

O-plane couplings at order α'^2 : one R-R field strength

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ABSTRACT: It is known that the anomalous Chern-Simons (CS) coupling of O_p -plane is not consistent with the T-duality transformations. Compatibility of this coupling with the T-duality requires the inclusion of couplings involving one R-R field strength. In this paper we find such couplings at order α'^2 .

By requiring the R-R and NS-NS gauge invariances, we first find all independent couplings at order α'^2 . There are 1, 6, 28, 20, 19, 2 couplings corresponding to the R-R field strengths $F^{(p-4)}$, $F^{(p-2)}$, $F^{(p)}$, $F^{(p+2)}$, $F^{(p+4)}$ and $F^{(p+6)}$, respectively. We then impose the T-duality constraint on these couplings and on the CS coupling $C^{(p-3)} \wedge R \wedge R$ at order α'^2 to fix their corresponding coefficients. The T-duality constraint fixes all coefficients in terms of the CS coefficient. They are fully consistent with the partial couplings that have been already found in the literature by the S-matrix method.

KEYWORDS: String Duality, D-branes, Superstrings and Heterotic Strings

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1 Introduction

The best candidate for quantum gravity is the superstring theory in which the graviton appears as a specific mode of a relativistic superstring at weak coupling [1, 2]. Superstring has massless and infinite tower of massive states which appear in the low energy effective action, as higher derivative corrections to the supergravity. Study of these higher derivative corrections are important because they signal the stringy nature of the quantum gravity.

One of the most exciting discoveries in perturbative string theory is the T-duality which has been observed first in the spectrum of string when one compactifies theory on a circle [3, 4]. This symmetry may be used to construct the effective action of string theory including its higher derivative corrections, in the Double Field Theory formalism in which the T-duality transformations are the standard $O(D, D)$ transformations whereas the gauge transformations are non-standard [5–7]. It has been also speculated that the invariance of the effective actions of string theory and its non-perturbative objects, i.e., D-branes and O-planes, under the standard gauge transformations and non-standard T-duality transformations may be used as a constraint to construct the effective actions [8]. In this approach, one first constructs the most general gauge invariant and independent couplings at a given order of α' with arbitrary parameters. Then the parameters may be fixed in the string theory by imposing the T-duality symmetry on the couplings. That is, one reduces the couplings on a circle and requires them to be consistent with the T-duality transformations which are the standard Buscher rules [9, 10] plus their α' -corrections [11–14]. Using this approach, the effective action of the bosonic string theory at order α' and α'^2 have been found in [14, 15]. It has been shown in [16, 17] that the leading order effective action of

type II superstring theories, including the Gibbons-Hawking-York boundary term [18, 19], can also be rederived by the T-duality constraint. The couplings involving metric and dilaton in both heterotic string and in superstring theories at order α'^3 have been also rederived by the T-duality constraint in [20]. There are many other approaches for constructing the effective actions including the S-matrix approach [21, 22], the sigma-model approach [23–25], and the supersymmetry approach [26–29]

The T-duality approach for constructing the effective action of D_p -brane (O_p -plane) is such that one first writes all gauge invariant and independent D_p -brane (O_p -plane) world-volume couplings at a specific order of α' with some unknown p -independent coefficients. Then one reduces the world-volume theory on the circle. There are two possibilities for the killing coordinate. Either it is along or orthogonal to the brane. The reduction of the world-volume theory when the killing coordinate is along the brane (the world-volume reduction), is different from the reduction of the world-volume theory when the killing coordinate is orthogonal to the brane (the transverse reduction). However, the T-duality transformation of the world-volume reduction of D_p -brane (O_p -plane) should be the same as the transverse reduction of the D_{p-1} -brane (O_{p-1} -plane) theory, up to some total derivative terms which have no physical effects for closed spacetime manifold [8].

Since O_p -planes are at the fixed points of spacetime, i.e., $X^i = 0$, some world-volume couplings are forbidden by orientifold projection [1]. The O_p -plane effective action has no open string couplings, no couplings that have odd number of transverse indices on metric and dilaton and their corresponding derivatives, and no couplings that have even number of transverse indices on B-field and its corresponding derivatives [1]. These O-plane conditions make the study of the O-plane couplings to be much easier than the D-brane couplings. The T-duality constraint has been used in [30, 31] to find the effective action of O_p -planes of type II superstring at order α'^2 for NS-NS fields. In this paper, we are interested in applying the T-duality constraint on the effective action of O_p -plane when there is one R-R field strength.

The O_p -plane CS action at the leading order of α' is given as [1]

$$S_{CS}^{(0)} = T_p \int_{M^{p+1}} C \tag{1.1}$$

where $C = \sum_{n=0}^8 C^{(n)}$ is the R-R potential and T_p is the O_p -plane tension. It is invariant under the R-R gauge transformation

$$\delta C = d\Lambda + H\Lambda \tag{1.2}$$

where $\Lambda = \sum_{n=0}^7 \Lambda^{(n)}$ and $H = dB$. Note that the last term in δC is zero for O_p -plane when all indices of the R-R potential are world-volume, as in (1.1). The curvature corrections to D_p -brane action has been found by requiring that the chiral anomaly on the world-volume of intersecting D-branes cancels with the anomalous variation of the CS action [32–34]. The corresponding corrections for O_p -plane has been found in [36] to be

$$S_{CS} = T_p \int_{M^{p+1}} C \sqrt{\frac{\mathcal{L}(\pi^2 \alpha' R_T)}{\mathcal{L}(\pi^2 \alpha' R_N)}} \tag{1.3}$$

where $\mathcal{L}(R_{T,N})$ is the Hirzebruch polynomials of the tangent and normal bundle curvatures respectively,

$$\sqrt{\frac{\mathcal{L}(\pi^2\alpha'R_T)}{\mathcal{L}(\pi^2\alpha'R_N)}} = 1 - \frac{\pi^2\alpha'^2}{48}(\text{tr}R_T^2 - \text{tr}R_N^2) + \dots \tag{1.4}$$

where $R_{T,N}$ are the curvature 2-forms of the tangent and normal bundles respectively. The corresponding curvature corrections to the CS action of D_p -brane is the same as (1.4) in which $\mathcal{L}(R/4)$ is replaced by the A-roof genus $\mathcal{A}(R)$ which produces up to a factor of -2 , the same curvature corrections at order α'^2 . However, the curvature corrections at higher orders of order α' are not the same in both cases.

The action (1.4) at order α'^2 in component form is¹

$$S_{\text{CS}}^{(2)} = -\frac{T_p\pi^2\alpha'^2}{48} \int d^{p+1}x \epsilon^{a_0 \dots a_p} \frac{1}{4(p-3)!} C_{a_4 \dots a_p}^{(p-3)} \left[R_{a_0 a_1}{}^{ab} R_{a_2 a_3 ab} - R_{a_0 a_1}{}^{ij} R_{a_2 a_3 ij} \right] \tag{1.5}$$

The above couplings have been confirmed by the S-matrix element calculations in [35–37]. Using the cyclic symmetry of the Riemann curvature, one can verify that the above diffeomorphism invariant action is also invariant under R-R gauge transformation. As it has been argued in [38, 39], the above couplings, however, are not consistent with the T-duality transformations.

On the other hand, there are many other gauge invariant couplings at this order which can not be found by the anomaly analysis. The R-R gauge symmetry requires all such couplings to be in terms of the nonlinear R-R field strength, i.e.,

$$F^{(n)} = dC^{(n-1)} + H \wedge C^{(n-3)} \tag{1.6}$$

which is invariant under the R-R gauge transformation (1.2). Some of these couplings involving one R-R field strength $F^{(p-2)}$ and two NS-NS fields have been found for D-brane in [40–43] by linear T-duality and by the disk-level S-matrix calculations. The complete couplings involving one R-R field strength $F^{(p)}$, $F^{(p+2)}$ or $F^{(p+4)}$ and one NS-NS field have been found in [44] by the S-matrix method and have been shown that they are invariant under the linear T-duality. However, these couplings are not invariant under the full nonlinear T-duality either. Hence, the T-duality of the CS coupling (1.5) may require adding couplings involving one R-R field strength and an arbitrary number of NS-NS fields at order α'^2 in which we are interested in this paper.

An outline of the paper is as follows: in section 2, we find the minimal gauge invariant couplings involving one R-R field strength. We use the Bianchi identities, total derivative terms and $\epsilon^{a_0 \dots a_p}$ -tensor identities to find the minimum number of gauge invariant couplings. We find there are 1, 6, 28, 20, 19, 2 such couplings corresponding to the R-R field strengths $F^{(p-4)}$, $F^{(p-2)}$, $F^{(p)}$, $F^{(p+2)}$, $F^{(p+4)}$, $F^{(p+6)}$, respectively. We then reduce them on a circle in section 3 to impose the T-duality constraint on them.

¹Our index convention is that A, B, \dots are 10-dimensional bulk indices, μ, ν, \dots are 9-dimensional base indices, y is killing index, a, b, \dots and a_0, \dots, a_p are world-volume indices and i, j, \dots are transverse space indices.

An appropriate method for reducing a gauge invariant coupling to 9-dimensional base space has been presented in [15]. In this method one keeps the $U(1) \times U(1)$ gauge invariant part in the reduction of the Riemann curvature and other components of a given coupling and removes all other terms. In section 3, we extend this method for the reduction of the couplings involving R-R fields as well, i.e., we find the $U(1) \times U(1)$ gauge invariant part of the reduction of R-R field strength and its first derivatives. In section 4, we impose the T-duality constraint on the independent gauge invariant couplings to fix their parameters. That is, we use the Bianchi identities, total derivative terms and $\epsilon^{a_0 \dots a_{p-1}}$ -tensor identities in the base space to write the T-duality constraint in terms of independent structures, and then solve them. In this section, we show that the T-duality can fix all parameters of the gauge invariant couplings in terms of an overall factor, and they are consistent with the partial couplings that have been already found in the literature by the S-matrix method. In section 5, we present the final form of the gauge invariant couplings and briefly discuss our results.

2 Minimal gauge invariant couplings

In this section we would like to find minimum number of gauge invariant couplings on the world-volume of O_p -plane involving one R-R field strength and an arbitrary number of NS-NS fields at order α'^2 , i.e.,

$$S_n = -\frac{T_p \pi^2 \alpha'^2}{48} \int d^{p+1}x \mathcal{L}^n \tag{2.1}$$

where \mathcal{L}^n is the Lagrangian which includes the minimum number of gauge invariant couplings involving one R-R field strength $F^{(n)}$. As it has been argued in [30], since we are interested in O_p -plane as a probe, it does not have back reaction on the spacetime. As a result, the massless closed string fields must satisfy the bulk equations of motion at order α'^0 . Using the equations of motion, one can rewrite the terms in the world-volume theory which have contraction of two transverse indices, e.g., $\nabla_i \nabla^i \Phi$, or $R_{iA}{}^i{}_B$ in terms of contraction of two world-volume indices, e.g., $\nabla_a \nabla^a \Phi$, or $R_{aA}{}^a{}_B$. This indicates that the former couplings are not independent. The O-plane couplings should also satisfy the orientifold projection.

The couplings involving the Riemann curvature and its derivative and the couplings involving derivatives of H and derivatives of R-R field strength satisfy the following Bianchi identities

$$\begin{aligned} R_{A[BCD]} &= 0 \\ \nabla_{[A} R_{BC]DE} &= 0 \\ dH &= 0 \\ dF^{(n)} + H \wedge F^{(n-2)} &= 0 \end{aligned} \tag{2.2}$$

Moreover, the couplings involving the commutator of two covariant derivatives of a tensor are not independent of the couplings involving the contraction of this tensor with the

Riemann curvature, i.e.,

$$[\nabla, \nabla]\mathcal{O} = R\mathcal{O} \tag{2.3}$$

This indicates that if one considers all gauge invariant couplings at a given order of α' , then only one ordering of the covariant derivatives is needed to be considered.

Using the symmetries of $\epsilon^{a_0 \dots a_p}$, the R-R field strength $F^{(n)}$, H and the Riemann curvature, one can easily verify that it is impossible to have non-zero contractions of one $F^{(n)}$ and some R , H , $\nabla\Phi$ at order α'^2 for $n < p - 4$ and $n > p + 6$. Moreover, the parity of the coupling (1.5) indicates that the couplings of the R-R field strength $F^{(p-2)}$ are non-zero when there are even number of B-field. The consistency with linear T-duality then indicates that the couplings of the R-R field strength $F^{(p-4)}$, $F^{(p)}$ and $F^{(p+4)}$ are non-zero when there are odd number of B-field, and the couplings of the R-R field strength $F^{(p+2)}$, and $F^{(p+6)}$ are non-zero when there are even number of B-field. There are similar parity selection rule for the corresponding S-matrix elements [48]. For $n = p - 4$ there is only one non-zero independent coupling,² i.e.,

$$\mathcal{L}^{p-4} = \epsilon^{a_0 \dots a_p} \left[\frac{a}{(p-5)!} F_{i a_6 \dots a_p} H^i{}_{a_0 a_1} H_{j a_2 a_3} H^j{}_{a_4 a_5} \right] \tag{2.4}$$

where we have used the O-plane conditions that there is no H term with even number of transverse indices. In above equation, the transverse indices are raised by the tensor $\perp^{ij} = G^{ij}$ (see next section for the definition of tensor \perp), and coefficient a is an arbitrary parameter at this point. This parameter may be fixed by studying the RP^2 -level S-matrix element of one R-R and three NS-NS vertex operators which is a very lengthy calculation. We expect this parameter to be fixed by the T-duality constraint.

There is no derivative on the R-R field strength and on the B-field strength in the above coupling. Hence, there is no Bianchi identity involved here. Since there is only one term, there would be no ϵ -tensor identity either. Moreover, there is no total derivative term here. This is not the case for $n > p - 4$ cases. Let us discuss each of the cases $n = p - 2$, $n = p$, $n = p + 2$, $n = p + 4$ and $n = p + 6$ separately.

2.1 $n = p - 2$ case

To find all gauge invariant and independent couplings corresponding to one R-R field strength $F^{(p-2)}$, we first consider all contractions of one $\epsilon^{a_0 \dots a_p}$, one F , ∇F or $\nabla\nabla F$, even number of H and ∇H , and any number of $\nabla\Phi$, $\nabla\nabla\Phi$, $\nabla\nabla\nabla\Phi$, R , ∇R at four-derivative order. Because of the relation (2.3), we consider only one ordering of the covariant derivatives. We then remove the forbidden couplings for O-plane, and remove the couplings in which two transverse indices in a term contracted, i.e., we impose the equations of motion. We call the remaining terms, with coefficients b'_1, b'_2, \dots , the Lagrangian L^{p-2} . Not all terms in this Lagrangian, however, are independent. Some of them are related by total derivative terms, by Bianchi identity and by ϵ -tensor identity.

To remove the total derivative redundancy, we write all total derivative terms at order α'^2 which involve the R-R field strength $F^{(p-2)}$. To this end we first write all contractions

²We have used the package “xAct” [46] for performing the calculations in this paper.

of one $\epsilon^{a_0 \dots a_p}$, one F , ∇F , even number of H and ∇H , and any number of $\nabla\Phi$, $\nabla\nabla\Phi$, R at three-derivative order. Then we remove the forbidden couplings and impose the equations of motion. We call the remaining terms, with arbitrary coefficients, the vector I_a^{p-2} . The total derivative terms are then

$$J^{p-2} = \int d^{p+1}x \tilde{g}^{ab} \nabla_a I_b^{p-2} \tag{2.5}$$

where $\tilde{g}^{ab} = G^{ab}$ is inverse of the pull-back metric (see next section for the definition of the pull-back metric). Adding the total derivative terms to L^{p-2} , one finds the same Lagrangian but with different parameters b_1, b_2, \dots . We call the new Lagrangian \mathcal{L}^{p-2} . Hence

$$\Delta^{p-2} - J^{p-2} = 0 \tag{2.6}$$

where $\Delta^{p-2} = \mathcal{L}^{p-2} - L^{p-2}$ is the same as L^{p-2} but with coefficients $\delta b_1, \delta b_2, \dots$ where $\delta b_i = b_i - b'_i$. Solving the above equation, one would find some linear relations between only $\delta b_1, \delta b_2, \dots$ which indicate how the couplings are related among themselves by the total derivative terms. The above equation would also give some relation between the coefficients of the total derivative terms and $\delta b_1, \delta b_2, \dots$ in which we are not interested.

However, to solve the above equation one has to impose the Bianchi identity and ϵ -tensor identities. To impose the Riemann curvature and H -field Bianchi identities (2.2), one may contract the term on the left-hand side of each Bianchi identity with appropriate couplings to produce terms at order α'^2 . The coefficients of these terms are also arbitrary. Adding these terms to the equation (2.6), then one could solve the equation to find the linear relations between only $\delta b_1, \delta b_2, \dots$. This method has been used in [30] to find the independent couplings involving only the NS-NS fields. Alternatively, to impose the Riemann curvature Bianchi identities, one may rewrite the terms in (2.6) in the local frame in which the first derivative of metric is zero. Similarly, to impose the H-field Bianchi identity, one may rewrite the terms in (2.6) which have derivatives of H in terms of B-field potential, i.e., $H = dB$. The last Bianchi identity in (2.2) relates the couplings involving derivative of $F^{(p-2)}$ to themselves and to the couplings involving $F^{(p-4)}$. However, the independent couplings involving $F^{(p-4)}$ have been already fixed in (2.4). Hence, the last Bianchi identity in (2.2) should relate only the couplings involving $F^{(p-2)}$, i.e., one should impose the identity $dF^{(p-2)} = 0$. To impose this identity on the couplings in (2.6) as well, one may rewrite the terms involving the derivatives of the R-R field strength $F^{(p-2)}$ in terms of the R-R potential, i.e., $F^{(p-2)} = dC^{(p-3)}$. In this way, all Bianchi identities satisfy automatically [47]. We find that this latter approach is easier to impose the Bianchi identities by computer. Moreover, in this approach one does not need to introduce a large number of arbitrary parameters to include the Bianchi identities to the equation (2.6). However, in this approach the gauge invariant equation (2.6) is written in terms of non-gauge invariant couplings. In this paper we use this approach for imposing the Bianchi identities.

After imposing the Bianchi identities, the non-gauge invariant couplings are not yet independent. To rewrite them in terms of independent couplings, one has to use the fact

that the number of world-volume indices in each coupling must be the same as the world-volume indices of $\epsilon^{a_0 \dots a_p}$. It has been observed in [44] that imposing this constraint, one may find some relations between couplings involving $\epsilon^{a_0 \dots a_p}$. Some of these ϵ -tensor identities for the simple case of two-field couplings, have been found in [44]. To impose this constraint on the couplings in (2.6) as well, we write the non-gauge invariant couplings explicitly in terms of the values that each world-volume index can take, e.g., $a_0 = 0, 1, 2, \dots, p$. It is easy to perform this step by computer using the “xAct” package [46].

Using the above steps, one can rewrite the different gauge invariant couplings on the left-hand side of (2.6) in terms of independent but non-gauge invariant couplings. The solution to the equation (2.6) then has two parts. One part is relations between only δb_i 's, and the other part is a relation between the coefficients of the total derivative terms and δb_i 's in which we are not interested. The number of relations in the first part gives the minimum number of gauge invariant couplings in \mathcal{L}^{p-2} . To write the independent couplings in a specific scheme, one must set some of the coefficients in L^{p-2} to zero. However, after replacing the non-zero terms in (2.6), the number of relations between only δb_i 's should not be changed. In the present case this number is 6. We set the coefficients of the terms that have world-volume derivative on the R-R field strength, to be zero. After setting this coefficients to zero, there are still 6 relations between δb_i 's. This means we are allowed to remove these terms. We choose some other coefficients to zero such that the remaining coefficients satisfy the 6 relations $\delta b_i = 0$. In this way one can find the minimum number of gauge invariant couplings. One particular choice for the 6 couplings is the following:³

$$\begin{aligned} \mathcal{L}^{(p-2)} = \epsilon^{a_0 \dots a_p} & \left[\frac{b_1}{(p-3)!} \nabla_i F_{j a_4 \dots a_p} H^i{}_{a_0 a_1} H^j{}_{a_2 a_3} + \frac{b_2}{(p-2)!} F_{a_3 \dots a_p} \nabla^a H_{i a a_0} H^i{}_{a_1 a_2} \right. \\ & + \frac{b_4}{(p-2)!} F_{a_3 \dots a_p} \nabla_{a_0} H_{i a a_1} H^{i a}{}_{a_2} + \frac{b_5}{(p-2)!} F_{a_3 \dots a_p} \nabla_a H_{i a_0 a_1} H^{i a}{}_{a_2} \\ & \left. + \frac{b_7}{(p-4)!} F_{i j a_5 \dots a_p} \nabla_{a_0} H^i{}_{a_1 a_2} H^j{}_{a_3 a_4} + \frac{b_9}{(p-2)!} F_{a_3 \dots a_p} H_{i a a_0} H^i{}_{a_1 a_2} \nabla^a \Phi \right] \quad (2.7) \end{aligned}$$

where the world-volume indices are raised by the first fundamental form $\tilde{G}^{ab} = G^{ab}$ (see next section for the definition of the first fundamental form), and the b 's are arbitrary coefficients. These coefficients do not depend on p . In fact the p -dependence of the couplings has been written explicitly by $1/n!$ where n is the number of indices of the R-R field strength that are contracted with $\epsilon^{a_0 \dots a_p}$. These couplings are consistent with the linear T-duality for the special case that the world-volume killing index of $\epsilon^{a_0 \dots a_p}$ contracts with the R-R field strength. That is,

$$\begin{aligned} \frac{1}{(p+1-m)!} \epsilon^{a_0 \dots a_m a_{m+1} \dots a_p} F_{\dots a_m a_{m+1} \dots a_p}(\dots) &= \frac{1}{(p-m)!} \epsilon^{a_0 \dots a_{p-1} y} F_{\dots a_m a_{m+1} \dots a_{p-1} y}(\dots) + \dots \\ &\rightarrow \frac{1}{(p-m)!} \epsilon^{a_0 \dots a_{p-1}} F_{\dots a_m a_{m+1} \dots a_{p-1}}(\dots) + \dots \end{aligned}$$

where the dots before the index a_m in the R-R field strength are the world-volume or transverse indices that contract with other parts of the coupling, i.e., contract with (\dots) .

³If one does not use the ϵ -tensor identities, then one would find 10 independent couplings.

In the first line we assume one of the world-volume indices is the killing index y , and in the second line we have used the linear T-duality transformation for the linearised R-R field strength, i.e., $F_{\dots y}^{(n)} = F_{\dots}^{(n-1)}$, and the identity $\epsilon^{a_0 \dots a_{p-1} y} = \epsilon^{a_0 \dots a_{p-1}}$. The couplings (2.7) for arbitrary coefficients, however, are not consistent with the linear T-duality when the killing index is not carried by the R-R field strength. We are interested in constricting these coefficients and the coefficients of other R-R field strengths that we will find in the subsequent subsections, by requiring the couplings to be consistent with nonlinear T-duality.

There is no term in (2.7) which involves only one NS-NS field. This indicates that the RP^2 -level S-matrix element of one R-R field strength $F^{(p-2)}$ and one NS-NS vertex operators should not have four-derivative terms. It has been observed in [44] that the disk-level S-matrix element of one R-R and one NS-NS vertex operators produce no such term at order α'^2 . On the other hand, it has been observed in [45] that the low energy expansion of RP^2 -level and disk-level S-matrix element of two massless closed string vertex operators are the same at order α'^2 , up to an overall factor.

The disk-level S-matrix element of one R-R potential $C^{(p-3)}$ and two B-field vertex operators has been calculated in [42, 43] from which the couplings of one $F^{(p-2)}$ and two H has been found for D_p -brane. The orientifold projection of the couplings found in [43] are the same as the above couplings with the following coefficients:

$$b_1 = b_7 = 0, \quad b_2 = -b_4 = b_5 = \frac{1}{2} \tag{2.8}$$

where we have also used the Bianchi identity $dH = 0$ to relate the couplings found in [43] to the couplings in (2.7). We will see that exactly the same coefficients (2.8) are reproduced by the T-duality constraint. This observation and the observation made in [45] may indicate that the orientifold projection of the disk-level S-matrix elements at order α'^2 are the same as the corresponding RP^2 -level S-matrix elements at order α'^2 , up to overall factors.

The independent couplings (2.7), however, are not the most general gauge invariant couplings because they do not include the Riemann curvature. The gauge invariant couplings involving the Riemann curvature are the couplings in the CS action (1.5) which are found by the anomaly cancellation mechanism. The T-duality constraint should reproduce these couplings as well. Hence, we include in this subsection the following gauge invariant couplings with arbitrary coefficients:

$$\mathcal{L}_{\text{CS}}^{(p-3)} = \epsilon^{a_0 \dots a_p} \left[\frac{\alpha_1}{(p-3)!} C_{a_4 \dots a_p}^{(p-3)} R_{a_0 a_1}{}^{ij} R_{a_2 a_3}{}_{ij} + \frac{\alpha_2}{(p-3)!} C_{a_4 \dots a_p}^{(p-3)} R_{a_0 a_1}{}^{ab} R_{a_2 a_3}{}_{ab} \right] \tag{2.9}$$

The two parameters α_1, α_2 which are known from the anomaly cancellation mechanism and also from the S-matrix calculation, should be fixed by the T-duality constraint as well.

2.2 $n = p$ case

To find all gauge invariant and independent couplings involving one R-R field strength $F^{(p)}$, we first consider all contractions of one $\epsilon^{a_0 \dots a_p}$, one $F, \nabla F$ or $\nabla \nabla F$, odd number of $H, \nabla H$ and $\nabla \nabla H$, and any number of $\nabla \Phi, \nabla \nabla \Phi, \nabla \nabla \nabla \Phi, R, \nabla R$ at four-derivative order.

We remove the terms which are forbidden for O-plane and impose the equations of motion. We then impose the total derivative terms, use the Bianchi identities and ϵ -tensor identities with the same strategy that is discussed in the previous subsection. In this manner one finds 28 independent couplings. One particular form for them is the following:⁴

$$\begin{aligned}
 \mathcal{L}^{(p)} = \epsilon^{a_0 \dots a_p} & \left[\frac{c_2}{(p-1)!} \nabla_a F_{ia_2 \dots a_p} \nabla_{a_0} H^{ia_1} + \frac{c_3}{(p-1)!} \nabla_a F_{ia_2 \dots a_p} \nabla^a H^i{}_{a_0 a_1} \right. \\
 & + \frac{c_5}{p!} \nabla_i F_{a_1 \dots a_p} H^{ia_0} \nabla_a \Phi + \frac{c_7}{(p-1)!} F_{ia_2 \dots a_p} \nabla^a H^i{}_{aa_1} \nabla_{a_0} \Phi \\
 & + \frac{c_8}{(p-1)!} F_{ia_2 \dots a_p} \nabla^a H^i{}_{a_0 a_1} \nabla_a \Phi + \frac{c_{10}}{(p-1)!} F_{ia_2 \dots a_p} H^i{}_{a_0 a_1} \nabla^a \nabla_a \Phi \\
 & + \frac{c_{12}}{(p-1)!} F_{ia_2 \dots a_p} H^i{}_{aa_1} \nabla^a \nabla_{a_0} \Phi + \frac{c_{13}}{(p-1)!} F_{ja_2 \dots a_p} H_{ia_0 a_1} \nabla^i \nabla^j \Phi \\
 & + \frac{c_{14}}{(p-1)!} F_{ia_2 \dots a_p} H^i{}_{a_0 a_1} \nabla^a \Phi \nabla_a \Phi + \frac{c_{16}}{(p-1)!} F_{ia_2 \dots a_p} H^i{}_{aa_1} \nabla_{a_0} \Phi \nabla^a \Phi \\
 & + \frac{c_{17}}{(p-1)!} F_{ja_2 \dots a_p} H^{iab} H_{iab} H^j{}_{a_0 a_1} + \frac{c_{21}}{(p-3)!} F_{jkl a_4 \dots a_p} H^{ikl} H_{ia_0 a_1} H^j{}_{a_2 a_3} \\
 & + \frac{c_{23}}{(p-3)!} F_{jkl a_4 \dots a_p} H_{ia_0 a_1} H^i{}_{a_2 a_3} H^{jkl} + \frac{c_{24}}{(p-1)!} F_{ka_2 \dots a_p} H_{iaa_0} H^{ijk} H_j{}^a{}_{a_1} \\
 & + \frac{c_{28}}{(p-1)!} F_{ja_2 \dots a_p} H^{iab} H_{iaa_0} H^j{}_{ba_1} + \frac{c_{30}}{(p-3)!} F_{iaba_4 \dots a_p} H^{iab} H^j{}_{a_0 a_1} H_{ja_2 a_3} \\
 & + \frac{c_{31}}{(p-1)!} F_{la_2 \dots a_p} H_{ia_0 a_1} H^i{}_{jk} H^{jkl} + \frac{c_{32}}{(p-1)!} F_{ia_2 \dots a_p} H^i{}_{a_0 a_1} H^{jkl} H_{jkl} \\
 & + \frac{c_{33}}{(p-1)!} F_{ia_2 \dots a_p} H^{iab} H^j{}_{aa_0} H_{jba_1} + \frac{c_{34}}{(p-3)!} F_{ijka_4 \dots a_p} H^i{}_{a_0} H^j{}_{aa_1} H^k{}_{a_2 a_3} \\
 & + \frac{c_{35}}{(p-1)!} F_{ka_2 \dots a_p} H^{ijk} R_{ia_0 j a_1} + \frac{c_{37}}{(p-1)!} F_{ja_2 \dots a_p} H^i{}_{aa_0} R_{ia_1}{}^{ja} \\
 & + \frac{c_{38}}{(p-2)!} F_{jaa_3 \dots a_p} H_{ia_0 a_1} R^{iaj}{}_{a_2} + \frac{c_{39}}{(p-1)!} F_{ja_2 \dots a_p} H_{iaa_0} R^{iaj}{}_{a_1} \\
 & + \frac{c_{40}}{(p-3)!} F_{ijka_4 \dots a_p} H^i{}_{a_0 a_1} R^j{}_{a_2}{}^k{}_{a_3} + \frac{c_{43}}{(p-1)!} F_{ia_2 \dots a_p} H^i{}_{a_0 a_1} R^{ab}{}_{ab} \\
 & \left. + \frac{c_{44}}{(p-1)!} F_{ia_2 \dots a_p} H^{iab} R_{aa_0 ba_1} + \frac{c_{46}}{(p-1)!} F_{ia_2 \dots a_p} H^i{}_{aa_1} R^{ab}{}_{a_0 b} \right] \quad (2.10)
 \end{aligned}$$

Note that in this case also we have set the coefficients of the terms that have world-volume derivative on the R-R field strength, to be zero. However, in the couplings in the first line we use an integration by part to remove one of the two derivatives on H because in imposing T-duality in the next section one needs to dimensionally reduce the couplings. The reduction of $\nabla F \nabla H$ is much easier to perform than the reduction of $F \nabla \nabla H$. In above equation, c_2, \dots, c_{46} are 28 arbitrary coefficients that do not depend on p . They may be found by the T-duality constraint.

The coefficients c_2, c_3 has been fixed by the tree-level S-matrix element of one R-R and one NS-NS vertex operators [44], i.e.,

$$c_2 = 2, \quad c_3 = -\frac{1}{2} \quad (2.11)$$

⁴If one does not use the ϵ -tensor identities, then one would find 46 independent couplings.

In finding this result we write the two-field terms in (2.10) and the couplings found in [44] in terms of independent structures, and then force them to be the same.

2.3 $n = p + 2$ case

To find all gauge invariant and independent couplings involving one R-R field strength $F^{(p+2)}$, we consider all contractions of one $\epsilon^{a_0 \dots a_p}$, one F , ∇F or $\nabla \nabla F$, even number of H and ∇H , and any number of $\nabla \Phi$, $\nabla \nabla \Phi$, $\nabla \nabla \nabla \Phi$, R , ∇R at four-derivative order. We then impose the equations of motion, the O-plane conditions, the total derivative terms, and use the Bianchi identities and ϵ -tensor identities with the same strategy that is discussed in the subsection 2.1. In this manner one finds 20 independent couplings. One particular form for them is the following:⁵

$$\begin{aligned}
 \mathcal{L}^{(p+2)} = & \epsilon^{a_0 \dots a_p} \left[\frac{d_2}{(p+1)!} \nabla_k F_{la_0 \dots a_p} H^{ijk} H_{ij}{}^l + \frac{d_3}{(p+1)!} \nabla_i F_{ja_0 \dots a_p} H^{iac} H^j{}_{ac} \right. \\
 & + \frac{d_9}{(p-1)!} \nabla_i F_{jkl a_2 \dots a_p} H^i{}_{a_0 a_1} H^{jkl} + \frac{d_{10}}{(p-1)!} \nabla_j F_{ikl a_2 \dots a_p} H^i{}_{a_0 a_1} H^{jkl} \\
 & + \frac{d_{11}}{(p+1)!} \nabla_i F_{ja_0 \dots a_p} \nabla^i \nabla^j \Phi + \frac{d_{12}}{(p+1)!} \nabla_i F_{ja_0 \dots a_p} R^{iaj}{}_{a_0} \\
 & + \frac{d_{15}}{p!} \nabla_a F_{ija_1 \dots a_p} R^{iaj}{}_{a_0} + \frac{d_{16}}{p!} F_{ija_1 \dots a_p} R^{iaj}{}_{a_0} \nabla_a \Phi \\
 & + \frac{d_{21}}{p!} F_{jka_1 \dots a_p} \nabla^a H^{ijk} H_{iaa_0} + \frac{d_{22}}{p!} F_{jka_1 \dots a_p} \nabla^i H^{jka} H_{iaa_0} \\
 & + \frac{d_{26}}{p!} F_{jka_1 \dots a_p} \nabla^a H_{iaa_0} H^{ijk} + \frac{d_{27}}{p!} F_{kla_1 \dots a_p} \nabla_{a_0} H^{ijk} H_{ij}{}^l \\
 & + \frac{d_{29}}{p!} F_{ija_1 \dots a_p} \nabla_a H^i{}_{ba_0} H^{jab} + \frac{d_{30}}{p!} F_{ija_1 \dots a_p} \nabla_{a_0} H^i{}_{ab} H^{jab} \\
 & + \frac{d_{36}}{p!} F_{ija_1 \dots a_p} \nabla_a H^{iab} H^j{}_{ba_0} + \frac{d_{41}}{(p-2)!} F_{ijkl a_3 \dots a_p} \nabla_{a_0} H^i{}_{a_1 a_2} H^{jkl} \\
 & + \frac{d_{42}}{p!} F_{ila_1 \dots a_p} \nabla^i H_{jka_0} H^{jkl} + \frac{d_{43}}{(p-2)!} F_{ijkl a_3 \dots a_p} H^i{}_{a_1 a_2} H^{jkl} \nabla_{a_0} \Phi \\
 & \left. + \frac{d_{47}}{p!} F_{ija_1 \dots a_p} H^{iab} H^j{}_{ba_0} \nabla_a \Phi + \frac{d_{48}}{p!} F_{jka_1 \dots a_p} H_{iaa_0} H^{ijk} \nabla^a \Phi \right] \quad (2.12)
 \end{aligned}$$

where the p -independent coefficients d_2, \dots, d_{48} may be found by the T-duality constraint.

The coefficients d_{11}, d_{12}, d_{15} have been fixed by the tree-level S-matrix element of one R-R and one NS-NS vertex operators [44]. They are

$$d_{11} = -2, \quad d_{12} = -2, \quad d_{15} = 2 \quad (2.13)$$

In finding the above result, we have imposed the first Bianchi identity in (2.2) on the two-field couplings found in [44]. Note that as observed in [44] the above results indicate that the curvature $R^{iaj}{}_{a_0}$ and $\nabla^i \nabla^j \Phi$ appear in the O-plane action as ij -component of the following combination:

$$\mathcal{R}^{AB} = R^{AaB}{}_{a_0} + \nabla^A \nabla^B \Phi \quad (2.14)$$

⁵If one does not use the ϵ -tensor identities, then one would find 53 independent couplings.

where A, B are 10-dimensional bulk indices. Note that the transverse contraction of the Riemann curvature, i.e., R^{AiB}_i has been removed at the onset by imposing the equations of motion. This dilaton-Riemann curvature appears also in NS-NS couplings of O-plane action at order α'^2 [31]. We speculate that the second derivative of dilaton appears in all O-plane and D-brane couplings in above combination.

2.4 $n = p + 4$ case

Performing the same steps as in subsection 2.1, one finds there are 19 independent couplings on the world-volume of O_p -plane that are not related to each other by the Bianchi identities, ϵ -tensor identities and the total derivative terms. One particular form for the couplings is the following:⁶

$$\begin{aligned}
 \mathcal{L}^{(p+4)} = & \epsilon^{a_0 \dots a_p} \left[\frac{e_1}{(p+1)!} \nabla_a F_{ijka_0 \dots a_p} \nabla^a H^{ijk} + \frac{e_3}{(p+1)!} F_{ijka_0 \dots a_p} \nabla^a H^{ijk} \nabla_a \Phi \right. \\
 & + \frac{e_6}{(p+1)!} F_{ijka_0 \dots a_p} H^{ijk} \nabla^a \nabla_a \Phi + \frac{e_8}{(p+1)!} F_{jkla_0 \dots a_p} H^{ikl} \nabla^j \nabla_i \Phi \\
 & + \frac{e_9}{(p+1)!} F_{ijka_0 \dots a_p} H^{ijk} \nabla^a \Phi \nabla_a \Phi + \frac{e_{12}}{(p+1)!} F_{kmna_0 \dots a_p} H^{ijk} H_i{}^{lm} H_j{}^n \\
 & + \frac{e_{13}}{(p-1)!} F_{iklmna_2 \dots a_p} H^i{}_{a_0 a_1} H^{jkl} H_j{}^{mn} + \frac{e_{17}}{(p+1)!} F_{jkla_0 \dots a_p} H^{iab} H_{iab} H^{jkl} \\
 & + \frac{e_{20}}{(p+1)!} F_{jkla_0 \dots a_p} H_{iab} H^{ikl} H^{jab} + \frac{e_{26}}{(p-1)!} F_{ijklma_2 \dots a_p} H^i{}_{a_0} H^j{}_{a_1} H^{klm} \\
 & + \frac{e_{28}}{(p+1)!} F_{ijka_0 \dots a_p} H^{iab} H^j{}_{ac} H^k{}_{bc} + \frac{e_{31}}{(p+1)!} F_{lmna_0 \dots a_p} H_{ijk} H^{ijl} H^{kmn} \\
 & + \frac{e_{32}}{(p-1)!} F_{jklmna_2 \dots a_p} H_{ia_0 a_1} H^{ijk} H^{lmn} + \frac{e_{33}}{(p+1)!} F_{lmna_0 \dots a_p} H^{ijk} H_{ijk} H^{lmn} \\
 & + \frac{e_{35}}{(p+1)!} F_{klma_0 \dots a_p} H_{ij}{}^k R^{ilmj} + \frac{e_{37}}{(p+1)!} F_{jkla_0 \dots a_p} H^{ijk} R^{la}{}_{ia} \\
 & + \frac{e_{42}}{(p+1)!} F_{ijka_0 \dots a_p} H^i{}_{ab} R^{jabb} + \frac{e_{44}}{(p-1)!} F_{ijklma_2 \dots a_p} H^{ijk} R^l{}_{a_0}{}^m{}_{a_1} \\
 & \left. + \frac{e_{47}}{(p+1)!} F_{ijka_0 \dots a_p} H^{ijk} R^{ab}{}_{ab} \right] \tag{2.15}
 \end{aligned}$$

where the p -independent coefficients e_1, \dots, e_{47} may be found by the T-duality constraint.

The coefficient e_1 has been fixed by the tree-level S-matrix element of one R-R and one NS-NS vertex operators [44], i.e.,

$$e_1 = -\frac{1}{3!} \tag{2.16}$$

The proposal that the combination (2.14) should appear in the world-volume couplings, dictates that the T-duality should fix the coefficient e_8 to be the same as e_{37} . As we will see in section 4, the T-duality indeed produces this relation.

⁶If one does not use the ϵ -tensor identities, then one would find 47 independent couplings.

2.5 $n = p + 6$ case

Similar calculation for the couplings involving one R-R field strength $F^{(p+6)}$ gives the following two independent coupling:

$$\mathcal{L}^{(p+6)} = \epsilon^{a_0 \dots a_p} \left[\frac{f_1}{p!} F_{ijklmna_1 \dots a_p} \nabla_{a_0} H^{ijk} H^{lmn} + \frac{f_2}{(p+1)!} \nabla_i F_{jklmna_0 \dots a_p} H^{ijk} H^{lmn} \right] \quad (2.17)$$

where f_1, f_2 are two arbitrary coefficients that may be found by the T-duality constraint. There are no couplings involving one NS-NS field which is consistent with the tree-level S-matrix element of one R-R and one NS-NS vertex operators [44]. The above two coefficients may be fixed by the low energy expansion of RP^2 -plane S-matrix element of one R-R and two NS-NS vertex operators at order α'^2 . The disk-level calculations have been fixed these coefficients to be zero [42]. We will see that the T-duality also fix these coefficients for O-plane to be zero which is consistent with the speculation that the orientifold projection of D-brane couplings at order α'^2 is the same as O-plane couplings at order α'^2 , up to overall factors.

Therefore, there are 76 independent couplings at order α'^2 which have one R-R field. These gauge invariant couplings are the appropriate couplings on the world-volume of O_p -plane for some specific values for the 76 parameters. They may be found by the S-matrix or other methods in string theory. We are going to find these parameters in this paper by the T-duality constraint. We will find that all 76 parameters are fixed up to an overall factor.

3 T-duality transformations

When compactifying the superstring theory on a circle with radius ρ and with the coordinate y , the full nonlinear T-duality transformations at the leading order of α' for the NS-NS and R-R fields are given in [9, 10, 49], i.e.,

$$\begin{aligned} e^{2\phi'} &= \frac{e^{2\phi}}{G_{yy}}; & G'_{yy} &= \frac{1}{G_{yy}} \\ G'_{\mu y} &= \frac{B_{\mu y}}{G_{yy}}; & G'_{\mu\nu} &= G_{\mu\nu} - \frac{G_{\mu y} G_{\nu y} - B_{\mu y} B_{\nu y}}{G_{yy}} \\ B'_{\mu y} &= \frac{G_{\mu y}}{G_{yy}}; & B'_{\mu\nu} &= B_{\mu\nu} - \frac{B_{\mu y} G_{\nu y} - G_{\mu y} B_{\nu y}}{G_{yy}} \end{aligned} \quad (3.1)$$

$$C'^{(n)}_{\mu \dots \nu \alpha y} = C^{(n-1)}_{\mu \dots \nu \alpha} - \frac{C^{(n-1)}_{[\mu \dots \nu | y} G_{|\alpha] y}}{G_{yy}};$$

$$C'^{(n)}_{\mu \dots \nu \alpha \beta} = C^{(n+1)}_{\mu \dots \nu \alpha \beta y} + C^{(n-1)}_{[\mu \dots \nu \alpha} B_{\beta] y} + \frac{C^{(n-1)}_{[\mu \dots \nu | y} B_{|\alpha] y} G_{|\beta] y}}{G_{yy}}$$

where μ, ν denote any direction other than y . Our notation for making antisymmetry is such that e.g., $C^{(2)}_{[\mu_1 \mu_2} B_{\mu_3] \nu} = C^{(2)}_{\mu_1 \mu_2} B_{\mu_3 \nu} - C^{(2)}_{\mu_3 \mu_2} B_{\mu_1 \nu} + C^{(2)}_{\mu_3 \mu_1} B_{\mu_2 \nu}$. In above transformations the metric is in the string frame. If one assumes fields are transformed covariantly under the coordinate transformations, then the above transformations receive corrections at order

α'^3 in the superstring theory [20] in which we are not interested because the couplings in this paper are at order α'^2 .

To impose the T-duality constraint on the effective action, one should first write all independent gauge invariant couplings of O_p -plane, as we have done in the previous section, and then reduce them on the circle when O_p -plane is along the circle. The T-duality transformation of the reduced action should be the same as the reduction of O_{p-1} -plane when it is orthogonal to the circle, up to some total derivative terms. To impose the T-duality constraint on the effective action, however, it is convenient to use the following reductions for the metric, B -field, dilaton and the R-R potentials [17, 50]:

$$\begin{aligned}
 G_{AB} &= \begin{pmatrix} \bar{g}_{\mu\nu} + e^\varphi g_\mu g_\nu & e^\varphi g_\mu \\ e^\varphi g_\nu & e^\varphi \end{pmatrix}, \quad B_{AB} = \begin{pmatrix} \bar{b}_{\mu\nu} + \frac{1}{2} b_\mu g_\nu - \frac{1}{2} b_\nu g_\mu & b_\mu \\ -b_\nu & 0 \end{pmatrix} \\
 \Phi &= \bar{\phi} + \varphi/4 \\
 C_{\mu_1 \dots \mu_n}^{(n)} &= \bar{c}_{\mu_1 \dots \mu_n}^{(n)} + \bar{c}_{[\mu_1 \dots \mu_{n-1}}^{(n-1)} g_{\mu_n]} \\
 C_{\mu_1 \dots \mu_{n-1} y}^{(n)} &= \bar{c}_{\mu_1 \dots \mu_{n-1}}^{(n-1)}
 \end{aligned} \tag{3.2}$$

where $\bar{g}_{\mu\nu}, \bar{b}_{\mu\nu}, \bar{\phi}$ and $\bar{c}^{(n)}$ are the metric, B-field, dilaton and the R-R potentials, respectively, in the 9-dimensional base space, and g_μ, b_μ are two vectors in this space. In this parametrization, inverse of metric becomes

$$G^{AB} = \begin{pmatrix} \bar{g}^{\mu\nu} & -g^\mu \\ -g^\nu & e^{-\varphi} + g_\alpha g^\alpha \end{pmatrix} \tag{3.3}$$

where $\bar{g}^{\mu\nu}$ is the inverse of the base metric which raises the indices of the vectors. The nonlinear T-duality transformations (3.1) in the parametrizations (3.2) then become remarkably the following linear transformations:

$$\varphi' = -\varphi, \quad g'_\mu = b_\mu, \quad b'_\mu = g_\mu \tag{3.4}$$

and all other 9-dimensional fields remain invariant under the T-duality transformation. Note that the T-duality transformation of the base space R-R potential $\bar{c}^{(n)}$ is trivial in the parametrization (3.2), however, the R-R gauge transformation of this potential in which we are not interested in this paper, seems to be non-trivial.

One can easily verify that the CS action at order α'^0 is invariant under the T-duality. If the killing coordinate y is a world volume, then the T-duality transformation of the reduction of O_p -plane action in the parametrization (3.2) becomes

$$T_{p-1} \int d^p x \epsilon^{a_0 \dots a_{p-1}} \frac{1}{p!} \bar{c}_{a_0 \dots a_{p-1}}^{(p)} \tag{3.5}$$

where we have used the relation $2\pi\rho T_p = T_{p-1}$ and $\epsilon^{a_0 \dots a_{p-1} y} = \epsilon^{a_0 \dots a_{p-1}}$. On the other hand, the reduction of the O_{p-1} -plane action in the parametrization (3.2) when the y -coordinate is transverse to the O_{p-1} -plane is

$$T_{p-1} \int d^p x \epsilon^{a_0 \dots a_{p-1}} \frac{1}{p!} \left(\bar{c}_{a_0 \dots a_{p-1}}^{(p)} + p \bar{c}_{[a_0 \dots a_{p-2}}^{(p-1)} g_{a_{p-1}] \right) \tag{3.6}$$

Using the fact that $g_{\alpha_{p-1}}$ is the component of the 10-dimensional metric which has one y -index and y is a transverse index in this case, the last term above is removed for the O-plane. The rest is the same as the action (3.5).

There is no such symmetry for the CS action at higher orders of α' because the Riemann curvature is not invariant under the T-duality transformations. As a result, one has to add some other terms to this action to make it T-duality invariant as in the leading order term. Since the new couplings involve R-R and NS-NS field strengths and their covariant derivatives, it is convenient to first find the reduction of these field strengths and then apply them to find the reduction of each gauge invariant coupling.

Using the reductions (3.2), it is straightforward to calculation reduction of the Riemann curvature, H , ∇H , $\nabla\Phi$ or $\nabla\nabla\Phi$. As it has been argued in [15], after writing the reductions in terms of \bar{H} which is defined as

$$\bar{H} = d\bar{b} - \frac{1}{2}g \wedge W - \frac{1}{2}b \wedge V \tag{3.7}$$

where $W = db$ and $V = dg$, they have two parts. One part includes terms which are invariant under $U(1) \times U(1)$ gauge transformations corresponding to the gauge fields g_μ, b_μ . They have been found in [15] (see eq. (35), eq. (36) and eq. (37) in this reference⁷). The other part which is not invariant under the $U(1) \times U(1)$ gauge transformations, includes the gauge fields g_μ, b_μ without derivative on them. Such terms are cancelled at the end of the day in the reduction of a 10-dimensional gauge invariant coupling. So one may keep only the $U(1) \times U(1)$ gauge invariant parts of the reduction of the Riemann curvature, H , ∇H , $\nabla\Phi$ and $\nabla\nabla\Phi$, and the following reduction of the inverse of the spacetime metric:

$$G^{AB} = \begin{pmatrix} \bar{g}^{\mu\nu} & 0 \\ 0 & e^{-\varphi} \end{pmatrix} \tag{3.8}$$

and removes all other terms in the reduction. In this way one can find the reduction of any gauge invariant bulk coupling. However, the metric G^{AB} is not used in constructing the O_p -plane couplings in the previous section. The world-volume couplings in fact are constructed by contracting the tensors with the first fundamental form $\tilde{G}^{AB} = \partial_a X^A \partial_b X^B \tilde{g}^{ab}$ which projects the spacetime tensors to the world-volume directions, and with $\perp^{AB} = G^{AB} - \tilde{G}^{AB}$ which projects the tensor to the transverse directions. In the first fundamental form, \tilde{g}^{ab} is inverse of the pull-back metric $\tilde{g}_{ab} = \partial_a X^A \partial_b X^B G_{AB}$.

In the static gauge where $X^a = \sigma^a$ and for the O_p -plane at $X^i = 0$, one has $\tilde{G}^{ij} = \tilde{G}^{ai} = \tilde{G}^{ia} = 0$, and $\tilde{G}^{ab} = \tilde{g}^{ab}$, $\tilde{g}_{ab} = G_{ab}$. When O_p -plane is orthogonal to the killing coordinate, the first fundamental form and world-volume components of the inverse of the spacetime metric have no component along the y -direction, because y is a transverse direction. Hence, in this case $\perp^{ab} = 0$. Moreover $\perp^{ai} = G^{ai} = 0$ by the orientifold projection. The non-zero components in this case are

$$\tilde{G}^{ab} = G^{ab} = \bar{g}^{ab}, \quad \perp^{ij} = G^{ij} = \begin{pmatrix} \bar{g}^{ij} & 0 \\ 0 & e^{-\varphi} \end{pmatrix} \tag{3.9}$$

⁷There is a typo in the reduction of $\nabla_\mu H_{\nu\alpha\gamma}$ in eq. (37) in the published version of [15]. The first term on the right hand side of this expression should be negative.

The gauge field $g_{\bar{a}}$ does not appear in \tilde{G}^{ab} , however, it appears in the reduction of \perp^{ij} . As in (3.8), we have ignored it because we have ignored the non-gauge invariant terms in the reduction of the Riemann curvature, H , ∇H , $\nabla\Phi$ and $\nabla\nabla\Phi$.

On the other hand, when O_p -plane is along the killing coordinate, both the first fundamental form and world-volume components of the inverse of the spacetime metric have component along the y -direction, however, because $\tilde{G}^{ab} = G^{ab}$ one again has $\perp^{ab} = 0$. In this case the non-zero components are

$$\tilde{G}^{ab} = G^{ab} = \begin{pmatrix} \bar{g}^{\bar{a}\bar{b}} & 0 \\ 0 & e^{-\varphi} \end{pmatrix}, \quad \perp^{ij} = G^{ij} = \bar{g}^{ij} \quad (3.10)$$

The gauge field $g_{\bar{a}}$ does not appear in \perp^{ij} , however, it appears in the reduction of \tilde{G}^{ab} that we have again removed it.

Using the reduction of the R-R potential in (3.2), one can find the reduction of R-R field strength and its first derivative which appear in the couplings in the previous section. They have again two parts. One part is not invariant under the $U(1) \times U(1)$ gauge transformations which is cancelled in the gauge invariant couplings, hence we ignore it. The $U(1) \times U(1)$ gauge invariant part of the reduction is

$$\begin{aligned} F_{\mu_1 \dots \mu_{n-1} y}^{(n)} &= \bar{F}_{\mu_1 \dots \mu_{n-1}}^{(n-1)} + (-1)^{(n-3)} W_{[\mu_1 \mu_2} \bar{c}_{\mu_3 \dots \mu_{n-1}}^{(n-3)} + \bar{H}_{[\mu_1 \mu_2 \mu_3} \bar{c}_{\mu_4 \dots \mu_{n-1}}^{(n-4)} \equiv F_{\mu_1 \dots \mu_{n-1}}^{W(n-1)} \\ F_{\mu_1 \dots \mu_n}^{(n)} &= \bar{F}_{\mu_1 \dots \mu_n}^{(n)} + (-1)^{(n-2)} V_{[\mu_1 \mu_2} \bar{c}_{\mu_3 \dots \mu_n}^{(n-2)} + \bar{H}_{[\mu_1 \mu_2 \mu_3} \bar{c}_{\mu_4 \dots \mu_n}^{(n-3)} \equiv F_{\mu_1 \dots \mu_n}^{V(n)} \\ \nabla_y F_{\mu_1 \dots \mu_{n-1} y}^{(n)} &= \frac{1}{2} e^\varphi \left[F_{\mu_1 \dots \mu_{n-1} \mu}^{V(n)} \nabla^\mu \varphi - F_{[\mu_2 \dots \mu_{n-1} \mu_1]}^{W(n-1)} V_{\mu_1}^\mu \right] \\ \nabla_y F_{\mu_1 \dots \mu_n}^{(n)} &= -\frac{1}{2} \left[(-1)^{(n-1)} F_{[\mu_2 \dots \mu_n}^{W(n-1)} \nabla_{\mu_1} \varphi + e^\varphi F_{[\mu \mu_2 \dots \mu_n}^{V(n)} V_{\mu_1}^\mu \right] \\ \nabla_\nu F_{\mu_1 \dots \mu_n}^{(n)} &= \frac{1}{2} \left[2 \nabla_\nu F_{\mu_1 \dots \mu_n}^{V(n)} - (-1)^{(n-1)} F_{[\mu_2 \dots \mu_n}^{W(n-1)} V_{\mu_1] \nu} \right] \\ \nabla_\nu F_{\mu_1 \dots \mu_{n-1} y}^{(n)} &= \frac{1}{2} \left[2 \nabla_\nu F_{\mu_1 \dots \mu_{n-1}}^{W(n-1)} + e^\varphi F_{\mu_1 \dots \mu_{n-1} \mu}^{V(n)} V^\mu_\nu - F_{\mu_1 \dots \mu_{n-1}}^{W(n-1)} \nabla_\nu \varphi \right] \end{aligned} \quad (3.11)$$

where the covariant derivatives on the right-hand side are 9-dimensional and $\bar{F} = d\bar{c}$. One can check that the reduction of ∇H found in [15] can be found from the above reduction when one uses $H^{W(2)} = W$ and $H^{V(3)} = \bar{H}$. Obviously, the $U(1) \times U(1)$ gauge invariant part of the reduction of the R-R potential C is

$$\begin{aligned} C_{\mu_1 \dots \mu_n}^{(n)} &= \bar{c}_{\mu_1 \dots \mu_n}^{(n)} \\ C_{\mu_1 \dots \mu_{n-1} y}^{(n)} &= \bar{c}_{\mu_1 \dots \mu_{n-1}}^{(n-1)} \end{aligned} \quad (3.12)$$

Using the above $U(1) \times U(1)$ gauge invariant part of the reductions, one can calculate the reduction of any 10-dimensional gauge invariant coupling. The result would be the same as writing the coupling in terms of ordinary derivatives of metric, B-field, dilaton and R-R potential and then using the reductions (3.2). For example, using the above reduction for the R-R field strength, one finds the following reduction for the gauge invariant coupling F^2 :

$$\frac{1}{n!} F^{(n)} \cdot F^{(n)} = \frac{1}{n!} F^{V(n)} \cdot F^{V(n)} + \frac{e^{-\varphi}}{(n-1)!} F^{W(n-1)} \cdot F^{W(n-1)} \quad (3.13)$$

which is the correct reduction that has been found in [17] by writing the R-R field strength in terms of R-R potential and using the reductions (3.2). It is obvious that the left-hand side is invariant under the 10-dimensional R-R gauge transformations, hence, the right-hand side should be also invariant under the 9-dimensional R-R gauge transformations. This might be used to define the gauge transformation of the base space R-R potential $\bar{c}^{(n)}$ in which we are not interested in this paper.

As another example, the O_p -plane world-volume reduction of the CS terms in (1.5) are

$$\begin{aligned}
 & \epsilon^{a_0 \dots a_p} \frac{1}{(p-3)!} C_{a_4 \dots a_p}^{(p-3)} R_{a_0 a_1 i j} R_{a_2 a_3}{}^{ij} = \tag{3.14} \\
 & \epsilon^{a_0 \dots a_{p-1}} e^{2\varphi} \left[\frac{1}{4(p-4)!} \bar{c}_{a_4 \dots a_{p-1}}^{(p-4)} V_{a_0 a_1} V_{a_2 a_3} V_{ij} V^{ij} \right. \\
 & \quad \left. - \frac{1}{(p-3)!} \bar{c}_{a_3 \dots a_{p-1}}^{(p-3)} \left(\nabla_{a_0} \varphi V_{a_1 a_2} V_{ij} V^{ij} + \nabla_{a_0} V_{ij} V_{a_1 a_2} V^{ij} \right) \right] \\
 & \epsilon^{a_0 \dots a_p} \frac{1}{(p-3)!} C_{a_4 \dots a_p}^{(p-3)} R_{a_0 a_1 a b} R_{a_2 a_3}{}^{ab} = \\
 & \epsilon^{a_0 \dots a_{p-1}} e^\varphi \left[\frac{1}{(p-3)!} \bar{c}_{a_3 \dots a_{p-1}}^{(p-3)} \left(e^\varphi \nabla_{a_0} V_{ab} V_{a_1}^a V_{a_2}^b - e^\varphi \nabla_{a_0} V_{cd} V^{cd} V_{a_1 a_2} - e^\varphi \nabla_a V_{a_0 a_1} V_b^a V_{a_2}^b \right. \right. \\
 & \quad - e^\varphi V_{cd} V^{cd} V_{a_1 a_2} \nabla_{a_0} \varphi - 2 e^\varphi V_{ab} V^a{}_{a_1} V^b{}_{a_2} \nabla_{a_0} \varphi - \nabla^a V_{a_1 a_2} \nabla_a \varphi \nabla_{a_0} \varphi \\
 & \quad - 2 V_{a_1 a_2} \nabla_a \nabla_{a_0} \varphi + 2 V^a{}_{a_2} \nabla_a \nabla_{a_1} \varphi \nabla_{a_0} \varphi - 2 V_{a_1 a_2} \nabla_a \nabla_{a_0} \varphi \nabla^a \varphi \\
 & \quad \left. - V_{a_1 a_2} \nabla_a \varphi \nabla^a \varphi \nabla_{a_0} \varphi \right) + \frac{1}{(p-4)!} \bar{c}_{a_4 \dots a_{p-1}}^{(p-4)} \left(\frac{1}{4} e^\varphi V_{ab} V^{ab} V_{a_0 a_1} V_{a_2 a_3} \right. \\
 & \quad + \frac{1}{2} e^\varphi V_{ab} V^a{}_{a_0} V^b{}_{a_1} V_{a_2 a_3} + \frac{1}{2} \nabla_a V_{a_2 a_3} \nabla^a V_{a_0 a_1} + \nabla_a V_{a_1 a_2} V^a{}_{a_3} \nabla_{a_0} \varphi \\
 & \quad \left. + \nabla^a V_{a_0 a_1} V_{a_2 a_3} \nabla \varphi + V^a{}_{a_1} V_{a_2 a_3} \nabla_a \varphi \nabla_{a_0} \varphi + \frac{1}{2} V_{a_0 a_1} V_{a_2 a_3} \nabla_a \varphi \nabla^a \varphi \right) \Big]
 \end{aligned}$$

In finding the above result we have separated the world-volume indices to y and the world indices which do not include the y -index, then we have used the reduction for each tensors. We have assumed the 9-dimensional base space is flat, and removed the terms that are projected out by the orientifold projection, e.g., we have removed V_{ai} because g_i is related to G_{iy} and y is world-volume index, hence, it is projected out. Note that the world-volume indices on the right-hand side do not include the y -index.

The O_{p-1} -plane transverse reduction of the CS terms are

$$\begin{aligned}
 & \epsilon^{a_0 \dots a_{p-1}} \frac{1}{(p-4)!} C_{a_4 \dots a_{p-1}}^{(p-4)} R_{a_0 a_1 i j} R_{a_2 a_3}{}^{ij} = \\
 & \quad \epsilon^{a_0 \dots a_{p-1}} \frac{e^\varphi}{(p-4)!} \bar{c}_{a_4 \dots a_{p-1}}^{(p-4)} \left[\frac{1}{2} \nabla_i V_{a_0 a_1} \nabla^i V_{a_2 a_3} - \nabla_i V_{a_1 a_2} V_{a_3}{}^i \nabla_{a_0} \varphi \right] \\
 & \epsilon^{a_0 \dots a_{p-1}} \frac{1}{(p-4)!} C_{a_4 \dots a_{p-1}}^{(p-4)} R_{a_0 a_1 a b} R_{a_2 a_3}{}^{ab} = 0 \tag{3.15}
 \end{aligned}$$

In finding the above result we have separated the transverse indices to y and the transverse indices which do not include the y -index, then we have used the reduction for each tensors.

Here, we have also removed the terms that are projected out for O-plane, e.g., we have removed V_{ab} because g_a is related to G_{ay} and y is transverse index, hence, it is projected out. Note that the transverse indices on the right-hand side do not include the y -index. Similar calculations as above can be done for all couplings in the previous section. Writing the reduced couplings in terms of the base fields \bar{c}, V, \dots , one can easily transform them under the T-duality transformations (3.4).

4 T-duality constraint on the couplings

It has been observed in [14, 15] that the T-duality constraints on the couplings in the bosonic string theory at order α' and α'^2 are the same whether or not the base space is flat. In fact, the constraints that one finds between the coefficients of effective action when base space is flat are exactly the same constraints as one finds for the curved base space. So it is convenient to consider the reduction of the couplings in section 2 on the flat base space, and then impose the T-duality constraint on them to find the unknown coefficients of the couplings.

The T-duality constraint is

$$\Delta - \mathcal{J} = 0 \tag{4.1}$$

where $\Delta = \text{O}_{(p-1)\text{-plane}} - (\text{O}_p\text{-plane})'$. The first term in Δ is transverse reduction of $\text{O}_{(p-1)\text{-plane}}$ and the second term is T-duality of world-volume reduction of $\text{O}_p\text{-plane}$. The \mathcal{J} in above equation represents some total derivative terms in the flat base space, i.e.,

$$\mathcal{J}^n = \int d^p \bar{g}^{ab} \partial_a \mathcal{I}_b^n \tag{4.2}$$

where the vector \mathcal{I}_a^n is made of $\epsilon^{a_0 \dots a_{p-1}}$ and the base space fields, $\bar{c}^{(n)}, V, W, \bar{H}, \partial\varphi, \partial\bar{\phi}$ and their derivatives at three derivative orders. Moreover, to produce the the same structures that appear in Δ , one should multiply each WW or its derivatives by factor $e^{-\varphi}$, each VV by factor e^{φ} , each extra W by factor $e^{-\varphi}$ and each extra V or VW with no such factor. These factors are traced to the parametrisation we have used in the reductions (3.2).

The T-duality constraint (4.1) is similar to the equation (2.6). Hence, to solve it one should use the following Bianchi identities for the field strengths V, W, \bar{H} :

$$dW = 0 ; dV = 0 ; d\bar{H} = -\frac{3}{2}W \wedge V \tag{4.3}$$

and should use the ϵ -tensor identities. Here also we find that it is easy to impose the above Bianchi identities by writing the field strengths W, V or \bar{H} , in terms of the potentials $b_\mu, g_\mu, \bar{b}_{\mu\nu}$. Moreover, to impose the ϵ -tensor identities, we write the resulting non-gauge invariant couplings explicitly in terms of the values that each world-volume index can take, e.g., $a_0 = 0, 1, 2, \dots, p-1$. Performing these steps, one rewrites the equation (4.1) in terms of independent structures. Solving them then one finds the parameters of the gauge invariant couplings found in section 2. This is the strategy that we follow in this section.

To impose the constraint (4.1), we note that the reduction of $F^{(n)}$, involves the base space fields $\bar{c}^{(n-1)}, \bar{c}^{(n-2)}, \bar{c}^{(n-3)}$ and $\bar{c}^{(n-4)}$. So the world-volume reduction of $\text{O}_p\text{-plane}$

and the transverse reduction of O_{p-1} -plane produces the following 9-dimensional R-R potentials:

$$\begin{aligned}
 F^{(p+6)} &\rightarrow \begin{cases} O_p & : \bar{c}^{(p+5)}, \bar{c}^{(p+4)}, \bar{c}^{(p+3)}, \bar{c}^{(p+2)} \\ O_{p-1} & : \bar{c}^{(p+4)}, \bar{c}^{(p+3)}, \bar{c}^{(p+2)}, \bar{c}^{(p+1)} \end{cases} \\
 F^{(p+4)} &\rightarrow \begin{cases} O_p & : \bar{c}^{(p+3)}, \bar{c}^{(p+2)}, \bar{c}^{(p+1)}, \bar{c}^{(p)} \\ O_{p-1} & : \bar{c}^{(p+2)}, \bar{c}^{(p+1)}, \bar{c}^{(p)}, \bar{c}^{(p-1)} \end{cases} \\
 F^{(p+2)} &\rightarrow \begin{cases} O_p & : \bar{c}^{(p+1)}, \bar{c}^{(p)}, \bar{c}^{(p-1)}, \bar{c}^{(p-2)} \\ O_{p-1} & : \bar{c}^{(p)}, \bar{c}^{(p-1)}, \bar{c}^{(p-2)}, \bar{c}^{(p-3)} \end{cases} \\
 F^{(p)} &\rightarrow \begin{cases} O_p & : \bar{c}^{(p-1)}, \bar{c}^{(p-2)}, \bar{c}^{(p-3)}, \bar{c}^{(p-4)} \\ O_{p-1} & : \bar{c}^{(p-2)}, \bar{c}^{(p-3)}, \bar{c}^{(p-4)}, \bar{c}^{(p-5)} \end{cases} \\
 F^{(p-2)} &\rightarrow \begin{cases} O_p & : \bar{c}^{(p-3)}, \bar{c}^{(p-4)}, \bar{c}^{(p-5)}, \bar{c}^{(p-6)} \\ O_{p-1} & : \bar{c}^{(p-4)}, \bar{c}^{(p-5)}, \bar{c}^{(p-6)}, \bar{c}^{(p-7)} \end{cases} \\
 F^{(p-4)} &\rightarrow \begin{cases} O_p & : \bar{c}^{(p-5)}, \bar{c}^{(p-6)}, \bar{c}^{(p-7)}, \bar{c}^{(p-8)} \\ O_{p-1} & : \bar{c}^{(p-6)}, \bar{c}^{(p-7)}, \bar{c}^{(p-8)}, \bar{c}^{(p-9)} \end{cases} \tag{4.4}
 \end{aligned}$$

We have to impose the T-duality constraint (4.1) for each potential $\bar{c}^{(n)}$.

Let us begin with the most simple case. It can easily be observed that the T-duality constraint fixes the coefficient of the coupling $F^{(p-4)}$ to be zero. We look at the term in the reduction which produces $\bar{c}^{(p-9)}$. This term is produced only by the reduction of the coupling (2.4) when one of the transverse indices of the R-R field strength carries the y -index. The reduction of this term, however, is zero after imposing the O-plane conditions. So this can not constraint the coefficient of the coupling (2.4). We consider instead the reductions which produce $\bar{c}^{(p-8)}$. When the O_p -plane is along the circle, it produces the following reduction:

$$\epsilon^{a_0 \dots a_{p-1}} \left[\frac{a}{(p-4)!} \bar{H}_{[ia_6 a_7} \bar{c}_{a_8 \dots a_{p-1}}^{(p-8)} \bar{H}^i{}_{a_0 a_1} \bar{H}_{ja_2 a_3} \bar{H}^j{}_{a_4 a_5} \right] + \dots \tag{4.5}$$

where dots represent some other terms which do not include $\bar{c}^{(p-8)}$. On the other hand, when O_{p-1} -plane is orthogonal to the circle, the reduction of the coupling (2.4) produces the following terms:

$$\epsilon^{a_0 \dots a_{p-1}} \left[\frac{a}{(p-4)!} \bar{H}_{[ia_6 a_7} \bar{c}_{a_8 \dots a_{p-1}}^{(p-8)} \bar{H}^i{}_{a_0 a_1} \left(\bar{H}_{ja_2 a_3} \bar{H}^j{}_{a_4 a_5} + W_{a_2 a_3} W_{a_4 a_5} \right) \right] + \dots \tag{4.6}$$

where dots represent some terms with other structures. The difference between this term and the T-duality transformation of (4.5) produces the following term which involves $\bar{c}^{(p-8)}$:

$$\Delta^{p-8} = \epsilon^{a_0 \dots a_{p-1}} \left[\frac{a}{(p-4)!} \bar{H}_{[ia_6 a_7} \bar{c}_{a_8 \dots a_{p-1}}^{(p-8)} \bar{H}^i{}_{a_0 a_1} W_{a_2 a_3} W_{a_4 a_5} \right] \tag{4.7}$$

This term can not be cancelled by total derivative terms, so the T-duality constraint predicts the coefficient of the coupling (2.4) to be zero, i.e.,

$$a = 0 \tag{4.8}$$

Hence, the T-duality constraint force the coupling (2.4) to be zero. It is a nontrivial result which would be very difficult to confirm with the S-matrix element of one R-R and three NS-NS vertex operators.

It can be also easily observed that the T-duality constraint fixes the coefficients of the $F^{(p+6)}$ -couplings to be zero. In this case we look at the term in the reduction which produces $\bar{c}^{(p+5)}$. This term is produced only by the world-volume reduction of the couplings in (2.17). The T-duality transformation of this term produces the following term for $\bar{c}^{(p+5)}$:

$$\Delta^{p+5} = \epsilon^{a_0 \dots a_{p-1}} \frac{(-1)^p e^\varphi}{2p!} (3f_1 - f_2) \bar{F}_{ijklmna_0 \dots a_{p-1}}^{(p+6)} W^i{}_o \bar{H}^{ojk} \bar{H}^{lmn} \quad (4.9)$$

which can not be cancelled by a gauge invariant total derivative term. Hence, the T-duality constraint forces the above term to be zero, i.e., $3f_1 - f_2 = 0$. To fix these coefficients completely, we look also at the terms in the reduction which produce $\bar{c}^{(p+4)}$. The difference between the O_{p-1} -plane and the T-duality of O_p -plane produces many terms involving $\bar{c}^{(p+4)}$. Here we focus on the terms involving $\bar{c}^{(p+4)}$ and $\nabla\varphi$. One can easily find that only the reduction of the second term in (2.17) produces such term. The T-duality of the reduction of O_p -plane produces $\bar{F}^{p+5} \nabla\varphi \bar{H} \bar{H}$, whereas, the reduction of O_{p-1} -plane produces $\bar{F}^{p+5} \nabla\varphi WW$. They can not cancel each other unless the coefficient of the second term in (2.17) to be zero, i.e., $f_2 = 0$. Combining with the previous constraint, one finds

$$f_1 = 0, f_2 = 0 \quad (4.10)$$

This is the result that the S-matrix calculation produces [42].

Since the coefficient of the $F^{(p-4)}$ -coupling is zero, the next simple case to look at is the terms involving $\bar{c}^{(p-7)}$. One finds $\bar{c}^{(p-7)}$ is produced only by the transverse reduction of the couplings $F^{(p-2)}$ in (2.7) which have R-R field strength with transverse indices. Since only the couplings with coefficients b_1, b_7 in (2.7) involves the R-R field strength with the transverse indices, and the transverse reduction of these terms produces non-zero results which are not total derivative terms, one finds that the T-duality constraint (4.1) fixes these coefficients to be zero, i.e.,

$$b_1 = 0, b_7 = 0 \quad (4.11)$$

The above result can also be found by looking at the terms involving $\bar{c}^{(p-6)}$. One finds that only the reductions of the terms with coefficients b_1, b_7 survived the O-plane conditions. The T-duality constraint then forces these coefficients to be zero. This result is consistent with the S-matrix calculation (2.8).

The surviving terms in (2.7) have R-R field strength with only world-volume indices. One finds that the reduction of these terms produce terms involving $\bar{c}^{(p-5)}$. However, they are removed by the O-plane conditions. Having no $\bar{c}^{(p-5)}$ -term from the reduction of $F^{(p-2)}$ -couplings, one concludes that the transverse reduction of $F^{(p)}$ -couplings on the O_{p-1} -plane which also produces $\bar{c}^{(p-5)}$, must be zero. So one has to consider the R-R field strengths $F^{(p)}, \nabla F^{(p)}$ in (2.10) which have transverse indices because only those terms produce $\bar{c}^{(p-5)}$. In fact all terms in (2.10) have such structure. However, the transverse

reduction of those terms that have only one transverse index, produce $\bar{H} \wedge \bar{c}^{(p-5)}$ with only world-volume indices which is removed by the O-plane condition. Therefore, they produce no non-zero term after reduction. The terms in (2.10) which have more than one transverse indices, i.e., $c_{21}, c_{23}, c_{34}, c_{40}$, however, produce non-zero result after imposing the O-plane conditions. The T-duality constraint (4.1) then requires these terms to be zero, i.e.,

$$c_{21} = 0, c_{23} = 0, c_{34} = 0, c_{40} = 0 \tag{4.12}$$

Since the reduced couplings involve only $\bar{c}^{(p-5)}$ there is no total derivative terms connecting the reduced couplings. Moreover, since they involve no derivative of field strength \bar{H} , there is no Bianchi identity relation between the reduced couplings. Hence, the coefficients of all terms must be zero, as we have set in above equation.

Since the coefficients of the couplings involving $F^{(p+6)}$ are zero, i.e., (4.10), the next simple case to consider is to look at the terms involving $\bar{c}^{(p+3)}$. One finds $\bar{c}^{(p+3)}$ is produced only by the world-volume reduction of the couplings in (2.15) which have R-R field strength with no y index. So all terms in (2.15), except the terms in which the R-R field strength carries the world-volume indices a_0, \dots, a_p , produce $\bar{c}^{(p+3)}$. The T-duality constraint (4.1) makes the coefficients of all these terms to be zero, i.e.,

$$e_{13} = 0, e_{26} = 0, e_{32} = 0, e_{44} = 0 \tag{4.13}$$

In finding the above result, we have added all possible total derivative terms and imposed the Bianchi identities and the ϵ -tensor identities. We find that there is no total derivative term involved here.

There are still further T-duality constraint on the non-zero couplings involving $F^{(p+4)}$. The T-duality constraint (4.1) produces the following relations for the other coefficients:

$$\begin{aligned} e_{17} = 0, \quad e_{31} = 0, \quad e_{33} = 0, \quad e_{35} = 0, \quad e_{47} = 0, \quad e_6 = 0, \quad e_9 = 0, \\ e_3 = e_1, \quad e_{37} = -3e_1, \quad e_{42} = -6e_1, \quad e_8 = -3e_1, \quad e_{12} = -\frac{1}{2}e_1, \quad e_{20} = \frac{3}{2}e_1, \quad e_{28} = \frac{1}{2}e_1 \end{aligned} \tag{4.14}$$

In this case we find that there is some total derivative term involved in which we are not interested in this paper. Up to an overall coefficient e_1 , then all terms in (2.15) are fixed by the T-duality constraint that we have considered so far.

It is interesting that the coefficients e_8, e_{37} are identical which is in accord with the proposal that the second derivative of dilaton appears in the world-volume action as the dilaton-Riemann curvature (2.14). Moreover, the first derivative of dilaton appears only in the term with coefficient e_3 . Using an integration by part on the first term in (2.15), and the relation $e_3 = e_1$, one finds that the first derivative of dilaton appears in the following extension of $\nabla_a \nabla^a H^{ABC}$:

$$\nabla_a \nabla^a H^{ABC} \rightarrow \mathcal{D}_a \nabla^a H^{ABC}; \quad \mathcal{D}_a \equiv \nabla_a - \nabla_a \Phi \tag{4.15}$$

We will see that this structure appears in all couplings that the T-duality produces. Note that the transverse contraction of two derivatives, i.e., $\nabla_i \nabla^i$ has been removed at the onset by imposing the equations of motion.

Imposing the constraints that we have found so far, i.e., (4.8), (4.10), (4.12), (4.13), and (4.14), the remaining reductions in (4.4) are

$$\begin{aligned}
 F^{(p+4)} &\rightarrow \begin{cases} O_p & : & \bar{c}^{(p+1)}, & \bar{c}^{(p)} \\ O_{p-1} & : & \bar{c}^{(p+1)}, & \bar{c}^{(p)}, & \bar{c}^{(p-1)} \end{cases} \\
 F^{(p+2)} &\rightarrow \begin{cases} O_p & : & \bar{c}^{(p+1)}, & \bar{c}^{(p)}, & \bar{c}^{(p-1)}, & \bar{c}^{(p-2)} \\ O_{p-1} & : & \bar{c}^{(p)}, & \bar{c}^{(p-1)}, & \bar{c}^{(p-2)}, & \bar{c}^{(p-3)} \end{cases} \\
 F^{(p)} &\rightarrow \begin{cases} O_p & : & \bar{c}^{(p-1)}, & \bar{c}^{(p-2)}, & \bar{c}^{(p-3)}, & \bar{c}^{(p-4)} \\ O_{p-1} & : & \bar{c}^{(p-2)}, & \bar{c}^{(p-3)}, & \bar{c}^{(p-4)} \end{cases} \\
 F^{(p-2)} &\rightarrow \begin{cases} O_p & : & \bar{c}^{(p-3)}, & \bar{c}^{(p-4)} \\ O_{p-1} & : & \bar{c}^{(p-4)} \end{cases} \tag{4.16}
 \end{aligned}$$

The next case that we are going to consider in the reductions (4.16), is $\bar{c}^{(p+1)}$. Since one part of the reduction involve the $F^{(p+2)}$ -couplings, the T-duality constraint should relate the remaining constant e_1 in $F^{(p+4)}$ -couplings to the d -parameters in (2.12). The T-duality constraint (4.1) in this case remarkably fixes e_1 and all d 's in terms of one overall parameter, i.e.,

$$\begin{aligned}
 d_9=0, \quad d_{10}=0, \quad d_{22}=0, \quad d_{36}=0, \quad d_{41}=0, \quad d_{42}=0, \quad d_{43}=0, \quad d_{47}=0, \\
 e_1 = \frac{1}{12}d_{11}, \quad d_{12}=d_{11}, \quad d_{15}=-d_{11}, \quad d_{16}=-d_{11}, \quad d_2 = \frac{1}{8}d_{11}, \quad d_{21} = -\frac{1}{4}d_{11}, \\
 d_{26} = -\frac{1}{4}d_{11}, \quad d_{27} = -\frac{1}{8}d_{11}, \quad d_{29} = -\frac{1}{2}d_{11}, \quad d_3 = -\frac{3}{8}d_{11}, \quad d_{30} = -\frac{1}{8}d_{11}, \quad d_{48} = \frac{1}{4}d_{11} \tag{4.17}
 \end{aligned}$$

In this case also, the T-duality constraint requires some total derivative terms in which we are not interested.

The coefficients d_{12}, d_{15} in (4.17) are consistent with the S-matrix result (2.13). Moreover, the relation between e_1 and d_{11} is also consistent with the S-matrix results (2.13) and (2.16). As pointed out before, since $d_{11} = d_{12}$ the second derivative of dilaton appears as the dilaton-Riemann curvature (2.14). The first derivative of dilaton also appears as dilaton-derivative extension of world-volume derivative contraction with Riemann curvature and with H , i.e.,

$$\begin{aligned}
 \nabla_a R^{aABC} &\rightarrow \mathcal{D}_a R^{aABC} \\
 \nabla_a H^{aAB} &\rightarrow \mathcal{D}_a H^{aAB} \tag{4.18}
 \end{aligned}$$

Note that the transverse derivative contraction with the Riemann curvature and with H have been removed by the equations of motion. We will see that this extension appears in other couplings that the T-duality produces.

Since all e -parameters and d -parameters are fixed up to the overall factor d_{11} , one does not need to consider $\bar{c}^{(p)}$ because this term is produced only by $F^{(p+4)}$ - and $F^{(p+2)}$ -couplings. In fact, we have checked that the T-duality constraint on $\bar{c}^{(p)}$ reproduces only

the relations in (4.14) and (4.17). Hence, for the next case we consider $\bar{c}^{(p-1)}$ in the reductions (4.16). The T-duality constraint on this term should give some relations between $F^{(p+4)}$ -, $F^{(p+2)}$ - and $F^{(p)}$ -couplings. Since the parameters in the first two set of couplings are fixed, this constraint should fix the c -parameters in (2.10). The T-duality constraint (4.1) in this case fixes d_{11} and all c 's in terms of one overall parameter c_{12} , i.e.,

$$\begin{aligned}
 c_{17}=0, \quad c_{32}=0, \quad c_{37}=0, \quad c_{10}=0, \quad c_{14}=0, \quad c_{16}=0, \quad c_{28}=0, \quad c_{43}=0, \quad c_7=0, \\
 d_{11}=2c_{12}, \quad c_{13}=\frac{1}{2}c_{12}, \quad c_2=-2c_{12}, \quad c_3=\frac{1}{2}c_{12}, \quad c_{33}=-\frac{1}{2}c_{12}, \quad c_{38}=\frac{1}{2}c_{12}, \quad c_{39}=-c_{12}, \quad c_{44}=2c_{12}, \\
 c_{46}=-c_{12}, \quad c_5=-2c_{12}, \quad c_8=\frac{1}{2}c_{12}, \quad c_{24}=\frac{1}{4}c_{12}, \quad c_{30}=-\frac{1}{32}c_{12}, \quad c_{31}=\frac{1}{8}c_{12}, \quad c_{35}=-c_{12}
 \end{aligned} \tag{4.19}$$

In this case also there are some total derivative terms in which we are not interested in this paper because we assumed the spacetime manifold has no boundary.

The coefficients c_2, c_3 in (4.19) are consistent with the S-matrix result (2.11). Moreover, the relation between d_{11} and c_2 is also consistent with the S-matrix results (2.11) and (2.13). The coefficients c_{12}, c_{46} are not identical, so one may conclude that the corresponding couplings in (2.10) are not in accord with the proposal that the second derivative of dilaton appears in the world-volume action as the dilaton-Riemann curvature (2.14). However, using the R-R Bianchi identity (2.2), one can write

$$\nabla_i F_{a_1 \dots a_p}^{(p)} = p \nabla_{a_1} F_{ia_2 \dots a_p}^{(p)} - \frac{p(p-1)}{2} H_{ia_1 a_2} F_{a_3 \dots a_p}^{(p-2)} \tag{4.20}$$

where we have used the O-plane conditions on H and the fact that there is an overall tensor $\epsilon^{a_0 \dots a_p}$. Then up to a total derivative term, one can write the term in (2.10) with coefficient c_5 as

$$\begin{aligned}
 \frac{1}{p!} \nabla_i F_{a_1 \dots a_p}^{(p)} H^{ia}{}_{a_0} \nabla_a \Phi &= \frac{1}{(p-1)!} F_{ia_2 \dots a_p}^{(p)} H^{ia}{}_{a_1} \nabla_a \nabla_{a_0} \Phi + \frac{1}{(p-1)!} F_{ia_2 \dots a_p}^{(p)} \nabla_{a_0} H^{ia}{}_{a_1} \nabla_a \Phi \\
 &\quad - \frac{1}{2(p-2)!} H_{ia_1 a_2} F_{a_3 \dots a_p}^{(p-2)} H^{ia}{}_{a_0} \nabla_a \Phi
 \end{aligned} \tag{4.21}$$

The first term on the right hand side then has the same structure as the term with coefficient c_{12} . Since $c_{12} + c_5 = c_{46}$, one can write the corresponding couplings in (2.10) as the dilaton-Riemann curvature (2.14). The second term on the right hand side can be combined with the first term in (2.10) to write them as dilaton-derivative combination (4.15). The last term should be added to the b_9 -coupling in (2.7).

The coefficients c_3, c_8 are identical, hence, the corresponding couplings can be combined as the dilaton-derivative (4.15). It seems, however, that the second derivative of dilaton in the coupling with coefficient c_{13} in (2.10) can not be combined with any coupling with structure FHR to be written as the dilaton-Riemann curvature. This stems from the fact that when we have written the independent couplings in (2.10), we had not paid attention on the proposal (2.14). Now that we have found the couplings we may use appropriate ϵ -tensor identities to write the couplings as the dilaton-Riemann curvature. In fact, writing

the world-volume indices explicitly as $0, 1, \dots, p$, one can find the following identity:

$$\begin{aligned} & \frac{1}{2(p-2)!} F_{j a a_3 \dots a_p} H_{i a_0 a_1} R^{i a j}{}_{a_2} - \frac{1}{(p-1)!} F_{j a_2 \dots a_p} H_{i a a_0} R^{i a j}{}_{a_1} \\ &= \frac{1}{2(p-1)!} F_{j a_2 \dots a_p} H_{i a_0 a_1} R^{i a j}{}_a \end{aligned}$$

Using this ϵ -tensor identity, one finds that the couplings in (2.10) with coefficients c_{13}, c_{38}, c_{39} can be written as the dilaton-Riemann curvature (2.14).

The T-duality constraint (4.1) for $\bar{c}^{(p-2)}$ should reproduce only the relations in (4.19). We have checked it explicitly.

Finally, to relate the constant c_{12} to the b -parameters in (2.7) and α -parameters in (2.9), one can consider the T-duality constraint on $\bar{c}^{(p-3)}$ or $\bar{c}^{(p-4)}$. We consider $\bar{c}^{(p-3)}$ in the reductions (4.16). The T-duality constraint on this term should give some relations between $F^{(p+2)}$ -, $F^{(p)}$ - and $F^{(p-2)}$ -couplings and the couplings in (2.9). Since the parameters in the first two sets of couplings are fixed, this constraint should fix the b -parameters in (2.7), α -parameters in (2.9) and c_{12} in terms of one overall parameter. The T-duality constraint in this case produces the following relations:

$$\alpha_2 = -\alpha_1, b_2 = -2\alpha_1, b_4 = 2\alpha_1, b_5 = -2\alpha_1, b_9 = -2\alpha_1, c_{12} = 4\alpha_1 \quad (4.22)$$

In this case also there are some total derivative terms in which we are not interested in this paper. The first relation above is consistent with CS coupling (1.5). The coefficients b_2, b_4, b_5 are consistent with the S-matrix result (2.8). The coefficient b_9 is consistent with the proposal that the first derivative of dilaton appears in the dilaton-derivative combination. To see this we note that the last term in (4.21) has the same structure as b_9 -coupling. Hence, this structure has coefficient $b_9 - c_5/2 = 2\alpha_1$ which is minus of b_2 . As a result they can be combined into the dilaton-derivative combination (4.15). This ends our illustrations that the T-duality constraint (4.1) can fix all parameters of the minimal gauge invariant couplings that we have found in section 2 up to an overall factor.

5 Discussion

In this paper, imposing only the gauge symmetry and the T-duality symmetry on the effective action of O_p -plane, we have found the following couplings at order α'^2 :

$$S = -\frac{\alpha_1 T_p \pi^2 \alpha'^2}{24} \int d^{p+1}x \left[\mathcal{L}_{\text{CS}}^{(p-3)} + \mathcal{L}^{(p-2)} + \mathcal{L}^{(p)} + \mathcal{L}^{(p+2)} + \mathcal{L}^{(p+4)} \right] \quad (5.1)$$

where α_1 is an overall constant that can not be fixed by the T-duality constraint. The gauge invariant Lagrangians are the following:

$$\begin{aligned} \mathcal{L}_{\text{CS}}^{(p-3)} &= \epsilon^{a_0 \dots a_p} \left[\frac{1}{(p-3)!} C_{a_4 \dots a_p}^{(p-3)} R_{a_0 a_1}{}^{ij} R_{a_2 a_3}{}_{ij} - \frac{1}{(p-3)!} C_{a_4 \dots a_p}^{(p-3)} R_{a_0 a_1}{}^{ab} R_{a_2 a_3}{}_{ab} \right] \\ \mathcal{L}^{(p-2)} &= 2\epsilon^{a_0 \dots a_p} \left[-\frac{1}{(p-2)!} F_{a_3 \dots a_p} \mathcal{D}^a H_{i a a_0} H^i{}_{a_1 a_2} \right. \\ &\quad \left. + \frac{1}{(p-2)!} F_{a_3 \dots a_p} \nabla_{a_0} H_{i a a_1} H^{i a}{}_{a_2} - \frac{1}{(p-2)!} F_{a_3 \dots a_p} \nabla_a H_{i a_0 a_1} H^{i a}{}_{a_2} \right] \quad (5.2) \end{aligned}$$

$$\begin{aligned}
 \mathcal{L}^{(p)} &= 4\epsilon^{a_0\dots a_p} \left[\frac{2}{(p-1)!} F_{ia_2\dots a_p} \mathcal{D}_a \nabla_{a_0} H^{ia_1} - \frac{1}{2(p-1)!} F_{ia_2\dots a_p} \mathcal{D}_a \nabla^a H^i{}_{a_0 a_1} \right. \\
 &\quad - \frac{1}{(p-1)!} F_{ia_2\dots a_p} H^i{}_{a a_1} \mathcal{R}^a{}_{a_0} + \frac{1}{2(p-1)!} F_{ja_2\dots a_p} H_{ia_0 a_1} \mathcal{R}^{ij} \\
 &\quad + \frac{1}{4(p-1)!} F_{ka_2\dots a_p} H_{iaa_0} H^{ijk} H_j{}^a{}_{a_1} - \frac{1}{32(p-3)!} F_{iaba_4\dots a_p} H^{iab} H^j{}_{a_0 a_1} H_{ja_2 a_3} \\
 &\quad + \frac{1}{8(p-1)!} F_{la_2\dots a_p} H_{ia_0 a_1} H^i{}_{jk} H^{jkl} - \frac{1}{2(p-1)!} F_{ia_2\dots a_p} H^{iab} H^j{}_{aa_0} H_{jba_1} \\
 &\quad \left. - \frac{1}{(p-1)!} F_{ka_2\dots a_p} H^{ijk} R_{ia_0 j a_1} + \frac{2}{(p-1)!} F_{ia_2\dots a_p} H^{iab} R_{aa_0 b a_1} \right] \\
 \mathcal{L}^{(p+2)} &= 8\epsilon^{a_0\dots a_p} \left[\frac{1}{8(p+1)!} \nabla_k F_{la_0\dots a_p} H^{ijk} H_{ij}{}^l - \frac{3}{8(p+1)!} \nabla_i F_{ja_0\dots a_p} H^{iac} H^j{}_{ac} \right. \\
 &\quad + \frac{1}{p!} F_{ija_1\dots a_p} \mathcal{D}_a R^{iaj}{}_{a_0} - \frac{1}{4p!} F_{jka_1\dots a_p} \mathcal{D}^a H_{iaa_0} H^{ijk} \\
 &\quad + \frac{1}{(p+1)!} \nabla_i F_{ja_0\dots a_p} \mathcal{R}^{ij} - \frac{1}{4p!} F_{jka_1\dots a_p} \nabla^a H^{ijk} H_{iaa_0} - \frac{1}{8p!} F_{kla_1\dots a_p} \nabla_{a_0} H^{ijk} H_{ij}{}^l \\
 &\quad \left. - \frac{1}{2p!} F_{ija_1\dots a_p} \nabla_a H^i{}_{ba_0} H^{jab} - \frac{1}{8p!} F_{ija_1\dots a_p} \nabla_{a_0} H^i{}_{ab} H^{jab} \right] \\
 \mathcal{L}^{(p+4)} &= \frac{2}{3} \epsilon^{a_0\dots a_p} \left[- \frac{1}{(p+1)!} F_{ijka_0\dots a_p} \mathcal{D}_a \nabla^a H^{ijk} - \frac{3}{(p+1)!} F_{jkla_0\dots a_p} H^{ikl} \mathcal{R}^j{}_i \right. \\
 &\quad - \frac{1}{2(p+1)!} F_{kmna_0\dots a_p} H^{ijk} H_i{}^{lm} H_{jl}{}^n + \frac{3}{2(p+1)!} F_{jkla_0\dots a_p} H_{iab} H^{ikl} H^{jab} \\
 &\quad \left. + \frac{1}{2(p+1)!} F_{ijka_0\dots a_p} H^{iab} H^j{}_{ac} H^k{}_{bc} - \frac{6}{(p+1)!} F_{ijka_0\dots a_p} H^i{}_{ab} R^{jabb} \right] \quad (5.3)
 \end{aligned}$$

The second derivative of dilaton appears in the dilaton-Riemann curvature (2.14) and the first derivative of dilaton appears in the dilaton-derivative (4.15). Most of the couplings in (5.1) are new couplings which have not been found by any other method in string theory. This action is fully consistent with the partial couplings that have been already found in the literature by the S-matrix method, i.e., the couplings of one arbitrary R-R field strength and one NS-NS, and also the couplings of one R-R field strength $F^{(p-2)}$ and two B-fields.

We have seen that the O-plane couplings at order α' , found by the T-duality constraint, are the same as the orientifold projection of the partial couplings that have been found in the literature from the disk-level S-matrix elements. However, the world-sheet corresponding to the tree-level S-matrix elements of O-plane is RP^2 . This may indicate that the orientifold projection of disk-level S-matrix elements and the RP^2 -level S-matrix elements should have the same low energy expansion at order α'^2 . In other words, up to overall factors, the orientifold projection of D_p -brane couplings at order α'^2 should produce the O_p -plane couplings at order α'^2 . This is not, however, the case for higher orders of α' which can be seen from the curvature expansion of the anomalous CS couplings, i.e.,

$$\begin{aligned}
 \sqrt{\mathcal{L}(\pi^2 \alpha' R)} &= 1 + \frac{(4\pi^2 \alpha')^2}{96} p_1(R) - (4\pi^2 \alpha')^4 \left(\frac{1}{10240} p_1^2(R) - \frac{7}{23040} p_2(R) \right) + \dots \\
 \sqrt{\mathcal{A}(4\pi^2 \alpha' R)} &= 1 - \frac{(4\pi^2 \alpha')^2}{48} p_1(R) + (4\pi^2 \alpha')^4 \left(\frac{1}{2560} p_1^2(R) - \frac{1}{2880} p_2(R) \right) + \dots \quad (5.4)
 \end{aligned}$$

where the first one is for O-plane and the second one is for D-brane [36]. The reason that the couplings are proportional at order α'^2 but not at the higher orders, may be rooted to the fact that the T-duality transformation at order α'^2 has no higher derivative correction whereas one expects corrections to the Buscher rules at higher orders of α' . If the T-duality transformations are the Buscher rules (3.4) which are linear, then the T-duality constraint would satisfy at each order of α' separately. The resulting couplings at a given order of α' then can be divided to two parts by the orientifold projection. One part would be the O-plane couplings. However, the corrections to the Buscher rules which are not linear, mix the constraints at different orders of α' . That is, the constraints at a given order of α' has contribution from the couplings at that order as well as couplings at lower orders of α' . Then the orientifold projection of the resulting T-duality invariant couplings at the given order of α' would not be the same as the couplings that one would find by imposing the orientifold projection at all orders of α' . Hence, the orientifold projection of the D-brane couplings at order α'^3 and higher would not produce the corresponding O-plane couplings.

The disk-level S-matrix elements of one arbitrary R-R and two NS-NS vertex operators have been calculated in [48, 51]. The low energy expansion of them should produce D-brane couplings at order α'^2 . The orientifold projection of those couplings should then be the same as the couplings that we have found in (5.1). It would be interesting to perform this calculation.

We have seen that the derivatives of dilaton appears only through the dilaton-Riemann curvature (2.14) and the dilaton-derivative (4.15). It has been shown in [44] that the dilaton-Riemann curvature is invariant under linear T-duality. The dilaton-derivative is also invariant under the linear T-duality. In fact one can write the contraction of the dilaton-derivative with an arbitrary vector at the linear order of metric perturbation as

$$\mathcal{D}_a A^a = \partial_a A^a + \frac{1}{2} A^a \eta^{bc} \partial_a h_{bc} - \partial_a \Phi A^a \tag{5.5}$$

where $G_{AB} = \eta_{AB} + h_{AB}$. Separating the world-volume indices to y -index and other world-volume indices, and using the linear T-duality transformations $h_{yy} \rightarrow -h_{yy}$ and $\Phi \rightarrow \Phi - h_{yy}/2$, then one finds the above expression is invariant under the linear T-duality. Similar analysis has been done in [44] to show that the dilaton-Riemann curvature is invariant under the linear T-duality. The invariance of the world-volume action under linear T-duality requires the derivatives of dilaton appear in the dilaton-Riemann and dilaton-derivative combinations. However, the invariance of the effective action under full nonlinear T-duality requires that the couplings of one R-R and an arbitrary number of NS-NS fields appear only through the combination (5.1).

The action (5.1) is complete action of O_p -plane at order α'^2 for $\alpha_1 = -1/4$. This action however has only one R-R field. The O_p -plane action for zero R-R field have been found in [30, 31]. This action should have couplings involving two, three and four R-R fields as well. Each set of couplings may be found by the T-duality constraint up to an overall factor. Then the S-duality may be used to relate the overall factor of three R-R couplings to the couplings (5.1), and the two and four R-R couplings to the couplings found in [30, 31]. It would be interesting to perform this calculation to find a gauge invariant action which is also invariant under the T-duality and the S-duality.

It would be also interesting to extend the calculation in this paper to find the D_p -brane couplings at order α'^2 . A difficulty in this calculation is that each coupling in the effective action at order α'^2 may have an arbitrary number of B_{ab} . They may also have world-volume derivative of this field, i.e., $\partial_a B_{bc}$ which does not appear in the field strength H_{abc} . They are consistent with the gauge symmetry because the D-brane has also open string gauge field strength f_{ab} and the combination $B_{ab} + f_{ab}$ is invariant under the gauge transformation. The T-duality does not relate the massless closed string fields to the massless open string fields. Hence, in the T-duality constraint for the massless closed string fields, one may have couplings that are not gauge invariant. The reduction of those couplings then would not be invariant under the $U(1) \times U(1)$ gauge transformations. That makes problem in using the trick used in section 3 to keep only the $U(1) \times U(1)$ gauge invariant part of reduction of the Riemann curvature and other field strengths.

In finding the parameters in section 4, we have ignored some total derivative terms in the base space. If O-plane are at the fixed point of closed spacetime, then there would be no boundary in the base space and the total derivative terms become zero by using the Stokes's theorem. However, if the spacetime has boundary, then the base space has boundary as well. In this case, the O-plane may end to the boundary. Hence, the total derivative terms in the base space can not be ignored. They produce some boundary terms in the boundary of the base space [16]. In that case, one should consider some couplings at the boundary of O-plane. The boundary terms in the boundary of the base space should be cancelled by the T-duality of the couplings on the boundary of O-plane. This constraint may fix the couplings at the boundary of the O-plane. It would be interesting to find the boundary terms in the effective action of O-plane.

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