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**Statics analysis of N-order nanoplates using modified couple stress theory with the Navier's solution method**

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**Abstract**

In this paper using the modified couple stress theory, to study the bending characteristics and buckling of N-order rectangular Nano plates with simply-supported boundary conditions was investigated. With the aim of considering the effects of small scales, the modified couple stress theory, which has only one parameter of length scale, was used. In the modified couple stress theory; the strain energy density is a function of the components of the strain tensor, curvature tensor, stress tensor and the symmetric part of the couple stress tensor. After obtaining the strain energy, external work, and buckling equation and placing them in the Hamilton's equation, the basic and auxiliary equations of the Nano plates were obtained. Then, by applying boundary and force conditions in the governing equations, the bending and buckling of the N-order rectangular Nano plates with thickness h and simply-supported conditions were explored. Moreover, the solution method was the Navier's solution.

**Keywords:** Modified couple stress theory, N-order Nanoplates, Navier's solution method, Bending, Buckling.

**Introduction**

For the study of materials in small-scale, the safest method is the atomic and molecular scale test. In this method, the nanostructures are studied in real dimensions. In this method, Atomic Force Microscopy (AFM) is used to determine the mechanical properties of nanostructures to apply different mechanical loads on nanoplates and measure response. The main problems with this method are the difficulty of controlling the test conditions at this scale, as well as the economic high-costs and time-consuming method. Therefore, this method is used only to validate other simple and low-cost methods. [1-6]

**N-order plate model**

The displacement equations for the N-order plate are defined as following:

|  |  |
| --- | --- |
|  | (1) |
| *- ,* n=3, 5, 7, 9, … |
|  |

Where in and are the normal vector rotation around the x-axis and y-axis, and also w is the plate midpoint at the z-axis direction. The symmetric part of the curvature tensor, strain tensor, stress tensor and rotational vector for the Mindlin's plate model is as following:

|  |  |
| --- | --- |
|  | (2) |
|  | (3) |
|  | (4) |
|  | (5) |
|  | (6) |
|  | (7) |
|  | (8) |
|  | (9) |
|  | (10) |
|  | (11) |
|  | (12) |
|  | (13) |
|  | (14) |
|  | (15) |
|  | (16) |
|  | (17) |
| + | (18) |
|  | (19) |
|  | (20) |
|  | (21) |
|  | (22) |
| The strain energy changes are as following: | |
|  | (23) |
| To simplify, the coefficients name of the variables can be denoted from F1 to F15 according to Equation (30) and they are obtained separately. | |
|  | (24) |

**Buckling force**

For a rectangular plate with length a and width b and thickness h, the axes forces (Pxy, Py, Px) as following:

Px: Axial force along x, Py: Axial force along y, Pxy: Plate shear force xy, and also q (x, y) is an out-of-plane force. The buckling force equation will be [7 and 8].

|  |  |  |
| --- | --- | --- |
|  | (25) | |
|  | (26) |
| Given that in this study only the external force qz was applied, virtual work is: | |
|  | (27) |
| Also using the Hamilton's principle, can be written: | |
|  | (28) |
| Where U is the strain energy, and W is the external force work. | |

**The final equation of the plate with the utilization of external and buckling forces**

Using Hamilton's principle (Equation 28), the main equations are obtained as following:

|  |  |
| --- | --- |
|  | (29) |
|  | (30) |
|  | (31) |

**N-order plate equations in the general state (including bending and buckling)**

Considering the following:

|  |  |
| --- | --- |
|  | (32) |
|  | (33) |
|  | (34) |
|  | (35) |
|  | (36) |
|  | (37) |
|  | (38) |
|  | (39) |
|  | (40) |
|  | (41) |
| The general equations of the N-order plate will be obtained as following: | |
|  | (42) |
|  | (43) |
|  | (44) |

**Results and Discussion**

The computational program was written in Matlab software, and the results were obtained using this program. All boundary conditions were also considered as simply-supported.

Figure (1) shows the N-order nanoplate bending rate under sinusoidal load effect for the length to width ratio and length to thickness ratio. That which is seen in the table, with increasing the length scale parameter than the thickness of the plate, the bending ratio decreased. Furthermore, the bending ratio increased with increasing aspect ratio of the plate.

Table (1) shows the bending value of the N-order nanoplate under the surface uniform load effect for the various ratio of length to width and different ratio of length to thickness. As shown in the table, with increasing length scale parameter than the thickness parameter, the bending value of the nanoplate decreased. Moreover, with increasing aspect ratio of the plate, the bending value increased.

Figure (2) shows the bending contours of the N-order nanoplate with simply-supported and under the force q. According to the figure, the value of bending at the plane center is highest.

Table (2) compares the dimensionless bending value of the n-order nanoplate under the sinusoidal load and surface uniform load effect for different length to width ratio. That which is seen in the table, the dimensionless bending value is equal for surface uniform load and sinusoidal load when the length scale parameter is not considered and in all other cases, the sinusoidal load bending is less than surface uniform load.

Figure (3) compares the bending rates of different nanoplates under the sinusoidal load effect for different length to width ratio. That which is seen in the table, the bending value is the lowest for the Kirchhoff's nanoplate and the highest for the third-order nanoplate.

Figure (4) Shows the critical force value of the N-order nanoplate under the effect of the bi-axial surface force in the x and y directions which with increasing the ratio of length scale to thickness increased, and with increasing length to thickness ratio of nanoplate decreased.

Table (3) compares the dimensionless critical buckling force value of nanoplate under the effect of the uniaxial and bi-axial forces. That which is seen, Mindlin's nanoplate has dimensionless critical buckling force value highest, and the Kirchhoff's nanoplate has the value lowest. Furthermore, the dimensionless critical buckling force of different nanoplates under the effect of the two-axis surface force in the x and y directions, with increasing the length to thickness ratio of the nanoplate behaved differently as following:

* The dimensionless critical force value increased with increasing the length to thickness ratio of the Mindlin's nanoplate.
* The dimensionless critical force value decreased slightly with increasing the length to thickness ratio of the third-order and fifth-order shear nanoplates.
* The dimensionless critical force value remained constant with increasing the length to thickness ratio of the Kirchhoff's nanoplate.

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| --- |
| Table 1. bending value comparison of the N-order nanoplate under the surface uniform load effect for the various ratio of length to width and different ratio of length to thickness (a/h=30, q=1e-18 N/nm2) |
| |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | a/b | l/h | | | | | | | | | | | | 0 | |  | 0.5 | |  | 1 | |  | 2 | | | n=3 | n=5 | n=3 | n=5 | n=3 | n=5 | n=3 | n=5 | | 1 | 10.7837 | 10.7822 |  | 5.3783 | 5.3763 |  | 2.1483 | 2.1500 |  | 0.6315 | 0.6313 | | 1.5 | 20.7713 | 20.7694 |  | 10.3671 | 10.3644 |  | 4.1428 | 4.1417 |  | 1.2179 | 1.2177 | | 2 | 28.0010 | 27.9987 |  | 13.9789 | 13.9757 |  | 5.5868 | 5.5856 |  | 1.6426 | 1.6422 | |

|  |
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| Figure 1. the N-order nanoplate bending rate comparison under sinusoidal load effect for the length to width ratio and length to thickness ratio (a/h=30, q=1e-18 N/nm2) |

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| Figure 2. the bending contours of the N-order nanoplate (a/h=30, q=1e-18 N/nm2,n=5, a/b=1, l/h=1) |

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| --- |
| Table 2. the dimensionless bending value comparison of the N-order nanoplate under the sinusoidal load and surface uniform load effect for different length to width ratio and diverse length to thickness ratio (n=5, a/h=30, q=1e-18 N/nm2) |
| |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | a/b | l/h | | | | | | | | | | | | 0 | |  | 0.5 | |  | 1 | |  | 2 | | | n=3 | n=5 | n=3 | n=5 | n=3 | n=5 | n=3 | n=5 | | 1 | 10.7837 | 10.7822 |  | 5.3783 | 5.3763 |  | 2.1483 | 2.1500 |  | 0.6315 | 0.6313 | | 1.5 | 20.7713 | 20.7694 |  | 10.3671 | 10.3644 |  | 4.1428 | 4.1417 |  | 1.2179 | 1.2177 | | 2 | 28.0010 | 27.9987 |  | 13.9789 | 13.9757 |  | 5.5868 | 5.5856 |  | 1.6426 | 1.6422 | |

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| Figure 3. the bending rates comparison of different nanoplates under the sinusoidal load effect for different length to width ratio (a/h=30, q=1e-18 N/nm2, l/h=1) |

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| Figure 4. the critical buckling force value of the N-order nanoplate under the effect of the bi-axial surface force in the x and y directions for different length to width ratio and various length to width ratio (n=5, a/b=1) |

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| --- |
| Table 3. the critical buckling force value comparison of different dimensionless nanoplate under the effect of the uniaxial and bi-axial forces for the various ratios of length to thickness ( a/b=1) |
| |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | a/h | Kirchhoff plate | |  | Mindlin plate | |  | Third order shear deformation plate | |  | N order shear deformation plate (n=5) | | | Biaxial buckling | Uniaxial buckling | Biaxial buckling | Uniaxial buckling | Biaxial buckling | Uniaxial buckling | Biaxial buckling | Uniaxial buckling | | 5 | 5.0000 | 5.0000 | 10.1594 | 10.1594 | 5.6521 | 5.6521 | 5.6937 | 5.6937 | | 10 | 5.0000 | 5.0000 | 12.8101 | 12.8101 | 5.1723 | 5.1723 | 5.1826 | 5.1826 | | 20 | 5.0000 | 5.0000 | 13.6820 | 13.6820 | 5.0437 | 5.0437 | 5.0463 | 5.0463 | | 30 | 5.0000 | 5.0000 | 13.8568 | 13.8568 | 5.0195 | 5.0195 | 5.0206 | 5.0206 | | 40 | 5.0000 | 5.0000 | 13.9191 | 13.9191 | 5.0110 | 5.0110 | 5.0116 | 5.0116 | | 50 | 5.0000 | 5.0000 | 13.9481 | 13.9481 | 5.0070 | 5.0070 | 5.0074 | 5.0074 | |

**Conclusion**

In this study, the bending and buckling of N-order nanoplate were investigated using the modified couple stress theory.  As observed in the tables and figures, the N-order nanoplate bending rate under surface uniform load effect, decreased with the increasing the length scale to thickness ratio parameter, however, the bending ratio increased with increasing aspect ratio. Also, the dimensionless bending value is equal for surface uniform load and sinusoidal load when the length scale parameter is not considered and in all other cases, the sinusoidal load bending is less than surface uniform load. Furthermore, the dimensionless critical buckling force value of the N-order nanoplate under uniaxial force in the x-direction increased with increasing length scale to thickness ratio and decreased slightly with increasing length to thickness ratio.Also, the fifth-order nanoplate dimensionless critical force value was higher than the third-order nanoplate.

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