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# Co-planning of electricity and gas networks considering risk level assessment

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**Abstract:** This study provides a risk-based gas and electricity expansion planning model to coordinate the expansion of electricity and gas networks in a multi-carrier energy network. Generally, electricity and gas networks have separate owners having no mechanisms to share information. In this study, a distributed algorithm based on alternative direction method of multipliers is developed to preserve the privacy of electricity and gas networks while maintaining a coordination link. Probabilistic outage of components is implemented into the expansion planning model to investigate the interactions between electricity and gas networks and evaluate the risk of contingencies in generating units, transmission lines, and pipelines. Second fuel of gas consuming generating units is modelled to have a holistic approach while studying electricity and gas interactions in the case of contingencies. Moreover, conditional value at risk is used to adjust a balance between risk and investment where each of energy parties can decide on the risk level of their expansion plans. The proposed expansion planning approach is applied to a realistic case study to evaluate its performance.

## Nomenclature

### *Indices and sets*

| | |
|---|---|
| $i, j$ | index for gas nodes |
| $m, n$ | index for electricity buses |
| $c$ | index for component condition, $c = 0$ for normal status and $c \geq 1$ for contingency states |
| $w$ | auxiliary index for contingency states $c \geq 1$ |
| $t$ | index for load period (off-peak, mid, peak) |
| $d$ | index for days |
| $y$ | index for years |
| $h$ | index for different units in a bus that are of different types or sizes |
| $\hat{h}$ | index for CGCUs of a bus that are of different sizes |
| $k$ | index for repetition |
| $im$ | index for linking bus $m$ of electricity network to node $i$ of gas network |
| $\mathscr{L}$ | set of nodes-buses that links gas and electricity networks |
| $\mathscr{B}/\mathscr{N}$ | set of buses/nodes of electricity/gas networks |
| $\mathscr{H}/\mathscr{K}/\mathscr{P}$ | set of generating units/transmission lines/pipelines including existing and new candidates |
| $\mathscr{H}_{\mathrm{N}}/\mathscr{K}_{\mathrm{N}}/\mathscr{P}_{\mathrm{N}}$ | set of new generation/transmission line/pipeline candidates |
| $\mathscr{H}_{\mathrm{G}}$ | set of GCGUs including existing and new candidates |
| $\mathscr{P}_{\mathrm{A}}/\mathscr{P}_{\mathrm{p}}$ | set of active/passive pipelines including existing and new candidates |
| $\mathscr{T}_D$ | set of daily load periods |
| $\Omega$ | set of non-contingent and single contingent states |
| $D_y$ | set of days |
| $Y_T$ | set of planning years |

### *Variables*

| | |
|---|---|
| $Q_{f_{ijydc}}$ | gas flow of pipeline $ij$ on day $d$ of year $y$ in contingency $c$ in MSCMD |
| $Q_{S_{iydc}}$ | gas injection at node $i$ in MSCMD |
| $Q^{pp}_{l_{iydc}}$ | gas demand of GCGUs at node $i$ in MSCMD |
| $Q^{\mathrm{comp}}_{l_{iydc}}$ | gas loss of compressor at node $i$ in MSCMD |
| $Q_{r_{iydc}}$ | curtailed gas demand at node $i$ in MSCMD |
| $pr^{g}_{iydc}/\pi^{g}_{iydc}$ | gas pressure in bar/squared pressure in bar$^2$ |
| $\mathrm{FC}_{mhydtc}$ | fuel consumption of unit $h$ in bus $m$ in MSCM per hour |
| $\mathrm{FC}^{\mathrm{Sec}}_{mhydtc}$ | second fuel consumption of unit $h$ in bus $m$ in MSCM per hour |
| $P_{f_{mnydtc}}$ | power flow of line $mn$ in MW |
| $P_{s_{mhydtc}}$ | generation power of unit $h$ of bus $m$ in MW |
| $P_{l_{mydtc}}$ | electricity load of bus $m$ in MW |
| $S^{F}_{mhydc}$ | binary variable indicating usage status of second fuel in generating unit $h$ of bus $m$ |
| $\theta_{mydtc}$ | voltage angle of bus $m$ in Rad |
| $P_{r_{mydtc}}$ | curtailed electric power at bus $m$ in MW |
| $u^{\mathrm{gen}}_{mh}/u^{\mathrm{trans}}_{mn}/u^{\mathrm{pipe}}_{ij}$ | binary variable indicating selection of generating unit $h$ of bus $m$/transmission line $mn$/pipeline $ij$ |
| $\zeta$ | value at risk in \$ |
| $\delta_c$ | auxiliary variable used to compute CVaR |
| $\zeta^{\mathrm{Elec}}/\zeta^{\mathrm{Gas}}$ | value at risk in electricity/gas networks in \$ |
| $\delta^{\mathrm{Elec}}_c/\delta^{\mathrm{Gas}}_c$ | auxiliary variable used to compute CVaR in electricity/gas networks in \$ |
| $\mathrm{CVaR}^{\mathrm{Elec}}_{\alpha}/\mathrm{CVaR}^{\mathrm{Gas}}_{\alpha}$ | CVaR at the confidence level $\alpha$ in electricity/gas networks in \$ |
| $\mu_{im}$ | Lagrangian multiplier of nodal balance equation at node $i$ |
| $\mathrm{TIC}^{\mathrm{Elec}}/\mathrm{TIC}^{\mathrm{Gas}}$ | TIC in electricity/gas networks in \$ |
| $\mathrm{TOC}^{\mathrm{Elec}}_c/\mathrm{TOC}^{\mathrm{Gas}}_c$ | TOC in electricity/gas networks in contingency $c$ in \$ |

### *Parameters*

| | |
|---|---|
| $K^{\mathrm{pipe}}_{ij}$ | Weymouth constant in MSCMD/bar |
| $R^{\mathrm{comp}}_{ij}$ | pressure ratio of compressor |
| $l^{\mathrm{comp}}_{ij}$ | compressor gas consumption constant in bar$^{-1}$ |
| $\lambda^{\mathrm{Gas}}_{l_{iydc}}$ | gas price at node $i$ in \$/MSCM |
| $\lambda^{\mathrm{Gas}}_{r_{iydc}}$ | gas curtailment price at node $i$ in \$/MSCM |
| $L^{\mathrm{Pipe}}_{ij}$ | length of pipeline $ij$ in km |

| | |
|---|---|
| $A_{ij}^{\text{Pipe}}$ | diameter of pipeline $ij$ in inch |
| $P_{s_{mh}}^{R}$ | rated power for unit $h$ of bus $m$ in MW |
| $Q_{l_{iydc}}^{\text{Npp}}$ | gas demand of non-power plant loads in MSCMD |
| $P_{l_{mydtc}}$ | electricity load at bus $m$ in MW |
| $\lambda_{r_{mydtc}}^{\text{Elec}}$ | load curtailment price at bus $m$ in \$/MW |
| $\lambda_{mhydc}^{\text{FC}}$ | fuel price of unit $h$ of bus $m$ in \$/MSCMD |
| $\lambda_{mhydc}^{\text{Sec-FC}}$ | second fuel price for unit $h$ of bus $m$ in \$/MSCMD |
| $y_{mn}$ | series admittance of line $mn$ |
| $L_{mn}^{\text{trans}}$ | length of transmission line $mn$ in km |
| $\partial_{mh}/\beta_{mh}/\gamma_{mh}$ | heat-rate curve parameters for generation unit $h$ of bus $m$ |
| $\partial_{mh}^{\text{Sec}}/\beta_{mh}^{\text{Sec}}/\gamma_{mh}^{\text{Sec}}$ | heat-rate curve parameters for generation unit $h$ of bus $m$ for second fuel |
| $\underline{P_{s_{mh}}}/\overline{P_{s_{mh}}}$ | lower/upper limits for generation unit $h$ of bus $m$ in MW |
| $\overline{P_{f_{mn}}}$ | upper limit for power flow through transmission line $mn$ in MW |
| $\underline{Q_{f_{ij}}}/\overline{Q_{f_{ij}}}$ | lower/upper limits for gas flow through pipeline $ij$ in MSCMD |
| $\underline{Q_{S_i}}/\overline{Q_{S_i}}$ | lower/upper limits for gas supply at node $i$ in MSCMD |
| $\underline{pr_i^{\text{g}}}/\overline{pr_i^{\text{g}}}$ | lower/upper limit for gas pressure in node $i$ in bar |
| $\text{GHV}_h/\text{GHV}_h^{\text{Sec}}$ | gross heating value of natural gas/second fuel in unit $h$ in MMBTU/MSCM |
| $P_{\text{b}}$ | base of power in MW |
| $UG_{mhydtc}/UT_{mnydtc}/UP_{ijydc}$ | binary parameter indicating contingency status of generating unit $h$ of bus $m$/pipeline $ij$/transmission line $mn$, during contingency $c$ |
| $\text{cost}_{ij}^{\text{Pipe}}/\text{cost}_{mn}^{\text{trans}}/\text{cost}_{mh}^{\text{gen}}$ | investment cost of pipeline $ij$ in \$/inch-km/transmission line $mn$ in \$/km/generating unit $h$ of bus $m$ in \$/MW |
| $\mathbb{P}_c$ | probability of occurrence in contingency $c$ |
| $\eta$ | weight of risk-averring or risk-seeking level |
| $\alpha^{\text{Elec}}/\alpha^{\text{Gas}}$ | per unit confidence level for electricity/gas networks used in CVaR |
| $T$ | planning period |
| $T^{\text{t}}/T^{\text{g}}/T^{\text{p}}$ | transmission/generation/pipeline useful life |
| $d_t$ | duration of period |
| $\tilde{\imath}$ | interest rate |
| $(P/A, \tilde{\imath}, T)/(P/F, \tilde{\imath}, T)/(A/P, \tilde{\imath}, T)$ | time value factors that convert an annual value to its equivalent present value/a future value to its equivalent present value/and a present value to its equivalent annual value over period $T$ with interest rate $\tilde{\imath}$ |

## 1 Introduction

Compared to other types of fossil fuel natural gas is an efficient source of energy that offers lower carbon emissions. Coal power plants are retiring by gas consuming generating units (GCGUs) as they have higher efficiency, lower capital cost and lower carbon emissions [1]. On the other hand, GCGUs in combination with energy storages have the advantage of the flexibility to mitigate growing renewable fluctuations [2].

Increasing the use of GCGUs makes electricity and gas networks tightly interdependent. While generally electricity and gas networks have autonomous operators and are designed separately [3]. Electricity network operator decides for the expansion planning of its subordinating grid ignoring the gas network expansion plan and the same procedure holds for the gas network operator. There is no effective link and no data exchange mechanism between the two networks which can lead to an unbalanced investment. Although centralised decision making can overcome the problem of unbalanced investment, but it is illegal with the concept of data privacy. To figure out the hierarchy of the proposed multi-carrier energy network, it is depicted in Fig. 1 considering the energy flow and information flow links.

On the other hand, uncertainties associated with unwanted outages of components would affect both electricity and gas networks. Pipeline outage imposes a risk to the fuel adequacy of GCGUs. Alternatively, transmission line and generation unwanted outages affect the demand in the gas network. Considering second fuel for GCGUs can decrease the impact of pipelines outages on the electricity network. However, a system investor is the one who must decide on the accepted risk level of its subordinating network. In this regard, this paper addresses the coordinated investment in electricity and gas infrastructures when there are risk resources as contingencies in both electricity and gas networks. Considering the interactions between electricity and gas networks, the proposed expansion planning model is capable of adjusting the risk level of either gas or electricity network.

According to the highlights of this paper, research studies in the area of gas–electricity systems can be classified in three categories: (i) researches that focus on the integrated expansion planning of electricity and gas networks [4–21]; (ii) researches that focus on decentralised optimisation methods in electricity and gas networks [3, 22, 23]; and (iii) researches that focus on modelling contingencies and their impacts on the coordinated operation and planning of gas–electricity systems [24–28]. In category 1, the proposed model in [4] considers that the electricity network makes the decision as a leader and gas network acts as a follower. Authors in [5] develop a multi-stage expansion planning model of electricity and gas networks from the viewpoint of a central decision maker. A similar method for distribution network of electricity and gas grids is presented in [6]. Authors in [7] optimise the expansion cost of electricity and gas networks where electricity network feasibility is satisfied by transmission line capacity increment and allocating the decided generating units. In the objective function of the model provided in [8], the social welfare of the integrated gas–electricity expansion planning problem is maximised. In which, the uncertainty of market price in electricity and gas networks is implemented into the proposed model. Authors in [9] introduce an integrated three-level framework based on the genetic algorithm to solve the electricity–gas expansion planning problem. Proposed model in [10] considers profit-to-cost maximisation of the electricity and gas networks expansion planning problem which seeks to reduce carbon emissions. Authors in [11] provide a model with optimal location, size and installation time of electricity and gas infrastructures. The proposed integrated model is decomposed into a master investment problem and two operational sub-problems representing technical constraints of electricity and gas networks. The centralised model introduced in [12] uses market prices as a guide in the expansion planning problem of electricity and gas networks. The proposed model considers electricity and gas market interactions in an iterative process. A similar method is presented in [13] that considers market outcomes during a high-stress situation. Authors in [14] present a stochastic planning model for an integrated electricity–gas system to deal with the uncertainty in demand growth. In a more detailed uncertainty analysis, renewable uncertainties, demand growth uncertainties and gas price uncertainties are also considered in the stochastic co-expansion planning model of [15]. A similar method is introduced in [16] that represents uncertainties in demand growth with non-anticipativity constraints. In this method, a multi-stage stochastic programming model is presented

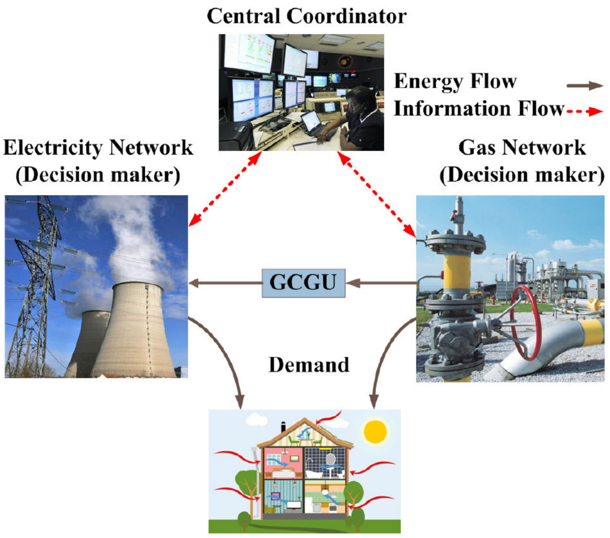


**Fig. 1** *Hierarchy of electricity and gas operators in a multi-carrier energy network*

wherein planning decisions are made in the first stage and operating constraints are satisfied in the second stage. Likewise, a multi-stage model is provided in [17] that considers bi-directional energy conversion among electricity and gas networks. A sequential planning model to the expansion planning of gas distribution pipelines, GCGUs, and capacitor banks is presented in [18]. Authors in [19] provide a chance-constrained model to manage uncertainties in demand while minimising the expansion cost of electricity and gas networks. A multi-attribute expansion planning model of electricity and gas networks is introduced in [20] that considers electricity network expansion cost, gas network expansion cost, robustness and maximum regret in the decision making the process of a central coordinator. Authors in [21] develop a dynamic co-planning model of electricity and gas networks considering uncertainties of renewable energy resources. The papers [4–21] draw the optimal expansion plan of the whole system from the viewpoint of a central decision maker having access to all the required technical data. However, generally, gas and electricity networks are separate entities in some countries.

To preserve the data privacy of private entities, some research studies focused on the decentralised optimisation of the gas–electricity system. Synergistic operation of electricity and gas networks is accomplished by alternative direction method of multipliers (ADMM) in [3]. Optimal scheduling of electricity and gas networks in a decentralised manner is provided by ADMM in [22] considering bi-directional energy conversion between electricity and gas networks. Authors in [23] propose the coordinated energy flow of a multi-area energy system by using ADMM. Despite the works in the coordinated operation of gas and electricity networks, data privacy is still a gap in the context of gas–electricity co-expansion planning.

On the other hand, reliability evaluation is of great major in analysing the interactions of electricity and gas networks. Reliability of an integrated gas–electricity system enhanced with power-to-gas devices and gas storages is evaluated in [29]. Authors compute reliability indices for the whole system to examine the interactions among different parts of an integrated energy system and identify its weak parts. The proposed model in [24] provides an optimal plan for an energy hub consists of combined heat and power, boiler, absorption chiller, compression chiller, electricity storage (Li-ion battery) and heat storage. It also takes energy supply availability into account. In the expansion planning model of [25], reliability and feasibility criterions are used to check the security of the gas–electricity system. The paper also does not perform contingency analysis. Authors in [26] plan multiple infrastructures of an energy hub including transmission lines and pipelines to satisfy the probabilistic reliability criteria. This paper considers a capacity outage probability table for the electricity network to measure the reliability criteria. The presented expansion planning model in [27] considers $N-1$ security criterion in electricity network components whereas supposes a highly reliable gas network. Also, a security-constrained planning model is introduced in [28] that proposes $N-1$ security criterion in both gas network components and transmission lines which can lead to over investment. Authors in [30] propose an expansion planning model for integrated gas–electricity system from the viewpoint of a central decision maker while taking into account the stochastic nature of wind power and $N-1$ network security criterion. In this paper, a heuristic method is used to solve the obtained complex model that captures the optimal Pareto front and enables decision makers to select the optimal plan based on their preference. In the current research studies, second fuel is not considered and risk level assessment is still a gap. Although GCGUs generally operate in gas-driven mode, sometimes they employ another type of fossil fuel rather than gas as a back-up plan in the case of emergencies [31], making it a necessary element in fuel-related contingency analysis. Also, $N-1$ security criterion does not take the probability of contingencies into account, which potentially increases investment cost. Hence, while a robust planning scheme leads to an over investment, considering probability of contingencies and second fuel of GCGUs contribute to a realistic approach and are of significance in contingency analysis of electricity and gas networks. In addition, conditional value at risk (CVaR) can play a key role in mitigating these shortages in the literature effectively as it offers flexibility in the accepted risk level. Besides, it can introduce a risk-oriented model in which, each of the network operators, i.e. gas and electricity networks, has freedom of expression in specifying their risk aversion level. In the meantime, using the co-planning model introduced in this paper, they are operated separately, maintaining their data privacy.

This paper introduces a probabilistic decentralised model for coordinating expansion planning of electricity and gas networks considering risk level assessment. In this model, component contingencies including unwanted outage in pipelines, transmission lines and GCGUs are included. Contingencies are considered according to their reliability indices as outage probability and unavailability time to prevent over-investment in electricity and gas networks. Second fuel is implemented into the electricity network expansion planning model to investigate the interactions between electricity and gas networks in the case of contingencies. CVaR is used to consider the risk level assessment of electricity and gas network operators. With the proposed risk-based gas and electricity expansion planning (RGEEP) model, capacity and location of new generating units, transmission lines and pipelines are determined. Moreover, ADMM is used for the coordinated expansion planning of electricity and gas networks with minimum data exchange in a decentralised manner. Hence, each network operator is tackled separately in which, electricity and gas networks are coordinated together through a master coordinator as Ministry of Energy (ME). The main contributions of this paper are as follows:

- Probabilistic indices are considered in the contingency analysis of different components to prevent over-investment in electricity and gas networks.
- Second fuel is implemented into the electricity network expansion planning model to fully investigate the interactions between electricity and gas networks.
- Considering the risk that is imposed by different contingencies, CVaR is used to adjust the risk level of electricity and gas network operators.
- ADMM is used as a decentralised method to coordinate the risk-based expansion planning of electricity and gas networks and preserve their privacy.

The remainder of this paper is structured as follows. Section 2 describes the proposed RGEEP model. In Section 3, a solution methodology based on the ADMM is used to solve the proposed RGEEP model in a decentralised manner. Section 4 evaluates the proposed RGEEP model on a real-world case study in Khorasan province of Iran. Finally, conclusions are provided in Section 5.

## 2 Mathematical formulation

To decompose the integrated problem into a decentralised problem, first, the centralised problem is introduced in this section. In the centralised expansion planning, it is assumed that a central entity as ME is responsible for the expansion of both gas and electricity networks that knows their requirements and limitations and minimises the total investment and operation cost. Considering the operation stage, gas and electricity networks in the unit commitment problem in [32] are coupled by combined-cycle units where adaptive dynamic programming algorithm is used to handle the computational complexity and dimensionality of renewable energy scenarios in the proposed problem. Authors in [33] represent a detailed formulation of gas network equipment. They provide a comprehensive model of gas–electricity operation in which gas compressors, as the power demand of electricity network, as well as GCGUs, are the joint point of the two energy networks. In the expansion problem of this paper, DC load flow is used to tackle feasible operation of the electricity network as it is accomplished in [33]. On the other hand, as the current paper focuses on the expansion planning side, detailed operation model of the gas network is simplified. Compressors have an important role in the operation of gas networks and neglecting them can cause a large error in non-radial gas networks [34]. Compressor energy usage can be known as loss in the gas network and it is estimated to be about 2–10% of the passing gas through the compressor [35]. In this paper, the presented model in [7, 29], which considers Weymouth gas flow equations, is expanded to tackle compressors' loss.

The centralised objective function, contingency analysis procedure, technical constraints and risk modelling of the proposed RGEEP model are provided in this section.

### 2.1 Objective function

The proposed RGEEP model considers new installation in transmission line, generating unit and pipeline facilities. A multi-objective framework is considered to accomplish the risk level assessment of the proposed RGEEP model. Second fuel of generating units is also considered to have a holistic approach in the contingency analysis of an interconnected gas–electricity system. In the proposed multi-objective framework, the main decision variables of the optimisation problem are binary variables $u_{mh}^{\text{gen}}$, $u_{mn}^{\text{trans}}$, and $u_{ij}^{\text{pipe}}$ that indicate selection of new candidates in generation, transmission line and pipeline facilities. These binaries are equal to 1 for existing facilities. Other variables of the optimisation are auxiliary decision variables that are used for modelling.

Objective function of the proposed RGEEP model comprises of the net present value (NPV) of total investment cost (TIC), NPV of expected total operation cost (ETOC) and CVaR; since the dimension of CVaR is \$ as well as the dimension of TIC and the total expected operation cost. Hence, these three objectives are added together as a single objective expressed in (1) (see (1)) The first term of (1) is the TIC. TIC in electricity and gas facilities are denoted by $\text{TIC}^{\text{Elec}}$ and $\text{TIC}^{\text{Gas}}$ and are defined in (2) and (3)

$$\begin{aligned}\text{TIC}^{\text{Elec}} = \left(\frac{P}{A}, \tilde{i}, T\right)\Bigg[&\sum_{mn \in \mathscr{K}_N} \left(u_{mn}^{\text{trans}}\, L_{mn}^{\text{trans}} \text{cost}_{mn}^{\text{trans}}\right)\left(\frac{A}{P}, \tilde{i}, T^t\right) \\ &+ \sum_{m \in \mathscr{B}, h \in \mathscr{H}_N} u_{mh}^{\text{gen}} P_{s_{mh}}^{R} \text{cost}_{mh}^{\text{gen}}\left(\frac{A}{P}, \tilde{i}, T_{mh}^{g}\right)\Bigg]\end{aligned} \tag{2}$$

$$\text{TIC}^{\text{Gas}} = \left(\frac{P}{A}, \tilde{i}, T\right) \sum_{ij \in \mathscr{P}_N} u_{ij}^{\text{Pipe}}\, L_{ij}^{\text{Pipe}}\, A_{ij}^{\text{Pipe}} \text{cost}_{ij}^{\text{Pipe}}\left(\frac{A}{P}, \tilde{i}, T^p\right) \tag{3}$$

NPV of investment in transmission lines and generating units are formulated by terms one and two of (2), respectively. NPV of investment in gas pipelines is formulated by (3). It is worth mentioning that the NPV of the investment cost of each facility is calculated according to its useful lifetime. Time value of money is modelled using time value equivalence factors [36].

The second term of (1) is total operation cost (TOC). TOC of electricity and gas in contingency state $c$ are denoted by $\text{TOC}_{\text{c}}^{\text{Elec}}$ and $\text{TOC}_{\text{c}}^{\text{Gas}}$, respectively, and are formulated by (4) and (5)

$$\begin{aligned}\text{TOC}_c^{\text{Elec}} = &\sum_{y \in Y_T} \left(\frac{P}{F}, \tilde{i}, y\right) \\ &\times \sum_{d \in D_y, t \in \mathscr{T}_D, m \in \mathscr{B}, h \in \mathscr{H}} d_t \begin{pmatrix} \left(1 - S_{mhydc}^{F}\right)\lambda_{mhydc}^{\text{FC}} \text{FC}_{mhydtc} \\ + S_{mhydc}^{F} \lambda_{mhydc}^{\text{sec-FC}} \text{FC}_{mhydtc}^{\text{Sec}} \end{pmatrix} \\ &+ \sum_{d \in D_y, t \in \mathscr{T}_D, m \in \mathscr{B}} P_{r_{mydtc}} \lambda_{r_{mydtc}}^{\text{Elec}} \quad \forall c \in \Omega\end{aligned} \tag{4}$$

$$\begin{aligned}\text{TOC}_c^{\text{Gas}} = &\sum_{y \in Y_T} \left(\frac{P}{F}, \tilde{i}, y\right) \\ &\times \sum_{d \in D_y, i \in \mathscr{N}} \left(\left(Q_{l_{iydc}}^{\text{Npp}} + Q_{l_{iydc}}^{\text{comp}}\right)\lambda_{l_{iydc}}^{\text{Gas}} + Q_{r_{iydc}} \lambda_{r_{iydc}}^{\text{Gas}}\right) \quad \forall c \in \Omega\end{aligned} \tag{5}$$

Term one of (4) represents the operation cost of the electricity network in both gas-driven and second fuel-driven modes. Cost of curtailed electricity loads is expressed by term two of (4). Terms one, two and three of (5) represent the TOC of non-power plant loads, total gas loss cost of compressors, and cost of curtailed gas loads, respectively.

The remaining term in (1) is weighted $\text{CVaR}_\alpha$ representing the risk of expected cost variability in different contingencies imposed to the gas–electricity system. $\text{CVaR}_\alpha$ is defined as the expected cost of the $(1-\alpha)$% highest-cost contingencies. More explanation on the concept of CVaR is provided in Appendix 1. $\text{CVaR}_\alpha$ for electricity and gas networks is described in (6) and (7), respectively [37]

$$\text{CVaR}_\alpha^{\text{Elec}} = \eta\left[\zeta^{\text{Elec}} + \frac{1}{\left(1 - \alpha^{\text{Elec}}\right)} \sum_{c \in \Omega} \mathbb{P}_c \delta_c^{\text{Elec}}\right] \tag{6}$$

$$\text{CVaR}_\alpha^{\text{Gas}} = \eta\left[\zeta^{\text{Gas}} + \frac{1}{\left(1 - \alpha^{\text{Gas}}\right)} \sum_{c \in \Omega} \mathbb{P}_c \delta_c^{\text{Gas}}\right] \tag{7}$$

### 2.2 Contingency analysis

In the proposed RGEEP model, outage probability of components are considered in security constraints to avoid over-investment in gas and electricity facilities [38]. To cover this issue, contingency states in transmission lines, generating units and gas pipelines are analysed in the proposed RGEEP model. Wherein, in each contingency, single outage of components is considered due to the higher probability of occurrence. Reliability indices as forced outage rate (FOR) and mean time to repair of components are used to model the effect of each contingency on the coordinated expansion planning. Gas pipeline outage can affect the fuel adequacy of generating units. Hence, both second fuel and load shedding opportunities are considered to study the interactions between electricity and gas networks in the case of contingencies and provide an expansion plan that can mitigate components unavailability with a minimum cost. Binary parameters

$$\begin{aligned}&\text{Min}\ \ C^{\text{RGEEP}} \\ &= \overbrace{\left(\text{TIC}^{\text{Elec}} + \text{TIC}^{\text{Gas}}\right)}^{\text{TINV}} + \overbrace{\sum_{c \in \Omega} \mathbb{P}_c\left(\text{TOC}_c^{\text{Elec}} + \text{TOC}_c^{\text{Gas}}\right)}^{\text{ETOC}} + \overbrace{\left(\text{CVaR}_\alpha^{\text{Elec}} + \text{CVaR}_\alpha^{\text{Gas}}\right)}^{\text{CVaR}_\alpha}\end{aligned} \tag{1}$$

$\mathrm{UP}_{ijydc}$, $\mathrm{UT}_{mnydtc}$ and $\mathrm{UG}_{mhydtc}$ are used to define the contingency status of each component in which, 0 and 1 indicate unavailability and availability of each component during a contingency, respectively. FOR is employed to calculate the probability of each contingency indicated by $\mathbb{P}_c$ in the objective function (1). Besides, mean time to repair affects unavailability of equipment for specified times of a year and it is dictated by the binary parameters $\mathrm{UP}_{ijydc}$, $\mathrm{UT}_{mnydtc}$ and $\mathrm{UG}_{mhydtc}$, remaining 0 during repair time.

Constraints of the proposed RGEEP model are described in the following subsections as constraints (8)–(28). Constraints (8)–(28) must be repeated for each day of each year of the planning period. So, expression $\forall d \in D_y, \forall y \in Y_T$ is omitted from the front of constraints (8)–(28) for the sake of simplicity.

### 2.3 Electricity system constraints

To ensure a feasible operation of the electricity network, DC load flow equations are considered as constraints. Error of DC load flow is acceptable for expansion planning in high voltage grids [39]. Operation constraints of electricity network are formulated as follows:

$$\sum_{h \in \mathscr{H}} P_{s_{mhydtc}} = \sum_{n\mathscr{B}} P_{f_{mnydtc}} + P_{l_{mydtc}} - P_{r_{mydtc}} \quad \forall t \in \mathscr{T}_D, m \in \mathscr{B}, c \in \Omega \tag{8}$$

$$P_{f_{mnydtc}} = UT_{mnydtc} u_{mn}^{\text{trans}} P_b \times y_{mn}(\theta_{mydtc} - \theta_{nydtc}) \quad \forall t \in \mathscr{T}_D,\ m \in \mathscr{B}, c \in \Omega \tag{9}$$

$$\theta_{\text{ref}} = 0 \tag{10}$$

$$UG_{mhydtc} u_{mh}^{\text{gen}} \underline{P_{s_{mh}}} \le P_{s_{mhydtc}} \le UG_{mhydtc} u_{mh}^{\text{gen}} \overline{P_{s_{mh}}} \quad \forall t \in \mathscr{T}_D, m \in \mathscr{B}, h \in \mathscr{H}, c \in \Omega \tag{11}$$

$$-UT_{mnydtc} u_{mn}^{\text{trans}} \overline{P_{f_{mn}}} \le P_{f_{mnydtc}} \le UT_{mnydtc} u_{mn}^{\text{trans}} \overline{P_{f_{mn}}} \quad \forall t \in \mathscr{T}_D, c \in \Omega, mn \in \mathscr{K} \tag{12}$$

$$0 \le P_{r_{mydtc}} \le P_{l_{mydtc}} \quad \forall t \in \mathscr{T}_D, m \in \mathscr{B}, c \in \Omega \tag{13}$$

$$\mathrm{FC}_{mhydtc} = \frac{u_{mh}^{\text{gen}} \partial_{mh} + \beta_{mh} P_{s_{mhydtc}} + \gamma_{mh} P_{s_{mhydtc}}^2}{\mathrm{GHV}_h} \quad \forall t \in \mathscr{T}_D, m \in \mathscr{B}, h \in \mathscr{H}, c \in \Omega \tag{14}$$

$$\mathrm{FC}_{mhydtc}^{\text{Sec}} = \frac{u_{mh}^{\text{gen}} \partial_{mh}^{\text{Sec}} + \beta_{mh}^{\text{Sec}} P_{s_{mhydtc}} + \gamma_{mh}^{\text{Sec}} P_{s_{mhydtc}}^2}{\mathrm{GHV}_h^{\text{Sec}}} \quad \forall t \in \mathscr{T}_D, m \in \mathscr{B}, h \in \mathscr{H}, c \in \Omega \tag{15}$$

Power balance is ensured by (8) in each bus of the electricity network. Wherein, binary parameter $UT_{mnydtc}$ specifies the contingency status of transmission line $mn$. Power flow through transmission lines is expressed by (9). Reference bus angle is set to zero by constraint (10). Power generation of different units is limited by constraint (11) in which, binary parameter $UG_{mhydtc}$ determines the contingency status of generating units. Power flow constraints are represented by (12). Load curtailment is limited by (13). Constraints (14) and (15) determine the consumption of gas and second fuel by generating units, respectively. $\mathrm{GHV}_h$ and $\mathrm{GHV}_h^{\text{Sec}}$ are gross heating value of natural gas and second fuel of unit $h$ expressing the amount of heat that is released from 1 Million Standard Cubic Meters (MSCM) of natural gas/second fuel, respectively.

### 2.4 Gas network constraints

Steady-state gas flow equations are considered to ensure feasible operation of the gas network. In this paper, the presented model in [7] is generalised to consider compressor losses. Gas network constraints are described as follows:

$$Q_{f_{ijydc}}^2 = UP_{ijydc} u_{ij}^{\text{Pipe}} K_{ij}^{\text{pipe}^2} \operatorname{sign}\left(Q_{f_{ijydc}}\right)\left(\pi_{iydc}^g - \pi_{jydc}^g\right)$$
$$\operatorname{sign}\left(Q_{f_{ijydc}}\right) = \begin{cases} 1, \ pr_{iydc}^g \ge pr_{jydc}^g & , \forall ij \in \mathscr{P}_p \\ -1, \ pr_{iydc}^g < pr_{jydc}^g & , c \in \Omega \end{cases} \tag{16}$$

$$Q_{f_{ijydc}}^2 \ge UP_{ijydc} u_{ij}^{\text{Pipe}} K_{ij}^{\text{pipe2}} \left(\pi_{jydc}^g - \pi_{iydc}^g\right) \quad \forall ij \in \mathscr{P}_A, c \in \Omega \tag{17}$$

$$UP_{ijydc} u_{ij}^{\text{Pipe}} \underline{Q_{f_{ij}}} \le Q_{f_{ijydc}} \le UP_{ijydc} u_{ij}^{\text{Pipe}} \overline{Q_{f_{ij}}} \quad \forall ij \in \mathscr{P}_P, c \in \Omega \tag{18}$$

$$0 \le Q_{f_{ijydc}} \le UP_{ijydc} u_{ij}^{\text{Pipe}} \overline{Q_{f_{ij}}} \quad \forall ij \in \mathscr{P}_A, c \in \Omega \tag{19}$$

$$\underline{Q_{S_i}} \le Q_{S_{iydc}} \le \overline{Q_{S_i}} \quad \forall i \in \mathscr{N}, c \in \Omega \tag{20}$$

$$\underline{pr_i^g} \le pr_{iydc}^g \le \overline{pr_i^g} \quad \forall i \in \mathscr{N}, c \in \Omega \tag{21}$$

$$0 \le Q_{r_{iydc}} \le Q_{l_{iyd}}^{Npp} \quad \forall i \in \mathscr{N}, c \in \Omega \tag{22}$$

$$\sum_{j \in \mathscr{N}} Q_{f_{ijydc}} = Q_{S_{iydc}} - \left(Q_{l_{iydc}}^{pp} + Q_{l_{iydc}}^{Npp} - Q_{r_{iydc}} + Q_{\mathrm{l}_{iydc}}^{\text{comp}}\right) \quad \forall i \in \mathscr{N}, c \in \Omega \tag{23}$$

$$pr_{iydc}^g \le pr_{jydc}^g \le R_{ij}^{\text{comp}} pr_{iydc}^g \quad \forall ij \in \mathscr{P}_A, c \in \Omega \tag{24}$$

$$Q_{l_{iydc}}^{\text{comp}} = l_{ij}^{\text{comp}} Q_{f_{ijydc}}\left(pr_{jydc}^g - pr_{iydc}^g\right) \quad \forall ij \in \mathscr{P}_A, c \in \Omega \tag{25}$$

Weymouth equations are expressed by constraints (16) and (17) [40], indicating the relation between gas flow and pressure difference in passive pipelines, i.e. pipelines without a compressor, and active pipelines, i.e. pipelines with a compressor, respectively. Gas flow restrictions in passive and active pipelines are specified by constraints (18) and (19), respectively. In which, constraint (19) specifies the unidirectional flow in active pipelines. Gas supply limitations at source nodes are set by (20). Gas pressure bounds at each node are expressed by (21). Non-power plant gas load curtailment is restricted by (22). Gas flow balance in each node of the gas network is expressed using (23). Binary parameter $UP_{ijy\mathrm{dc}}$ imposes the contingency status of pipeline $ij$. One node before and one node after of each compressor are considered to model pressure ratio limitation of each compressor by (24). Gas consumption of compressor is approximated using (25) [41].

### 2.5 Gas–electricity coupling constraint

All the constraints in the expansion planning of electricity and gas networks are local, except the constraints that limit the requested gas fuel at each electricity bus from the related gas node. These constraints couple gas and electricity networks and determine the upper limit of gas quantity that gas nodes can deliver to the GCGU of the related bus. The coupling constraints can be formulated as follows:

$$\sum_{\hat{h} \in \mathscr{H}_G, t \in \mathscr{T}_D} d_t FC_{m\hat{h}ydtc} \le Q_{l_{iydc}}^{pp} \quad \forall im \in \mathscr{L}, c \in \Omega \tag{26}$$

### 2.6 Risk modelling

Contingencies occurring in transmission lines, generating units and pipelines impose risk to the expansion planning decisions of electricity and gas networks. In the proposed model, risk of expansion cost variability in different contingencies is minimised as well as the total expected cost. CVaR is used as a criterion for measuring the risk level. CVaR at the confidence level $\alpha$ ($\mathrm{CVaR}_\alpha$)

is calculated by the last term of the objective function (1) and constraints (27) and (28) [37]

$$\delta_c \geq 0 \quad \forall c \in \Omega \tag{27}$$

$$\delta_c \geq \mathrm{TIC} + \mathrm{TOC}_c - \zeta \quad \forall c \in \Omega \tag{28}$$

## 3 Solution methodology

ADMM method is used to decompose the optimisation problem (1) into two independent optimisation problems for gas and electricity networks. In the following, ADMM algorithm for decomposing the integrated optimisation problem is described.

### 3.1 ADMM method

Convex optimisation problems in the following separable format can be decomposed by the ADMM method [42]:

$$\mathrm{Min}\ f(x) + g(y) \quad x \in X, y \in Y \tag{29}$$

$$\begin{aligned} &\mathrm{s.t.} \\ &Ax + By = c \end{aligned} \tag{30}$$

Constraint (30) can be relaxed and augmented in the objective function as follows:

$$\begin{aligned} &\mathrm{Min}\ L_\rho = f(x) + g(y) + \boldsymbol{\mu}^{\mathrm{T}}(Ax + By - c) + \frac{\rho}{2} \parallel Ax + By - c \parallel_2^2 \\ &x \in X, y \in Y \end{aligned} \tag{31}$$

Wherein, $\boldsymbol{\mu}$ is the Lagrangian multiplier vector corresponding to the coupling constraint (30), constant $\rho$ is a penalty factor which is obtained through experiences, and $\|.\|_2$ is $l_2$-norm of supposed vector.

Optimisation problem (31) can be decomposed into following problems:

$$x^{k+1} = \arg\min \left(L_\rho, y^k, \mu^k\right) \quad x \in X \tag{32}$$

$$y^{k+1} = \arg\min \left(L_\rho, x^k, \mu^k\right) \quad y \in Y \tag{33}$$

$$\mu^{k+1} = \mu^k + \rho\left(Ax^{k+1} + By^{k+1} - c\right) \tag{34}$$

Local variables of each sub-problem are specified with $k+1$ and variables of the other sub-problem, which are considered as constants, are specified with k. In the decentralised method based on ADMM, (32) and (33) are solved separately based on the initial conditions assumption. Then, $x^{k+1}$ and $y^{k+1}$ are sent to the coordinator, coordinator updates $\mu$ using (34), $\mu^{k+1}$ and $y^{k+1}$ are sent to the first optimisation handler, $\mu^{k+1}$ and $x^{k+1}$ are sent for the second optimisation handler, $k$ increases by 1, and the process is repeated till the repetitive process reaches to the optimum values of the problem (29)–(30) [42]. Stop criterion for the convergence of ADMM method is [42]:

$$\parallel Ax^{k+1} + By^{k+1} - c \parallel_2 \ \leq \varepsilon \tag{35}$$

The ADMM method is originated from dual methods while it has advantages such as better convergence properties alongside with its parallel computation that makes it flexible in many applications. The ADMM method is reported to have a satisfactory computational performance [43] for non-convex problems. Also, heuristic algorithms as relax-round-polish [22] and tractable alternating optimisation procedure [44] have provided compromising results if the model cannot converge in a finite number of iterations. However, this matter is out of the scope of this paper.

### 3.2 Detailed formulation based on ADMM

So far, co-planning of electricity and gas networks is drawn from the viewpoint of a central entity as ME that has access to the data required for the planning process. The central entity performs the expansion planning of electricity and gas networks by solving the proposed optimisation problem, i.e. the objective function (1) subject to the constraints (2)–(28). Although electricity and gas systems are financially sub-organisations of ME, in some countries ME has no authority on operation and expansion of these systems. In these countries, electricity and gas systems are expanded independently. To coordinate the expansion of electricity and gas networks in such countries, the proposed RGEEP model is decomposed into two optimisation problems for electricity and gas networks and they are solved with limited data exchange, preserving their privacy.

In the proposed RGEEP model, the objective function can be decomposed into two independent terms for electricity and gas networks, and all the variables and constraints are local except constraint (26) that relates electricity and gas networks together. In other words, the RGEEP model is in the form of problems (29) and (30). Hence, ADMM can be used as a decomposition method to handle the interdependency of electricity and gas networks [42]. ADMM coordinates the expansion planning of electricity and gas networks with minimum data exchange and takes into account contingencies and risk constraints. This way, coupling constraint (26) is relaxed and augmented into the objective functions of electricity and gas networks as a penalty. Electricity and gas network operators solve their optimisation problems in a repetitive process until they converge with an agreement. A central coordinator as ME helps the network operators to minimise disagreements. Based on the ADMM method, decomposed optimisation for electricity network is as follows:

$$\begin{aligned} &\mathrm{Min}\ \mathrm{TIC}^{\mathrm{Elec}} + \sum_{c \in \Omega} \mathbb{P}_c \times \mathrm{TOC}_c^{\mathrm{Elec}^{k+1}} + \mathrm{CVaR}_\alpha^{\mathrm{Elec}^{k+1}} \\ &+ \sum_{c \in \Omega} \mathbb{P}_c \sum_{y \in Y_T} \left(\frac{P}{F}, \tilde{i}, y\right) \\ &\times \overbrace{\begin{aligned} &\sum_{d \in D_y, im \in \mathscr{L}} \mu_{imydc}^k \left( Q_{l_{iydc}}^{pp\ k} - \sum_{\hat{h} \in \mathscr{H}_G, t \in \mathscr{T}_D} d_t \mathrm{FC}_{m\hat{h}ydtc}^{k+1} \right) \\ &+ \frac{\rho}{2} \sum_{d \in D_y, im \in \mathscr{L}} \left\| Q_{l_{iydc}}^{pp\ k} - \sum_{\hat{h} \in \mathscr{H}_G, t \in \mathscr{T}_D} d_t \mathrm{FC}_{m\hat{h}ydtc}^{k+1} \right\|_2^2 \end{aligned}}^{\text{Augmented term according to the ADMM method}} \end{aligned} \tag{36}$$

$$\begin{aligned} &\mathrm{s.t.} \\ &(8) - (15) \end{aligned} \tag{37}$$

$$\delta_c^{\mathrm{Elec}k+1} \geq 0 \quad \forall c \in \Omega \tag{38}$$

$$\delta_c^{\mathrm{Elec}^{k+1}} \geq \mathrm{TIC}^{\mathrm{Elec}} + \mathrm{TOC}_c^{\mathrm{Elec}^{k+1}} - \zeta^{\mathrm{Elec}^{k+1}} \quad \forall c \in \Omega \tag{39}$$

In the same manner, decomposed optimisation for the gas network is as follows:

$$\begin{aligned} &\mathrm{Min}\ \mathrm{TIC}^{\mathrm{Gas}} + \sum_{c \in \Omega} \mathbb{P}_c \times \mathrm{TOC}_c^{\mathrm{Gas}^{k+1}} + \mathrm{CVaR}_\alpha^{\mathrm{Gas}^{k+1}} \\ &+ \sum_{c \in \Omega} \mathbb{P}_c \sum_{y \in Y_T} \left(\frac{P}{F}, \tilde{i}, y\right) \\ &\times \overbrace{\begin{aligned} &\sum_{d \in D_y, im \in \mathscr{L}} \mu_{imydc}^k \left( Q_{l_{iydc}}^{pp\ k+1} - \sum_{\hat{h} \in \mathscr{H}_G, t \in \mathscr{T}_D} d_t\ FC_{m\hat{h}ydtc}^k \right) \\ &+ \frac{\rho}{2} \sum_{d \in D_y, im \in \mathscr{L}} \left\| Q_{l_{iydc}}^{pp\ k+1} - \sum_{\hat{h} \in \mathscr{H}_G, t \in \mathscr{T}_D} d_t FC_{m\hat{h}ydtc}^k \right\|_2^2 \end{aligned}}^{\text{Augmented term according to the ADMM method}} \end{aligned} \tag{40}$$

$$
\begin{aligned}
&\text{s.t.}\\
&(16)-(25)
\end{aligned} \tag{41}
$$

$$
\delta_c^{\mathrm{Gas}^{k+1}} \geq 0 \quad \forall c \in \Omega \tag{42}
$$

$$
\delta_c^{\mathrm{Gas}^{k+1}} \geq \mathrm{TIC}^{\mathrm{Gas}} + \mathrm{TOC}_c^{\mathrm{Gas}^{k+1}} - \zeta^{\mathrm{Gas}^{k+1}} \quad \forall c \in \Omega \tag{43}
$$

$\mathrm{CVaR}_\alpha$ constraints (27) and (28) are generalised and repeated as constraints (38)–(39) and constraints (42)–(43) for electricity and gas networks, respectively. Lagrangian multipliers are used to coordinate electricity and gas networks in the proposed ADMM method. In this regard, the dual variable $\mu_{im}$ is updated by the coordinator using the sub-gradient method as follows [42]:

$$
\begin{aligned}
&\mu_{imydc}^{k+1} = \mu_{imydc}^{k} + \rho\left(Q_{l_{iydc}}^{pp\ k+1} - \sum_{\hat{h} \in \mathscr{H}_G, t \in \mathscr{T}_D} d_t FC_{m\hat{h}ydtc}^{k+1}\right)\\
&\forall im \in \mathscr{L}, c \in \Omega
\end{aligned} \tag{44}
$$

The resulted sub-problem optimisations, i.e. (36)–(39) for the electricity network operator and (40)–(43) for gas network operator, are mixed integer non-linear programming (MINLP) problems and can be solved using the branch-and-reduce algorithm. In electricity and gas sub-problems, local variables of each sub-problem are specified with $k+1$ and variables of the other sub-problem, which are considered as constants, are specified with k. Consequently, in the electricity network sub-problem, $Q_{l_{iydc}}^{pp}$ and $\mu_{imydc}$ are supposed to be constants that are gathered from coordinator. On the other hand, in the gas network sub-problem, $\mathrm{FC}_{m\hat{h}ydtc}$ and $\mu_{imydc}$ are supposed as constants and are gathered from the coordinator. The difference between $\mathrm{FC}_{m\hat{h}ydtc}$ and $Q_{l_{iydc}}^{pp}$ is minimised in a repetitive process between electricity and gas sub-problems.

Coordinator stops the repetitive process considering two criteria: (i) disagreement between the energy parties on the required gas of electricity network in each bus and the gas that gas network can deliver to the related node is insignificant; and (ii) changes in the shared variable between gas and electricity networks – gas consumption of GCGUs – at each bus in two consecutive iterations is small enough. These criteria are defined as follows:

$$
\begin{aligned}
&\left\| \sum_{\hat{h} \in \mathscr{H}_G, t \in \mathscr{T}_D} d_t FC_{m\hat{h}ydtc}^{k+1} - Q_{l_{iydc}}^{ppk+1} \right\|_2^2 \leq \varepsilon_1 \quad \forall im \in \mathscr{L}, c\\
&\in \Omega
\end{aligned} \tag{45}
$$

$$
\begin{aligned}
&\left\| \sum_{\hat{h} \in \mathscr{H}_G, t \in \mathscr{T}_D} d_t \mathrm{FC}_{m\hat{h}ydtc}^{k+1} - \sum_{\hat{h} \in \mathscr{H}_G, t \in \mathscr{T}_D} d_t \mathrm{FC}_{m\hat{h}ydtc}^{k} \right\|_2^2 \leq \varepsilon_2\\
&m \in \mathscr{B}, c \in \Omega
\end{aligned} \tag{46}
$$

Constraints (44)–(46) are applied for each day $d$ of each year $y$ of the planning period. Flowchart of the proposed ADMM method is shown in Fig. 2.

## 4 Case study

This section examines the proposed method on a realistic case study in Khorasan province of Iran.

### 4.1 Data

Proposed RGEEP model is examined on a real-world case study in Khorasan province of Iran. Because of local gas resources, the proposed case study has a high penetration level of GCGUs. Khorasan electricity network includes 19 transmission lines that are connecting 17 buses together. On the other hand, there are 33 GCGUs that are placed in 7 buses. Also, there are 16 nodes in the gas network that are connected together through 15 pipelines. A planning period of 15 years is supposed wherein, demand in both electricity and gas sectors grow 3% annually. Current load level in electricity network is 3129 MW and there is a maximum generation capacity of 3880 MW. On the other hand, there is a consumption rate of 39.133 MSCM per Day (MSCMD) in other gas consuming sectors than GCGUs. Existing pipelines, transmission lines, and generating units and their expansion candidates are depicted in Fig. 3. Different regions of the Khorasan case study are specified with letters A to T in Fig. 3. Name of gas network nodes and electricity network buses are the name of the related zone. Further information of the proposed electricity and gas networks are given in [9, 45], respectively. Investment cost of transmission lines, GCGUs, and pipelines are given in [4]. Reliability indices of the components are provided in Appendix 2.

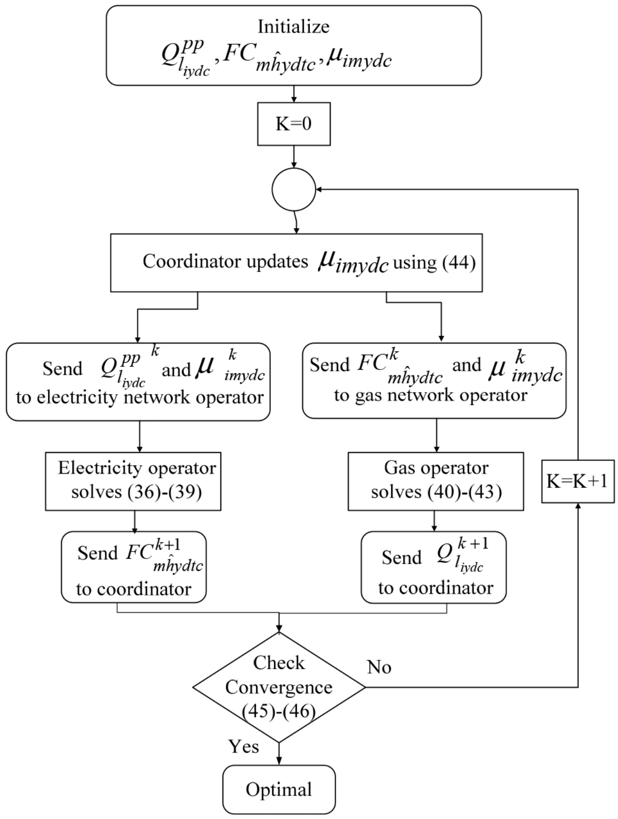


**Fig. 2** *Flowchart of the proposed method for coordinated expansion of electricity and gas networks*

### 4.2 Results

The proposed ADMM algorithm is implemented in GAMS software where the Baron solver is used to solve the MINLP optimisation problem using the branch-and-reduce algorithm. The solving procedure is handled in a desktop computer with 2.5 GHz processor and 4 GB RAM [46].

We examine five cases on the Khorasan case study to evaluate the effectiveness of proposed RGEEP model. The proposed cases are supposed as follows to see how contingencies, second fuel, and risk-based planning can affect the expansion planning of electricity and gas networks.

- *Case 1:* RGEEP model considering second fuel and CVaR in both electricity and gas networks.
- *Case 2:* RGEEP model without second fuel while considering CVaR in both electricity and gas networks.
- *Case 3:* RGEEP model considering second fuel and risk-averse electricity network investor with $\eta = 1$, and risk-seeking gas network investor with $\eta = 0$.
- *Case 4:* RGEEP model considering second fuel and a risk-averse gas network investor with $\eta = 1$ and a risk-seeking electricity network $\eta = 0$.

- *Case 5:* Deterministic case without considering contingencies.

Expansion planning of the proposed case study is performed using the central and decentralised methods based on ADMM. Comparing the results of centralised and decentralised methods show that both the ADMM and central methods produce the same results, while, in the proposed decentralised expansion planning method, data privacy of electricity and gas networks is preserved. The convergence process of the proposed RGEEP model using the ADMM method is shown in Fig. 4, where, the convergence process for electricity and gas networks is represented according to the total cost (TC) of case 1. TC* stands for the TC using the centralised method. It can be seen that consensus is achieved after 11 iterations using the decentralised expansion planning method. In Fig. 4, because of gas fuel unavailability in the first iterations, there is a large amount of operation cost for electricity network that leads to a high peak in the electricity network graph. However, this is not the case for the gas network.

Although expansion planning is an offline task, processing times as well as the number of variables and equations in different cases are given in Table 1 to give a view of the size and complexity of the problem.

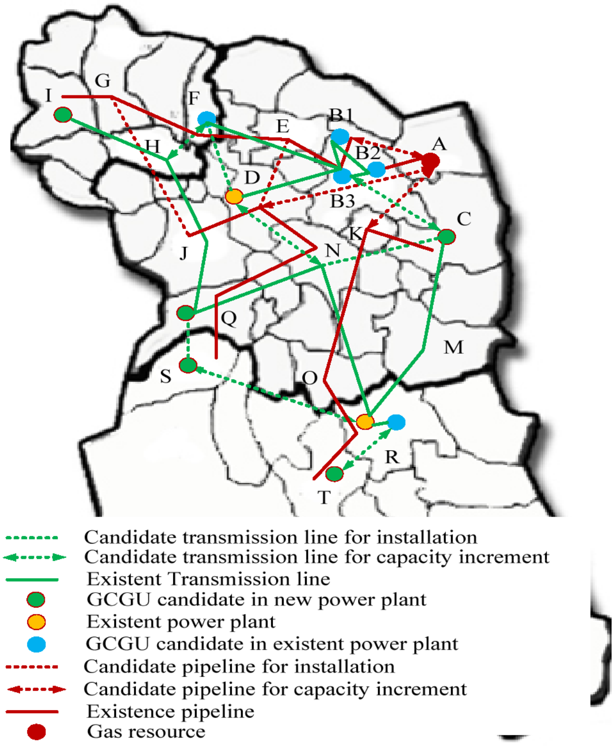


**Fig. 3** *Khorasan gas–electricity networks*

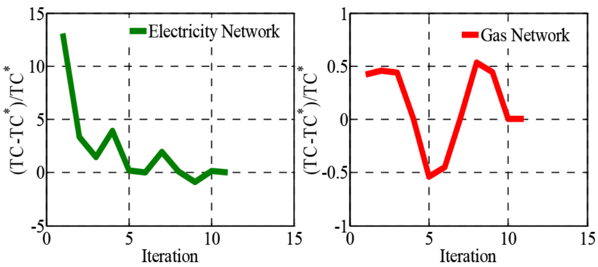


**Fig. 4** *Convergence process in the proposed decentralised method*

Expansion planning results of the proposed RGEEP model in case 1 including NPV of TC, investment cost, selected transmission line/pipeline candidates, selected generation candidates, CVaR and total capacity/length of new generation/pipeline installations are given in Table 2. In the radial gas network of the Khorasan case study, by installing new pipelines from the main source, i.e. A-B1, A-K and A-D pipelines, the system becomes robust against any single outage of the mentioned pipelines. Although E-D pipeline is an expensive choice, the risk-averse gas network operator installs the new pipeline E-D to become more reliable as a ring network and possessing a lower risk level, as shown in Table 2. Expansion planning results for the electricity network also shows that the proposed method seeks for an expansion plan with lower risk, by installing new generating units in different regions of the case study. Table 2 shows that selected transmission lines remain unchanged in all cases.

TC of electricity and gas networks in different cases are compared in Fig. 5. In Fig. 5, it is apparent that considering contingencies have increased the TC of both electricity and gas networks in case 1 comparing to case 5. On the other hand, omitting the second fuel in case 2 has led to a peak in both electricity and gas TC. By comparing cases 1, 3 and 4 in Fig. 5, it is shown that considering CVaR in the expansion planning studies has increased the TC of risk-averse planners. To better investigate the proposed RGGEP model, cases 1 to 5 are studied from different viewpoints in more detail in the following.

*Comparing cases 1 and 5 in term of probabilistic contingencies:* To evaluate the effect of contingencies on the case study, results of the probabilistic case 1 are compared with those of the deterministic case 5 in Table 2. Results show that in the deterministic case 5, by installing new generations in the populated region B2 and also in the corner part of the system, i.e. F and S, the need for new generating units is satisfied. However, by considering contingencies, electricity network planner prefers to add new installations in different regions of the system to avoid load curtailment in the contingency scenarios. Also, obtained results in the deterministic case show that gas network planner prefers to remain radial to avoid extra expansion cost. Comparing the TC of either gas or electricity network in cases 1 and 5 indicates that considering contingencies lead to more cost due to the load curtailments.

*Comparing cases 1 and 2 in term of reserve fuel:* To better investigate the effect of reserve fuel in the proposed case study, expansion planning results with and without considering reserve fuel, i.e. cases 1 and 2, are compared. Case 1 as a base case shows minimum new candidates that are required to satisfy the capacity adequacy. Obtained results in Table 2 show that considering reserve fuel reduces the expansion cost and capacity of generation candidates in the electricity network. By considering reserve fuel, almost 1600 MW lower capacity is needed in the electricity network to satisfy the capacity adequacy, even taking into account contingencies. However, in the gas network, as electricity network's reserve fuel is used only in short times, it does not affect the gas network expansion plans and they are the same in both cases 1 and 2. Results in Table 2 show that in case 2, operation cost of gas network is higher than case 1. Since, by considering reserve fuel in case 1, the fuel demand of electricity network is supplied by reserve fuel that avoids more load curtailment in the gas network. Also in case 2 comparing to case 1, the extra capacity requirement of the electricity network is almost concentrated in locations B1, B2 and I and capacity requirement in C is vanished; because, with

**Table 1** Numbers of variables, discrete variables, equations, and processing time of RGEEP model in different cases

| Case | Case 1 | | Case 2 | | Case 3 | | Case 4 | | Case 5 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Elec. | Gas | Elec. | Gas | Elec. | Gas | Elec. | Gas | Elec. | Gas |
| variables | 33,540 | 7830 | 33,540 | 7830 | 33,540 | 7830 | 33,540 | 7830 | 269 | 108 |
| discrete variables | 1027 | 16 | 811 | 16 | 1027 | 16 | 1027 | 16 | 17 | 6 |
| equation | 37,775 | 1837 | 37,775 | 1837 | 37,775 | 1837 | 37,775 | 1837 | 299 | 22 |
| processing time, s | 12,240 | | 8301 | | 11,836 | | 11,505 | | 3405 | |

**Table 2** Results of the proposed method in different cases

| Case | Case 1 | | Case 2 | | Case 3 | | Case 4 | | Case 5 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Elec. $\eta = 1$ | Gas $\eta = 1$ | Elec. $\eta = 1$ | Gas $\eta = 1$ | Elec. $\eta = 1$ | Gas $\eta = 0$ | Elec. $\eta = 0$ | Gas $\eta = 1$ | Elec. | Gas |
| total cost ($\$10^9$) | 5.65 | 25.43 | 6.35 | 25.59 | 5.64 | 24.35 | 5.43 | 25.25 | 4.49 | 21.64 |
| investment cost ($\$10^9$) | 1.56 | 1.9 | 2.46 | 1.9 | 1.56 | 0.57 | 1.35 | 1. 9 | 0.7 | 0.02 |
| selected trans. line and pipeline candidates | F-H | E-D | F-H | E-D | F-H | A-B1 | F-H | E-D | F-H | A-B1 |
| | | A-B1 | | A-B1 | | | | A-B1 | | |
| | B3-C | A-K | B3-C | A-K | B3-C | A-K | B3-C | A-K | B3-C | |
| | D-N | A-D | D-N | A-D | D-N | A-D | D-N | A-D | D-N | |
| selected generation candidates (location/capacity (MW)) | I/600 | — | I/1000 | — | I/600 | — | I/600 | — | F/600 | — |
| | B2/400 | | B2/800 | | B2/400 | | B2/400 | | S/400 | |
| | C/1000 | | C/800 | | C/1000 | | C/400 | | B2/600 | |
| | Q/800 | | Q/800 | | Q/800 | | Q/800 | | | |
| | R/400 | | R/800 | | R/400 | | R/400 | | | |
| | B1/400 | | B1/1000 | | B1/400 | | B1/400 | | | |
| new installation | 3600 MW | 298 km | 5200 MW | 298 km | 3600 MW | 151 km | 3000 MW | 298 km | 1600 MW | 32 km |
| CVaR ($\$10^9$) | 11.55 | 37.71 | 11.49 | 38.96 | 11.54 | 76.19 | 16.69 | 42.48 | — | — |

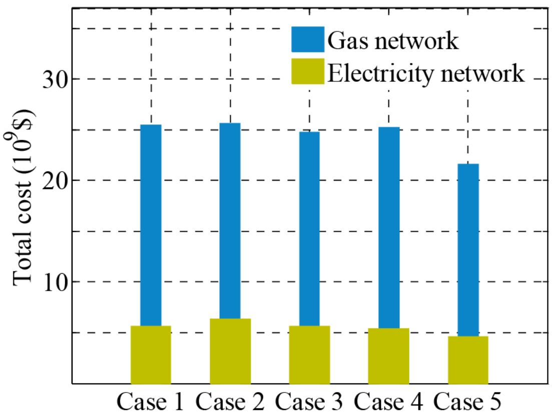


**Fig. 5** *TC of electricity and gas networks in different cases*

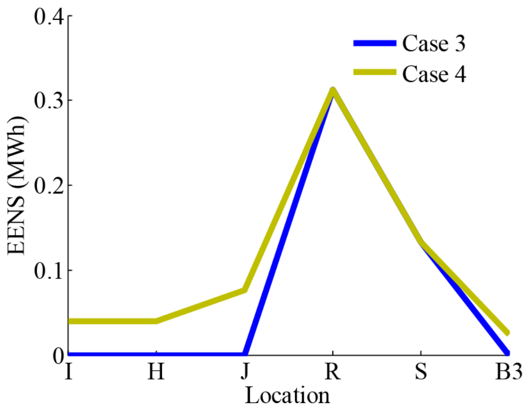


**Fig. 6** *EENS of electricity network in cases 3 and 4*

new selected pipelines, the upper corridor of gas network is more reliable as a ring network.

*Comparing cases 3 and 4 in term of* expected energy not supplied (*EENS):* By probabilistic contingency analysis of electricity and gas networks, system reliability can be studied in term of EENS [38]. EENS of the proposed electricity and gas networks is shown in Figs. 6 and 7, respectively. As Fig. 6 shows, considering a risk-averse planner in the electricity network, in case 3 comparing to case 4, leads to an expansion plan with lower EENS. Similarly, Fig. 7 illustrates that considering a risk-averse gas planner in case 4 comparing to case 3, leads to an expansion plan with lower EENS. However, some regions of electricity network are subjected to EENS in both cases 3 and 4, as shown in Fig. 6. The reason for this is that by considering contingencies in transmission lines R1-R2 and also R2-S, load curtailment occurs in these regions of electricity network when a contingency occurs.

On the other hand, results in Table 2 show that the gas network prefers to keep the radial network in case 3 and accepts more risk. Fig. 7 shows that in case 3, some regions of the gas network such as F, G, T and J are subjected to high risk because of a radial network. However, by choosing a ring network in case 4, EENS of the gas network decreases considerably, as it is shown in Fig. 7. Results indicate that the proposed RGEEP model provides a trade-off between EENS and investment cost.

*Comparing cases 3 and 4 in term of CVaR:* In this part, the impact of risk-averse investors and resultant interactions between electricity and gas networks are investigated. Using the methodology introduced in this paper, each of the network operators can adjust their risk aversion level independently. It means that they have the freedom to choose their risk aversion level $\eta$ in their decision making process. Cases 3/4 refer to a condition in which electricity/gas networks operator decides to be risk averse ($\eta = 1$) and gas/electricity networks operator decides to be risk seeking ($\eta = 0$). Indeed, in these cases, interactions between gas and electricity networks are studied when only one of the network operators decides to consider CVaR. Values of CVaR in Table 2 show that in case 4 comparing to case 3, CVaR of electricity/gas networks and consequently its risk level increases/ decreases. This means that considering CVaR in the objective function decreases the system risk. Results show that by decreasing the risk of each network, its TC increases. In other words, by considering risk in the planning process of each network, the need for installing new facilities increases and consequently its TC increases. Results show that a risk-averse gas network planner in case 4 seeks to add more expansion candidates into the gas network comparing to case 3. In case 4, the gas network is more reliable as a ring network by installing the E-D pipeline. On the other hand, electricity network planner accepts more risk in case 4 by installing less expansion candidates comparing to case 3.

*Evaluation of confidence level's effect*: In the previous case studies the same confidence level of 0.95 was taken into account for both gas and electricity network operators. In the methodology described in this paper, however, they can adjust their risk level independently. Hence, although the typical value of $\alpha$ is 95% [37], here, two levels of 0.95 and 0.85 are examined as the alternatives for each of the network operators. In Table 3 it is demonstrated that how confidence level variations can affect the selected plan for generation, transmission and pipeline expansion of gas and electricity networks. Obtained results show that similar to the studied cases 3 and 4, this would change the expansion plans of both networks accordingly. When electricity/gas networks admit more risk by setting a lower level of confidence level, some of expansion candidates would be omitted. This, in turn, reveals that variation in the $\alpha$ changes the interval of worst-case scenarios over the expansion plans, meaning that by a lower/higher confidence

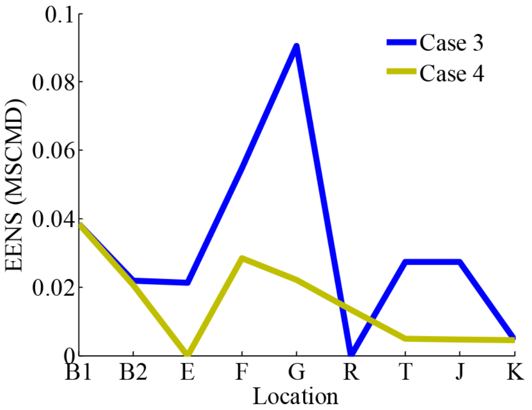


**Fig. 7** *EENS of gas network in cases 3 and 4*

**Table 3** Effect of Confidence level variations on expansion candidates

| Case | Elec. $\alpha = 0.95$ | Gas $\alpha = 0.85$ | Elec. $\alpha = 0.85$ | Gas $\alpha = 0.95$ |
|---|---|---|---|---|
| selected trans. line and pipeline candidates | F-H | A-B1 | F-H | E-D |
| | | | | A-B1 |
| | B3-C | A-K | B3-C | A-K |
| | D-N | A-D | D-N | A-D |
| selected generation candidates (Location/Capacity (MW)) | I/600 | — | I/600 | — |
| | B2/400 | | B2/400 | |
| | C/1000 | | C/800 | |
| | Q/800 | | Q/800 | |
| | R/400 | | R/400 | |
| | B1/400 | | B1/400 | |
| new installation | 3600 MW | 151 km | 3400 MW | 298 km |

level, more/less contingency states and consequently more/less risk are accepted by the decision maker.

## 5 Conclusion

The proposed RGEEP model provides a practical method in the expansion planning of electricity and gas networks in a multi-carrier energy network. Contingencies in transmission lines, generating units and pipelines as well as second fuel of GCGUs are considered to see how the interactions between electricity and gas network can affect the expansion plan. CVaR, as a risk measurement tool, is implemented into the objective function of electricity and gas networks to provide a risk-averse expansion plan. Using the proposed ADMM method, expansion planning of electricity and gas networks, which are performed separately, are coordinated and their privacies are preserved. Moreover, with the ADMM method, each of the network operators can consider their desired level of risk aversion.

Proposed RGGEP model is examined on a realistic case study in Iran in five cases. Simulation results show that the proposed decentralised RGEEP model converges to the solution of the central RGEEP model in a few iterations. On the other hand, by investigating different cases, it is shown that considering contingencies increase the need for new installations in both electricity and gas networks. In this regard, effect of second fuel in the case of contingencies is also considered. Second fuel in the electricity network decreases the TC of both electricity and gas networks as well as reducing the need for new generation installations. Analysing the effect of CVaR on the proposed case study shows that a risk-averse investor can decrease the risk of different contingencies by providing a trade-off between investment cost and EENS. Moreover, it is shown that decreasing the confidence level drives decision makers towards accepting more risk in their expansion plans.

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## 7 Appendix

### 7.1 Appendix 1: CVaR concept

To consider the risks in the objective function, the technical risks occurred by different contingencies in pipelines, generation units and transmission lines are transformed into an economic risk. In economy, value at risk (VaR) is a financial metric that measures the amount of potential cost that could happen in some scenarios over a given time period. In other words, $\text{VaR}_\alpha$ is the smallest value of the cost such that the probability that the actual cost exceeds or equals it is less than or equal to $1–\alpha$. $\text{CVaR}_\alpha$, however, is an extension of $\text{VaR}_\alpha$ and calculates average VaR associated to $(1-\alpha)\%$ contingencies with greatest costs [37]. These values are illustrated in Fig. 8. The objective in a cost minimisation expansion planning problem is to minimise the $\text{CVaR}_\alpha$ which can be defined as:

$$\text{CVaR}_\alpha = \text{Min}\ \zeta + \frac{1}{(1-\alpha)}\sum_{c \in \Omega}\mathbb{P}_c\delta_c \tag{47}$$

Subject to:

$$\delta_c \geq 0 \quad \forall c \in \Omega \tag{48}$$

$$\delta_c \geq \text{TIC} + \text{TOC}_c - \zeta \quad \forall c \in \Omega \tag{49}$$

The optimal value of $\zeta$ is the VaR which is in \$. Additionally, $\delta_c$ provides the excess of the cost in scenario $c$ over $\zeta$, which has a positive value and it is also in \$. Hence, the dimension of CVaR is \$.

### 7.2 Appendix 2: Reliability parameters

Equipment's FORs are given in Table 4. Repair time of pipelines is set to be 6 days and repair time of transmission lines as well as generation units are 10 and 45 h, respectively.

According to the FORs provided in Table 4, outage probabilities can be calculated using Bernoulli distribution as follows [38]:

$$\mathbb{P}_c = \text{FOR}_c \prod_{\substack{w \in \Omega \\ w \neq c}} (1-\text{FOR}_w) \quad \forall c \in \Omega \tag{50}$$

$$\mathbb{P}_0 = \prod_{w \in \Omega} (1-\text{FOR}_w) \tag{51}$$

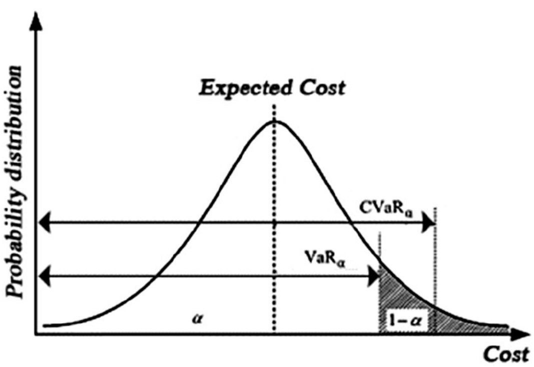


**Fig. 8** *CVaR description*

**Table 4** FOR for pipelines, transmission lines and generation units

| Gen. | FOR% | Trans. | FOR% | Pipe. | FOR% |
|---|---|---|---|---|---|
| F | 1.17 | I-H | 1.30 | A-B1 | 2.26 |
| B3 | 1.17 | F-H | 3.33 | A-B2 | 1.25 |
| B1 | 0.81 | F-B3 | 2.85 | B1-B3 | 2.23 |
| D | 1 | B3-D | 0.90 | B3-E | 2.13 |
| R | 0.47 | B3-C | 1.30 | E-F | 1.99 |
| B2 | 0.42 | A-K | 1.04 | F-G | 1.55 |
| | | B3-B2 | 3.33 | K-O | 1.51 |
| | | B1-B2 | 0.90 | O-R | 2.26 |
| | | H-J | 2.33 | R-T | 1.49 |
| | | D-J | 1.63 | A-D | 1.57 |
| | | D-N | 1.60 | D-J | 2.33 |
| | | C-M | 1.59 | D-N | 2.33 |
| | | J-Q | 1.57 | N-Q | 1.63 |
| | | N-Q | 1 | Q-S | 1.31 |
| | | N-R | 0.2 | G-I | 1.75 |
| | | M-R | 2.21 | G-I | 1.75 |
| | | R-S | 1.8 | | |
| | | R-T | 2.04 | | |