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Surface terms in the effective actions via duality constraints

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ABSTRACT

The effective action of string theory on a spacetime manifold with boundary has both bulk and boundary terms. We propose that both bulk and boundary actions, may be found by imposing the effective action to be invariant under the gauge transformations and under the string dualities. Using this proposal at the leading order of α', the standard Gibbons-Hawking-York boundary term is reproduced.

String theory is a quantum theory of gravity with a finite number of massless fields and a tower of infinite number of massive fields reflecting the stringy nature of the gravity. This theory on the spacetime manifolds with boundary is conjectured to be dual to a gauge theory on the boundary [1]. The string theory is usually explored by studying its effective action which includes the massless fields and their higher derivative terms. For the spacetime manifolds with boundary, the effective action has both bulk and boundary terms, *i.e.*, $\mathbf{S}_{\rm eff} + \partial\mathbf{S}_{\rm eff}$. At the leading order of the derivative, the bulk action should include the Einstein action and the boundary action should include the Gibbons-Hawking-York term [2,3]. These terms and their appropriate higher derivative corrections should be produced by specific techniques in string theory.

There are various approaches for calculating the bulk actions *e.g.*, the S-matrix approach [4,5], the sigma-model approach [6,7], the Double Field Theory [8,9] and duality approach [10–15]. In the duality approach, the consistency of the effective actions with T- and S-duality transformations is imposed to find the higher derivative couplings. In the T-duality approach, in particular, the T-duality [16,17] is imposed as a constraint on the reduction of the effective action on a circle which we call it $S_{\rm eff}$. That is, the effective action satisfies the following constraint:

$$S_{\rm eff}(\psi) - S_{\rm eff}(\psi') = {\rm TD} \tag{1}$$

where ψ represents all massless fields in the base space and ψ' represents their transformations under the T-duality transformations which are the Buscher rules [18,19] and their higher derivative corrections. On the right-hand side, TD represents some total derivative terms in the base space which become zero using the Stokes's theorem because the base space has no boundary. This approach has been used in [20,21] to find the effective action of string theory at orders α'^0, α', α'^2 in the bosonic string theory on the closed manifolds. In the superstring theory, there are S-duality as well as T-duality. Imposing S-duality as well as T-duality, one may find couplings in the superstring theory [12–14]. In imposing the S-duality constraint, one should first transform the couplings to the Einstein frame and then enforcing them to be invariant under the S-duality transformations. In transforming the metric from the string frame to the Einstein frame, one finds some total derivative terms that are again ignored for the closed spacetime.

It is desirable to extend the above techniques such that they would calculate the boundary action $\partial\mathbf{S}_{\rm eff}$ as well. In this paper, we are going to illustrate that a simple extension in the duality approach [14] enables one to calculate both the bulk and the boundary actions.

When spacetime has boundary, the base space in the reduction of the spacetime on a circle, has also boundary. As a result, the total derivative terms on the right-hand side of (1) do not vanish using the Stokes's theorem. The total derivative terms resulting from the T-duality of the bulk action should be cancelled by the T-duality of the boundary action. Calling the reduction of the bulk action on the circle $S_{\rm eff}$ and the reduction of the boundary action on the same circle $\partial S_{\rm eff}$, then the T-duality constraint on the effective action (1) is extended as the following:

$$S_{\rm eff}(\psi) + \partial S_{\rm eff}(\psi) = S_{\rm eff}(\psi') + \partial S_{\rm eff}(\psi') \tag{2}$$

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Unlike (1), there are no total derivative terms in the base space. The above relation constrains both bulk and boundary actions.

Similarly the combination of bulk and boundary actions, *i.e.*, $\mathbf{S}_{\rm eff}+\partial\mathbf{S}_{\rm eff}$, should be written in a S-duality invariant form without ignoring any total derivative term in the bulk. In other words, using the Stokes's theorem, the total derivative terms resulting from transforming the string frame metric to the Einstein frame metric, produce some boundary terms. They should be combined with the boundary action $\partial\mathbf{S}_{\rm eff}$ to be written in a S-duality invariant form. This S-duality also constrains both bulk and boundary actions.

Using appropriate gauge transformations corresponding to the massless fields, one may write the most general couplings in $\mathbf{S}_{\rm eff}$ and in $\partial\mathbf{S}_{\rm eff}$, up to Bianchi identities and field redefinitions [22]. The arbitrary parameters in the gauge invariant couplings may be fixed by imposing the above duality constraints. We are going to examine this proposal to find the effective action of superstring theory at the leading order of derivatives, *i.e.*, $\mathbf{S}_0+\partial\mathbf{S}_0$ and for NS-NS fields. In particular, we are going to show that the duality constraints are satisfied only when the Gibbons-Hawking-York term [2,3] is included.

We now construct the most general D-dimensional bulk action and $(D-1)$-dimensional boundary action at the leading order of α' which are invariant under the coordinate transformations and under the standard gauge transformation of B-field, *i.e.*, $B_{\mu\nu}\to B_{\mu\nu}+\partial_{[\mu}\lambda_{\nu]}$. Using the fact that in the bulk action the total derivative terms can be transformed to the boundary action by the Stokes's theorem, one finds that in the bulk there are only three terms and in the boundary there are two terms, *i.e.*,

$$\mathbf{S}_0=-\frac{2}{\kappa^2}\int d^Dxe^{-2\Phi}\sqrt{-G}\left(a_1R+a_2\nabla_\mu\Phi\nabla^\mu\Phi+a_3H^2\right)\,. \tag{3}$$

$$\partial\mathbf{S}_0=-\frac{2}{\kappa^2}\int d^{D-1}ye^{-2\Phi}\sqrt{\pm g}\,\left(a_4G^{\mu\nu}K_{\mu\nu}+a_5n^\mu\nabla_\mu\Phi\right) \tag{4}$$

where the three-form H is field strength of the two-form B, *i.e.*, $H_{\mu\nu\zeta}=\partial_\mu B_{\nu\zeta}+\partial_\zeta B_{\mu\nu}+\partial_\nu B_{\zeta\mu}$. In the second equation, the plus (minus) sign in the square root apply for a spacelike (timelike) boundary, $K_{\mu\nu}$ is the extrinsic curvature of the boundary and $g_{\alpha\beta}$ is induced metric, *i.e.*,

$$\begin{aligned}K_{\mu\nu}&=\nabla_\mu n_\nu\;=\partial_\mu n_\nu-\Gamma^\zeta{}_{\mu\nu}n_\zeta\\ g_{\alpha\beta}&=\partial_\alpha x^\mu\partial_\beta x^\nu G_{\mu\nu}\;=\frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta}G_{\mu\nu}\end{aligned} \tag{5}$$

where the boundary is specified by the functions $x^\mu=x^\mu(y^\alpha)$ and n^μ is the unit vector orthogonal to the boundary. Up to this point the parameters a_1,a_2,a_3,a_4,a_5, are arbitrary and above actions are valid for any theory which has massless fields metric, B-field and dilaton. For string theory, however, these parameters should be fixed to specific numbers. We are going to find them by the T-duality constraint (2) and the S-duality constraint.

To impose the T-duality constraint on the bulk action, we have to consider a background with $U(1)$ isometry. It is convenient to use the following background for metric, Kalb-Ramond and dilaton fields:

$$G_{\mu\nu}=\begin{pmatrix}\bar{g}_{ab}+e^{\varphi}g_ag_b & e^{\varphi}g_a\\ e^{\varphi}g_b & e^{\varphi}\end{pmatrix},\;B_{\mu\nu}=\begin{pmatrix}\bar{b}_{ab}+\frac{1}{2}b_ag_b-\frac{1}{2}b_bg_a & b_a\\ -b_b & 0\end{pmatrix},\;\Phi=\bar{\phi}+\varphi/4 \tag{6}$$

where $\bar{g}_{ab},\bar{b}_{ab},\bar{\phi}$ are the metric, the B-field and the dilaton in the base space, and g_a, b_b are two vectors in this space. Inverse of the above D-dimensional metric is

$$G^{\mu\nu}=\begin{pmatrix}\bar{g}^{ab} & -g^a\\ -g^b & e^{-\varphi}+g_cg^c\end{pmatrix} \tag{7}$$

where $\bar{g}^{ab}$ is the inverse of the base metric which raises the index of the vectors. The Buscher rules [18,19] in this parametrization become the following linear transformations:

$$\varphi'=-\varphi\,,\;g'_a=b_a\,,\;b'_a=g_a\;,\;\bar{g}'_{ab}=\bar{g}_{ab}\;,\;\bar{b}'_{ab}=\bar{b}_{ab}\;,\;\bar{\phi}'=\bar{\phi} \tag{8}$$

There are higher derivative corrections to these transformations [23,20] in which we are not interested in this paper.

The proposal (2) at the leading order of α' can be written as

$$S_0(\psi)-S_0(\psi')=\partial S_0(\psi')-\partial S_0(\psi) \tag{9}$$

where S_0 is the reduction of the bulk action (3) and ∂S_0 is the reduction of the boundary action (4) on the circle. Since the bulk actions on the left-hand side are $(D-1)$-dimensional and the boundary actions on the right-hand side are $(D-2)$-dimensional, one expects the left-hand side to be zero up to some boundary terms which should be cancelled by the T-duality transformation of the boundary actions on the right-hand side.

Reduction of different scalar terms in $\mathbf{S}_0$ is the following (see *e.g.*, [20]):

$$\begin{aligned}e^{-2\Phi}\sqrt{-G}&\Rightarrow e^{-2\bar{\phi}}\sqrt{-\bar{g}}\\ R&\Rightarrow\bar{R}-\bar{\nabla}^a\bar{\nabla}_a\varphi-\frac{1}{2}\bar{\nabla}_a\varphi\bar{\nabla}^a\varphi-\frac{1}{4}e^{\varphi}V^2\\ \nabla_\mu\Phi\nabla^\mu\Phi&\Rightarrow\bar{\nabla}_a\bar{\phi}\bar{\nabla}^a\bar{\phi}+\frac{1}{2}\bar{\nabla}_a\bar{\phi}\bar{\nabla}^a\varphi+\frac{1}{16}\bar{\nabla}_a\varphi\bar{\nabla}^a\varphi\\ H^2&\Rightarrow\bar{H}_{abc}\bar{H}^{abc}+3e^{-\varphi}W^2\end{aligned} \tag{10}$$

where V_{ab} is field strength of the $U(1)$ gauge field g_a, *i.e.*, $V_{ab}=\bar{\nabla}_a g_b-\bar{\nabla}_b g_a$, and $W_{\mu\nu}$ is field strength of the $U(1)$ gauge field b_a, *i.e.*, $W_{ab}=\bar{\nabla}_a b_\nu-\bar{\nabla}_b b_a$. The three-form $\bar{H}$ is defined as $\bar{H}_{abc}=\tilde{H}_{abc}-g_a W_{bc}-g_c W_{ab}-g_b W_{ca}$ where the three-form $\tilde{H}$ is field strength of the two-form $\bar{b}_{ab}+\frac{1}{2}b_a g_b-\frac{1}{2}b_b g_a$ in (6). The three-form $\bar{H}$ is invariant under the Buscher rules (8) and satisfies an anomalous Bianchi identity [23].

Using the reductions in (10), one can calculate the reduced action $S_0(\psi)$ and its transformation $S_0(\psi')$ under the Buscher rules (2). Their difference then becomes

$$S_0(\psi)-S_0(\psi')=-\frac{2}{\kappa^2}\int d^{D-1}x\sqrt{-\bar{g}}e^{-2\bar{\phi}}\Big[a_2\bar{\nabla}^a\bar{\phi}\bar{\nabla}_a\varphi-2a_1\bar{\nabla}^a\bar{\nabla}_a\varphi+(\frac{1}{4}a_1+3a_3)(e^{-\varphi}W^2-e^{\varphi}V^2)\Big] \tag{11}$$

One can easily observe that for the following relations between the parameters:

$$a_3=-\frac{1}{12}a_1\ ;\ a_2=4a_1 \tag{12}$$

The right-hand side of (11) becomes a total derivative term in the base space, *i.e.*,

$$S_0(\psi)-S_0(\psi')=\frac{4a_1}{\kappa^2}\int d^{D-1}x\sqrt{-\bar{g}}\bar{\nabla}^a(e^{-2\bar{\phi}}\bar{\nabla}_a\varphi) \tag{13}$$

The general form of the Stokes's theorem relating total derivative of a vector V^A in a bulk to its value at the boundary is

$$\int_M d^dx\sqrt{|G|}\nabla_A V^A=\int_{\partial M} d^{d-1}y\sqrt{|g|}n_A V^A \tag{14}$$

where G_{AB} is the bulk metric, g_{IJ} is the induced metric and the boundary is specified by the functions $x^A=x^A(y^I)$. In above relation, n^A is normal vector to the boundary. It is outward-pointing (inward-pointing) is the boundary is spacelike (timelike).

It is convenient to use the Gaussian normal coordinates in the Stokes's theorem. Using the normal coordinates $\{z,y^1,\cdots,y^{D-2}\}$ in the base space, *i.e.*,

$$ds^2\ =\ \bar{g}_{ab}dx^a dx^b\ =\sigma d^2z+\widetilde{g}_{\tilde{a}\tilde{b}}(z,y^{\tilde{c}})dy^{\tilde{a}}dy^{\tilde{b}}\ ;\ or,\quad \bar{g}_{ab}=\begin{pmatrix}\sigma & 0\\ 0 & \widetilde{g}_{\tilde{a}\tilde{b}}(z,y^{\tilde{c}})\end{pmatrix} \tag{15}$$

where $\sigma=\pm 1$, and specifying the boundary as $x^a=(z_*,y^{\tilde{a}})$ where boundary is at $z=z_*$, one can write the induced metric in the base space as

$$\begin{aligned}g_{\tilde{a}\tilde{b}}&=\frac{\partial x^a}{\partial y^{\tilde{a}}}\frac{\partial x^b}{\partial y^{\tilde{b}}}\bar{g}_{ab}\\&=\frac{\partial z_*}{\partial y^{\tilde{a}}}\frac{\partial z_*}{\partial y^{\tilde{b}}}\sigma+\frac{\partial y^{\tilde{c}}}{\partial y^{\tilde{a}}}\frac{\partial y^{\tilde{d}}}{\partial y^{\tilde{b}}}\widetilde{g}_{\tilde{c}\tilde{d}}\\&=\widetilde{g}_{\tilde{a}\tilde{b}}(z_*,y^{\tilde{c}})\end{aligned} \tag{16}$$

The total derivative term in the base space then can be written as the following boundary term:

$$S_0(\psi)-S_0(\psi')=\pm\frac{4a_1}{\kappa^2}\int d^{D-2}y\sqrt{\pm\widetilde{g}(z_*,y^{\tilde{a}})}e^{-2\bar{\phi}}\bar{\nabla}^a\varphi n_a \tag{17}$$

where $n^a=(1,0,\cdots,0)$ is the outward-pointing unit vector orthogonal to the boundary, the plus (minus) sign is when the boundary is spacelike (timelike).

We now turn to the T-duality constraint on the boundary term (4). In the D-dimensional Gaussian normal coordinates $\{z,y^1,\cdots,y^{D-1}\}$, the bulk metric takes the form

$$ds^2=G_{\mu\nu}dx^\mu dx^\nu\ =\sigma d^2z+\gamma_{\alpha\beta}(z,y^\delta)dy^\alpha dy^\beta\ ;\ or,\quad G_{\mu\nu}=\begin{pmatrix}\sigma & 0\\ 0 & \gamma_{\alpha\beta}(z,y^\delta)\end{pmatrix} \tag{18}$$

Inverse of this metric is

$$G^{\mu\nu}=\begin{pmatrix}\sigma & 0\\ 0 & \gamma^{\alpha\beta}(z,y^\delta)\end{pmatrix} \tag{19}$$

The unit vector orthogonal to the boundary in the normal coordinates is $n^\mu=(1,0,\ldots,0)$. In using the Stokes's theorem in the base space, we have specified the boundary of the base space as $x^a=(z_*,y^{\tilde{a}})$, hence, the boundary in the original D-dimensional spacetime is specified as $x^\mu=(z_*,y^\delta)$ where $y^\delta=(y^{\tilde{a}},y)$ and y is the circle along which we have used the T-duality transformation of the bulk action. The induced metric in (4) then becomes

$$\begin{aligned}g_{\alpha\beta}&=\frac{\partial x^\mu}{\partial y^\alpha}\frac{\partial x^\nu}{\partial y^\beta}G_{\mu\nu}\\&=\gamma_{\alpha\beta}(z_*,y^\delta)\end{aligned} \tag{20}$$

Since one of the y^δ directions is the circle along which we have used the T-duality transformation in the bulk action, the reduction of the boundary metric and its inverse are

$$\gamma_{\alpha\beta}=\begin{pmatrix}\tilde{g}_{\tilde{a}\tilde{b}}+e^{\varphi}g_{\tilde{a}}g_{\tilde{b}} & e^{\varphi}g_{\tilde{a}}\\ e^{\varphi}g_{\tilde{b}} & e^{\varphi}\end{pmatrix};\ \gamma^{\alpha\beta}=\begin{pmatrix}\tilde{g}^{\tilde{a}\tilde{b}} & -g^{\tilde{a}}\\ -g^{\tilde{b}} & e^{-\varphi}+g_{\tilde{c}}g^{\tilde{c}}\end{pmatrix} \tag{21}$$

The reduction of different terms in the boundary action (4) then becomes

$$\begin{aligned} e^{-2\Phi}\sqrt{\pm g} &\Rightarrow e^{-2\bar{\phi}}\sqrt{\pm\tilde{g}}\\ G^{\mu\nu}K_{\mu\nu} &\Rightarrow \bar{g}^{ab}\bar{K}_{ab}+\frac{1}{2}n^a\bar{\nabla}_a\varphi\\ n^{\mu}\nabla_{\mu}\Phi &\Rightarrow n^a\bar{\nabla}_a\bar{\phi}+\frac{1}{4}n^a\bar{\nabla}_a\varphi \end{aligned}\tag{22}$$

where $\bar{K}_{ab}$ is extrinsic curvature of the boundary in the base space, and we have used the fact that in the Gaussian normal coordinate $n^a=(1,0,\ldots,0)$. The reduction of the boundary action (4) then becomes

$$\partial S_0(\psi)=-\frac{2}{\kappa^2}\int d^{D-2}y\sqrt{\pm\tilde{g}(z_*,y^{\tilde{a}})}e^{-2\bar{\phi}}\Big[a_4(\bar{g}^{ab}\bar{K}_{ab}+\frac{1}{2}n^a\bar{\nabla}_a\varphi)+a_5(n^a\bar{\nabla}_a\bar{\phi}+\frac{1}{4}n^a\bar{\nabla}_a\varphi)\Big]$$

Then under the Buscher rules (8), it transforms as

$$\partial S_0(\psi)-\partial S_0(\psi')=-\frac{2}{\kappa^2}\Big[a_4+\frac{1}{2}a_5\Big]\int d^{D-2}y\sqrt{\pm\tilde{g}(z_*,y^{\tilde{a}})}e^{-2\bar{\phi}}n^a\bar{\nabla}_a\varphi \tag{23}$$

where we have used the fact that n^a is invariant under the T-duality.

Replacing the T-duality transformations of the bulk action, *i.e.*, (17), and the T-duality transformation of the boundary action, *i.e.*, (23), into the constraint (9), one finds

$$a_4=\pm 2a_1-\frac{1}{2}a_5 \tag{24}$$

This relation as well as the relations in (12) fixes the effective action up to two parameters a_1, a_5, *i.e.*,

$$\begin{aligned}\mathbf{S}_0+\partial\mathbf{S}_0=&-\frac{2a_1}{\kappa^2}\Big[\int d^Dx\sqrt{-G}e^{-2\Phi}\left(R+4\nabla_a\Phi\nabla^a\Phi-\frac{1}{12}H^2\right)\pm 2\int d^{D-1}y\sqrt{\pm g}e^{-2\Phi}K\Big]\\ &-\frac{2a_5}{\kappa^2}\int d^{D-1}ye^{-2\Phi}\sqrt{\pm g}\left(-\frac{1}{2}K+n^{\mu}\nabla_{\mu}\Phi\right)\end{aligned}\tag{25}$$

While there is only one bulk action, there are two boundary actions. In the bosonic theory there is no further constraint that should be imposed to fix the parameter a_5. In the superstring theory however there is still another duality which should be imposed.

We now impose the S-duality constraint for $D=10$ to fix a_5. To show that the bulk action for type IIB superstring theory can be rewritten in S-duality invariant form, one should first change the string frame metric to the Einstein frame metric, *i.e.*, $G_{\mu\nu}=e^{\Phi/2}G^E_{\mu\nu}$. Ignoring a total derivative term resulting from this change of the frames, one finds that after including R-R couplings in which we are not interested in this paper, the Einstein frame couplings can be written in a S-duality invariant from (see *e.g.*, [24]). However, for the spacetime manifolds with boundary, the total derivative term must be cancelled with the corresponding terms in the boundary. In fact the total derivative term is produced by transforming the scalar curvature in the bulk action to the Einstein frame, *i.e.*,

$$R\to e^{-\Phi/2}\Big(R-\frac{9}{2}\nabla_{\mu}\nabla^{\mu}\Phi-\frac{9}{2}\nabla_{\mu}\Phi\nabla^{\mu}\Phi\Big) \tag{26}$$

The second term above, when replacing it into the bulk action (25), produces the following total derivative term in the Einstein frame:

$$\frac{9a_1}{\kappa^2}\int d^{10}x\sqrt{-G^E}\nabla_{\mu}\nabla^{\mu}\Phi=\pm\frac{9a_1}{\kappa^2}\int d^9y\sqrt{\pm g^E}\nabla^{\mu}\Phi n^E_{\mu} \tag{27}$$

where on the right-hand side we have used the Stokes's theorem as well. In this equation, n^E is unite vector orthogonal to the surface in the Einstein frame. This Einstein frame boundary term is not invariant under the S-duality.

To fully study the S-duality of the boundary terms, we should also transform the boundary terms in (25) to the Einstein frame. Different terms in this action transform as the following:

$$\begin{aligned}\sqrt{\pm g} &\to e^{9\Phi/4}\sqrt{\pm g^E}\\ K &\to e^{-\Phi/4}(K^E+\frac{9}{4}n^E_{\mu}\nabla^{\mu}\Phi)\\ n_{\mu}\nabla^{\mu}\Phi &\to e^{-\Phi/4}n^E_{\mu}\nabla^{\mu}\Phi\end{aligned}\tag{28}$$

where $K^E=\nabla_{\mu}(n^E)^{\mu}$ is the trace of the extrinsic curvature of the boundary in the Einstein frame, and we have used the fact that the unite vector n^{μ} in the string frame and in the Einstein frame should be related as $n^{\mu}=e^{-\Phi/4}(n^E)^{\mu}$, because their lengths are one in both frames, *i.e.*, $G_{\mu\nu}n^{\mu}n^{\nu}=1=G^E_{\mu\nu}(n^E)^{\mu}(n^E)^{\nu}$.

The total boundary terms in the Einstein frame are then

$$\frac{2}{\kappa^2}\int d^9y\sqrt{\pm g^E}\Big[(\mp 2a_1+\frac{1}{2}a_5)K^E+\frac{a_5}{8}\nabla^\mu\Phi n^E_\mu\Big] \tag{29}$$

The first term is invariant under S-duality. On the other hand, it has been observed in [25] that the odd number of dilaton terms in the Einstein frame can not be combined with the corresponding R-R scalar to be written in a S-duality invariant form. Hence the S-duality constrains the coefficient of the last term above to be zero, *i.e.*,

$$a_5=0 \tag{30}$$

Therefore, the NS-NS part of the low energy effective action of type II string theories on the spacetime manifolds with boundary can be fixed by the gauge transformations and by the string duality, up to an overall factor a_1. To have the standard Einstein term, this parameter must be $a_1=1$ as well. So the effective action is

$$\mathbf{S}_0+\partial\mathbf{S}_0=-\frac{2}{\kappa^2}\int d^Dx\sqrt{-G}e^{-2\Phi}\left(R+4\nabla_a\Phi\nabla^a\Phi-\frac{1}{12}H^2\right)\mp\frac{4}{\kappa^2}\int d^{D-1}y\sqrt{\pm g}e^{-2\Phi}K \tag{31}$$

where K is the trace of the extrinsic curvature. For zero dilaton and B-field, it is the standard action that its boundary term has been found by York, Hawking and Gibbons [2,3] by other means. It would be interesting to extend the above calculations to the R-R couplings as well as to the higher orders of α'.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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