+,im} F_1^{im} + Z_{kj}^{+,re} F_2^{re} - Z_{kj}^{+,im} F_2^{im} \tag{8}$$

$$\Delta V_k^{+,im} = Z_{ki}^{+,im} F_1^{re} + Z_{ki}^{+,re} F_1^{im} + Z_{kj}^{+,im} F_2^{re} + Z_{kj}^{+,re} F_2^{im} \tag{9}$$

The measurement residual at buses equipped with PMUs, denoted by $r^{\Delta V}$, is expressed by (10), where $\Delta V^{+,re,m}$ and $\Delta V^{+,im,m}$ are the positive-sequence of real and imaginary parts of measured voltage change, respectively.

$$r^{\Delta V} = |\Delta V^{+,re,m} - \Delta V^{+,re}| + |\Delta V^{+,im,m} - \Delta V^{+,im}| \tag{10}$$

Superscript m denotes the measured value of the variable. Fig.2 illustrates a healthy line in a network between buses k and n. The relationships between the pre-fault and post-fault currents, $I_{kn}^{+,p}$ and $I_{kn}^{+,f}$, and the voltages at buses k and n are expressed as in (11) and (12), in both of them the first term is constant.

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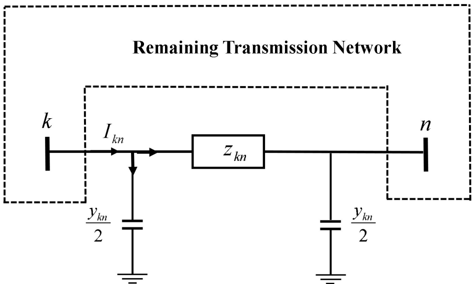

Fig. 2. A healthy line in a network

$$I_{kn}^{+,p} = \frac{y_{kn}^{+}}{2} V_k^{+,p} + \frac{V_k^{+,p} - V_n^{+,p}}{z_{kn}^{+}} \tag{11}$$

$$I_{kn}^{+,f} = \frac{y_{kn}^{+}}{2} V_k^{+,f} + \frac{V_k^{+,f} - V_n^{+,f}}{z_{kn}^{+}} \tag{12}$$

By transferring the first terms of equations (11) and (12) to the left-hand-side for more simplicity, as considered in [15], $I_{kn}^{',+,p}$ and $I_{kn}^{',+,f}$ are obtained as (13) and (14).

$$I_{kn}^{',+,p} = I_{kn}^{+,p} - \frac{y_{kn}^{+}}{2} V_k^{+,p} = \frac{V_k^{+,p} - V_n^{+,p}}{z_{kn}^{+}} \tag{13}$$

$$I_{kn}^{',+,f} = I_{kn}^{+,f} - \frac{y_{kn}^{+}}{2} V_k^{+,f} = \frac{V_k^{+,f} - V_n^{+,f}}{z_{kn}^{+}} \tag{14}$$

Subtracting (14) from (13), and Assuming $a = b = \frac{1}{z_{kn}}$, $Z_{kni} = Z_{ki} - Z_{ni}$ and $Z_{knj} = Z_{kj} - Z_{nj}$ and using (5), positive-sequence current change through branch kn is obtained as following:

$$\Delta I_{kn}^{',+} = \left[a Z_{kni}^{+} f_1(x) + b Z_{knj}^{+} f_2(x) \right] I_f^{+} \tag{15}$$

We should mention that, only equations corresponding to currents for healthy lines have been used in our estimations. After that, (15) is separated into the real and imaginary parts as in (8) and (9). Notice that bus k is equipped with PMU.

The measurement residuals of the current change is written as (16).

$$r_{kn}^{\Delta I} = \left| \Delta I_{kn}^{',+,re,m} - \Delta I_{kn}^{',+,re} \right| + \left| \Delta I_{kn}^{',+,im,m} - \Delta I_{kn}^{',+,im} \right| \tag{16}$$

where, $\Delta I_{kn}^{',+,re,m}$ and $\Delta I_{kn}^{',+,im,m}$ are the measured real and imaginary parts of the current change, respectively. The objective function of the optimization problem is a function of the measurement residuals $r_k^{\Delta V}$ and $r_{kn}^{\Delta I}$ as stated in (17).

$$\rho(r) = \sum_{k=1}^{N_P} r_k^{\Delta V} + \sum_{k=1}^{N_P} \sum_{n=1}^{N_{Lk}} r_{kn}^{\Delta I} = \rho\left(r^{\Delta V}\right) + \rho\left(r^{\Delta I}\right) \tag{17}$$

N_p represents the number of PMUs and N_{Lk} lines are connected to bus k. The objective function consists of two parts $\rho\left(r^{\Delta V}\right)$ and $\rho\left(r^{\Delta I}\right)$ each of which can be used individually in the proposed fault location method. The optimization problem with the constraint of fault distance boundary is expressed in (18).

$$\begin{gathered} \min \rho(r) \\ \text{subject to } 0 \leqslant x \leqslant 1 \end{gathered} \tag{18}$$

III. Proposed method

III.A. The Primary FL Problem

To provide a proper initial point for the main FL problem, the method introduced in [15] has been used. This satisfies the convergence criteria, which is of great importance in our FL problem. In that method, the goal is to calculate the location and the current of the fault by the help of superposition and state estimation equations. This paper employs the PMU measurements to estimate the faulted line parameters as well as the fault location and current. In the optimization process, due to the change in faulted line parameters and network impedance matrix in each step, it may be very much probable to converge to an incorrect answer. Therefore, it is very important to choose a proper initial point. Moreover, the PMUs containing bad data, will also be detected.

III.B. Detection of Leverage Measurements

The concept of leverage measuring unit is stated in [17]. Leverage measuring units, are those in which the measured parameter significantly affects the unknown state variables. Therefore, in cases where the measurement is erroneous, the estimated parameters may be strongly influenced. According to [17], the residual of a leverage measurement could be very small even when it is contaminated with a large error. To resolve this issue, the proposed method uses a combination of superposition and state estimation equations for detection and elimination of leverage measuring units in order to achieve accurate FL estimation results. To detect leverage measurement units hat symmetric matrix mapping measured values to the estimated values named K in calculated as follows:

$$K = H(H^T H)^{-1} H^T \tag{19}$$

H denotes Jacobian matrix which is expressed in the next subsection. Since K is both symmetric ($K = K^T$) and idempotent ($K \cdot K = K$), the diagonal elements of K, K_{ii}, can be expressed as follows:

$$K_{ii} = K_{ii}^2 + \sum_{i \neq j} K_{ij}^2 \tag{20}$$

The value K_{ii}, ranging from 0 to 1 ($0 \leqslant K_{ii} \leqslant 1$), corresponds to influence level of i^{th} measurement z_i on its estimation $\tilde{z}_i$ where 0 and 1 imply low and high influence, respectively. Then in cases, where K_{ii} is close to 1, the measurement is considered as a leverage point. The expected value of K_{ii} can be found as (21).

$$E\left[K_{ii}\right] = \overline{K} = \frac{1}{m} \sum_{i=1}^{m} K_{ii} = \frac{n}{m} \tag{21}$$

where, n and m in this equation correspond to the number of estimated and measured variables, respectively. In cases where K_{ii}, which correlates to i^{th} measurement unit, is very much different from $\overline{k}$, the unit can be considered as leverage. As a rule of thumb, if $K_{ii} \geq 2\frac{n}{m}$, the unit will be assumed as a leverage measuring unit. As mentioned before, if a leverage measuring unit is erroneous, estimation of variables may be strongly influenced. Therefore, it may be hard to find the

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error via conventional methods.

III.C. Solving FL Problem by LAV EstimationMethod

To solve the main FL estimation problem, the optimization relations in (18), are rewritten as an optimization problem represented by (25). Actually FL algorithms may result a high deviated solution due to the probable errors associated with the PMU measurements or network parameters. To detect and modify the error level, we have used the Least Absolute Value (LAV) estimation method, which is a robust state estimator. The optimization problem expressed in (18) is a non-linear problem. The nonlinear problem has been modeled as a LAV estimation problem and the new optimization becomes a successive linear problem. Now, as mentioned before, since the parameters of the faulted line have been considered as a new set of free parameters in the FL estimation problem, it is very essential to choose a proper point for the main FL problem to preserve probable divergence.

The relation between the state variables vector λ, and measurements vector, Z, is expressed by a measurement model in (22).

$$Z = g(\lambda) + r \tag{22}$$

$$Z_{d\times 1} = \left[\Delta V^{+,re,m} \cdots \Delta V_{N_P}^{+,im,m}\ \Delta I_{1,1}^{+,re,m} \cdots \Delta I_{N_P,N_{LN_p}}^{',+,im,m}\right]^T \tag{23}$$

where, Z represents measurement values having the dimension of $d \times 1$ ($d = 2(N_p + \sum_{k=1}^{N_p} N_{L_k})$). λ refers to the vector of state variables containing x, x_1, x_2 and p, where, p, itself, refers to parameters of the faulted line including X, R or Y. $g(\lambda)$ corresponds to equations (8), (9) and (17) in which, except to the fault current and its location, a new state variable, p, is also included. The parametrical effect of the faulted line parameters will be imposed on bus impedance matrix elements. r stands for the residual vector of the voltage and current measurements similar to the residuals in equations (10) and (16). H in (24), is the Jacobian matrix of the function $g(\lambda)$ regarding the state variables λ.

$$H_{d\times 4} = \begin{bmatrix} \frac{\partial \Delta V_1^{+,re}}{\partial x} & \frac{\partial \Delta V_1^{+,re}}{\partial x_1} & \frac{\partial \Delta V_1^{+,re}}{\partial x_2} & \frac{\partial \Delta V_1^{+,re}}{\partial p} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \Delta V_{N_P}^{+,im}}{\partial x} & \frac{\partial \Delta V_{N_P}^{+,im}}{\partial x_1} & \frac{\partial \Delta V_{N_P}^{+,im}}{\partial x_2} & \frac{\partial \Delta V_{N_P}^{+,im}}{\partial p} \\ \frac{\partial \Delta I_{1,1}^{',+,re}}{\partial x} & \frac{\partial \Delta I_{1,1}^{',+,re}}{\partial x_1} & \frac{\partial \Delta I_{1,1}^{',+,re}}{\partial x_2} & \frac{\partial \Delta I_{1,1}^{',+,re}}{\partial p} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \Delta I_{N_P,N_{Lk}}^{',+,im}}{\partial x} & \frac{\partial \Delta I_{N_P,N_{Lk}}^{',+,im}}{\partial x_1} & \frac{\partial \Delta I_{N_P,N_{Lk}}^{',+,im}}{\partial x_2} & \frac{\partial \Delta I_{N_P,N_{Lk}}^{',+,im}}{\partial p} \end{bmatrix} \tag{24}$$

The optimization LAV estimation problem is modeled by (25) [17].

$$\begin{aligned} &\min_{\lambda} \mathbf{c^T Y} \\ &\text{subject to } \mathbf{MY} = \mathbf{Z} \\ &\mathbf{Y} \geqslant 0 \\ &\mathbf{c^T} = [0_{1\times 2n}\ 1_{1\times 2d}] \\ &\mathbf{Y_{2(n+d)\times 1}} = \left[\boldsymbol{\Delta} X_{a_{1\times n}}\ \boldsymbol{\Delta} X_{b_{1\times n}}\ \mathbf{U}_{1\times d}\ \mathbf{V}_{1\times d}\right]^{\mathbf{T}} \\ &\mathbf{M_{d\times 2(n+d)}} = [\mathbf{H}_{d\times n}\ -\mathbf{H}_{d\times n}\ \mathbf{I}_{d\times d}\ -\mathbf{I}_{d\times d}] \end{aligned} \tag{25}$$

In (25), $\mathbf{X_a}$, $\mathbf{X_b}$, $\mathbf{U}$ and $\mathbf{V}$ are non negative auxiliary vectors and $\mathbf{I}$ is the identity matrix with the order of d. n denotes the number of state variables. According to the above optimization, $\boldsymbol{\Delta}\lambda$ and $\mathbf{r}$ can be calculated as $\boldsymbol{\Delta}\lambda = \mathbf{\Delta X_a^T} - \mathbf{\Delta X_b^T}$ and $\mathbf{r} = \mathbf{U^T} - \mathbf{V^T}$. After solving the optimization problem, the normalized residual for each measurement (r_i^N) is obtained using (26). Since r_i is a complex number, to attain a constant value for r_i^N, the norm 2 of r_i is used in (26).

$$\mathbf{r_i^N} = \frac{|\mathbf{r_i}|}{\sqrt{\boldsymbol{\Omega}_{\mathbf{ii}}}} \tag{26}$$

where, Ω_{ii} denotes the diagonal elements of the residual covariance matrix Ω, as expressed in (27).

$$\Omega = R - H(H^T R^{-1} H)^{-1} H^T \tag{27}$$

To identify the erroneous measurements, the value of normalized residuals will be compared with a pre-defined threshold c, for instance $c = 3$ ([17]). After eliminating the erroneous measurements, the optimization problem is solved repeatedly until the acceptable accuracy is attained.

III.D. Proposed Algorithm

The FL problem is divided into some steps as bellow:

- Step 1: Provide the vector of PMUs measurements, Z, as introduced in (23).
- Step 2: Set the initial point (x, x_1, x_2) for primary FL problem. The initial point of the primary FL problem is randomly selected.
- Step 3: Solve the primary FL problem as introduced in [15] and obtain x, x_1 and x_2. Calculate the PMU measurements containing bad data and reject bad data from the measurement set.
- Step 4: Use the output of step 3, as the initial point for main FL problem. Parameter p, as explained before, refers to the faulted line parameters X, R, or Y. For the initial point of the main FL problem, p is selected based on the incoming information of the network, which may contain error.
- Step 5: Calculate H using (24). According to [17] and [18] detecting the leverage measurement units containing error by normalized residual test is too difficult and may not be possible. Finding the leverage measurements and eliminating the ones overlapped with the bad data measurements detected in the primary problem from the measurement set will resolve this issue. The procedure of leverage measurement detection is explained in the part B of this section.

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- Step 6: Solve the main FL problem in order to modify R, X and Y one by one. By "one by one", we mean that, one of the faulted line parameters is considered as parameter p and is modified irrespective of the two others.
- Step 7: Check the conditions $\|\Delta\lambda\|_2 < \epsilon$. If the condition is true, go to the next step, if not, go to step 6.
- Step 8: Check if any normalized residual exceeds the threshold c that has been defined previously. In this step, the PMUs contaminated with bad data as can be detected by their normalized residual, are eliminated. If bad data measurement is not detected, go to next step, if detected, return to step 4.
- Step 9: Record the correct values of λ.

IV. PMU Placement Strategy

In this work, same PMU placement circumstances as in [12] and [15] are considered as:

- Networks formed by two sub-networks which have been connected by a single line
- Presence of one-joint loops
- Loop conflicts possibility

In addition to the above mentioned circumstances, this paper considers the constraint of maximum distance which is defined as the distance at which a PMU may be effective in estimation and optimization of line parameters. In this paper, maximum distance takes the value of 4 sections [17].This condition means that the distance of each line from at least one PMU should be less than 4 sections. According to the above description, PMUs related to the circumstances will be considered first. Then if the maximum distance condition does not fulfill buses with the most number of connected lines are best candidates for PMU installation.

V. Simulation Results

The accuracy and precision of the proposed method is evaluated on IEEE 14-bus and IEEE 57-bus test networks. The main advantages of the proposed method, as mentioned before, are fault location and parameter estimation of the faulted line utilizing only a few number of measurement units in presence of other possible errors. For convenience, the results are mostly presented as faulted line reactance estimation since reactance values outweigh of other parameters. In cases where the resistance and admittance of the faulted line are high, the optimization problem is solved in several steps. According to the previous section , and taking into consideration that the distance of each line from at least one PMU should be less than 4 sections, the number of PMUs in the IEEE 14-bus and IEEE 57-bus test systems was set at 2 and 5 units, respectively.

The absolute value of percentage error (PE%) is defined as the difference between the estimated and actual fault locations according to equation (28):

$$\mathrm{PE\%} = \left| \frac{\text{estimated FL} - \text{actual FL}}{\text{line length}} \right| \times 100 \tag{28}$$

Initially, the proposed method has been evaluated by simulating phase to ground fault with the impedance of 10 at

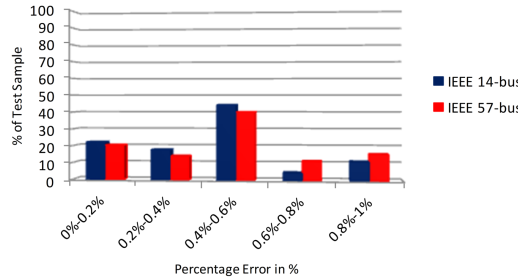

Fig. 3. Percentage error of simulated cases

0.1pu, 0.5pu and 0.9pu of all lines in IEEE 14 bus (45 sample test) and IEEE 57 bus (186 sample test) test systems. In this study, 15% error in the amount of faulted line reactance is considered. In IEEE 14 bus test system, 4%, 8% and 2% error has been considered in units V2, I(2-4) and I(6-13), respectively. Also, In IEEE 57 bus test system, 7%, 3%, 5% and 2% error has been considered in units V9, V38, I(5-6) and I(12-13), respectively. The results presented in Fig.3 show the acceptable accuracy of the proposed method in different cases. The error considered in voltage and current values includes phasor measurement unit error (acceptable TVE) and the measuring transformers.

$V.A.\ IEEE\ 14 - bus\ Network$

In this network the PMUs are assumed to be installed at buses 2 and 6. Such layout of PMUs ensures having a distance of less than four sections between all lines and at least one measurement unit. The results of the fault location estimation considering fault resistances of 0, 50 and 100 Ω without existence of any other errors are listed in Table I. C^TY, $PE\%$ and $X^{estimate}$ are objective function value, percentage of fault location error and estimated reactance of the faulted line, respectively. Voltage measurements are related to the PMUs placed on buses 2 and 6. Current measurements are related to branches 2-3, 2-4, 2-5, 1-2, 6-11, 5-12 and 6-13. As can be seen, the percentage of fault location estimation errors ($< 1\%$ of fault line length in per-unit) and the estimated reactances are in their acceptable ranges. The minor but acceptable impact of fault resistance on the percentage of fault location estimation error can be seen in the presented results as well.

Table II shows the results of fault location estimation and the estimated reactance of the faulted lines aiming to put forward the necessity to eliminate any leverage unit that involves error, which can be calculated by Jacobean matrix (according to (24)) and the primary answer [15]. The results are presented considering both pre-fault and post-fault errors in voltage and current measurements. According to [18] and the results presented in Table II, detecting the inaccurate leverage units by normalized residual test may not be possible. Therefore, in case of not eliminating inaccurate leverage units, the optimization is unlikely to converge or converges with an unacceptable accuracy in several cases. So, removing the leverage units involved in error may increase the probability of achieving an acceptable answer in the proposed method.

TABLE I
FAULT LOCATION ESTIMATION CONSIDERING FAULT RESISTANCES OF 0, 50 AND 100 Ω WITHOUT EXISTENCE OF ANY OTHER ERRORS

Line, fault type, fault point	C^TY	$PE\%$	X^{real}	$X^{estimate}$
$R_f = 0$				
4-3, AG, 20%	0.0067	0.2974	0.171	0.1709
6-13, BG, 80%	0.0015	0.4943	0.1303	0.1321
12-13, BCG, 95%	0.0068	0.0053	0.1999	0.1999
2-5, ABCG, 30%	0.016	0.5896	0.1739	0.1741
2-4, ABCG, 5%	0.0073	0.2492	0.1763	0.1755
$R_f = 50$				
4-3, AG, 20%	0.0017	0.2877	0.171	0.1709
6-13, BG, 80%	0.0006	0.49	0.1303	0.1313
12-13, BCG, 95%	0.0032	0.0049	0.1999	0.1999
2-5, ABCG, 30%	0.0001	0.5896	0.1739	0.1742
2-4, ABCG, 5%	0.0001	0.249	0.1763	0.1756
$R_f = 100$				
4-3, AG, 20%	0.0009	0.2857	0.171	0.1709
6-13, BG, 80%	0.0004	0.49	0.1303	0.1313
12-13, BCG, 95%	0.0031	0.0054	0.1999	0.1999
2-5, ABCG, 30%	0.0016	0.5859	0.1739	0.1749
2-4, ABCG, 5%	0.0004	0.2603	0.1763	0.176

TABLE II
FAULT LOCATION ESTIMATION ERROR BY CONSIDERING ERRORS IN MEASURED VALUES OF THE PMUS AND FAULTED LINE REACTANCE

Line	**4-3**	**12-13**	**2-5**	**2-4**	**4-5**
Fault point	20%	95%	30%	5%	70%
R_f (Ω)	0	0	0	0	0
X^{real}	0.1710	0.1998	0.1738	0.1763	0.0421
Fault type	BG	ACG	ABCG	ABCG	CG
Leverage units	I(2-3) I(2-4)	I(6-12) I(6-13)	I(2-4) I(6-11)	I(1-2) I(6-11)	I(2-4) I(2-5)
Measur-ment errors %	V2:1 pre I(2-3):-2 I(6-11):2	V6:2 pst I(2-4):1 I(6-13):3	I(2-4):5 I(6-13):2	V2:2 pre I(2-5):-2 I(1-2):1	I(2-4):-2 I(6-11):5
PE% without elimina-tion	*Unac-ceptable*	0.3523	0.7394	13.1368	1.997
$X^{estimate}$	*Unac-ceptable*	0.1998	0.1736	0.1958	0.0421
PE% with elim-ination	0.6746	0.3523	0.6733	0.39	0.956
$X^{estimate}$	0.1705	0.1998	0.1739	0.176	0.0421

TABLE III
ELIMINATION OF LEVERAGE UNITS OVERLAPPED WITH UNITS CONTAINING MEASUREMENTS ERRORS AND ESTIMATION OF FAULT LOCATION AND FAULTED LINE REACTANCE

Line	**4-3**	**12-13**	**2-5**	**6-13**
Fault point	20%	95%	30%	80%
R_f (Ω)	0	0	0	0
X^{real}	0.171	0.199	0.174	0.130
Fault type	AG	BCG	ABCG	ABCG
Faulted line reactance error in %	-15%	10%	7%	-15%
Leverage units	I(2-3) I(2-4)	I(6-12) I(6-13)	I(2-4) I(6-11)	I(1-2) I(6-12)
Measur-ment errors %	V2:1 pre I(2-3):-2 I(6-11):2	V6:2 post I(2-4):1 I(6-13):3	I(2-4):8 I(6-13):10	V2:10 post V6:-10 pre I(6-12):10
Overlapped measurment units	I(2-3)	I(6-13)	I(2-4)	I(6-12)
Bad data units detected in proposed method	V2 I(6-11)	V6 I(2-4)	I(6-13)	V2 V6
$PE\%$ pre-answer [15]	1.4151	0.6840	0.6925	4.2197
$PE\%$ **proposed method**	0.6816	0.3524	0.6728	0.3584
$X^{estimate}$	0.1709	0.1998	0.1738	0.1341

The FL estimation error, while considering errors in PMU measured values and faulted line reactance, can be seen in Table III. The results are presented considering both pre-fault and post-fault errors in voltage and current measurements. At first, as outlined in III, the fault point and erroneous measurement data were calculated using the method in [15]. Next, the leverage units overlapped with erroneous units were eliminated and then the percentage of fault location error and faulted line reactance were estimated using the proposed method, shown in the 11th and 12th rows of Table III, respectively. As can be seen in the results, inaccurate faulted line reactance usually lead to lower accuracy of estimation in the primary answer [15], which can be improved by applying the proposed method.

The normalized residual vectors for one phase to ground fault at 45% of line 3-4 with 10 Ω fault resistance, is shown in Table IV. In this case, the pre-fault voltage measured values of bus 2 had 1% error and the measured current magnitudes of the lines between buses 2-4 and 6-11 had -2% and 2% errors, as well. In the proposed optimization, the equation corresponding the voltage of bus 2 was eliminated because of the overlap with bad data measurements calculated in the primary answer. The two other equations corresponding erroneous units were removed with respect to the remaining normalized residual values in the proposed method. Normalized residual of bad data measurements in the first and second steps were 5.1488 and 4.152, respectively. As can be seen, the normalized residuals corresponding to $\Delta I(2-4)$ and $\Delta I(6-11)$ are greater than 3, so they were detected as erroneous measurements. Improvement in FL estimation after removing erroneous units is evident in this table.

$V.B.\ IEEE\ 57-bus\ Network$

The other network that was used to confirm and validate the proposed method is the IEEE 57-bus test system. With regard

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRD.2019.2897402, IEEE Transactions on Power Delivery

TABLE IV
THE NORMALIZED RESIDUAL VECTORS FOR ONE PHASE TO GROUND FAULT AT 45% OF LINE 3-4 WITH 10 Ω FAULT RESISTANCE

Measurments	First step	Second step	Third step
$\Delta V(2)$	1.69	**4.152**	*(Removed in step 8)*
$\Delta V(6)$	0.1136	0.279	0.0859
$\Delta I(2-3)$	*(Removed in step 5)*	—	—
$\Delta I(2-4)$	0	0	0
$\Delta I(2-5)$	0.0679	0.1668	0.0333
$\Delta I(1-2)$	0	0	0
$\Delta I(6-11)$	**5.1488**	*(Removed in step 8)*	—
$\Delta I(5-12)$	0.1993	0.4894	0.09794
$\Delta I(6-13)$	0.7788	1.9121	0.03826
PE%	3.98%	3.15%	0.62%

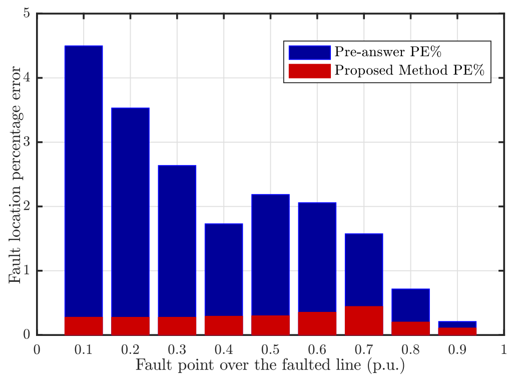

Fig. 4. Comparison between proposed method and pre-answer [15]

to the circumstances mentioned in IV, phasor measurement units in this network were installed at buses 6, 9, 13, 33 and 38. In order to check the accuracy of the proposed method, a number of points were randomly investigated as faulted points in the presence of different errors in parameters and measuring units values, as described in Table V.
Table VI shows the effect of erroneous faulted line resistance and admittance while the two parameters are small in comparison with the faulted line reactance. As can be seen, in such cases, these two parameters had no much effect on fault point estimation error, and an accurate answer could be obtained by estimating just the faulted line reactance. In contrast, Table VII shows the results in the case where the faulted line resistance is comparable to its reactance. In such cases, the resistance and the reactance of faulted line are corrected in 3 and 2 optimization steps, respectively. An acceptable accuracy in estimating faulted line parameters and faulted point for both possible cases is evident.

Another important issue in fault location methods is the minimum sensitivity of the methods to the fault point along the line. In order to study this, two fault points were selected: (1) one phase to ground fault between buses 54 and 55 with 15% error in the faulted line reactance, for which the results are presented in Figs. 4 and 5; and (2) three phase to ground fault between buses 56 and 42 with 15% error in the faulted line reactance, presented in Figs. 6 and 7.

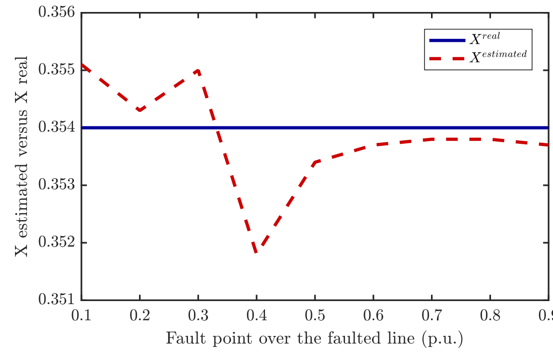

Fig. 5. The accuracy of the proposed method to estimate X

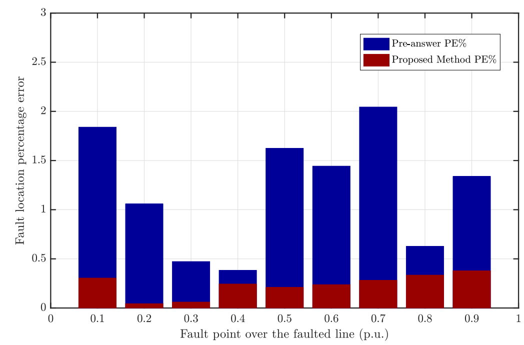

Fig. 6. Comparison between the proposed method and pre-answer [15]

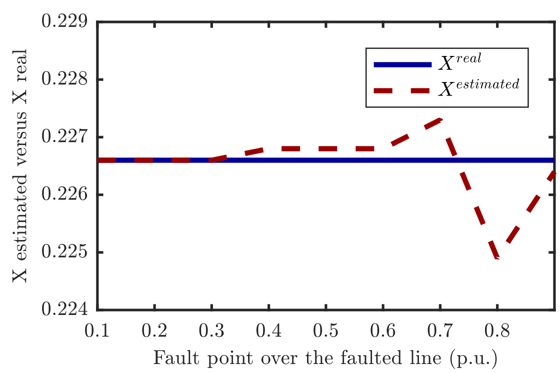

Fig. 7. The accuracy of the proposed method to estimate X

VI. CONCLUSION

In this paper, we addressed errors associated with parameters of the faulted line in addition to the errors related to the measured voltages and currents of PMUs in fault location problem. To derive the equations, we have used equations developed by [15]. Leverage measurement units, in which the measured parameter significantly affect the unknown state variables, were identified and their intersection with bad data units have been omitted. The Main fault location problem is solved using least absolute value estimation method. Our proposed FL algorithm, besides estimating the faulted line parameters more accurately, has the advantage of detecting and eliminating the leverage measurements which may be hard to be recognized in conventional FL methods. The accuracy and precision of the proposed method has been evaluated and

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TABLE V
FAULT POINT IN THE PRESENCE OF DIFFERENT KINDS OF ERRORS, PARAMETERS AND MEASURING UNITS VALUES

Line	**4-6**	**9-12**	**1-16**	**3-15**	**37-38**	**31-32**
Fault type	ABCG	BG	ABG	CG	BCG	ABCG
Fault point	80%	35%	95%	5%	25%	95%
R_f (Ω)	0	36	18	0	0	50
X^{real}	0.148	0.295	0.206	0.053	0.1009	0.755
Measurement error (in %)	V9=2 post V38=-3 pre I(5-6)=-3 I(6-7)=2	V9=-2 pre V38=1 pre I(12-13)= 3 I(22-38)= 3	V9=-2 post V13=2 pre I(13-15)=-3 I(8-9)=2 I(6-4)=-2	V9=-2 pre V38=1% post I(8-9)=-3 I(5-6)=1	V6=3 post V38=2 post I(4-6)=-2 I(22-38)=3	V6=2 pre V9=4 pre I(37-38)=-3 I(8-9)=2
Line errors of parameters (%) $[R, X, Y]$	**4-6**: [10, 10, 0] ——— **4-5**: [0, 10, 0] ——— **6-8**: [0, 10, -10]	**9-12**: [10, 15, 10] ——— **8-9**: [-5, 0, -10] ——— **12-16**: [0, 5, -10]	**1-16**: [0, -15, 10] ——— **12-17**: [10, 0, -10] ———- **8-9**: [0, 10, 0] ——— **4-5**: [-5, 0, -10]	**9-12**: [0, -20, -10] ——— **12-13**: [-5, 10, 5] ——— **3-15**: [10, -15, 0]	**37-38**: [-5, 10, 0] ——— **22-38**: [5, -5, 0] ——— **6-7**: [8, -10, 10] ——— **18-19**: [5, -5, 0]	**31-32**: [0, -5, 0] ——— **34-35**: [10, 0, 10] ——— **6-7**: [-5, 15, -10] ——— **22-23**: [-8, -10, 0]
$PE\%$ pre-answer [15]	1.66	2.84	4.189	0.7738	0.6733	1.463
$X^{estimate}$	0.1483	0.2965	0.2069	0.0536	0.1032	0.7586
PE% proposed method	0.0122	0.3754	0.9296	0.2525	0.1032	0.8271

TABLE VI
THE EFFECT OF ERRONEOUS FAULTED LINE RESISTANCE AND INDUCTANCE WITH NEGLIGIBLE R/X

Line	**1-16**	**9-12**
Fault type	3PhG	2PhG
Fault point	30% from bus 1	80% from bus 9
R_f (Ω)	50	18
$X^{real}_{faulted\ line}$	0.206	0.295
$R^{real}_{faulted\ line}$	0.0454	0.0648
$Y^{real}_{faulted\ line}$	0.0546	0.0772
Measurement error	V13=3% pre I(4-6)=-2% I(13-14)=3% I(38-44)=-2%	V13=-2% post V38=3% post I(11-13)=-2% I(9-8)=-5%
Line errors of parameters (%) $[R, X, Y]$	**1-16**: [10, 10, -15]	**9-12**: [-10, 10, 15]
PE% $X^{estimate}$	0.7609 0.216	0.5173 0.2962

TABLE VII
THE EFFECT OF ERRONEOUS FAULTED LINE RESISTANCE AND INDUCTANCE WITH SIGNIFICANT R/X

Line	**31-32**	**23-24**
Fault point	30% from bus 31	30% from bus 23
Fault type	ABCG	AG
R_f (Ω)	0	50
X^{real}	0.755	0.256
R^{real}	0.507	0.166
Y^{real}	0	0.0084
Line errors of line parameters in %	V33=3% pre V38=3% pre I(9-12)=-2% I(22-38)=3%	V6=2% pre V38=-1% post I(4-6)=2% I(32-33)=-2%
Line errors of parameters (%) $[R, X, Y]$	**31-32**: [-15, 15, 0] ————- **13-15**: [10, -5, 0] ————- **11-13**: [-10, 5, 0]	**23-24**: [15, 0, 0] ————- **34-35**: [0, 0, 10] ————- **22-23**: [-10, 5, 0]
Steps $PE\%$ $X^{estimate}$ $R^{estimate}$	**Step 1**: $PE\%$= 9.4623 $X^{est} = 0.7253$ R^{est}= Not estimated ————- **Step 2**: $PE\%$= 1.9219, X^{est}= Not estimated R^{est}=0.5113 ————- **Step 3**: $PE\%$= 0.2523, X^{est}=0.7427 R^{est}= Not estimated	**Step 1**: $PE\%$= 3.3849, $X^{est} = 0.2546$ R^{est}= Not estimated ————- **Step 2**: $PE\%$= 0.6581, X^{est}= Not estimated $R^{est} = 0.1663$

tested on the IEEE 14-bus and IEEE 57-bus systems. The results reveals that:

- Refusing to eliminate the overlap of bad data units with leverage measurements, might lead to significant increase in error of FL estimation.
- Both the impedance and the type of the fault, have low impact on the accuracy of FL estimation.
- The accuracy of the proposed method in the absence of faulted line parameter errors, despite being in acceptable range (<1%), is lower than that in similar methods. This is due to increased number of optimization parameters. However, the proposed method significantly outperforms other methods, when there is error in faulted line parameters, which is usually the case.

REFERENCES

[1] J. D. L. Ree, V. Centeno, J. S. Thorp, and A. G. Phadke, "Synchronized phasor measurement applications in power systems," *IEEE Transactions on Smart Grid*, vol. 1, no. 1, pp. 20–27, June 2010.

[2] Y. Liao, "Fault location for single-circuit line based on bus-impedance matrix utilizing voltage measurements," *IEEE Transactions on Power Delivery*, vol. 23, no. 2, pp. 609–617, April 2008.

[3] T. Takagi, Y. . Yamakoshi, M. Yamaura, R. Kondow, and T. Matsushima, "Development of a new type fault locator using the one-terminal voltage and current data," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-101, no. 8, pp. 2892–2898, Aug 1982.

[4] J. Izykowski, E. Rosolowski, and M. M. Saha, "Locating faults in parallel transmission lines under availability of complete measurements at one end," *IEE Proceedings - Generation, Transmission and Distribution*, vol. 151, no. 2, pp. 268–273, March 2004.

[5] Y.-H. Lin, C.-W. Liu, and C.-S. Yu, "A new fault locator for three-terminal transmission lines using two-terminal synchronized voltage and current phasors," *IEEE Transactions on Power Delivery*, vol. 17, no. 2, pp. 452–459, Apr 2002.

[6] D. Novosel, D. G. Hart, E. Udren, and J. Garitty, "Unsynchronized two-terminal fault location estimation," *IEEE Transactions on Power Delivery*, vol. 11, no. 1, pp. 130–138, Jan 1996.

[7] J. Izykowski, R. Molag, E. Rosolowski, and M. M. Saha, "Accurate location of faults on power transmission lines with use of two-end unsynchronized measurements," *IEEE Transactions on Power Delivery*, vol. 21, no. 2, pp. 627–633, April 2006.

[8] G. Manassero, E. C. Senger, R. M. Nakagomi, E. L. Pellini, and E. C. N. Rodrigues, "Fault-location system for multiterminal transmission lines," *IEEE Transactions on Power Delivery*, vol. 25, no. 3, pp. 1418–1426, July 2010.

[9] K.-P. Lien, C.-W. Liu, C.-S. Yu, and J. A. Jiang, "Transmission network fault location observability with minimal pmu placement," *IEEE Transactions on Power Delivery*, vol. 21, no. 3, pp. 1128–1136, July 2006.

[10] W. Bo, Q. Jiang, and Y. Cao, "Transmission network fault location using sparse pmu measurements," in *2009 International Conference on Sustainable Power Generation and Supply*, April 2009, pp. 1–6.

[11] Q. Jiang, X. Li, B. Wang, and H. Wang, "Pmu-based fault location using voltage measurements in large transmission networks," *IEEE Transactions on Power Delivery*, vol. 27, no. 3, pp. 1644–1652, July 2012.

[12] S. H. Mortazavi and J. Sadeh, "An analytical fault location method based on minimum number of installed pmus," *International Transactions on Electrical Energy Systems*, vol. 26, no. 2, pp. 253–273, 2016, eTEP-14-0490.R2. [Online]. Available: http://dx.doi.org/10.1002/etep.2075

[13] A. S. Dobakhshari and A. M. Ranjbar, "A novel method for fault location of transmission lines by wide-area voltage measurements considering measurement errors," *IEEE Transactions on Smart Grid*, vol. 6, no. 2, pp. 874–884, March 2015.

[14] ——, "A wide-area scheme for power system fault location incorporating bad data detection," *IEEE Transactions on Power Delivery*, vol. 30, no. 2, pp. 800–808, April 2015.

[15] S. A. Hosseini, J. Sadeh, and B. Mozafari, "Robust wide-area impedance-based fault location method utilising lav estimator," *IET Generation, Transmission Distribution*, vol. 10, no. 10, pp. 2475–2485, 2016.

[16] Y. Liao, "Fault location for single-circuit line based on bus-impedance matrix utilizing voltage measurements," *IEEE Transactions on Power Delivery*, vol. 23, no. 2, pp. 609–617, April 2008.

[17] A. Abur and A. Expósito, *Power System State Estimation: Theory and Implementation*, ser. Power Engineering (Willis). CRC Press, 2004. [Online]. Available: https://books.google.com/books?id=NQhbtFC6_40C

[18] J. Zhao and L. Mili, "Vulnerability of the largest normalized residual statistical test to leverage points," *IEEE Transactions on Power Systems*, vol. 33, no. 4, pp. 4643–4646, July 2018.