Reliability Analysis of Available Stress Models for FRP Confined Rectangular Concrete Columns

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Abstract

Using fiber reinforced polymers (FRP) in engineering applications has grown rapidly in the past decades due to their desirable mechanical properties such as high tensile strength and corrosion resistance. One of these applications is to wrap FRP sheets around the concrete columns for strengthening and rehabilitation. Many investigations are performed in this field and various relations are proposed to predict ultimate stress of FRP confined concrete columns. Most of these models are developed for circular columns, because the previous researches and experiences demonstrated that the confinement effect of FRP wraps in circular sections is more than the rectangular ones. This is due to non-uniform distribution of confinment pressure in rectangular sections. The main goal of this paper is to perform a reliability analysis on the available stress models for FRP confined rectangular concrete columns. For this purpose, after a brief review of the existing relations, accuracy and reliability of the reviewed models is investigated based on a database of experimental results collected from previous researches. In this process, the Monte Carlo simulation technique will be used to produce sample data necessary for calculation of uncertainty index. A novel efficiency index is defined for comparing performance of different models. Based on the obtained results, the reviewed methods are ranked according to their accuracy and reliability.

Keywords: Reliability analysis, Stress Model, Strain Model, FRP Confined Concrete Column, Rectangular section.

1. INTRODUCTION

In early stages, methods such as metallic tubes were used for strengthening structures, but these approaches leads to increase in cross section area of members. A suitable alternative for the traditional methods is to utilize FRP wraps. Nowadays, the FRPs are being widely used in practical engineering applications due to their positive characteristics such as high tensile strength, corrosion resistance, durability, light weight, ease of application and electromagnetic neutrality. It must be noted that FRP wraps increases strength and ductility of concrete in structural components such as columns. Due to higher efficiency of confinement effect in circular sections, most of the available studies are performed on circular columns. It is obvious that for reliable and economic design of FRP confinement, an accurate relation to predict strength of FRP confined columns is required. Chaallal and Shahawy showed that due to non-uniform distribution of confinement pressure in rectangular columns, the relations of confined concrete strength of circular columns is not applicable for the rectangular ones [1]. The sharp corners of rectangular sections results in stress concentration and reduces efficiency of FRP confinement on strength improvement. As mentioned previously, there are fewer available strength models for FRP confined rectangular columns in comparison with circular ones. However, it is not clear that which if the dozen existing relations are more accurate and reliable. The main objective of this study is to evaluate accuracy and reliability of the available relations for the stress of FRP confined rectangular concrete. According to the obtained results, the best relations are specified.

2. THE STRESS MODELS FOR RECTANGULAR COLUMNS

As mentioned previously, there are various relations for confined strength of FRP confined concrete columns, which most of them are valid for circular columns under concentric axial force. The most well-known relation for strength of confined concrete is proposed by Richart [2]:

$$\frac{f'_{cc}}{f'_{co}} = 1 + k_1 \frac{f_l}{f'_{co}} \quad (1)$$

$$\frac{\varepsilon_{cc}}{\varepsilon_{co}} = 1 + k_2 \frac{f_l}{f'_{co}} \quad (2)$$

$$f_l = \frac{2 f_{frp} t_{frp}}{d} = \frac{\rho_{frp} f_{frp}}{2} \quad (3)$$

$$\rho_{frp} = \frac{\pi d t_{frp}}{\frac{\pi d^2}{4}} = \frac{4 t_{frp}}{d} \quad (4)$$

In these relations, k_1 and k_2 are empirical coefficients, f_l is the lateral confining pressure. f'_{co} and f'_{cc} are compressive strength of unconfined and confined concrete, respectively. f_{frp}, ρ_{frp} and t_{frp} are ultimate stress, volumetric ratio and thickness of FRP wraps, respectively. Many the other strength models are modification of the Richart model. In these modified relations, the coefficient k_1 is replaced by another constants or relations. Twelve different relations for strength of FRP confined rectangular concrete columns are presented in Table (1).

Table 1- confined concrete strength equations used in this study

No.	Model	Strength equation
1	Mirmiran et al. [3]	$f'_{cc} = f'_{co}(1 + 6(\frac{2R}{D})(\frac{f_l}{f'_{co}})^{0.7})$
2	ACI 440.2R-17 [4]	$f'_{cc} = f'_{co}(-1.254 + 2.254\sqrt{1 + \frac{7.94 k_s f_l}{f'_{co}}} - 2\frac{k_s f_l}{f'_{co}})$
3	Shehata et al. [5]	$f'_{cc} = f'_{co}(1 + 0.7 \times \left(\frac{f_l}{f'_{co}}\right))$
4	Teng et al.[6]	$f'_{cc} = f'_{co}(1 + 2.98 K_s \left(\frac{f_l}{f'_{co}}\right))$
5	Lam and Teng [7]	$f'_{cc} = f'_{co}(1 + 3.3(\frac{A_e}{A_c})(\frac{f_l}{f'_{co}}))$
6	Campion and Miraglia [8]	$f'_{cc} = f'_{co}(1 + 2K_s \left(\frac{f_l}{f'_{co}}\right))$
7	Ilki et al. [9]	$f'_{cc} = f'_{co}(1 + 2.4\left(\frac{f_l}{f'_{co}}\right)^{1.2})$
8	Kumutha et al. [10]	$f'_{cc} = f'_{co}(1 + 0.93 \times \frac{f_l}{f'_{co}})$
9	Toutanji et al. [11]	$f'_{cc} = f'_{co}(1 + 4(\frac{2R}{D})^{0.1}(\frac{D}{b})^{.12}(\frac{A_e}{A_c})(\frac{f_l}{f'_{co}}))$

10	Youssef et al. [12]	$f'_{cc} = f'_{co}(0.5 + 1.225(\frac{K_s f_l}{f'_{co}})^{0.6})$
11	Al-Salloum [13]	$f'_{cc} = f'_{co}(1 + 3.14k_s(\frac{b}{D})(\frac{f_l}{f'_{co}}))$
12	Arabshahi [14]	$f'_{cc} = f'_{co}(1 + ((\frac{A_e}{A_c})^{2.5} \times (\frac{t_{frp}}{D})^{0.7} \times (\frac{f_{frp}}{f_{co}})^{0.7})^{0.7})^{2.5}$

In these relations, *h* and *b* are length and width of the cross sections and *D* is the equivalent diameter which is derived using the following relation:

$$D = \sqrt{b^2 + h^2} \quad (5)$$

R is the radius of curvature for the corners of the section and K_s is a shape factor which is derived using the succeeding equation:

$$k_S = \left(\frac{b}{h}\right)^2 \frac{A_e}{A_c} \quad (6)$$

where $\frac{A_e}{A_c}$ is the ratio of confined area to the gross concrete area and can be computed by taking advantage of the following formulae:

$$\frac{A_e}{A_c} = \frac{1 - \frac{\left[\left(\frac{b}{h}\right)(h-2R)^2 + \left(\frac{h}{b}\right)(b-2R)^2\right]}{3A_g} - \rho_{sc}}{1 - \rho_{sc}} \quad (7)$$

$$A_g = bh - (4 - \pi)R^2 \quad (8)$$

In the present models, ρ_{sc} stands for the ratio of the longitudinal bars area to the gross concrete area. The other parameters are those which were introduced in the previous section. The only difference is that the confining pressure is computed using the following relation:

$$f_l = \frac{2 f_{frp} t_{frp}}{D} = \frac{2 f_{frp} t_{frp}}{\sqrt{h^2 + b^2}} \quad (9)$$

3. EVALUATION METHODS

In order to assess accuracy of the relations which are listed in Table (1), a database including 120 specimens from the previous experimental studies on rectangular confined columns are collected. The utilized references and number of specimens taken from each study are listed in Table (2). To evaluate accuracy of the confined stress relations, four different error measures, namely MARE, RMSE, MSE and MAPE, are used. The relations of these error measures are presented by equations (9) to (12):

$$MARE = \frac{(\sum_{i=1}^{n} \frac{|f_{ccT} - f_{ccE}|}{f_{ccE}})}{n} \quad (10)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (f_{ccT} - f_{ccE})^2}{n}}) \quad (11)$$

$$MSE = \frac{\sum_{i=1}^{n} (f_{ccT} - f_{ccE})^2}{n}) \quad (12)$$

$$MAPE = \frac{2 f_{frp} t_{frp}}{D} = \frac{2 f_{frp} t_{frp}}{\sqrt{h^2 + b^2}} \quad (13)$$

In this equation, f_{ccT} and f_{ccE} are the computational and experimental strength, respectively:

Table 2- Experiments on rectangular confined concrete

Experiment	No. of test data	Experiment	No. of test data
Parvin and Wang[15]	2	De Luca[21]	9
Harajli and Rteil[16]	18	Zhenyu[22]	14
Rousakis et al[17]	15	Hadi et al[23]	3
Pan et al[18]	6	Belouar et al[24]	16
Kumutha et al[19]	6	Bambole and Joshi[25]	17
Ilki et al[20]	14		

In addition to the previous error criteria, a new performance criteria based on reliability concepts are proposed. This criteria is based on this fact that the relation is reliable and safe provided that it predict strength of the confined concrete accurately, but the predicted value should not exceed the experimental strength, which is actual strength of the confined concrete. Moreover, the influential parameters on the available relations are random variables with probabilistic nature. Accordingly, a relation is considered safe provided that it predict values that are close but lower than the experimental value. Based on this definition, a safety index is proposed for comparison of the reviewed relations. For this purpose, dimensions of specimens, strength of unconfined concrete, and thickness and tensile strength of FRP are considered as random variables, which their distribution properties are presented in Table 3 [26, 27]:

Table 3- Variables distribution statistics

Variable	Distribution Type	COV
b	Normal	0.0417
h	Normal	0.0417
t_{frp}	Lognormal	0.05
f_{co}	Lognormal	0.15
f_{frp}	Normal	0.05

Then, the Monte Carlo simulation technique is used to produce 10000 random variables for each effective parameter, and based on these produced data; strength of confined concrete is computed using the relations listed in Table 1. Then the relations presented in Table 4 are utilized to compute safety and efficiency indexes. A method is considered more reliable, provided that its safety index or efficiency index are higher.

Table 4- Relations for reliability analysis

$\overline{Er} = \frac{\sum_{i=1}^{121} Er_i}{121}$	Average error criterion
$Er_i = \frac{\sum_{i=1}^{10000} \left\lvert \frac{f'_{cc}theo - f'_{cc}exp}{f'_{cc}exp} \right\rvert}{10000} \times 100$	Average error of random data
$\overline{F} = \frac{\sum_{i=1}^{121} \frac{F_i}{10000}}{121}$	Misgiving criterion
$SI = 1 - \overline{F}$	Safety index

In addition to the mentioned criteria, the ratio of $\frac{f_{ccT}}{f_{ccE}}$ is also used to assess performance of the methods. A method is deemed better if its average ratio is closer to 1. Moreover, for reliability of the method, its average ratio should be less than 1. It means that the predicted values by the methods are less than the experimental strength.

4. OBTAINED RESULTS

The computed errors for each relation and the calculated reliability and safety indexes according to the experimental results are listed in Table 5. Based on these results, it is evident that the relation proposed by Arabshahi has lower error in comparison with the other strength models. Moreover, the safety index of this method is about 52 percent. It means that the probability that the strength estimated by this method be lower than the real strength of confined concrete is 52 percent. It can be seen that the Yussef et al. method is more reliable according to the safety index criteria with the index equal to 76.56 percent, but its error is higher. In other words, the mentioned model predicts values which are considerably lower than the experimental values. The diagrams in figures 1 and 2 provide a better approach for comparing the reviewed methods.

Table 5- Errors and Reliability index

No.	Model	MARE	RMSE (10^2)	NMSE (10^3)	MAPE (10^2)	Safety index(I)	$\frac{f_{ccT}}{f_{ccE}}$ average
1	Mirmiran et al. [3]	0.23	0.12	0.15	0.23	0.59	1.20
2	ACI 440.2R-17 [4]	0.30	0.16	0.27	0.30	0.47	0.82
3	Shehata et al. [5]	0.24	0.12	0.15	0.24	0.57	0.90
4	Teng et al.[6]	0.23	0.12	0.15	0.23	0.52	1.02
5	Lam and Teng [7]	0.25	0.16	0.24	0.25	0.48	1.13
6	Campion and Miraglia [8]	0.22	0.11	0.13	0.22	0.52	0.93
7	Ilki et al. [9]	0.30	0.22	0.50	0.30	0.47	1.2
8	Kumutha et al. [10]	0.23	0.13	0.16	0.23	0.55	0.96
9	Toutanji et al. [11]	0.34	0.21	0.43	0.34	0.43	1.18
10	Youssef et al. [12]	0.40	0.20	0.40	0.40	0.76	0.62
11	Al-Salloum [13]	0.22	0.11	0.13	0.22	0.55	0.93
12	Arabshahi [14]	0.21	0.11	0.12	0.21	0.520	1.002

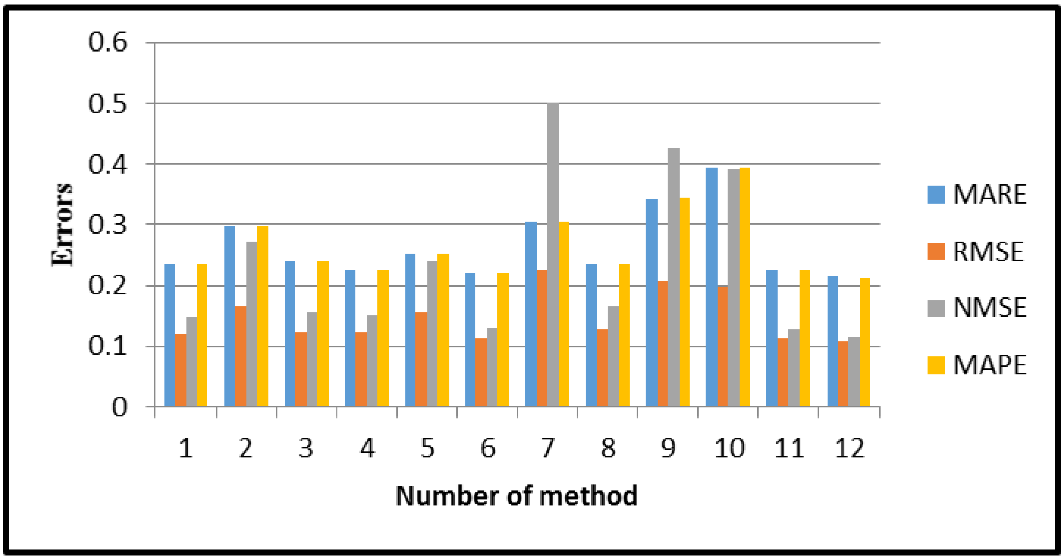

Figure 1. Four types of Error for each method

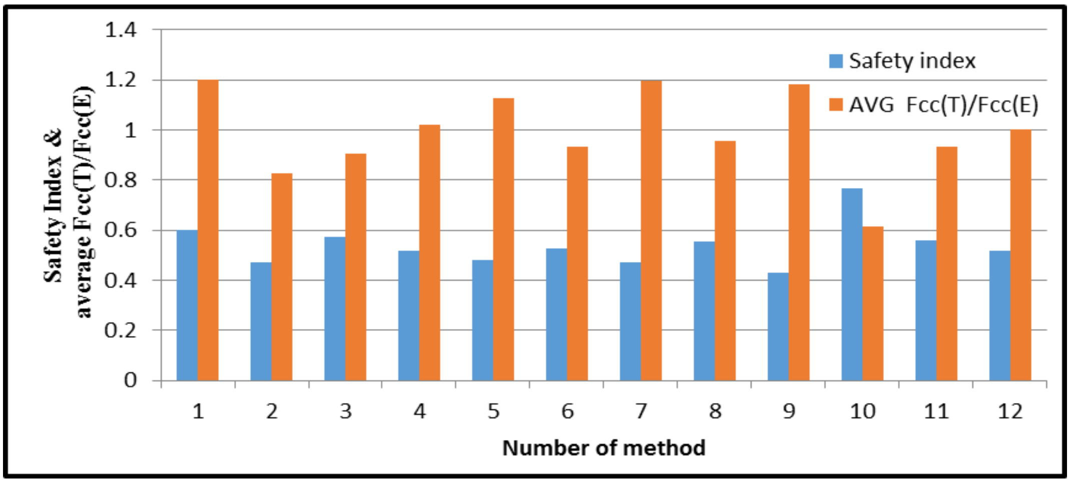

Figure 2. Safety index for each method

5. CONCLUSIONS

In this paper, performance of the available models for the strength of FRP confined rectangular concrete columns is investigated. For this purpose, a data base of experimental results is collected from the previous experimental studies on rectangular columns. Then, twelve different available models for the strength of FRP confined rectangular columns are assessed by different error estimation methods. In order to incorporate the probabilistic nature of influential parameters in the evaluation method, a new probabilistic criteria is proposed, that indicate reliability of the results attained by each model. Based on the overall attained results, it is evident that the model proposed by Arabshahi presents the better results with lower error measures and acceptable safety and efficiency indexes.

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