Capacity Region for Wireless More Capable Broadcast Channel with Channel State Available at the Receivers

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Abstract— **In this paper, we study a more capable broadcast channel with channel state information at receivers. We derive capacity region for discrete memoryless 2-receiver of this type of channels, then; this region is possibly extended to discrete time and continuous alphabet fading Gaussian 2-receiver more capable broadcast channel. Finally, as a result, the ergodic capacity of this fading channel is derived.**

Keywords— More capable broadcast channel, Fading channel, Ergodic capacity, State information.

I. INTRODUCTION

An information-theoretic study of broadcast channel (BC) was first introduced by T. M. Cover in [1]. This channel is of wide interest owing to its fundamental nature of broadcast communication in which one transmitter sends k independent messages to k non-cooperative receivers. Determining the capacity region of discrete memoryless general BC is still an unsolved problem, except for few special cases in which one channel is superior to the other channel, including the degraded channel, the more capable and the less noisy channels, deterministic and semi-deterministic channels. The best known capacity bounds for general 2-receiver BC (k=2) has been derived by Marton [2] and Nair and El Gamal [3]. Then, the authors in [4] obtained an achievable rate region for degraded 2-receiver BC using superposition coding; Gallager [5] and Ahlswede and Korner [6] showed that this rate region is optimal. Less noisy and more capable were introduced by Korner and Marton in [7]. In [8] and [9], the optimality of the superposition coding scheme for these classes of the 2-receiver case have been established respectively.

Shannon studied channels with side information (SI) [10], when causal SI is available at the transmitter. Single user channel, when non-causal SI is available only at the transmitter, was studied by Gel'fand and Pinsker [11]. The results of [11] to the case where non-causal SIs are available at transmitter and receiver was generalized by Cover and Chiang in [12]. The Gaussian versions of [11] were studied in [13] and [14]. Moreover, single user channel with partial channel state information at the transmitter (CSIT) was studied in [15].

BC with SI received considerable attention recently. Several works have been done upon this setting of channels. This type of channels were investigated in [16]. Steinberg and Shamai in [17] investigated general BC in presence of non-causal SI at the transmitter, where Marton's achievability scheme has been extended to state dependent channels. Later, Farsani and Marvasti considered SI non-causally available at the transmitter in multiuser channels in [18] and obtained their capacity bound. Fading in wireless communications has been widely studied but there exist still many unsolved problems regarding fundamental limits of communications in these scenarios. Some works have been studied this phenomenon such as [19-22] in communication theory. One of the recent works on capacity bound for the fading BC with partial CSIT is [17]. As described in [19], this approach has some advantages such as a full control over how much SI is available from perfect to no information and also, no need for necessarily separating the channel into parallel sub-channels to analyse it. Notwithstanding many works have been done on the capacity region of BCs in presence of fading phenomenon such as [23]-[24], however outage capacity region for wireless ergodic fading BC is only known when perfect channel state information is available at both transmitter and receivers [20].

In this paper we focus on the capacity of discrete and continuous alphabet versions of more capable BC when channel state is only available at the receivers, and derive capacity region for discrete memoryless 2-receiver, and then, extend this region to continuous alphabet fading Gaussian version of this channel. This work has been done for less noisy BC in [25].

This paper is organized as follows. In Section II, we present channel model, some definitions and preliminaries. In Section III, we obtain capacity region of discrete memoryless 2-receiver more capable BC when CSI is known only at the receivers. Then extending obtained capacity region to ergodic capacity region of continuous alphabet fading Gaussian channel was devoted in this Section. Finally, Section IV concludes the paper.

II. CHANNEL MODEL, DEFINITIONS AND PRELIMINARIES

In this section, we describe the channel model and some basic definitions and preliminaries necessary for continuation of the study. Discrete random variables (RVs) and their realizations are denoted with uppercase and lowercase letters, respectively. We assume that X_i^n is the sequence of the random variable $(X_i, X_{i+1}, \dots, X_n)$, also let $\mathcal{H}, \mathcal{X}$, and $\mathcal{Y}_1, \mathcal{Y}_2$ be finite sets which denote alphabets of a rv of the fading coefficient, channel input, and channel outputs, respectively. Expectation operator is denoted by $E\{.\}$.

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A. *Definitions*

Definition 1: A discrete memoryless more capable BC with CSI available only at the receivers, $p(y_1, y_2|x, h)$, which X, H, and Y_1, Y_2 denote channel input, channel state, and channel outputs, respectively, is a channel which we have for that following inequality:

$$I(X; Y_2|H = h) \leq I(X; Y_1|H = h);$$
$$\forall h \in \mathcal{H} \text{ and } \forall p(x, y_1, y_2) = p(x)p(y_1, y_2|x, h).$$

Here, receiver Y_1 is said to be more capable than receiver Y_2.

In this configuration, the transmitter sends messages $m_2 \in \mathcal{M}_2$ and $m_1 \in \mathcal{M}_1$ where $\mathcal{M}_1$ and $\mathcal{M}_2$ denote message sets and m_2 is received by both receivers (common message) and m_1 is received only by better receivers (private message).

Definition 2: An $(n, 2^{nR_1}, 2^{nR_2}, \epsilon)$ code for discrete memoryless 2-receiver more capable BC with message sets $\mathcal{M}_1$ and $\mathcal{M}_2$, introduced in previous definition, and channel states $H = (H_1, H_2)$ available at the receivers consists of three maps: An encoder at the transmitter and a decoder at each receiver consists of following map:

$$enc.: \{1, 2, \dots, 2^{nR_1}\} \times \{1, 2, \dots, 2^{nR_2}\} \rightarrow \mathcal{X}^n,$$
$$dec.(y_2): \mathcal{Y}_2^n \times (\mathcal{H}_1^n, \mathcal{H}_2^n) \rightarrow \{1, 2, \dots, 2^{nR_2}\},$$
$$dec.(y_1): \mathcal{Y}_1^n \times (\mathcal{H}_1^n, \mathcal{H}_2^n) \rightarrow \{1, 2, \dots, 2^{nR_1}\} \times \{1, 2, \dots, 2^{nR_2}\}$$

such that for private message m_1 and common message m_2 we have $m_1 \in \{1, 2, \dots, 2^{nR_1}\}$ and $m_2 \in \{1, 2, \dots, 2^{nR_2}\}$, and average probabilities of error are defined as follow:

$$P_{Error}^{(n)} \triangleq \frac{1}{2^{nR_1} \times 2^{nR_2}} \sum_{m_2=1}^{2^{nR_2}} \sum_{m_1=1}^{2^{nR_1}} \sum_{(h_1^n, h_2^n) \in (\mathcal{H}_1^n, \mathcal{H}_2^n)} p(h_1^n, h_2^n)$$
$$\Pr\{dec.(y_1)(y_1^n, h_1^n, h_2^n) \neq (m_1, m_2) \text{ or } dec.(y_2)(y_2^n, h_1^n, h_2^n) \neq m_2 | h_1^n, h_2^n, x^n(m_1, m_2)\} \leq \epsilon.$$

A rate pair (R_1, R_2) is said to be achievable for $p(y_1, y_2|x, h)$ if we have an integer n_0 for any $\mu > 0$ such that for all $n \geq n_0$ an $(n, 2^{n(R_1-\mu)}, 2^{n(R_2-\mu)}, \epsilon)$ code exists. The capacity region is defined as the closure of the union of all ϵ-achievable rates.

Definition 3: The model of fading Gaussian 2-receiver more capable BC with fading coefficient available at the receivers is as follows:

$$Y_1 = H_1 X + Z_1$$
$$Y_2 = H_2 X + Z_2,$$

where for this fading channel, X and Y_1, Y_2 denote channel input and channel outputs, respectively. We denote Channel state by $H = (H_1, H_2)$ in which H_1 and H_2 are related to ergodic stationary fading processes. Complex Gaussian noises with zero means and unit variances are denoted by Z_1 and Z_2. Z_1 and $Z_2$are independent of each other, channel state and auxiliary RVs. In figure 1 this channel is depicted. Here, messages m_2 and m_1 are sent and also decoded versions of them at the receivers are illustrated by $\hat{\hat{m}}_2$, $\hat{m}_2$ and $\hat{m}_1$.

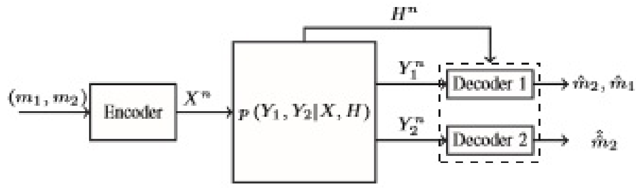

Fig. 1. Fading Gaussian 2-receiver BC with two degraded message sets and fading coefficient available at the receivers.

An $(n, 2^{nR_1}, 2^{nR_2}, \epsilon)$ code for this channel consists of following encoder map where C denotes complex number set:

$$enc._i.: \{1, 2, \dots, 2^{nR_1}\} \times \{1, 2, \dots, 2^{nR_2}\} \rightarrow C.$$

For private message m_1 and common message m_2 we have $m_1 \in \{1, 2, \dots, 2^{nR_1}\}$ and $m_2 \in \{1, 2, \dots, 2^{nR_2}\}$. Current channel input is produced as $X_i = enc._i(m_1, m_2), i = 1, \dots, n$, and also we have average power constraint at the transmitter as follows:

$$\frac{1}{n} E\left\{\sum_{i=1}^{n} |X_i|^2\right\} \leq P, \quad (1)$$

also, the code consists of two maps at decoders as follows:

$$dec.(y_2): \mathcal{Y}_2^n \times (\mathcal{H}_1^n, \mathcal{H}_2^n) \rightarrow \{1, 2, \dots, 2^{nR_2}\},$$
$$dec.(y_1): \mathcal{Y}_1^n \times (\mathcal{H}_1^n, \mathcal{H}_2^n) \rightarrow \{1, 2, \dots, 2^{nR_1}\} \times \{1, 2, \dots, 2^{nR_2}\}$$

Such that for average error probability we have $P_{Error}^{(n)} \leq \epsilon$. The definition of average error probability, achievable rates, and capacity region are identical to what mentioned in definition 2.

B. *Preliminaries*

Scaling: $H(aX) = H(X) + \log|a|^2$, in which a is a complex number and X is a continuous complex RV.

Entropy Power Inequality: For independent complex rv X_1 and X_2 we have:

$$2^{H(X_1)} + 2^{H(X_2)} \leq 2^{H(X_1+X_2)}. \quad (2)$$

III. Capacity Results

In this section, we present our main results which consists of the capacity region for channel defined in definition 1, i.e. discrete memoryless 2-receiver more capable BC with CSI available only at the receivers and extension of these results to discrete time and continuous alphabet fading channels. Finally ergodic capacity of this fading channel is derived. For a discrete memoryless BC without any knowledge about CSI at the transmitter and receivers, if for all $p(u, x)$ we have $I(X; Y_2) \leq I(X; Y_1)$ then Y_1 is more capable than Y_2. U, X, and Y_1, Y_2 are auxiliary rv related to common message, channel input, and channel outputs, respectively. In [9], the capacity region is obtained for this channel as follows:

$$R_1 \leq I(X; Y_1|U),$$
$$R_2 \leq (U; Y_2),$$
$$R_1 + R_2 \leq I(X; Y_1) \quad (3)$$

for all $p(u, x)$, where $|\mathcal{U}| \leq \min\{|\mathcal{X}|, |\mathcal{Y}_1|.|\mathcal{Y}_2|\} + 2$.

Theorem 1. Capacity region for a discrete memoryless 2-receiver more capable BC where Y_1 is more capable than Y_2 with CSI available at the receivers is as follows:

$$\begin{aligned} R_1 &\le I(X;Y_1|U,H), \\ R_2 &\le I(U;Y_2|H), \\ R_1+R_2 &\le I(X;Y_1|H). \end{aligned} \tag{4}$$

for some $p(u,x)p(y_1,y_2|x,h)$.

Proof. Achievability: Rate region (4) is achievable by replacing Y_i by $(Y_i,H), i\in\{1,2\}$ in achievability part of a capacity theorem of discrete memoryless 2-receiver more capable BC (3).

Converse: We prove the converse for the alternative region consisting of the set of rate pairs (R_1,R_2) such that:

$$\begin{aligned} R_2 &\le I(U;Y_2|H), \\ R_1+R_2 &\le I(U;Y_2|H)+I(X;Y_1|U,H), \\ R_1+R_2 &\le I(X;Y_1|H). \end{aligned} \tag{5}$$

Let m_1 and m_2 be independent RVs. R_2 can be bounded as follows:

$$\begin{aligned} nR_2 &= H(m_2) = H(m_2|Y_2^n,H^n)+I(m_2;Y_2^n|H^n) \\ &\le n\epsilon_{2n}+I(m_2;Y_2^n|H^n) \qquad (6) \\ &= n\epsilon_{2n}+\sum_{i=1}^{n} I\big(m_2;Y_{2,i}\big|Y_{2,i+1}^n,H^n\big) \\ &\le n\epsilon_{2n}+\sum_{i=1}^{n} I\big(m_2,Y_{2,i+1}^n,H^{i-1},H_{i+1}^n;Y_{2,i}\big|H_i\big) \\ &\le n\epsilon_{2n}+\sum_{i=1}^{n} I\big(m_2,Y_1^{i-1},Y_{2,i+1}^n,H^{i-1},H_{i+1}^n;Y_{2,i}\big|H_i\big) \\ &\quad -\sum_{i=1}^{n} I\big(Y_1^{i-1};Y_{2,i}\big|m_2,Y_{2,i+1}^n,H^n\big) \\ &\le n\epsilon_{2n}+\sum_{i=1}^{n} I\big(m_2,Y_1^{i-1},Y_{2,i+1}^n,H^{i-1},H_{i+1}^n;Y_{2,i}\big|H_i\big) \\ &= n\epsilon_{2n}+\sum_{i=1}^{n} I\big(U_i;Y_{2,i}\big|H_i\big), \qquad (7) \end{aligned}$$

where (6) and (7) follows from Fano's inequality and $U_i \triangleq (m_2,Y_1^{i-1},Y_{2,i+1}^n,H^{i-1},H_{i+1}^n)$, respectively. To bound the sum rate R_1+R_2, we use the fact that m_1 and m_2 are independent messages. In first, R_1+R_2 can be bounded as follows:

$$\begin{aligned} n(R_1+R_2) &= H(m_1,m_2) = H(m_2)+H(m_1|m_2) \\ &= H(m_2|Y_2^n,H^n)+I(m_2;Y_2^n|H^n) \\ &\quad +H(m_1|m_2,Y_1^n,H^n)+I(m_1;Y_2^n|m_2,H^n) \\ &\le n\epsilon_{12n}+I(m_2;Y_2^n|H^n)+I(m_1;Y_2^n|m_2,H^n) \qquad (8) \\ &= n\epsilon_{12n}+\sum_{i=1}^{n} I\big(m_2;Y_{2,i}\big|Y_{2,i+1}^n,H^n\big) \\ &\quad +\sum_{i=1}^{n} I\big(m_1;Y_{1,i}\big|m_2,Y_1^{i-1},H^n\big) \\ &\le n\epsilon_{12n}+\sum_{i=1}^{n} I\big(m_2,Y_{2,i+1}^n;Y_{2,i}\big|H^n\big) \\ &\quad +\sum_{i=1}^{n} I\big(m_1,Y_{2,i+1}^n;Y_{1,i}\big|m_2,Y_1^{i-1},H^n\big) \\ &= n\epsilon_{12n}+\sum_{i=1}^{n} I\big(m_2,Y_{2,i+1}^n,Y_1^{i-1};Y_{2,i}\big|H^n\big) \\ &\quad -\sum_{i=1}^{n} I\big(Y_1^{i-1};Y_{2,i}\big|m_2,Y_{2,i+1}^n,H^n\big) \\ &\quad +\sum_{i=1}^{n} I\big(m_1,Y_{2,i+1}^n;Y_{1,i}\big|m_2,Y_1^{i-1},H^n\big) \\ &= n\epsilon_{12n}+\sum_{i=1}^{n} I\big(m_2,Y_{2,i+1}^n,Y_1^{i-1};Y_{2,i}\big|H^n\big) \\ &\quad -\sum_{i=1}^{n} I\big(Y_1^{i-1};Y_{2,i}\big|m_2,Y_{2,i+1}^n,H^n\big) \\ &\quad +\sum_{i=1}^{n} I\big(m_1;Y_{1,i}\big|m_2,Y_1^{i-1},Y_{2,i+1}^n,H^n\big) \\ &\quad +\sum_{i=1}^{n} I\big(Y_{2,i+1}^n;Y_{1,i}\big|m_2,Y_1^{i-1},H^n\big) \\ &\le n\epsilon_{12n}+\sum_{i=1}^{n} I\big(m_1;Y_{1,i}\big|m_2,Y_1^{i-1},Y_{2,i+1}^n,H^{i-1},H_{i+1}^n,H_i\big) \\ &\quad +\sum_{i=1}^{n} I\big(m_2,Y_1^{i-1},Y_{2,i+1}^n,H^{i-1},H_{i+1}^n;Y_{2,i}\big|H_i\big) \qquad (9) \\ &= n\epsilon_{12n}+\sum_{i=1}^{n} I\big(m_1;Y_{1,i}\big|U_i,H_i\big)+\sum_{i=1}^{n} I\big(U_i;Y_{2,i}\big|H_i\big) \\ &\le n\epsilon_{12n}+\sum_{i=1}^{n} I\big(X_i;Y_{1,i}\big|U_i,H_i\big)+\sum_{i=1}^{n} I\big(U_i;Y_{2,i}\big|H_i\big) \qquad (10) \end{aligned}$$

where (8) follows from Fano's inequality and the fact that m_1 is also conditionally independent of H^n when m_2 is given. (9) Follows from Csiszar-Korner identity. (10) Is results from data processing inequality [26]. Also. for sum rate R_1+R_2 we have:

$$\begin{aligned} n(R_1+R_2) &= H(m_1,m_2) = H(m_1)+H(m_2|m_1) \\ &= H(m_1|Y_1^n,H^n)+I(m_1;Y_1^n|H^n) \\ &\quad +H(m_2|m_1,Y_2^n,H^n)+I(m_2;Y_2^n|m_1,H^n) \\ &\le n\epsilon_{12n}+I(m_1;Y_1^n|H^n)+I(m_2;Y_2^n|m_1,H^n) \qquad (11) \\ &\le n\epsilon_{12n}+I(m_1;Y_1^n|H^n) \\ &\quad +\sum_{i=1}^{n} I\big(X_i;Y_{2,i}\big|m_1,Y_{2,i+1}^n,H^n\big) \qquad (12) \\ &\le n\epsilon_{12n}+\sum_{i=1}^{n} I\big(m_1;Y_{1,i}\big|Y_1^{i-1},H^n\big) \\ &\quad +\sum_{i=1}^{n} I\big(X_i;Y_{2,i}\big|m_1,Y_1^{i-1},Y_{2,i+1}^n,H^n\big) \\ &\quad +\sum_{i=1}^{n} I\big(Y_1^{i-1};Y_{2,i}\big|m_1,Y_{2,i+1}^n,H^n\big) \\ &= n\epsilon_{12n}+\sum_{i=1}^{n} I\big(m_1;Y_{1,i}\big|Y_1^{i-1},H^n\big) \end{aligned}$$

$$+\sum_{i=1}^{n} I\left(X_i; Y_{2,i} \middle| m_1, Y_1^{i-1}, Y_{2,i+1}^{n}, H^n\right)$$

$$+\sum_{i=1}^{n} I\left(Y_{2,i+1}^{n}; Y_{1,i} \middle| m_1, Y_1^{i-1}, H^n\right) \quad (13)$$

$$= n\epsilon_{12n} + \sum_{i=1}^{n} I\left(X_i; Y_{2,i} \middle| m_1, Y_1^{i-1}, Y_{2,i+1}^{n}, H^n\right)$$

$$+\sum_{i=1}^{n} I\left(m_1, Y_{2,i+1}^{n}; Y_{1,i} \middle| Y_1^{i-1}, H^n\right)$$

$$\leq n\epsilon_{12n} + \sum_{i=1}^{n} I\left(X_i; Y_{1,i} \middle| m_1, Y_1^{i-1}, Y_{2,i+1}^{n}, H^n\right)$$

$$+\sum_{i=1}^{n} I\left(m_1, Y_{2,i+1}^{n}; Y_{1,i} \middle| Y_1^{i-1}, H^n\right) \quad (14)$$

$$= n\epsilon_{12n} + \sum_{i=1}^{n} I\left(X_i; Y_{1,i} \middle| Y_1^{i-1}, H^n\right) \quad (15)$$

where (11) follows from Fano's inequality and the fact that m_2 is also conditionally independent of H^n when m_1 is given. (12) and (13) follows from data processing inequality [27] and Csiszar-Korner identity, respectively. Finally (14) is results from the definition on more capable BC with channel state available at the receivers

In above inequalities $\epsilon_{1n}, \epsilon_{2n}$ and ϵ_{12n} tend to zero as $n \to \infty$. Considering relations (7), (10) and (15), and using time-sharing RV Q, inequalities in (4) are a capacity region for discrete memoryless 2-receiver more capable BC with channel state available at the receivers. ■

IV. Ergodic Capacity Region Of Continuous Alphabet Fading Gaussian Channel

In this section, CSI as fading coefficients in fading Gaussian version of this channel has been considered. We extend capacity result (4) to the ergodic capacity of fading Gaussian 2-receiver more capable BC average power constraint (1) at the transmitter and fading coefficients available at the receivers.

Theorem 2. Ergodic capacity region for a fading Gaussian 2-receiver more capable BC with fading coefficient available at the receivers is as follow:

$$R_1 \leq \log(|H_1|^2 P\bar{a} + 1),$$
$$R_2 \leq \log\left[1 + \frac{|H_2|^2 Pa}{1 + |H_2|^2 P\bar{a}}\right],$$
$$R_1 + R_2 \leq \log(|H_1|^2 P + 1) \quad (16)$$

for all constants $a \in [0,1]$, where $a + \bar{a} = 1$. Here, we have average power constraint $\frac{1}{n} E\{\sum_{i=1}^{n} |X_i|^2\} \leq P$.

Remark 1: In the fading Gaussian BC considered we must have [17]:

$$|H_2| \leq |H_1|. \quad (17)$$

Proof. It is sufficient to extend the proved theorem 1 into continuous alphabet fading Gaussian case to prove theorem 2.

Achievability: We use inequalities in (4) to prove achievability of (16). First, we extend $p(u,x)p(y_1,y_2|x,h)$ to the continuous alphabet version:

$$X = \sqrt{P}\left(\sqrt{a}U + \sqrt{\bar{a}}X'\right), \quad (18)$$

where X denotes defined signaling. Auxiliary RVs U and X' are independent RVs with unit variances and zero means, and related to common message m_2 and private message m_1, respectively. a and $\bar{a}$ are positive constant showing power contributions of each message considered at the transmitter so that $a + \bar{a} = 1$. From first inequality in (4) we have:

$$R_1 \leq I(X; Y_1|U, H) = \int_h I(X; Y_1|U, H = h) f(h) dh, \quad (19)$$

where $f(h)$ denotes channel state distribution. From the above relation for showing the achievability of first inequality we have:

$$I(X; Y_1|U, H = h) = H(Y_1|U, H = h) - H(Y_1|X, U, H = h)$$
$$= H(Y_1|U, H = h) - \log \pi e \quad (20)$$
$$= H\left(H_1\left[\sqrt{P}\left(\sqrt{a}U + \sqrt{\bar{a}}X'\right) + Z_1\right] \middle| U, H = h\right) - \log \pi e \quad (21)$$
$$= \log(|H_1|^2 P\bar{a} + 1), \quad (22)$$

where, (20) and (21) are resulted from definition 3 and (18), and by choosing U and X' Gaussian for maximizing the achievable rate (22) is obtained. So, by (19) and (22) we have following inequality:

$$R_1 \leq \log(|H_1|^2 P\bar{a} + 1). \quad (23)$$

For showing the achievability of second inequality in (16), we have:

$$R_2 \leq I(U; Y_2|H) = \int_h I(U; Y_2|H = h) f(h) dh. \quad (24)$$

In (24), $I(U; Y_2|H = h)$ can be written as follows:

$$I(U; Y_2|H = h) = H(Y_2|H = h) - H(Y_2|U, H = h)$$
$$= \log \pi e(|H_2|^2 P + 1) - \log \pi e(|H_2|^2 P\bar{a} + 1) \quad (25)$$
$$= \log \pi e(|H_2|^2 P(a + \bar{a}) + 1) - \log \pi e(|H_2|^2 P\bar{a} + 1)$$
$$= \log\left[1 + \frac{|H_2|^2 Pa}{1 + |H_2|^2 P\bar{a}}\right], \quad (26)$$

where (25) is resulted from definition 3 and (18). By (24) and (26) we have:

$$R_2 \leq \log\left[1 + \frac{|H_2|^2 Pa}{1 + |H_2|^2 P\bar{a}}\right]. \quad (27)$$

For showing the achievability of third inequality in (16) we have:

$$R_1 + R_2 \leq I(X; Y_1|H) = \int_h I(X; Y_1|H = h) f(h) dh. \quad (28)$$

In (29), $I(X; Y_1|H = h)$ can be written as follows:

$$I(X; Y_1|H = h) = H(Y_1|H = h) - H(Y_1|X, H = h)$$
$$= \log \pi e(|H_1|^2 P + 1) - \log \pi e \quad (29)$$
$$= \log(|H_1|^2 P + 1) \quad (30)$$

where (29) is resulted from definition 3 and (18). By (28) and (30) we have:

$$R_1 + R_2 \leq \log(|H_1|^2 P + 1). \quad (31)$$

Considering (23), (27) and (31) it is readily seen that (16) is achievable.

Convers: For first inequality in (16), we write $I(X;Y_1|U,H=h)$ as follows:

$$I(X;Y_1|U,H=h) = (Y_1|U,h) - H(Y_1|X,U,h)$$
$$= H(Y_1|U,h) - \log \pi e \quad (32)$$

where (32) is resulted from (18). Under average power constraint, Gaussian rv maximizes entropy. Hence, according to this fact, we can write following inequality for $H(Y_1|U,h)$:

$$H(Y_1|U,h) \leq \log(|H_1|^2 P + 1). \quad (33)$$

There exist a $\bar{\beta} \in [0,1]$ such that:

$$H(Y_1|U,h) = \log\left(|H_1|^2 P\, \bar{\beta} + 1\right). \quad (34)$$

Considering (19), (32) and (34) we have following inequality:

$$R_1 \leq \log\left(|H_1|^2 P \bar{\beta} + 1\right). \quad (35)$$

For second inequality in (16), we can write $I(U;Y_2|H=h)$ according to (24) as follows:

$$I(U;Y_2|H=h) = H(Y_2|h) - H(Y_2|U,h)$$
$$= H(Y_2|h) - H\left(\frac{H_2}{H_1}Y_1 + \sqrt{1 - \left|\frac{H_2}{H_1}\right|^2}\,\tilde{Z}\,\middle|\,U,h\right), \quad (36)$$

where (36) is resulted from definition 3 and (18). Considering $\tilde{Z}$ as a complex Gaussian virtual noise with zero mean and unit variance which is independent of noise, auxiliary random variable, and channel state. In addition, from entropy maximizing property of Gaussian random variable under average power constraint for $H(Y_2|h)$ and (18), we have:

$$H(Y_2|h) \leq \log \pi e(|H_2|^2 P + 1), \quad (37)$$

also, from (2) and scaling we have:

$$2^{H(Y_2|U,h)} = 2^{H(\frac{H_2}{H_1}Y_1 + \sqrt{1-\left|\frac{H_2}{H_1}\right|^2}\tilde{Z}|U,h)}$$
$$\geq 2^{H\left(\frac{H_2}{H_1}Y_1|U,h\right)} + 2^{H(\sqrt{1-\left|\frac{H_2}{H_1}\right|^2}\tilde{Z}|U,h)}$$
$$= \pi e\left|\frac{H_2}{H_1}\right|^2 \left(|H_1|^2 P\, \bar{\beta} + 1\right) + \pi e\left(1 - \left|\frac{H_2}{H_1}\right|^2\right) \quad (38)$$
$$= \pi e\left(|H_2|^2 P\, \bar{\beta} + 1\right). \quad (39)$$

So, consequently, we have the following inequality:

$$H(Y_2|U,h) \geq \log \pi e\left(|H_2|^2 P \bar{\beta} + 1\right). \quad (40)$$

Finally, by choosing a constant β such that $\beta + \bar{\beta} = 1$, the following inequality is resulted:

$$I(U;Y_2|H=h) \leq \log\left[1 + \frac{|H_2|^2 P\beta}{1 + |H_2|^2 P\bar{\beta}}\right], \quad (41)$$

so, by considering (24) and (41) we derive an inequality as follows:

$$R_2 \leq \log\left[1 + \frac{|H_2|^2 P\beta}{1 + |H_2|^2 P\bar{\beta}}\right]. \quad (42)$$

For third inequality in (16), we can write $I(X;Y_1|H=h)$ according to (28) as follows:

$$I(X;Y_1|H=h) = H(Y_1|h) - H(Y_1|X,h)$$
$$= H(Y_1|h) - \log \pi e \quad (43)$$

Similar to the previous description, we can write following inequality for $H(Y_1|h)$:

$$H(Y_1|h) \leq \log \pi e(|H_1|^2 P + 1). \quad (44)$$

The following inequality is resulted by considering (43) and (44).

$$I(X;Y_1|H=h) \leq \log(|H_1|^2 P + 1). \quad (45)$$

So, we derive an inequality as follows from (28) and (45):

$$R_1 + R_2 \leq \log(|H_1|^2 P + 1). \quad (46)$$

Consider (35), (42) and (46). Let constant α and β defined in achievable part and converse part be equal. Therefore, the ergodic capacity region for fading Gaussian 2-receiver more capable BC with fading coefficient available at the receiver is obtained as (16). ■

V. Conclusion

In this work, we derived capacity results for the more capable broadcast channel when state information is available at the receivers. We obtained capacity region for discrete memoryless 2-receiver of these channels, then, considering channel state as a fading coefficient, we extended this discrete alphabet result to the continuous alphabet fading version of the channel and obtained ergodic capacity.

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