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Rule-based joint fuzzy and probabilistic networks

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Abstract

One of the important challenges in Graphical models is the problem of dealing with the uncertainties in the problem. Among graphical networks, fuzzy cognitive map is only capable of modeling fuzzy uncertainty and the Bayesian network is only capable of modeling probabilistic uncertainty. In many real issues, we are faced with both fuzzy and probabilistic uncertainties. In these cases, the proposed method of this paper can take into account both types of uncertainty with a new and different approach. In this method, we avoid fuzzy transformations to probabilities and vice versa, and fuzzy uncertainties and probabilities are considered jointly. For this purpose, in the original graphical model, first, the type of uncertainty of each node is identified, and accordingly two separate fuzzy and probabilistic networks are constructed. In these networks, relations between nodes are expressed in terms of a set of rules. In each network, fuzzy and probabilistic inference is individually constructed and ultimately the values obtained from each network are combined. This method has been tested on a real problem of localization in wireless sensor networks. In this case, a sensor with uncertain location should be able to predict its location from the received power of its adjacent sensors. In the given scenario, 60 sensors with uncertain locations and 121 sensors with a specific location are considered. Meanwhile, the average location error of sensors has been used to evaluate the methods. The simulation results show the efficiency of the proposed method well.

Keywords: Graphical models, fuzzy cognitive map, Bayesian network, fuzzy and probabilistic uncertainty, rules, wireless sensor network.

1 Introduction

One of the major goals of a mechanized and intelligent system is to replace a system with one or more human beings and to carry out the respective duties. In order to achieve this goal, the task and the role of man must be conceptualized into a machine or system and here an important issue called modeling is raised [22].

For modeling a system, various methods have been proposed such as StateSpace modeling, parametric and nonparametric models, neural networks, etc. [27].

One of the most widely used methods in this area is graphic models [1, 14]. In these models, concepts, states, inputs, outputs, etc. are represented as a series of nodes. The relationship between these nodes is determined by a series of link lines. Fuzzy cognitive maps (FCMs) in fuzzy models [13, 28] and Bayesian networks (BN) in probabilistic models [26] are among the most important graphical models.

Graphical models have the advantage that each node has a physical concept and reality and one can have an inference and decision making on each node. However, in some models, such as the neural network, some nodes have only mathematical aspects and do not have an external meaning. On the other hand, networks such as the neural network require learning data for construction and inference, but graphical networks can be created without learning data. Also, an expert knowledge and data is implemented in a much simpler and more understandable graphical model.

One of the most important issues in modeling a real system is the collision and modeling of uncertainties in the system. Uncertainties arise from various factors such as misunderstanding of a system, different expert opinions, various

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physical and environmental conditions of the system, sensor errors, etc. [17]. So far, various methods have been proposed to show various types of uncertainty, most notably the Fuzziness and Randomness [43].

In a real system, a variety of uncertainties such as incompleteness, Imprecision and uncertainty of data may occur jointly [11]. In such a situation, we are looking for a way to model all of these cases.

In graphical models, uncertainty is an important issue. So far, in most graphical modeling, fuzzy and probabilistic uncertainties have been modeled separately. Specifically, fuzzy cognitive maps have been used for fuzzy uncertainty [28] and Bayesian networks have been applied for probabilistic uncertainty [26]. Only few limited methods with serious limitations are proposed to consider both uncertainties. In these methods, or in a series of unrealistic assumptions that we encounter some problems in practice and in synchronous systems, or in the same model, we must use fuzzy and probabilistic transformations that cause transformation error and ignore a part of the uncertainty.

In this paper, an effective method with a different idea for dealing with both fuzzy and probabilistic uncertainties, called "Rule based joint fuzzy and probabilistic networks", is proposed. In this method, the fuzzy and probabilistic uncertainties are considered jointly and the relations between the concepts and network nodes are based on a series of rules. Since there are different types of uncertainties in the real systems, especially fuzzy and probabilistic uncertainties jointly, the proposed method is very popular.

In the following, a review of literature and other researcher's methods in this area are investigated. In the third section, the structure of the proposed method is presented and in the fourth section the efficiency of the proposed method is shown by simulation. Finally, conclusions and references are also presented.

2 A review of related works

In this section, the research of others in the field of fuzzy and probabilistic networks is quickly reviewed. At first graphic models are investigated and then some types of uncertainty and methods for considering both fuzzy and probabilistic uncertainties are reviewed by researchers.

2.1 Graphical models

Graphical models, model the real system with a series of nodes and lines. The nodes represent the concepts, variables, states, inputs, or outputs of the system and the liens show the relationship of the nodes. A graphical model is illustrated in Figure 1.

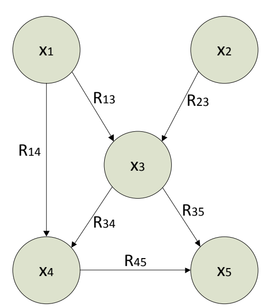

Figure 1: A view of a graphic model.

This mode of modeling has advantages like modeling of diverse systems, the capability of implementing human knowledge, no need to learning data, interpretability and decision making, flexibility, Information fusion and the capability to represent uncertainty.

These issues are well modeled in two well-known graphic models, fuzzy cognitive maps and Bayesian networks. In fact, FCM and BN are two important categories of graphical models that are well used for extracting information, displaying information, inference and modeling [9].

Fuzzy cognitive maps based on Lotfi A. Zadeh fuzzy logic and Robert Axelrod cognitive models by Bart Kosko were first introduced in 1986 [18].

In this method, a system with a series of nodes and directed edge is shown and modeled. The nodes represent the states, properties, inputs, outputs and effective parameters of system. The lines also show causal relationships between the nodes.

In fact, the FCM seeks to model a system as understood. These maps have received much attention from the researchers due to the ease of understanding and construction, flexibility, broad application and adaptability with a variety of issues. However, fuzzy cognitive maps have only the capability to model fuzzy uncertainty [35].

Bayesian networks based on the works of Thomas Base (ca. 1702-1762) is named on probability theory. Bayesian networks as a strong modeling tool were formed based on the Mr. Pearl's work and researches [31] and since then have been used in a variety of applications.

Bayesian Networks (also known as Bayesian Belief Network) is a graphical model that turns a system as probabilistic and can only model probabilistic uncertainty [31, 10]. In this model, nodes are random variables whose event in each state is expressed with a probability. The nodes represent the concepts and parameters of the system. In Bayesian networks, the relationship between the states of two nodes is expressed as probabilistic and by the conditional probability table (CPT) attributed to the connected nodes. This table is usually expressed by one or more experts or, in case of the availability of the system data, it can be obtained through learning [32].

2.2 Uncertainty

Uncertainty is one of the serious issues of the current systems. Uncertainty should be considered in addition to modeling a system. Moreover, uncertainty itself affects the choice of the model structure. Misunderstanding of a system, different expert opinions, various physical and environmental conditions of the system, sensor errors, data imperfection, different data modality, Outliers and spurious data, conflicting data, etc. are of the reasons of uncertainty in systems [38].

A classification of existing uncertainties and theories corresponding to each uncertainty is illustrated in Figure 2.

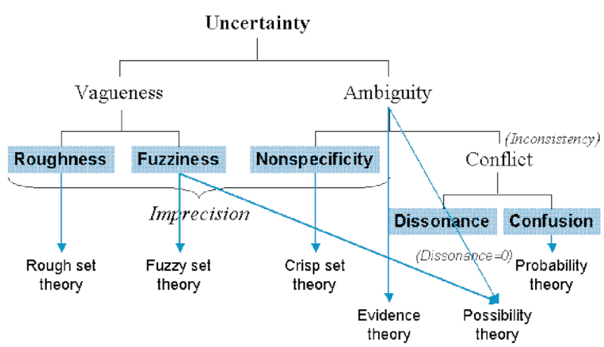

Figure 2: Types of uncertainty and corresponding theories [2].

In this form, the uncertainty is divided into two types: vagueness and ambiguity. To model vagueness uncertainty, one of the most important used theories is the theory of fuzzy sets. On the other hand, to model the uncertainty of ambiguity type, one of the most important theories is the theory of probabilities. Another form of uncertainty classification that is more practical in practice is shown in Figure 3 [11].

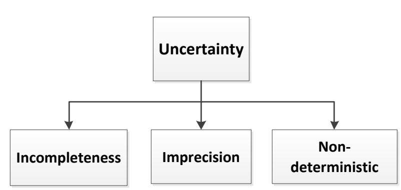

Figure 3: A practical classification of uncertainties [11].

In this classification, the "imperfect" uncertainty is when one part of the information is not expressed. "Inaccurate" is when a definitive but inaccurate information is expressed and "uncertain" is when an uncertain and unreliable information is expressed.

To display the inaccurate information, one can well use fuzzy theory and probability theory to display unreliable information.

As a result, it can be seen that both in the classification of Figure 2 and in the classification of Figure 3, fuzzy theory, along with probability theory, models most of the existing uncertainties.

In general, uncertainty can be divided into random and non-random uncertainties [25]. In many applications probability modeling is suitable for random uncertainty and fuzzy modeling is suitable for non-random uncertainty [20, 39].

2.3 Considering both fuzzy and probabilistic uncertainties

The problem of considering both fuzzy and probabilistic uncertainties has long been taken into consideration and many efforts have been made in this regard [41]. In many systems, both uncertainties exist and one system must model both types [23, 19].

In graphic models, given the fact that cognitive and Bayesian networks are suitable for modeling many systems, but each of them models two different types of uncertainty; some researchers have contused these two networks jointly. But all these efforts have been made that either the cognitive network becomes a Bayesian type, or vice versa, or that the two networks will be merged with a series of constraints and transformations in a network and all these methods lead to the loss of a part of information.

In some studies, at first, a causal network is created and then a Bayesian network called Bayesian causal map is created or the cognitive Bayesian network [8]. In some other studies, a fuzzy network is added to a probabilistic uncertainty fuzzy network [34].

Another set of methods, called fuzzy Bayesian network, is based on methods that are based on a Bayesian network in which fuzzy logic in the network is converted into probabilities [12] or fuzzy logic is used as a discrete in the input of Bayesian network [7, 36, 3].

3 The proposed structure

In this section, the proposed method of this paper to consider both fuzzy and probabilistic uncertainties in a graphical network is raised. First, the structure of rule based networks is investigated as a base structure, then the modeling of uncertainty in network nodes and the steps of the proposed method are discussed.

3.1 Rule-based networks

In this paper, rules are used to express the relationship between network nodes. By using rules, nonlinear relationships between nodes can also be described. The rule based FCM structure was first proposed by Carvalho and Tom called Rule Based Fuzzy Cognitive Map (RBFCM) [5, 4, 6].

The inference steps between two nodes C_1 and C_3 and the RBFCM method are shown in Figure 4.

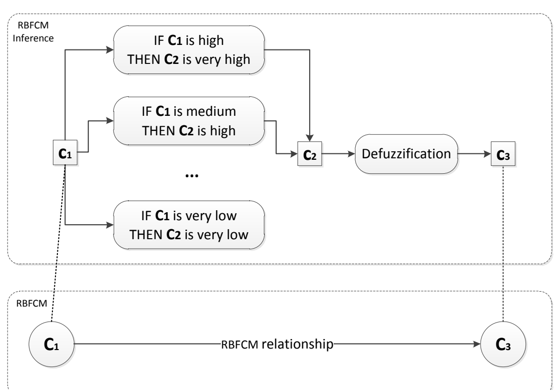

Figure 4: Inference procedure in the RBFCM method [29].

The structure of Bayesian network is very close to the RBFCM network structure. We can consider conditional CPT probability table between nodes as the binding rules of the nodes.

A simple method for using fuzzy rules in the production of CPT is the direct mapping of each rule to a member of the CPT table [21, 40]. By this method, one can build CPT based on the rules, and vice versa, via the CPT, we can reach fuzzy rules. As a result, this method can be used to generate two FCM and similar Bayesian networks.

3.2 Uncertainty of the node

In the proposed method with the purpose of making two separate fuzzy and Bayesian models, at first we should determine the type of uncertainty of each node. If we use two fuzzy and probabilistic uncertainties to represent uncertainty, the information of a node can be represented in Figure 5 in fuzzy, probabilistic, or both types.

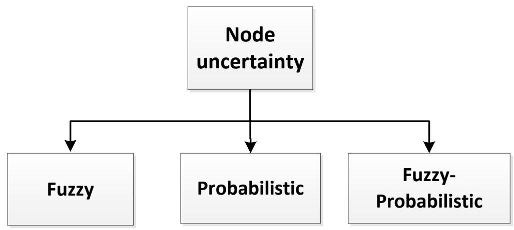

Figure 5: Different types of node uncertainty.

For better representation of the uncertainty of each node in Figure 6, fuzzy node is shown with square, probability node with circle and a fuzzyprobabilistic node is shown with hexagonal form.

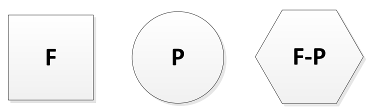

Figure 6: Symbols for Fuzzy, Probabilistic and Fuzzy-Probabilistic nodes.

It should be noted that the value of a node is derived from its parent nodes in graphical networks. As a result, fuzziness and randomness are inherited from parents to children.

3.3 Steps of the proposed method

In the proposed approach, to consider fuzzy and probabilistic uncertainties jointly, a hybrid system consisting of a fuzzy cognitive map and a Bayesian network is used. But unlike other combined methods in which two methods are combined together and a unified form is created, in the proposed method, a different and more efficient method is applied. In this method, both networks are used separately. Fuzzy variables and uncertainties are modeled by fuzzy cognitive map and probabilistic variables and uncertainties are modeled by Bayesian network.

With this method, both the nature of the fuzzy and probabilistic uncertainties is maintained and the system is modeled as it is. In the proposed model, different types of nodes and links can be implemented. There is also the possibility of dynamic nature of link so that the degree and value of relationship is dependent on the value of the nodes over time.

The stepwise illustration of the proposed method is shown in Figure 7.

Figure 7: Steps of the proposed method.

3.3.1 Determine the states and membership functions of each node

First, we represent each node of the network with x_k, $k = 1, 2, \ldots, m$. Each node may be characterized by n_k states, i.e. $X_k^{v_k}$, $v_k = 1, 2, \ldots, n_k$. Each state is assigned a membership function that is represented by $f_{X_k^{v_k}}(x_k)$. As a result, the following should be specified in the graphic network that was presented:

1- The type of uncertainty of each node (fuzzy, probabilistic or fuzzy-probabilistic).
2- States assigned to each node $X_k^{v_k}$, $v_k = 1, 2, \ldots, n_k$.
3- Membership functions assigned to each state $f_{X_k^{v_k}}(x_k)$, $v_k = 1, 2, \ldots, n_k$.
4- The rules for linking the nodes.

For example, for the network shown in Figure 8 it is assumed that the node x_k has three states with Gaussian membership functions.

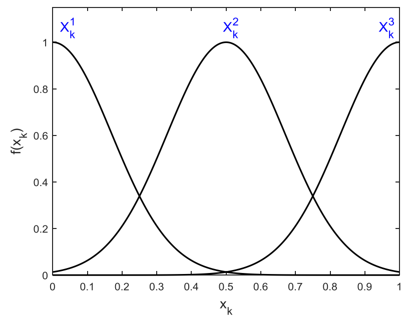

Figure 8: Membership functions assigned to a node's states.

3.3.2 Construction of fuzzy and Bayesian networks separately

In this section, two separate networks are constructed on a network that includes both fuzzy and probabilistic uncertain nodes. A FCM network in which all fuzzy nodes are present and a BN network in which all probability nodes are located.

For root nodes, if the node is fuzzy, this node is added to the fuzzy model and if it is random, it is added to Bayesian model and if the node is fuzzy-probabilistic, it is added to both models. In other words, this node is turned into two nodes. In this case, the number and range of states nodes considered for each node is similar in two models.

For non-root nodes it is clear that if the node is fuzzy-probabilistic, as before, a node is added to each model. If the node is fuzzy and its parent is just fuzzy, the node will only be added to the fuzzy model. But if only one parent has probabilistic state, as this probabilistic state of parent is effective on child, a node is added to the Bayesian model.

Similarly, if the node is probabilistic and its parent is only probabilistic, the node will only be added to the Bayesian model. But if only one of its parents has the fuzzy state, as this parents fuzzy state also affects the child, one node is added to the fuzzy model.

For an example, in Figure 9 it is assumed that the parent node is probabilistic-fuzzy, and with this assumption is made based on the main model of two fuzzy and Bayesian models.

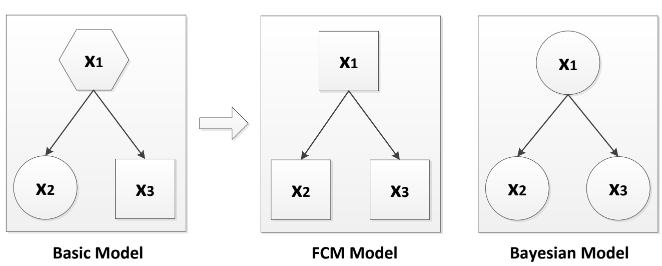

Figure 9: Making two models with a fuzzy-probabilistic parent node.

Since the sum of probabilities must always be one, the membership functions of the Bayesian network are normalized according to equation 1.

$$f^N_{X_k^{v_k}}(x_k) = \frac{f_{X_k^{v_k}}(x_k)}{\sum_{v_k=1}^{n_k} f_{X_k^{v_k}}(x_k)} \tag{1}$$

We use fuzzy rules to convert CPT and Vice versa in order to obtain the relationships between the nodes, according to section 3.1. For example, if the parent nodes of x_3 are the two nodes x_1 and x_2, then, the procedure to convert a fuzzy rule to the CPT or vice versa is shown in Table 1.

Table 1: The procedure to convert a fuzzy rule to a CPT and vice versa.

Rule : **IF** x_1 is X_1^i AND x_2 is X_2^j **THEN** x_3 is X_3^q $\Longleftrightarrow$ CPT

	x_1	$X_1^1 \dots$			X_1^i			$\dots\ X_1^{v_1}$		
	x_2	X_2^1 ..	X_2^j	.. $X_2^{v_2}$	X_2^1 ..	X_2^j	.. $X_2^{v_2}$	X_2^1 ..	X_2^j	.. $X_2^{v_2}$
	X_3^1	0	0	0	0	0	0	0	0	0
	$\dots$	0	0	0	0	0	0	0	0	0
x_3	X_3^q	0	0	0	0	1	0	0	0	0
	$\dots$	0	0	0	0	0	0	0	0	0
	$X_3^{v_3}$	0	0	0	0	0	0	0	0	0

For further discussion of the subject, a situation is discussed that is very practical. In this case, it is assumed that one of the FCM network nodes has probabilistic noise. As a result, this node has both fuzzy and probabilistic aspects. We assume that the probability density function of noise is $g_{X_k}(x_k)$. It is also assumed for ease that the membership functions assigned to the states of each node, as well as the probability density function of the noise, are Gaussian. In equation 2 and Figure 10, the probability density function of noise and its Gaussian shape is given, respectively.

$$g_{X_k}(x_k) = \frac{1}{\sqrt{2\pi\sigma_g^2}}\, e^{-\frac{(x_k - x_{0k})^2}{2\sigma_g^2}} \tag{2}$$

where x_{0k} is the mean of the probability density function and σ_g^2 is its variance.

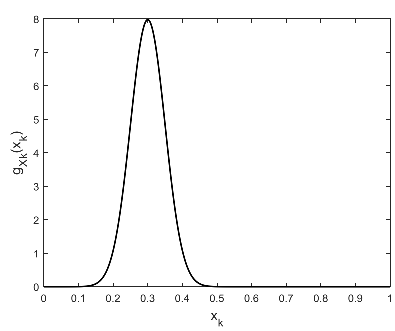

Figure 10: Gaussian probability density function of noise.

By having the density function of the noise and normalized functions in the Bayesian network, we can obtain the joint probability density function $h_{X_k^{v_k}, X_k}(X_k^{v_k}, x_k)$ as equation 3.

$$h_{X_k^{v_k}, X_k}(X_k^{v_k}, x_k) = P(X_k^{v_k} | x_k)\, g_{X_k}(x_k) = f^N_{X_k^{v_k}}(x_k)\, g_{X_k}(x_k) \tag{3}$$

Figure 11 shows the joint probability density function of each state.

In the Bayesian network, probability degrees are calculated based on the joint probability density function. In other words, we have soft evidences $p_k^{v_k}$, $v_k = 1, 2, \dots, n_k$ from $h_{X_k^{v_k}, X_k}(X_k^{v_k}, x_k)$ according to equation 4.

$$p_k^{v_k} = P(X_k^D = X_k^{v_k}) = \int h_{X_k^{v_k}, X_k}(X_k^{v_k}, x_k)\, \mathrm{d}x_k = \int f^N_{X_k^{v_k}}(x_k)\, g_{X_k}(x_k)\, \mathrm{d}x_k \tag{4}$$

where X_k^D is a discrete random variable whose possible values are $X_k^{v_k}$.

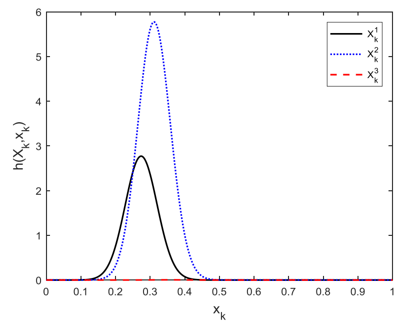

Figure 11: Joint probability density function.

3.3.3 Inference in FCM and BN

After constructing two separate FCM and BN models, inference is performed in each network. The node to be deduced is called the decision node.

In the FCM network, the fuzzy membership degrees $\mu_k^{v_k}$, $v_k = 1, 2, \ldots, n_k$ are calculated for the decision node based on the functions $f_{X_k^{v_k}}(x_k)$ and according to the fuzzy inference relationships (such as Max-Min or Max-Product). To avoid information loss, it should be noted that defuzzification operation should be avoided except for the decision node and the degrees of membership are transferred from the root nodes to the decision nodes.

In Bayesian networks, after determining the degrees of probability for the noisy node, the degrees of probability $p_k^{v_k}$, $v_k = 1, 2, \ldots, n_k$ are calculated for the decision node based on the Bayes relations and conditional probability tables according to equation 4.

Thus, the degrees of fuzzy membership and probability are obtained for each state of decision node. These values can be used to make decisions.

3.3.4 Final node value

After the inference operation, the final value of the decision node can be obtained. As we have seen, using inference in each model for the node x_k, the fuzzy membership degrees $\mu_k^{v_k}$ are obtained on the fuzzy set $\widetilde{x}_k$ and the probability degrees $p_k^{v_k}$ on the random set $\overline{x}_k$. $\widetilde{x}_k$ constitute fuzzy set with degrees of membership $\mu_k^{v_k}$ and $\overline{x}_k$ is a random set with probability degrees $p_k^{v_k}$ defined on the values of node x_k with the states $X_k^{v_k}$, $v_k = 1, 2, \ldots, n_k$.

Since probability degrees are considered to be very useful for noise-induced random variations in the network, we use them to correct the degrees of membership in the decision node. The modified fuzzy membership degrees are obtained from the fuzzy membership degrees $\mu_k^{v_k}$ and the probability degrees $p_k^{v_k}$ according to equation 5:

$$\acute{\mu}_k^{v_k} = \mu_k^{v_k}\, p_k^{v_k} \tag{5}$$

In the next step, an aggregated fuzzy function $M_{x_k}(\widetilde{x}_k)$ is constructed for the x_k node and in the FCM network based on the modified membership degrees $\acute{\mu}_k^{v_k}$ and using the membership functions $f_{X_k^{v_k}}(\widetilde{x}_k)$ and the aggregation operation as follows.

From this aggregated fuzzy function and center of gravity defuzzification (COG), a final value of decision node is obtained according to equation 6 [42, 37].

$$x_k^r = \frac{\int M_{x_k}(\widetilde{x}_k)\, x_k \, \mathrm{d}x_k}{\int M_{x_k}(\widetilde{x}_k)\, \mathrm{d}x_k} \tag{6}$$

3.4 Analytical proof of the proposed method

To prove the effectiveness of the proposed method, we consider a simple graphical network with two nodes. This network is shown in Figure 12. In this network, node x_1 is a fuzzy node whose measured values have probabilistic noise. As a result, the uncertainty of this node is fuzzy-probabilistic. According to the proposed method, this network is divided into a FCM model and a BN model. For ease of relations, assume that all membership functions and probability density functions are Gaussian and normalized. We also assume that the number of states of two nodes is equal and that the rules connecting the parent node and the child are uniform. In this Figure, x_1 is the input node and x_2 is the output node or decision node.

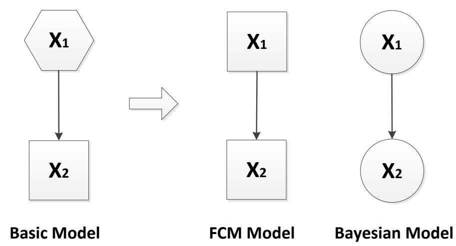

Figure 12: A graphical network with two nodes.

According to the assumptions and based on the fuzzy inference relationships based on the Max-Product method, the output value is shown in equation 7 without considering the proposed method.

$$x_2^r = \frac{\int max\{\prod_{v_k=1}^{n_k}\left[f_{X_1^{v_k}}(x_{01}+d)\, f_{X_2^{v_k}}(x_2)\right]\}\, x_2\, \mathrm{d}x_2}{\int max\{\prod_{v_k=1}^{n_k}\left[f_{X_1^{v_k}}(x_{01}+d)\, f_{X_2^{v_k}}(x_2)\right]\}\, \mathrm{d}x_2} \tag{7}$$

In this case, x_{01} is the real and actual value of the node x_1, and d is the value that changes the real value due to probabilistic noise. Given the fact that the noise input changes the membership value of the input functions, we can obtain the absolute error of each main term of equation 7 based on the Taylor series in the form of equation 8.

$$e_{v_k}^{Noisy} = \left|f_{X_1^{v_k}}(x_{01}+d) - f_{X_1^{v_k}}(x_{01})\right| \cong \left|\sum_{i=1}^{n} \frac{f_{X_1^{v_k}}^{(i)}(x_{01})}{i!}\, d^i\right| \tag{8}$$

where $f_{X_1^{v_k}}^{(i)}(x_1)$ is the i-th order derivative of $f_{X_1^{v_k}}(x_1)$ and n is the number of Taylor expansions. Now, using the proposed method, the output value is obtained from equation 9.

$$x_2^{Proposed} = \frac{\int max\{\prod_{v_k=1}^{n_k}\left[\int f_{X_1^{v_k}}(x_1) g_{X_1}(x_1)\mathrm{d}x_1\, f_{X_2^{v_k}}(x_2)\right]\}\, x_2\, \mathrm{d}x_2}{\int max\{\prod_{v_k=1}^{n_k}\left[\int f_{X_1^{v_k}}(x_1) g_{X_1}(x_1)\mathrm{d}x_1\, f_{X_2^{v_k}}(x_2)\right]\}\, \mathrm{d}x_2} \tag{9}$$

And the absolute error of each main term of equation 9 in the proposed method is given by equation 10.

$$e_{v_k}^{Proposed} = \left|\int f_{X_1^{v_k}}(x_1) g_{X_1}(x_1)\, \mathrm{d}x_1 - f_{X_1^{v_k}}(x_{01})\right| \tag{10}$$

Using the Taylor expansion of $f_{X_1^{v_k}}(x_1)$ around the point x_{01}, we have equation 11.

$$f_{X_1^{v_k}}(x_1)\, g_{X_1}(x_1) \cong f_{X_1^{v_k}}(x_{01})\, g_{X_1}(x_1) + \sum_{i=1}^{n} \frac{f_{X_1^{v_k}}^{(i)}(x_{01})}{i!}\, (x_1 - x_{01})^i\, g_{X_1}(x_1) \tag{11}$$

By obtaining the integral of the two sides of equation (11, we arrive at equation 12.

$$\int f_{X_1^{v_k}}(x_1)\, g_{X_1}(x_1)\, \mathrm{d}x_1 \cong f_{X_1^{v_k}}(x_{01}) + \sum_{i=1}^{n} \left[\frac{f_{X_1^{v_k}}^{(i)}(x_{01})}{i!} \int (x_1 - x_{01})^i\, g_{X_1}(x_1)\, \mathrm{d}x_1\right] \tag{12}$$

As a result, the relation is obtained in the form of equation 13.

$$e_{v_k}^{Proposed} = \left|\sum_{i=1}^{n} \frac{f_{X_1^{v_k}}^{(i)}(x_{01})}{i!}\, M^i\right| \tag{13}$$

where M^i is the i-th central moment of the probability density function of noise according to equation 14.

$$M^i = \int (x_1 - x_{01})^i \, g_{X_1}(x_1)\, \mathrm{d}x_1 \tag{14}$$

Given the fact that the probability density function of noise is Gaussian, the central moments can be easily calculated from equation 15 [30].

$$M^i = \begin{cases} 0 & i \text{ is odd} \\ \sigma_g^i (i-1)!! & i \text{ is even} \end{cases} \tag{15}$$

Here $q!!$ denotes the double factorial, that is, the product of all numbers from q to 1 that have the same parity as q. The value of the deviation from the actual value is shown with d. The probability of occurrence of any value d is obtained from the probability density function $g_{X_1}(x_1)$. As a result, the general absolute error of each of the two methods can be calculated from equation 16.

$$\text{Error} = \int \Big[\sum_{v_k=1}^{n_k} e_{v_k} \Big] \, g_{X_1}(x_1)\, \mathrm{d}x_1 \tag{16}$$

Due to the Gaussian characterization of the probability density function, we have $Pr\{|x_1 - x_{01}| \leqslant 3\sigma_g\} = 0.9973$ [33]. Therefore, equation 16 can be approximated by equation 17.

$$\text{Error} \cong \sum_{v_k=1}^{n_k} e_{v_k} \sum_{j=\frac{-3\sigma_g}{\Delta x_1}}^{\frac{+3\sigma_g}{\Delta x_1}} g_{X_1}(x_{01} + j\Delta x_1)\, \Delta x_1 \tag{17}$$

Using equation 17 and also the two equations 8 and 13, we can examine the advantage of the proposed method. For this purpose, the errors of the two methods are calculated and compared. This comparison is shown in Figure 13.

In this Figure, the error value of the two methods is calculated based on changes in the variance of noise and the actual changes in input value. The input and output membership functions are considered in accordance with Figure 8.

Considering the Figure, it is clear that in most cases, the proposed method has less error than noisy mode and without considering the proposed method. By increasing the variance of noise, the probabilistic uncertainty of the input node increases. Since the proposed method models both fuzzy and probabilistic uncertainties, in this case the proposed method shows better performance than the conventional method.

4 Simulation: Sensor location estimation in wireless sensor networks

One of the most important issues in wireless sensor networks is sensor location estimation. The objective in this case is to determine the uncertain position of a number of sensors based on a number of other sensors whose position is known [24]. One of the common methods for this problem is to use power. In this way, a sensor with an unknown location should estimate its location based on the amount of Received Signal Strength (RSS) from a given number of sensors with known position.

The conditions used for simulation in this paper are identical to the ones applied by M. Kadkhoda and M.R. Akbarzadeh, [16]. In this simulation, a 100×100 m^2 region with 121 anchor nodes is used. The anchor nodes are placed regularly within 10 m distance from each other and 60 sensor nodes are randomly placed across the area as shown in Figure 14. A sensor node can receive signals from the adjacent anchor node if it stands at a distance smaller than the transmission range which is assumed to be 8.94 m in this case. Each anchor node knows its exact position within this particular setup, either through GPS or by other means such as pre-configuration.

While anchor nodes send out beacon signals, each sensor node listens for a fixed time period and collects the RSS information in the beacon signals received from adjacent anchor nodes in order to locate itself. For the purpose of simulation, the following RSS model is used which also takes noise into account [15].

$$R_{ij} = (\varphi\, d_{ij}^{-\alpha}) + \delta\,(\mathrm{SNR}) \tag{18}$$

Where R_{ij} is the RSS value between the i-th adjacent anchor node and the j-th sensor node, φ is a constant which takes carrier frequency and transmitted power into account, d_{ij} is the distance between the i-th adjacent anchor and the j-th sensor node and α is the attenuation exponent. Here, we use $\varphi = 50$ and $\alpha = 1$ and δ (additive white Gaussian

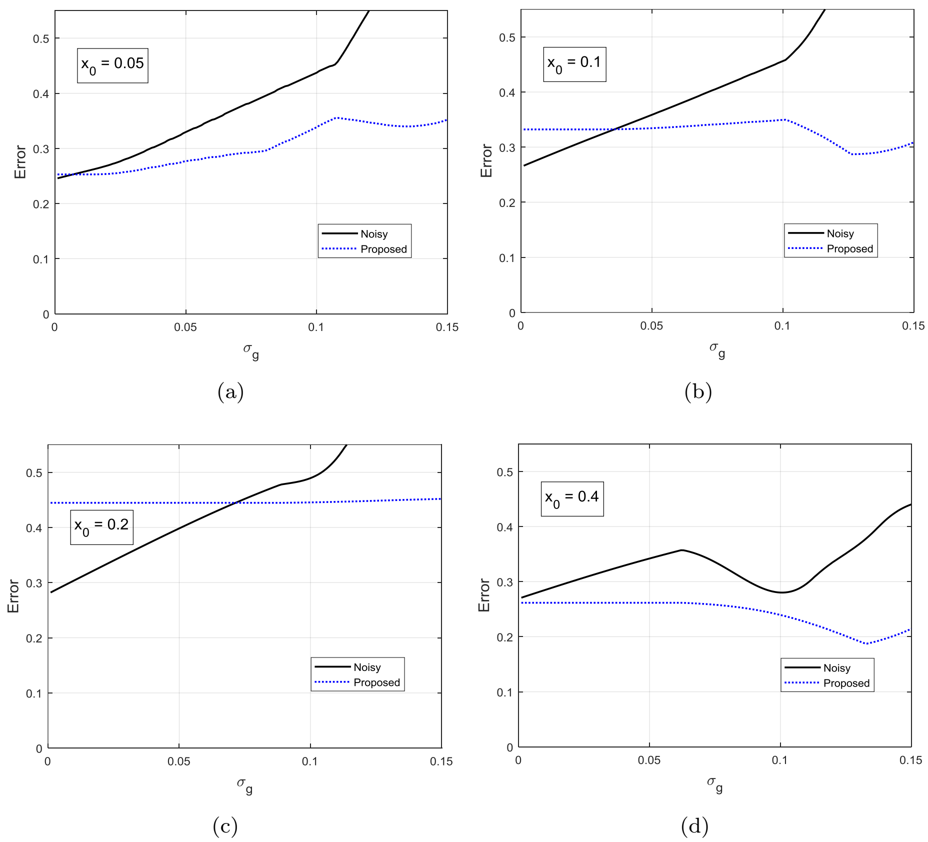

Figure 13: Error changes in conventional and proposed method.

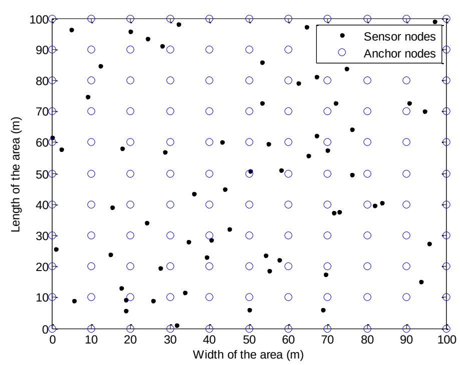

Figure 14: Distribution of nodes in the simulated area.

noise) to simulate more a realistic environment. SNR (signal to noise ratio) as a parameter of δ is the ratio of signal power to noise power and it is used to generate various noise levels in the simulations. The Average Location Error criteria (ALE) has been selected in order to evaluate the proposed method. Location Error is the distance between the estimated location (X_{est}, Y_{est}) and the actual position (X, Y) of sensor node. Therefore, ALE is defined as the average error over all sensor nodes according to equation 19.

$$\text{ALE} = \frac{\sum \sqrt{(X_{est} - X)^2 + (Y_{est} - Y)^2}}{N} \tag{19}$$

Where N is the number of sensor nodes.

Two methods for obtaining (X_{est}, Y_{est}) are introduced and compared in [15]. The first method is to use a fuzzy inference system called FLS. In this method, a number of weight coefficients is obtained based on fuzzy rules and the received power of each sensor. Then, the position of the unknown sensors is estimated using equation 20 and based on the coefficients and position of the specific sensors.

$$(X_{est}, Y_{est}) = \left(\frac{\sum_{i=1}^{K} w_i\, X_i}{\sum_{i=1}^{K} w_i}, \frac{\sum_{i=1}^{K} w_i\, Y_i}{\sum_{i=1}^{K} w_i} \right) \tag{20}$$

These rules are as follows.

$$\textbf{If } RSS \text{ is } A_i \textbf{ Then } w \text{ is } B_i \tag{21}$$

The second method is to use a Probabilistic Fuzzy Logic System (PFLS). In this method, the procedure of the FLS method is performed with the exception being that probabilities are also involved in fuzzy rules based on the probabilistic fuzzy system method. In this way, the form of expression of the rules is as follows.

$$\begin{aligned} &\textbf{If } RSS \text{ is } A_i \textbf{ Then } w \text{ is} \\ &\qquad B_{i1} \text{ with probability } p_1 = P(B_{i1}|A_i) \\ &\quad \text{and } B_{i2} \text{ with probability } p_2 = P(B_{i2}|A_i) \\ &\quad \text{and} \ldots \\ &\quad \text{and } B_{iM} \text{ with probability } p_M = P(B_{iM}|A_i) \end{aligned} \tag{22}$$

The problem of locating wireless sensors can be modelled with a simple graphical network as shown in Figure 15 by using the stated concepts. Since the RSS is described by a number of fuzzy membership functions and is affected by Gaussian white noise with a random aspect, it is a node with fuzzy and probability modes. As a result, the weight node $\mathbf{W}$ has probabilistic uncertainty in addition to fuzzy uncertainty. Using the proposed method, the FCM and BN models will be as shown in Figure 15.

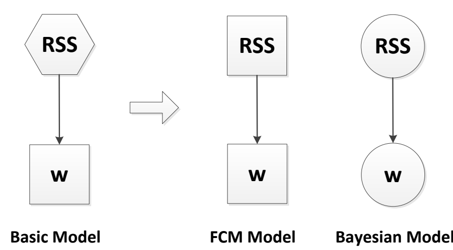

Figure 15: The graphical model of the problem of locating wireless sensors.

The results of running the algorithms with different SNRs using FLS, PLFS and the proposed method are shown in Figure 16.

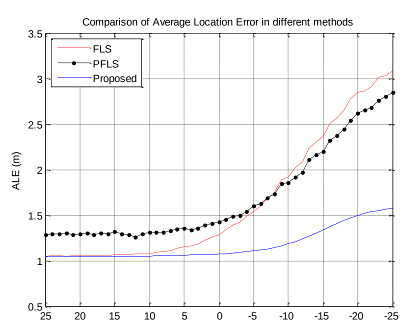

Figure 16: Average Location Error for different methods for various noise levels.

It can be seen that the average error of the proposed method from both the FLS and PLFS is lower for all SNRs. It is also noteworthy that the advantage of the proposed method becomes more significant when the SNR has a small value. In low SNRs, noise power increases, and thus the randomness aspect increases. Therefore, both fuzzy and probabilistic uncertainties are significant in the network. In this case, the proposed method can model both types of uncertainty very well.

The minimum, maximum and average values of ALE are shown for the three methods in Table 2. These results are obtained by averaging the results of implementation of each algorithm 50 times.

Table 2: Minimum, Maximum and Average values of ALE

Method	**Min ALE**	**Max ALE**	**Avg ALE**
FLS	1.05	3.08	1.64
PFLS	1.25	2.84	1.70
Proposed	1.04	1.57	1.17

5 Conclusions

One of the important issues in the construction and use of graphical networks is the issue of dealing with and taking into account uncertainty. In graphical networks, considering the fuzzy and probabilistic uncertainty, a large set of existing uncertainties is considered. If any one of these two uncertainties is present alone in the problem, then a fuzzy cognitive network can be used to model the fuzzy uncertainty and the Bayesian network to model probabilistic uncertainty.

But the challenging issue is the presence of both types of uncertainty jointly in practical and applied issues. For example, it was observed that there is a probabilistic uncertainty associated with the fuzzy uncertainty in wireless sensors localization in a network of sensors due to the presence of noise in measuring the power given to each sensor.

In most of the methods being used to consider both types of uncertainties, fuzzy uncertainty becomes probabilistic uncertainty or vice versa, which causes transformation error in the problem and false assumption of the type of uncertainty.

In this paper, we proposed an efficient and useful method for considering both fuzzy and probabilistic uncertainties in rule based graphical networks. The proposed method considers the uncertainties of the problem as it is with the new approach. To do this, first, the uncertainty type of each node in the graphical model is identified, and then from the main network, two separate fuzzy and probabilistic networks are made. The fuzzy cognitive network is used to model fuzzy uncertainty and the Bayesian network is applied for modeling probabilistic uncertainty. Inference in each network is done separately. Finally, the decision node can be used to combine the results of the two networks. This avoids converting a fuzzy network to Bayesian or vice versa, and both types of uncertainties are not included in the problem. This method was used for wireless sensors localization. The simulation results show the efficiency of the proposed method well.

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