Research Article

Decision support system for optimal location of HIFDs in real distribution network using an integrated EPSO-fuzzy AHP model

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Abstract: Distribution overhead line network faults with the potential threats to public safety are of extreme importance. Highimpedance faults (HIFs) occur on distribution feeders with close proximity to the population. Advanced technology has enabled power distribution utilities to detect a high percentage of HIFs. However, the application of HIF detectors (HIFDs) should meet both operational and economic requirements due to expense and a large number of distribution feeders. Installation of these devices on overhead feeders among several candidate feeders seems to be a challenging issue. Therefore, ranking of feeders is essential for equipping them with HIFDs. This study proposes a practical approach which is an improved fuzzy analytic hierarchy process (AHP), accompanied by a constrained non-linear optimisation model to extract optimal crisp priorities from fuzzified inputs, considering various criteria with a degree of inconsistency in their real data. This improved integrated model is also able to adequately evaluate both qualitative and subjective parts of the location-allocation problems. In this study, enhanced particle swarm optimisation (EPSO) is used to deal with the optimisation problem for criteria weight ratio and also finding the individual feeder preference weighting to optimally locate HIFDs in distribution feeders. The proposed model is applied on a real network and then sensitivity analysis is performed in the optimal solution.

1 Introduction

High-impedance fault (HIF) detection problem has been extensively studied in literature and it has received a lot of attention since late 1970s and early 1980s [1]. HIFs are considered as faults that cannot be detected by conventional technology under certain conditions such as downed conductor on dry asphalt or dry sand in which arcing is not produced. The resulting current may be too low and insufficient to trigger conventional protective devices (PDs), namely overcurrent relays, reclosers or cutouts. HIFs occur when an energised conductor makes undesired contact with a quasi-insulating object, such as a tree limb or a road. Based on the report by the IEEE PSRC standard [2], faults are labelled as HIFs, premising that the fault current falls below the pickup threshold of overcurrent relays. The current level of HIFs depends mainly on the surface of contact. In addition, the detection of HIFs is even worsened when covered conductors are used in overhead distribution lines. Typical fault currents range from 10 to 50 A, with a very erratic waveform [2]. In solidly grounded distribution networks where the value of the residual current under normal conditions is considerable, overcurrent and earth fault relays do not protect against HIFs. In some cases even sensitive earth fault protection cannot accurately detect such low levels of fault current flow and may mistakenly trip the entire feeder. Analysis of past faults has given rise to identify different indications of HIFs on distribution overhead lines. The large-scale reasons behind HIFs can be generally divided into two categories, i.e. broken conductor and downed conductor.

Dealing with HIF problems not only implies technical and operational points, but also it more importantly involves safety and economic matters. Therefore, application of HIF detectors (HIFDs) in response to different types of HIFs is inevitable because of improving power quality and human safety [3]. Nevertheless, it is evident that deployment of this protective relay on each feeder in a distribution network would, in most cases, not be economically justifiable [4]. In case of downed conductor either on the load side or the source side, typical fault currents level vary depending on the surface of contact, length of feeder and so on [5].

Some extensive studies have been conducted in solving optimal placement problems, involving various optimisation techniques which are categorised into classic [6] and heuristic methods [7]. Authors in [8, 9] used heuristic methods to solve the switch placement problem. Beside heuristic algorithms, authors in [10, 11] determined the optimal placement problem by formulating the problem in mixed integer programming (MIP) format. Farajollahi et al. [11] formulate a MIP-based model to integrate malfunction probability into switch placement problem. Moreover, iterative mixed integer non-linear programing method is proposed in [12] to find the location and size of fault current limiters for mesh configured transmission and distribution systems. In [13], a developed analytical reliability model is proposed for optimal allocation of PDs and fault detectors in smart distribution network to improve reliability. Shahsavari et al. [14] proposed a multiobjective fault indicator (FI) placement problem by considering the impacts of protective and automatic devices. Particle swarm optimisation (PSO)-based algorithm in company with fuzzy decision-making method is adopted to solve the problem. In [15], optimal FI allocation problem is converted to a mixed integer nonlinear programming model that considers N-1 contingencies. Mathematically speaking, optimal placement problem may be stated in either single-objective or multi-objective formulation. Authors in [16, 17] proposed a multi-objective approach to optimally place controlling devices (CDs) and PDs in distribution network, respectively, whereas the authors in [18, 19] presented a single objective mathematical model to solve joint PD and CD placement problem. In [20], the CD placement problem is solved via a MIP model wherein network interruption and remotecontrolled switch (RCS) costs are minimised. The RCS allocation problem in [21] seeks a cost-effective solution to reach a trade-off among the RCS cost, reliability improvement and other objectives. Authors in [10, 22] extend a model to consider the financial risk caused by the stochastic nature of faults and its impact on the RCS allocation problem. The mathematical programming model involves two conflicting objectives, i.e. maximisation of the expected profit and minimisation of the financial risk. However, in real-world practical systems when we encounter objective

functions that cannot be quantified; the aforementioned models are

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Fig. 1 Triangular fuzzy number; \tilde{M}

limited and ineffective. Therefore, inevitably, the decision makers (DMs) should be asked about the relative importance of decision criteria that are neither originally quantifiable nor scalable. Furthermore, intrinsic computational complexity of multi-objective programming (MOP) mostly restricts considerations of many attributes, involved in placement problems. In MOP problem formulation, many of the objectives are regarded as constraints, but this can be avoided by forming a hierarchical structure and converting the priorities into the ratings with respect to each criteria using pairwise comparison. Besides, MOP cannot incorporate subjective criteria into the model as in [23]. In [24], the use of effective methodology for allocating RCS is discussed. However, Bernardon et al. [24] does not consider drawbacks of the analytic hierarchy process (AHP) method in handling the uncertainties and imprecision of multi-criteria decision-making (MCDM) systems [25]. Therefore, the model should be further developed to include the superiorities of fuzzy MCDM techniques [25, 26]. The problem of HIF detection techniques on distribution networks has been widely investigated in [27]. To the best of our knowledge, however, there are limited studies which introduce technical and non-technical issues with regards to applying HIFDs [5]. Aucoin and Jones [5] briefly state criteria which make distribution feeders more prone to HIF. In this regard, distribution utilities face major challenges. On the one hand, purchase and installation of HIFD on every feeder are not economically viable and sometimes practical aspects do not allow this to happen due to the large number of distribution feeders and limited budgets. On the other hand, the significant importance of HIF identification is undeniable. Thus, this paper is aimed primarily at the task of optimal HIFDs placement in distribution feeders. It is important to note that installation of HIFDs on every feeder at once is not possible or appropriate [5]. Therefore, ranking of feeders is essential for equipping them with HIFDs from the economic and technical viewpoints. To do so, it is prudent to develop a model for selecting optimal locations for application of HIFDs. To sum up, the main contributions of this paper are:

(1) The HIFD placement problem for distribution feeders has been converted to an integrated fuzzy optimisation model. Fuzzy MCDM strategies [28, 29] are ideally suited to this application when selection is to be made based on hierarchical relationship of the feeder selection problem.

(2) The improved enhanced particle swarm optimisation (EPSO)fuzzy AHP model is proposed, which shows better performance in terms of consistency index (CI) and function value (fval), compared to other existing methods and approaches [30–34] (this matter is presented in Section 4).

(3) Unlike other placement problems as in [23], subjective-oriented part of placement problem is considered in the model. Both qualitative and quantitative features are incorporated into the model through defining criteria in the hierarchical structure.

(4) A comprehensive sensitivity analysis for characterising the optimal solution of the proposed model has been performed.

The remainder of the paper is organised as follows. Section 2 explains the preliminaries for the fuzzy AHP method [31] and the related work. In Section 3, an integrated EPSO-fuzzy AHP model for solving the placement problem of HIFDs is introduced. In Section 4, required comparative analysis is carried out to show the superiority of the proposed model to the other existing methods and approaches [30–34]. Section 5 elaborates the case study and related criteria in detail. To verify the practicality and effectiveness

IET Gener. Transm. Distrib., 2020, Vol. 14 Iss. 9, pp. 1616-1626 © The Institution of Engineering and Technology 2020 of the final outcome of the proposed methodology, sensitivity analysis is carried out in Section 6. Section 7 concludes the paper.

2 Preliminary bases

In this section, basic concepts correlated to the fuzzy-AHP, triangular fuzzy numbers (TFNs), are briefly introduced.

2.1 Fuzzy AHP

The conventional AHP, developed by Saaty [33], cannot handle the complexity and uncertainty involved in real-world decision problems. Hence, fuzzy judgments instead of crisp judgements are used [35]. Fuzzy judgements can be described by any type of membership functions to construct relative degree of fuzziness in which objectives have different importance. Distinguishing characteristics of fuzzy AHP are as follows: (i) it has the ability to combine linguistic terms, knowledge and experience with mathematics and data [25, 26], (ii) it provides a flexible approach to handle bias and conflict objectives as different criteria, (iii) it allows DMs enjoy the benefits of descriptive analytical approaches [28].

8 TFNs and representation of preferences

In this work, TFNs [36] are used to represent the relative importance in criteria and relative preference in alternatives. A leading contribution of fuzzy set theory [26] is to supply a systematic plan to transform a set of vague or estimated data into a non-linear mapping to be easily employed. Each TFN has linear representations on its left and right side in a way that its membership function can be specified as follows [36]:

$$\mu(x|\tilde{M}) = \begin{cases} 0, & x < l, \\ \frac{x-l}{m-l}, & l \le x \le m, \\ \frac{x-l}{m-l}, & m \le x \le u, \\ 0, & x > u. \end{cases}$$
(1)

A fuzzy number is a special fuzzy set, such that $\tilde{M} = \{(x, \mu_M(x)), x \in R\}$, where the value of x lies on the real line R, i.e. $-\infty < x < \infty$ and $\mu_{\tilde{M}}(x)$ is a continuous mapping from R to the close interval [0,1]. The TFN 'M' is shown in Fig. 1. In (1), l, m and u, respectively, denote the smallest possible value, the most promising value and the largest possible value that describe a fuzzy term. A fuzzy number can always be specified by its corresponding left and right representation of each degree of membership

$$\tilde{M} = \left(M^{l(y)}, M^{r(y)}\right) = (l + (m - l)y, u + (m - u)y), \quad y \in [0, 1]$$
(2)

In (2), *y* is considered as the value of $\mu_{\tilde{M}}(x)$, where l(y) and r(y) denote the left side representation and the right side representation of a fuzzy number, respectively. The TFN \tilde{M} is often represented as (l, m, u). In this regard, if there are n (i, j = 1, 2, ..., n) decision elements, the DMs are required to make (n(n + 1))/2 judgments. A fuzzy reciprocal judgment matrix \tilde{A} , comprised of real and positive elements, $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, $\tilde{a}_{ji} = 1/\tilde{a}_{ij} = (1/u_{ij}, 1/m_{ij}, 1/l_{ij})$, $\tilde{a}_{ii} = (1, 1, 1)$ and can be created as follows:

$$\tilde{A} = \{\tilde{a}_{ij}\} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{nn} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nn} \end{bmatrix}$$
(3)

Wang's method [36] is applied to determine the relative preference weights for both the criteria and the alternatives. The basic steps of the fuzzy AHP are as follows:

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Step 1: obtaining the hierarchical structure: The problem is structured as a family tree in this step. At the highest level is the overall goal of this decision-making problem, and the alternatives are at the lowest level. Between them are criteria and sub-criteria.

Step 2: Development of judgment matrices by pairwise comparisons: DMs compare the criteria or alternatives via fuzzy judgment scores, as illustrated in Table 1. In other words, the fuzzy comparison judgment matrices are decided according to the suggestions of a group of experts during decision-making process. The imprecision and uncertain evaluations of them are translated to triangular fuzzy judgment scores according to Table 1.

Step 3: Consistency check and calculating local priorities from judgment matrices: This step investigates for consistency and extracts the priorities from the pairwise comparison matrices (PCMs). In fuzzy AHP method, degree of consistency in PCMs is of high importance. Perfect consistency is said to be satisfied if the following equation holds [33]:

$$r_{ik} * r_{kj} = r_{ij}, \quad \forall \ i, \ j, \ k = 1, 2, \ \dots, \ n.$$
 (4)

A few studies [34, 37] have addressed the issue of checking for inconsistencies in PCMs. Then, the priority vector $J = (w_1, w_2, ..., w_n)$, where *n* is the size of the PCM, can be achieved from the PCM by applying a prioritisation method [30]. Step 4: Ranking of the alternatives: Global ranking or final ranking can be computed from the local priorities as in the conventional AHP.

In what follows, due to dimensions of problem, a constrained nonlinear optimisation model is introduced which help to ease the computational burden and eliminate step 3 of the fuzzy AHP procedure. This model also extracts exact discrete weights with a very acceptable degree of accuracy and precision from both consistent and inconsistent fuzzy comparison matrices. Additionally, it yields CI, as a measurement of inconsistency in DMs' judgments [37]. Fuzzy preference programming (FPP) was firstly introduced in [37], and then it was compared with two-stage logarithmic goal programming in [31].

3 Fuzzy optimisation model

In a prioritisation problem, elements of PCM are represented by TFNs $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$. Each set of comparisons fill the upper diagonal elements, and the reciprocal \tilde{a}_{ji} fill the lower diagonal elements. Hence, the following holds [26]:

$$\begin{cases} l_{ij} \stackrel{\sim}{\leq} m_{ij} \stackrel{\sim}{\leq} u_{ij} & \text{if } i \neq j \\ \tilde{a}_{ij} = \tilde{a}_{ji} = (1, 1, 1) & \text{if } i = j \end{cases}$$
(5)

Symbol ~ along with any operator indicates operations in fuzzy domain. Fuzzy arithmetic operations are performed using conventional interval arithmetic according to the α -cut representation [37]. Note that μ_{ij} shows the degree for satisfaction for different crisp ratios. μ_{ij} is expressed as CI. This is given by [36]

$$\mu_{ij}(w_i, w_j) = \begin{cases} \frac{m_{ij} - (w_i/w_j)}{m_{ij} - l_{ij}}, & 0 < \frac{w_i}{w_j} \le m_{ij} \\ \frac{(w_i/w_j) - m_{ij}}{u_{ij} - m_{ij}}, & \frac{w_i}{w_j} \le m_{ij} \end{cases}$$
(6)

This function is linearly decreasing over the interval $(0, m_{ij}]$ and linearly increasing over the interval $[m_{ij}, \infty)$. The smaller the value of this function, the more acceptable the crisp ratio w_i/w_j becomes. Therefore, the following non-linear mathematical model is presented to calculate the priority vector by taking into account different criteria. The model minimises non-differentiable vector J, which is defined as follows: $\min \boldsymbol{J}(w_1, w_2, \ldots, w_n)$

$$= \min \sum_{i}^{n} \sum_{j}^{n} \left[\mu_{ij}^{2} \left(\frac{w_{i}}{w_{j}} \right) \right]$$

$$= \min \sum_{i}^{n} \sum_{j}^{n} \left[\delta \left(m_{ij} - \frac{w_{i}}{w_{j}} \right) \left(\frac{m_{ij} - (w_{i}/w_{j})}{m_{ij} - l_{ij}} \right)^{2} + \delta \left(\frac{w_{i}}{w_{j}} - m_{ij} \right) \left(\frac{(w_{i}/w_{j}) - m_{ij}}{u_{ij} - m_{ij}} \right)^{2} \right]$$
(7)

Subject to

$$l_{ij} \leq \frac{w_i}{w_j} \leq u_{ij}, \quad i, \ j = 1, \ 2, \ ..., \ n, \quad i \neq j$$
 (8)

$$\sum_{k=1}^{n} w_k = 1, \quad w_k > 0, \quad k = 1, 2, \dots, n$$
(9)

where δ is the heaviside function given by

$$\delta(x) = \begin{cases} 0, & x < 0\\ \frac{1}{2}, & x = 0\\ 1, & x > 0 \end{cases}$$
(10)

This fuzzy model is to yield the optimal crisp priority vector $J^* = (w_1^*, w_2^*, ..., w_n^*)$ on the fuzzy feasible area, *P* specified by the intersection of all fuzzy constraints on the (n - 1) dimensional simplex Q^{n-1}

$$Q^{n-1} = \left\{ (w_1, \dots, w_n) \middle| \sum_{i=1}^n w_i = 1, \ w_i > 0 \right\}$$
(11)

in which compared fuzzy ratios on Q^{n-1} described by the following triangular membership functions:

$$\mu_{\tilde{P}}(w) = \min \left\{ \mu_{ij} \left(\frac{w_i}{w_j} \right) \middle| i = 1, 2, ..., n-1; j = 2, 3, ..., n \right\} (12)$$

Also, J^* corresponds to the maximum fuzzy feasible area as follows [37]:

$$\lambda^* = \mu_{\tilde{P}}(\mathbf{w}^*) = \max\left\{ \min_{ij} \left\{ \mu_{ij} \left\{ \frac{w_i^*}{w_j^*} \right\} \middle| w \in Q^{n-1} \right\} \right\}$$
(13)

Subsequently, λ^* is calculated and it indicates the level of satisfaction from the optimised priority vector. In this regard, the λ^* computed for fuzzy PCM can take the values ranging between 1 for perfect consistency to 0 for inconsistency, negative value of λ^* implies that pairwise judgments are profoundly inconsistent. Due to non-linearity and combinatorial properties of the problem, EPSO algorithm is used. It falls under the category of population-based optimisation methods. In this paper, EPSO is formed by adding mutation operator to conventional PSO algorithm [38] to prevent premature convergence and increase the exploration.

Table	1 Fuzz	y scales	transl	ation	36	

Fuzzy judgments	Membership function
equally preferred	(1/2, 1, 2)
about x times more important	(x-1, x, x+1)
about x times less important	(1/(x+1), 1/x, 1/(x-1))
between y and z times more important	(y, (y + z)/2, z)
between y and z times less important	(1/z, 2/(y+z)), 1/y)

x = 2, 3, ..., 9 and y, z = 1, 2, ..., 9 and y < z.

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1 begin 2 insert TFNs of comparison matrix 3 Initialize parameters EPSO (MaxIteration, mutation rate, population size, constriction coefficients) 4 for i=1: population size 5 Initialize and evaluate the particle 6 update personal best of particles and global best 7 end for 8 for j=1: MaxIteration 9 for k=1: population size 10 update velocity and apply velocity limits update position of particle and evaluate it 11 update personal best of particles and global best 12 13 end for 14 apply mutation on particles 15 update personal best of particles and global best 16 end for 17 export position of global best as priority vector $(w_1, w_2, \dots, w_n)^T$ 18 calculate of Lambda

Fig. 2 Algorithm 1: Main steps of the proposed algorithm

Table 2	Comparative anal	vsis for crisp values,	CI and fval in 4×4 fuzzy	y PCM [32]
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Existing FAHP approaches	<i>W</i> ₁	<i>W</i> ₂	W ₃	W_4	CI	fval		
extent analysis method [30]	0.13	0.41	0.03	0.43	_	_		
CI method [33]	0.2025	0.3242	0.1465	0.3268	—	—		
PSO-fuzzy AHP [32]	0.1962	0.2988	0.1417	0.3633	0.1140	30.8186		
genetic algorithm method [34]	0.1807	0.3312	0.1461	0.3420	0.0419	27.5220		
proposed model	0.1801	0.3323	0.1463	0.3413	0.286	26.9950		

Table 3	Comparative analy	sis for crisp values,	CI and fval in 5×5	fuzzy PCM [32]
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Existing FAHP approaches	<i>W</i> ₁	W_2	W ₃	W_4	W_5	CI	fval
non-linear FPP [31]	0.0871	0.4141	0.0472	0.1582	0.2934	0.4523	
PSO-fuzzy AHP [32]	0.0909	0.4104	0.0472	0.1568	0.2947	0.446	16.5556
genetic algorithm method [34]	0.0869	0.4151	0.0474	0.1576	0.2940	0.4504	12.2360
proposed model	0.0898	0.4165	0.0482	0.1556	0.2889	0.5061	6.5209

The proposed improved EPSO-fuzzy AHP model is developed for determining preference order of placement problem in distribution system by Algorithm 1 (see Fig. 2).

4 Model verification

In this paper, additional modifications have been made to the model in [32] to improve CI and fval. For instance, (8) is added to define bounds for fuzzy priority vector in every iteration of the algorithm. This implies that J^* has to satisfy all fuzzy judgments. There is also a change in heaviside function definition, given as (10). The improved version of the model together with application of mutation operator in conventional PSO [38] reduce the computational time and chance of difficulties in finding a feasible/ optimal solution. This way, not only fval is better, also CI reaches a greater value. The validity of the proposed model has been verified by comparative analyses with existing methods [30-34]. In this regard, two different examples are considered for evaluation of the proposed model. As seen in Tables 2 and 3, comparing to the other methods, the proposed model leads to the crisp values of same priorities from the fuzzy PCMs presented in [32]. Moreover, the proposed model is superior from two perspectives: (i) greater CI and (ii) lower fval. As seen in Table 2, CI obtained by the proposed model in the 4×4 fuzzy PCM is 0.286 which is greater than the consistency in the other methods, whereas fval is reduced to 26.995. It is notable that the proposed model in the 5×5 fuzzy PCM is able to minimise fval more, while CI reaches 0.5061 which is the most among other methods [31, 32, 34], as observed in

Table 3. The results are demonstrated in Tables 2 and 3 in detail. The corresponding PCMs of Tables 2 and 3 are presented in [32].

5 Case study

The 20 kV studied distribution system is part of Mashhad Electric Energy Distribution Company (MEEDC) Network, located in Razavi-Khorasan province in Iran. It is employed to study the applicability of the proposed model. The GIS ready map of the under study network is shown in Fig. 3. This network consists of \sim 5345 km of lines operating at 20 kV. The poles of aerial distribution network are made up mainly of wood (27,677), concrete (227,390) and steel (11,009). Required network data is provided in Table 4.

5.1 Model implementation

One of the most essential components in the decision-making process is appointing the right criteria against which alternatives will be evaluated. Detailed discussions are held to ensure that all the aspects involved in the placement problem are considered. Hence, the main criteria relevant to the feeder selection and the hierarchy of the selection criteria and candidate feeders belonging to multiple 132 kV substations can be observed in Fig. 4.

5.2 Feeder selection

Table 4 shows historical data of one year of past HIF incidents in ten feeders with their related energy not supplied (ENS) and



Fig. 3 Real-life radial distribution network of the studied system

Table 4 Recorded HIF in	ncidents during a year
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Feeder name	Feeder code	Number of HIF incident	ENS, MWh	Interruption duration, min
Таууеb	F1	2	2.2297	18
Abozar	F2	1	1.3348	31
Afsaneh	F3	2	0.2086	17
Pedram	F4	1	0.638	10
Parasto	F5	2	1.2367	9
Toseeh	F6	3	3.3357	41
Damavand	F7	2	3.2146	18
Soroush	F8	1	1.1483	26
Kardeh	F9	4	7.055	32
Mazdavand	F10	3	3.2389	37



Fig. 4 Hierarchical relationship of the feeder selection problem

interruption duration in MEEDC. The experience of distribution network with the soil conditions, type of circuit construction, past experience with HIFs and the nature of the load on the circuit can give an indication of the priority for the application of HIFDs [2]. Underground feeder circuits need no HIFDs, since they pose few public safety concerns. Newer overhead circuits with large conductor cross-section are less susceptible to HIFs due to broken conductor. Older circuits are more likely to have small crosssection are more susceptible to HIFs. Moreover, circuits with conductors in poor condition or those that have experienced severe storms or overloads, significant tree contact or with histories of excessive broken conductors may benefit from HIFDs. Hence, due to the fact that relative importance of decision criteria is neither quantifiable nor scalable, the proposed mathematical programming model is applied to rank the candidate feeders in terms of HIFD allocation. Therefore, criteria for feeder weighing process, in prioritisation point of view, are mainly comprised of the following: (i) past HIF events, (ii) length of feeder, (iii) population density on feeder, (iv) fire prone areas, (v) ageing infrastructure particularly in urban areas, (vi) size of conductor in overhead main line and its laterals at 20 kV voltage level. The fuzzy comparison judgments of six main criteria with respect to the overall goal are shown in Table 5.

Applying the fuzzy prioritisation method, introduced in Section 3, the exact weights of main criteria are obtained as

- $w_1 = 0.2562$ (past HIF events)
- $w_2 = 0.1336$ (length of feeder)
- $w_3 = 0.2015$ (population on density feeder)
- $w_4 = 0.0830$ (fire prone areas)
- $w_5 = 0.1513$ (ageing of overhead line)
- $w_6 = 0.1745$ (conductor size)

The graphical representation of main criteria involved in feeder ranking is shown in Fig. 5. Similarly, the fval convergence of the proposed optimisation model for the second level PCM, related to Table 5, is shown in Fig. 6. fval is equal to 1.0547. Additionally, the corresponding CI of the fuzzy judgment matrix shown in

Table 5 Evaluation of criteria with respect to goal

Criteria	Past HIF events	Length of feeder	Population density	Fire prone areas	Ageing of overhead	Conductor size
		C C	on feeder		line	
past HIF events	(1,1,1)	(1,2,3)	(1/2,3/2,5/2)	(2,3,4)	(3/4,7/4,11/4)	(1/2,3/2,5/2)
length of feeder	(1/3,1/2,1)	(1,1,1)	(0.43,0.75,3.03)	(1/2,3/2,5/2)	(0.47,0.87,6.99)	(0.43,0.75,3.03)
population density on feeder	(2/5,2/3,2)	(1/3,4/3,7/3)	(1,1,1)	(2,3,4)	(1/6,7/6,13/6)	(1/2,1,2)
fire prone areas	(1/4,1/3,1/2)	(2/5,2/3,2)	(1/4,1/3,1/2)	(1,1,1)	(0.37,0.58,1.4)	(1/3,1/2,1)
ageing of overhead line	e (4/11,4/7,4/3)	(0.14,1.14,2.14)	(6/13,6/7,6)	(0.71,1.71,2.71)	(1,1,1)	(6/13,6/7,6)
conductor size	(2/5,2/3,2)	(1/3,4/3,7/3)	(1/2,1,2)	(1,2,3)	(1/6,7/6,13/6)	(1,1,1)



Fig. 5 Criteria weights

Table 5 is computed using the proposed method and the result is as follows:

$$\mu_{1} = \min \left\{ \mu_{1j} \left(\frac{w_{1}^{*}}{w_{j}^{*}} \right) \right\} = 0.7709;$$

for $j = 2, 3, 4, 5, 6;$
$$\mu_{2} = \min \left\{ \mu_{2j} \left(\frac{w_{2}^{*}}{w_{j}^{*}} \right) \right\} = 0.7282; \quad \text{for } j = 3, 4, 5, 6;$$

$$\mu_{3} = \min \left\{ \mu_{3j} \left(\frac{w_{3}^{*}}{w_{j}^{*}} \right) \right\} = 0.4297; \quad \text{for } j = 4, 5, 6;$$

$$\mu_{4} = \min \left\{ \mu_{4j} \left(\frac{w_{4}^{*}}{w_{j}^{*}} \right) \right\} = 0.8375; \quad \text{for } j = 5, 6;$$

$$\mu_{5} = \min \left\{ \mu_{5j} \left(\frac{w_{5}^{*}}{w_{j}^{*}} \right) \right\} = 0.9981; \quad \text{for } j = 6;$$

According to (13), the value of λ^* becomes

$$\lambda^* = \max \{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5\} = 0.9981$$

The achieved value indicates an approximately perfect consistency. Correspondingly, the local scores of the feeders with regard to all main criteria are then obtained (for simplicity, PHE, LF, PDF, FPA, AOL, CS are abbreviations for past HIF events, length of feeder, population density on feeder, fire prone areas, ageing of overhead line, conductor size, respectively). Fig. 7 illustrates changes in weights of decision-making criteria in 80 iterations. The common characteristics of these feeders which are critical in terms of prioritising them in HIFDs installation point of view are depicted in Table 6. Hence, triangular fuzzy judgments of different feeders, pertaining to the each criterion, are made based on Tables 5 and 6. Then, corresponding local weight of each feeder is calculated according to the proposed fuzzy optimisation model in the same manner. The numerical simulations are done in MATLAB on a computer with one processor Intel (R) Core (TM) i7-2600 CPU 3.40 GHz with 4 GB of RAM. In terms of computational efficiency, simulations of each of the criteria took about 10-15 s. It

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Fig. 6 Convergence characteristics of EPSO for the proposed objective function



Fig. 7 Convergence characteristics corresponding to main criteria

should be noticed that the fuzzification is utilised in order to eliminate or reduce cognitive biases in decision making. Furthermore, this method enables related experts to be directly involved in the placement problem more common sensibly with their prior knowledge, experience and recorded data. Criteria are defined subsequent to consultation with several experts and based on manufacturers' specifications, product description, device functionality and network topology. So, each criterion is clarified preceding the deployment of the proposed method.

- *Past HIF events*: First and foremost, feeders with respective HIF event data record have to be considered since this shows the overall state of candidate feeders. Hence, this criterion may apparently play a key role in HIFD placement problem. Additionally, this is also consistent with the manufacturers' recommendations [4]. The fuzzy evaluation matrix of feeders with respect to past HIF events, and the corresponding weight of each feeder are shown in Table 7.
- Length of feeder and population density on feeder: These criteria imply the fact that improvement of service reliability and reduction in capital and maintenance expenditure, coupled with

electrical safety provision cannot be realised unless feeder length and number of customers are considered. These criteria play key roles in HIFD allocation to candidate feeders. Hence, they are translated to problem objectives. FCMs are constructed to determine the weights of the feeders with respect to aforementioned criteria and shown in Tables 8 and 9, respectively.

• *Fire prone areas*: Over the years, the distribution networks have been an infamous cause of fire in rural areas, then identification of HIF in remote areas plays a vital role in fire prevention.

Table 6 Candidate feeders characteristi

Feeder	Practical line loadability, A	Load characteristics	Line length including laterals, km	Cross- sectional area, mm ²	Conductor ageing, years	Population density on feeder	
F1	135	residential = 89%	9.38 (100% aerial)	120 70	33	4694	
F2	120	residential = 75%	12 (100% aerial)	120 70	29	10,013	
F3	110	residential = 93% industrial = 7%	77.54 (100% aerial)	120	25	3540	
F4	150	residential = 87% commercial = 13%	8.84 aerial, 0.55 underground	120 70	29	9400	
F5	140	residential	13.16 (100% aerial)	120	29	7065	
F6	115	agricultural = 52% residential = 48%	72.63 (100% aerial)	70	38	1512	
F7	95	residential = 60%	23.96 (100% aerial)	120 70	33	4895	commercial = 21%.
industrial = 19%							,
F8	60	industrial	11.42 (11 aerial, 0.42 underground)	120	37	89	
F9	140	residential = 77% agricultural = 18% industrial = 5%	34 (100% aerial)	120\1/3 70\2/3	8 29	3997	
F10	135	industrial = 60%, agricultural = 30%,	49.37 (94% aerial, 6% underground)	120 70	34	586	residential = 10%

Table 7 Evaluation of feeders with respect to past HIF events

PHE	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
F1	1,1,1	1,2,3	1/2,1,2	1,2,3	1/2,1,2	2/5,2/3,2	1/2,1,2	1,2,3	1/3,1/2,1	2/5,2/3,2
F2	1/3,1/2,1	1,1,1	1/3,1/2,1	1/2,1,2	1/3,1/2,1	1/4,1/3,1/2	1/3,1/2,1	1/2,1,2	1/5,1/4,1/3	1/4,1/3,1/2
F3	1/2,1,2	1,2,3	1,1,1	1,2,3	1/2,1,2	2/5,2/3,2	1/2,1,2	1,2,3	1/3,1/2,1	2/5,2/3,2
F4	1/3,1/2,1	1/2,1,2	1/3,1/2,1	1,1,1	1/3,1/2,1	1/4,1/3,1/2	1/3,1/2,1	1/2,1,2	1/5,1/4,1/3	1/4,1/3,1/2
F5	1/2,1,2	1,2,3	1/2,1,2	1,2,3	1,1,1	2/5,2/3,2	1/2,1,2	1,2,3	1/3,1/2,1	2/5,2/3,2
F6	1/2,3/2,5/2	2,3,4	1/2,3/2,5/2	2,3,4	1/2,3/2,5/2	1,1,1	1/2,3/2,5/2	2,3,4	3/7,3/4,3	1/2,1,2
F7	1/2,1,2	1,2,3	1/2,1,2	1,2,3	1/2,1,2	2/5,2/3,2	1,1,1	1,2,3	1/3,1/2,1	2/5,2/3,2
F8	1/3,1/2,1	1/2,1,2	1,2,3	1/2,1,2	1/3,1/2,1	1/4,1/3,1/2	1/3,1/2,1	1,1,1	1/5,1/4,1/3	1/4,1/3,1/2
F9	1,2,3	3,4,5	1,2,3	3,4,5	1,2,3	1/3,4/3,7/3	1,2,3	3,4,5	1,1,1	1/3,4/3,7/3
F10	1/2,3/2,5/2	2,3,4	1/2,3/2,5/2	2,3,4	1/2,3/2,5/2	1/2,1,2	1/2,3/2,5/2	2,3,4	3/7,3/4,3	1,1,1
W _{PHE}	0.0953	0.0477	0.0941	0.0477	0.0953	0.1430	0.0953	0.0481	0.1907	0.1430

Table 8 Evaluation of feeders with respect to LF

F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
1,1,1	0.44, 0.8,3.6	0.1, 0.12, 0.14	0.06,1,2.1	0.4, 0.7,2.	5 0.1, 0.13, 0.15	0.28, 0.4, 0.64	0.45, 0.8,4.5	0.22, 0.3, 0.4	0.16, 0.2, 0.23
0.3,1.3,2.3	1,1,1	0.13, 0.15, 0.18	0.4,1.4,2.4	0.5, 0.9,1	0 0.14, 0.16, 0.2	0.33, 0.5,1	0.05,1,2	0.26, 0.35, 0.54	0.2, 0.24, 0.32
7,8,9	5.5,6.5,7.5	1,1,1	8,9,10	5,6,7	0.1,1.1,2.1	2.1,3.2,4	5.8,6.8,7.8	1.3,2.3,3.3	0.6,1.6,2.6
0.5, 0.9,16	0.42, 0.7,2.8	0.1, 0.11, 0.13	1,1,1	0.4, 0.67,	2 0.1, 0.12, 0.14	0.27, 0.37, 0.6	0.4, 0.8,3.3	0.2, 0.26, 0.35	0.15, 0.18, 0.22
0.4,1.4,2.4	0.1,1.1,2.1	0.14, 0.17, 0.2	0.5,1.5,2.5	1,1,1	0.15, 0.2, 0.22	0.35, 0.5,1.2	0.15,1.15,2.1	0.28, 0.4, 0.62	0.21, 0.27, 0.37
6.7,7.7,8.7	5,6,7	0.5, 0.9,14.3	7.2,8.2,9.2	4,5,6	1,1,1	2,3,4	5.4,6.4,7.4	1.14,2.14,3.14	0.5,1.5,2.5
1.5,2.5,3.5	1,2,3	0.23, 0.31, 0.44	1.7,2.7,3.7	1,2,3	0.25, 0.33, 0.5	1,1,1	1.1,2.1,3.1	0.4, 0.7,2.4	0.33, 0.5, 0.94
0.2,1.2,2.2	0.5, 0.95,20	0.13, 0.15, 0.17	0.3,1.3,2.3	0.5, 0.9,6.	7 0.13, 0.16, 0.2	0.3, 0.5, 0.9	1,1,1	0.25, 0.33, 0.5	0.19, 0.23, 0.3
2.6,3.6,4.6	1.8,2.8,3.8	0.3, 0.43,3.3	3,4,5	2,3,4	0.32, 0.5, 0.9	0.4,1.4,2.4	2,3,4	1,1,1	0.4, 0.7,2.2
4.3,5.3,6.3	3.1,4.1,5.1	0.4, 0.64,1.7	5,6,7	3,4,5	0.4, 0.7,2.1	1,2,3	3.3,4.3,5.3	0.45,1.45,2.45	1,1,1
0.0300	0.0384	0.2484	0.0283	0.0421	0.2325	0.0767	0.0365	0.1089	0.1581
	F1 1,1,1 0.3,1.3,2.3 7,8,9 0.5, 0.9,16 0.4,1.4,2.4 6.7,7.7,8.7 1.5,2.5,3.5 0.2,1.2,2.2 2.6,3.6,4.6 4.3,5.3,6.3 : 0.0300	F1 F2 1,1,1 0.44, 0.8,3.6 0.3,1.3,2.3 1,1,1 7,8,9 5.5,6.5,7.5 0.5, 0.9,16 0.42, 0.7,2.8 0.4,1.4,2.4 0.1,1.1,2.1 6.7,7.7,8.7 5,6,7 1.5,2.5,3.5 1,2,3 0.2,1.2,2.2 0.5, 0.95,20 2.6,3.6,4.6 1.8,2.8,3.8 4.3,5.3,6.3 3.1,4.1,5.1 0.0300 0.0384	F1 F2 F3 1,1,1 0.44, 0.8,3.6 0.1, 0.12, 0.14 0.3,1.3,2.3 1,1,1 0.13, 0.15, 0.18 7,8,9 5.5,6.5,7.5 1,1,1 0.5, 0.9,16 0.42, 0.7,2.8 0.1, 0.11, 0.13 0.4,1.4,2.4 0.1,1.1,2.1 0.14, 0.17, 0.2 6.7,7.7,8.7 5,6,7 0.5, 0.9,14.3 1.5,2.5,3.5 1,2,3 0.23, 0.31, 0.44 0.2,1.2,2.2 0.5, 0.95,20 0.13, 0.15, 0.17 2.6,3.6,4.6 1.8,2.8,3.8 0.3, 0.43,3.3 4.3,5.3,6.3 3.1,4.1,5.1 0.4, 0.64,1.7 0.0300 0.0384 0.2484	F1 F2 F3 F4 1,1,1 0.44, 0.8,3.6 0.1, 0.12, 0.14 0.06,1,2.1 0.3,1.3,2.3 1,1,1 0.13, 0.15, 0.18 0.4,1.4,2.4 7,8,9 5.5,6.5,7.5 1,1,1 8,9,10 0.5, 0.9,16 0.42, 0.7,2.8 0.1, 0.11, 0.13 1,1,1 0.4,1.4,2.4 0.1,1.1,2.1 0.14, 0.17, 0.2 0.5,1.5,2.5 6.7,7.7,8.7 5,6,7 0.5, 0.9,14.3 7.2,8.2,9.2 1.5,2.5,3.5 1,2,3 0.23, 0.31, 0.44 1.7,2.7,3.7 0.2,1.2,2.2 0.5, 0.95,20 0.13, 0.15, 0.17 0.3,1.3,2.3 2.6,3.6,4.6 1.8,2.8,3.8 0.3, 0.43,3.3 3,4,5 4.3,5.3,6.3 3.1,4.1,5.1 0.4, 0.64,1.7 5,6,7 0.0300 0.0384 0.2484 0.0283	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $

Table 9 Evaluation of feeders with respect to PD

PD	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
F1	1,1,1	0.32, 0.5, 0.9	0.3,1.3,2.3	0.3, 0.5,1	0.4, 0.7,2	2,3,4	0.5,1,2	51,52,53	0.2,1.2,2.2	7,8,9
F2	1.1,2.1,3.1	1,1,1	1.8,2.8,3.8	0.1,1.1,2.1	0.4,1.4,2.4	5.6,6.6,7.6	1,2,3	110,111,112	1.5,2.5,3.5	16,17,18
F3	0.4, 0.7,3	0.26, 0.35, 0.5	1,1,1	0.3, 0.4, 0.6	0.33, 0.5,1	0.3,2.3,3.3	0.4, 0.7,2.3	38,39,40	0.47, 0.9,7	5, 6,7
F4	1,2,3	0.5, 0.9,10	1.7,2.7,3.7	1,1,1	0.3,1.3,2.3	5,6,7	1,2,3	103,104,105	1.3,2.3,3.3	15,16,17
F5	0.5,1.5,2.5	0.4, 0.7,2.5	1,2,3	0.43, 0.76,3.1	1,1,1	4,5,6	0.4,1.4,2.4	78,79,80	1,2,3	11,12,13
F6	0.25, 0.33, 0.5	0.13, 0.15, 0.18	0.3, 0.43,3	0.14, 0.16, 0.19	0.17, 0.2, 0.27	1,1,1	0.23, 0.3, 0.43	14,15,16	0.3, 0.4, 0.6	1.5,2.5,3.5
F7	0.5,1,2	0.3, 0.5,1	0.4,1.4,2.4	0.34, 0.5,1.1	0.42, 0.7, 2.5	2.3,3.3,4.3	1,1,1	49,50,51	0.25,1.25,2.25	7,8,9
F8	0.018, 0.019, 0.02	0.008, 0.009, 0.01	0.025, 0.026, 0.027	0.009, 0.01, 0.011	0.01, 0.012, 0.013	0.06, 0.067, 0.07	0.019, 0.02, 0.021	1,1,1	0.02, 0.023, 0.024	.13, 0.15, 0.18
F9	0.45, 0.8,5	0.3, 0.4, 0.7	0.14,1.14,2.1	0.3, 0.4, 0.74	0.36, 0.6,1.2	1.7,2.7,3.7	0.45, 0.8,4	43,44,45	1,1,1	6,7,8
F10	0.1, 0.12, 0.14	0.05, 0.06, 0.064	0.15, 0.17, 0.2	0.06, 0.064, 0.07	0.08, 0.09, 0.1	0.3, 0.4, 0.7	0.1, 0.12, 0.14	5.7,6.7,7.7	0.12, 0.14, 0.17	1,1,1
W _{PC}	0.1035	0.2208	0.0775	0.2068	0.1571	0.0311	0.0997	0.0020	0.0884	0.0131

Table 10 Evaluation of feeders with respect to FPA

FPA	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
F1	1,1,1	1/2,1,2	2/7,2/5,2/3	1,2,3	1,2,3	2/7,2/5,2/3	1/3,1/2,1	1/3,1/2,1	2/5,2/3,2	2/5,2/3,2
F2	1/2,1,2	1,1,1	2/7,2/5,2/3	1,2,3	1/2,1,2	2/7,2/5,2/3	1/3,1/2,1	1/3,1/2,1	2/5,2/3,2	2/5,2/3,2
F3	3/2,5/2,7/2	3/2,5/2,7/2	1,1,1	3,4,5	4,5,6	1/2,1,2	1/4,5/4,9/4	1/4,5/4,9/4	2/3,5/3,8/3	2/3,5/3,8/3
F4	1/3,1/2,1	1/3,1/2,1	1/5,1/4,1/3	1,1,1	1/2,1,2	1/6,1/5,1/4	1/5,1/4,1/3	1/5,1/4,1/3	1/4,1/3,1/2	1/4,1/3,1/2
F5	1/3,1/2,1	1/2,1,2	1/6,1/5,1/4	1/2,1,2	1,1,1	1/6,1/5,1/4	1/5,1/4,1/3	1/5,1/4,1/3	1/4,1/3,1/2	1/4,1/3,1/2
F6	3/2,5/2,7/2	3/2,5/2,7/2	1/2,1,2	4,5,6	4,5,6	1,1,1	1/4,5/4,9/4	1/4,5/4,9/4	2/3,5/3,8/3	2/3,5/3,8/3
F7	1,2,3	1,2,3	4/9,4/5,4	3,4,5	3,4,5	4/9,4/5,4	1,1,1	1/2,1,2	1/3,4/3,7/3	1/3,4/3,7/3
F8	1,2,3	1,2,3	4/9,4/5,4	3,4,5	3,4,5	4/9,4/5,4	1/2,1,2	1,1,1	1/3,4/3,7/3	1/3,4/3,7/3
F9	1/2,3/2,5/2	1/2,3/2,5/2	3/8,3/5,3/2	2,3,4	2,3,4	3/7,3/5,3/2	3/7,3/4,3	3/7,3/4,3	1,1,1	1,2,3
F10	1/2,3/2,5/2	1/2,3/2,5/2	3/8,3/5,3/2	2,3,4	2,3,4	3/8,3/5,3/2	3/7,3/4,3	3/7,3/4,3	1/3,1/2,1	1,1,1
W _{FPA}	0.0672	0.0648	0.1573	0.0350	0.0339	0.1686	0.1352	0.1352	0.1053	0.0976

Table 11 Evaluation of feeders with respect to AOL

AOL	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
F1	1,1,1	1/6,7/6,13/6	2/5,7/5,12/5	1/6,7/6,13/6	1/6,7/6,13/6	7/15,7/8,7	1/2,1,2	7/15,7/8,7	1/6,7/6,13/6	1/2,1,2
F2	6/13,6/7,6	1,1,1	1/5,6/5,11/5	1/2,1,2	1/2,1,2	3/7,3/4,3	6/13,6/7,6	3/7,3/4,3	1/2,1,2	6/13,6/7,6
F3	5/12,5/7,5/2	5/11,5/6,5	1,1,1	5/11,5/6,5	5/11,5/6,5	5/13,5/8,5/3	5/12,5/7,5/2	5/13,5/8,5/3	5/11,5/6,5	5/12,5/7,5/2
F4	6/13,6/7,6	1/2,1,2	1/5,6/5,11/5	1,1,1	1/2,1,2	3/7,3/4,3	6/13,6/7,6	3/7,3/4,3	1/2,1,2	6/13,6/7,6
F5	6/13,6/7,6	1/2,1,2	1/5,6/5,11/5	1/2,1,2	1,1,1	3/7,3/4,3	6/13,6/7,6	3/7,3/4,3	1/2,1,2	6/13,6/7,6
F6	1/7,8/7,15/7	1/3,4/3,7/3	3/5,8/5,13/5	1/3,4/3,7/3	1/3,4/3,7/3	1,1,1	7/15,7/8,7	1/2,1,2	3/7,3/4,3	7/15,7/8,7
F7	1/2,1,2	1/6,7/6,13/6	2/5,7/5,12/5	1/6,7/6,13/6	1/6,7/6,13/6	1/7,8/7,15/7	1,1,1	7/15,7/8,7	6/13,6/7,6	1/2,1,2
F8	1/7,8/7,15/7	1/3,4/3,7/3	3/5,8/5,13/5	1/3,4/3,7/3	1/3,4/3,7/3	1/2,1,2	1/7,8/7,15/7	1,1,1	3/7,3/4,3	7/15,7/8,7
F9	6/13,6/7,6	1/2,1,2	1/5,6/5,11/5	1/2,1,2	1/2,1,2	1/3,4/3,7/3	1/6,7/6,13/6	1/3,4/3,7/3	1,1,1	6/13,6/7,6
F10	1/2,1,2	1/6,7/6,13/6	2/5,7/5,12/5	1/6,7/6,13/6	1/6,7/6,13/6	1/7,8/7,15/7	1/2,1,2	1/7,8/7,15/7	1/6,7/6,13/6	1,1,1
WAOL	0.1070	0.0930	0.0768	0.0923	0.0916	0.1124	0.1068	0.1140	0.0974	0.1087

Although covered aerial conductor is superior to bare conductor with regard to its impact on considerable reduction in fire risk, it often intensifies the identification of HIF as it increases the overall line impedance. In this regard, fuzzy judgments are made on the basis of feeder operating zone, i.e. rural, suburban and urban aerial distribution networks and shown in Table 10.

• Ageing of overhead line: The evidence and logic imply that the other important criterion/attribute, contributing to the frequency of HIF is considered as ageing infrastructure. In this regard, failures resulted from ageing of conductors, cross-arms, insulators and poles increase the percentage of HIF incidents in distribution networks. As a result, a great deal of attention to

IET Gener. Transm. Distrib., 2020, Vol. 14 Iss. 9, pp. 1616-1626 © The Institution of Engineering and Technology 2020 ageing infrastructure as well as cost-related issues [20–22] should be paid when organising the critical aspects of a placement problem. In this study, it is supposed that failure rate increases with age. The associated fuzzy judgments are shown in Table 11.

• *Conductor size:* Undersized overhead conductors are extremely vulnerable to wear and tear during years, and also substantial load growth in feeders can deteriorate the condition. Hence, feeders with undersized conductor either in primary section or in its laterals increase the probability of HIF. The result is shown in Table 12.

Table 12 Evaluation of feeders with respect to CS

CS	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
F1	1,1,1	1/2,1,2	2,3,4	1/2,3/2,5/2	2,3,4	3/7,3/4,3	1/2,3/2,5/2	2,3,4	1/2,3/2,5/2	1/2,3/2,5/2
F2	1/2,1,2	1,1,1	2,3,4	1/2,3/2,5/2	2,3,4	3/7,3/4,3	1/2,3/2,5/2	2,3,4	1/2,3/2,5/2	1/2,3/2,5/2
F3	1/4,1/3,1/2	1/4,1/3,1/2	1,1,1	1/3,1/2,1	1/2,1,2	1/5,1/4,1/3	1/3,1/2,1	1/2,1,2	1/2,1,2	1/3,1/2,1
F4	2/5,2/3,2	2/5,2/3,2	1,2,3	1,1,1	1,2,3	1/3,1/2,1	1/2,1,2	1,2,3	1/2,1,2	1/2,1,2
F5	1/4,1/3,1/2	1/4,1/3,1/2	1/2,1,2	1/3,1/2,1	1,1,1	1/5,1/4,1/3	1/3,1/2,1	1/2,1,2	1/3,1/2,1	1/3,1/2,1
F6	1/3,4/3,7/3	1/3,4/3,7/3	3,4,5	1,2,3	3,4,5	1,1,1	1,2,3	3,4,5	1,2,3	1,2,3
F7	2/5,2/3,2	2/5,2/3,2	1,2,3	1/2,1,2	1,2,3	1/3,1/2,1	1,1,1	1,2,3	1/2,1,2	1/2,1,2
F8	1/4,1/3,1/2	1/4,1/3,1/2	1/2,1,2	1/3,1/2,1	1/2,1,2	1/5,1/4,1/3	1/3,1/2,1	1,1,1	1/3,1/2,1	1/3,1/2,1
F9	2/5,2/3,2	2/5,2/3,2	1/2,1,2	1/2,1,2	1,2,3	1/3,1/2,1	1/2,1,2	1,2,3	1,1,1	1/2,1,2
F10	2/5,2/3,2	2/5,2/3,2	1,2,3	1/2,1,2	1,2,3	1/3,1/2,1	1/2,1,2	1,2,3	1/2,1,2	1,1,1
W _{CS}	0.1432	0.1432	0.0494	0.0958	0.0478	0.1915	0.0950	0.0477	0.0910	0.0954

Table 13	Function values	and CIs related to fuzz	y PCMs constructed above
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W	fval	CI (lambda)
W _{Criteria}	1.0547	0.9981
WPHE	2.234	0.9997
W _{LF}	0.0026782	0.998
WPDF	3.3609	0.9433
W _{FPA}	4.6993	0.9480
W _{AOL}	1.0024	0.9924
W _{CS}	1.9753	0.9567



Fig. 8 Priority of feeders in case of HIFDs installation

The proposed framework based on EPSO is capable of leading to more consistent crisp weights in which ultimately greater CI values are achieved. Therefore, the optimal placement is obtained by ranking the feeders based on preserving their feasibility of solution over relative importance among the criteria and relative preference among feeders by employing the proposed scheme. The calculated results of the optimised values of the objective function and related CI values, presented in (7) and (13), respectively, are illustrated in Table 13.

As can be seen in Table 13, achieved consistencies validate the efficiency of the proposed model. Unlike methods presented in [39–41], local aggregation is eliminated in this approach. Having computed the weight of each criterion with respect to goal and each feeder with respect to each criterion, the final global order of feeders is obtained. The final global order of feeders is achieved according to the following relation:

Feeder Rank =
$$\sum_{j} w_{ij} v_j$$
 (14)

where w_{ij} is the weight of feeder *i* against criterion *j* and v_j is the weight of criterion *j*. As a result, the optimal placement of HIFDs considering the criteria selected in a trade-off between relay

manufacturer and experts in the distribution sector are achieved in a hierarchical manner. Consequently, overall ranking of feeders after evaluation based on the proposed method are depicted in Fig. 8.

Table 14 represents descending order of feeder ranks with the top one indicating the best place for HIFD installation and the preference declines by climbing down the third column. As can be seen, F6 ('Tosseh' feeder) has the maximum preference.

6 Sensitivity analysis

Sensitivity analysis is a fundamental concept for the effective use and implementation of quantitative and qualitative decision models, and also for studying perturbations in optimisation problems [42]. In this paper, a stability index (*L*) is proposed to evaluate the sensitivity analysis of the ranking against uncertainty features, existed in DMs' judgments in PCMs. In other words, *L* is introduced to assess to what extent the results achieved from the proposed model in HIFDs placement are reliable. The reverse of stability index is defined as instability index (L' = 1/L), multiplied by the upper triangular elements above the main diagonal of criteria-to-goal matrix. Subsequently, the lower triangular elements below the main diagonal are divided by *L'*. The value of *L'* decreases gradually from unity until the ranking is changed or CI

Table 14 Placement rank for ten distribution feeders

Feeder code	Weight	Rank
F6	0.1384	1
F9	0.1206	2
F2	0.1063	3
F3	0.1062	4
F10	0.1016	5
F7	0.0987	6
F1	0.0960	7
F4	0.0912	8
F5	0.0867	9
F8	0.0544	10



Fig. 9 *General schema of sensitivity analysis*

0.17453

W6

Table 15 Changes in the weight values associated with							
different criteria							
	L = 1	L = 1.1364					
<i>w</i> ₁	0.25615	0.22925					
<i>w</i> ₂	0.13355	0.12608					
W3	0.20155	0.1956					
W4	0.082952	0.085136					
W5	0.15126	0.1672					

0.19674



Fig. 10 *Global ranking of feeders for the case where* L = 1.1364

IET Gener. Transm. Distrib., 2020, Vol. 14 Iss. 9, pp. 1616-1626 © The Institution of Engineering and Technology 2020 turns into a negative value. If DM makes their judgments in a distinguishable and reliable manner, either global ranking remains unchanged or changes in global ranking occur in alternatives which are close to each other in terms of priority. The general strategy for implementation of sensitivity analysis can be done according to the flowchart seen in Fig. 9.

Therefore, having conducted sensitivity analysis with respect to the main criteria, the value of L, and its impact on the solution of this HIFDs allocation model is obtained.

As can be seen in Table 15, the ranking for L' = 0.88 is changed, i.e. conductor size becomes more important than population on density feeder. Additionally, according to Table 15, past HIF events still has the absolute dominant importance in the case study. However, as can be seen in Fig. 10, when the input data (preference judgments and degrees of fuzziness) are changed into new values, the resultant weight values undergo trivial changes, while the global ranking of the feeders remains unchanged which shows that the proposed model is stable and robust to limited variations of main criteria.

7 Conclusion

In this paper, the HIFDs placement problem in distribution feeders is converted to a fuzzy MCDM model. An improved integrated/ hybrid model is employed to optimally select distribution feeders to be equipped with HIFDs. It has been shown that if only quantitative feature like number of past HIF events as the most dominant criterion is considered, then it is most likely that feeder with the maximum number of recorded HIF becomes the first and foremost priority. However, the proposed model sorts preferences by taking into account both quantitative and qualitative features. Hence, preference ranking notably differs from the expected priorities. The present model simultaneously deals with six main objectives as criteria to optimise the objective function in a fuzzy environment. The obtained results have marked that the fuzzy rankings are reflecting the subjective viewpoint of the related experts. In addition, the validity of the proposed model has been verified by a comparative analysis with existing approaches and a case study. It is also proved that consistency in decision-making process has been significantly improved. Finally, the proposed scheme has shown robustness to variations of the main criteria.

8 References

- Aucoin, B.M., Russell, B.D.: 'Distribution high impedance fault detection utilizing high frequency current components', *IEEE Trans. Power Appar.* Syst., 1982, PAS-101, (6), pp. 1596–1606
- [2] Tengdin, J., Westfall, R., Stephan, K., et al.: 'High impedance fault detection technology', PSRC Working Group D15, Piscataway, NJ, USA, 1996. Available at http://grouper.ieee.org/groups/td/dist/documents/highz.pdf
- [3] Soudi, F., Tomsovic, K.: 'Optimized distribution protection using binary programming', *IEEE Trans. Power Deliv.*, 1998, 13, (1), pp. 218–224
 [4] Wester, C.G.: 'High impedance fault detection on distribution systems,
- Wester, C.G.: 'High impedance fault detection on distribution systems, general electric company', 2008 Available at http://pm.geindustrial.com/faq/ documents/f60/. Accessed 11 September 2009
- [5] Aucoin, B.M., Jones, R.H.: 'High impedance fault detection implementation issues', *IEEE Trans. Power Deliv.*, 1996, 11, (1), pp. 139–148
- [6] Chen, D., Jing, Z., Tan, H.: 'Optimal siting and sizing of used battery energy storage based on accelerating benders decomposition', *IEEE. Access.*, 2019, 7, pp. 42993–43003
- [7] Xu, Y., Liu, C.-C., Schneider, K.P., et al.: 'Placement of remote-controlled switches to enhance distribution system restoration capability', *IEEE Trans. Power Syst.*, 2016, 31, (2), pp. 1139–1150
- [8] Chen, C., Lin, C., Chuang, H., et al.: 'Optimal placement of line switches for distribution automation systems using immune algorithm', *IEEE Trans. Power Syst.*, 2006, 21, (3), pp. 1209–1217
 [9] Bezerra, J.R., Barroso, G.C., Leão, R.P.S., et al.: 'Multiobjective optimization
- [9] Bezerra, J.R., Barroso, G.C., Leão, R.P.S., et al.: 'Multiobjective optimization algorithm for switch placement in radial power distribution networks', *IEEE Trans. Power Deliv.*, 2014, **30**, (2), pp. 545–552
- [10] Izadi, M., Safdarian, A.: 'A MIP model for risk constrained switch placement in distribution networks', *IEEE Trans. Smart Grid*, 2019, **10**, (4), pp. 4543– 4553
- [11] Farajollahi, M., Fotuhi-Firuzabad, M., Safdarian, A.: 'Optimal placement of sectionalizing switch considering switch malfunction probability', *IEEE Trans. Smart Grid*, 2017, **10**, (1), pp. 403–413
- Yu, P., Venkatesh, B., Yazdani, A., et al.: 'Optimal location and sizing of fault current limiters in mesh networks using iterative mixed integer nonlinear programming', *IEEE Trans. Power Syst.*, 2016, **31**, (6), pp. 4776–4783
 Kapourchali, M.H., Sepehry, M., Aravinthan, V.: 'Fault detector and switch
- [13] Kapourchali, M.H., Sepehry, M., Aravinthan, V.: 'Fault detector and switch placement in cyber-enabled power distribution network', *IEEE Trans. Smart Grid*, 2018, 9, (2), pp. 980–992

- [14] Shahsavari, A., Mazhari, S.M., Fereidunian, A., et al.: 'Fault indicator deployment in distribution systems considering available control and Protection devices: a multi-objective formulation approach', *IEEE Trans. Power Syst.*, 2014, **29**, (5), pp. 2359–2369 Acosta, J.S., López, J.C., Rider, M.J.: 'Optimal multi-scenario, multi-objective allocation of fault indicators in electrical distribution systems using
- [15] a mixed-integer linear programming model', IEEE Trans. Smart Grid, 2019, 10, (4), pp. 4508-4519
- Pombo, A.V., Murta-Pina, J., Pires, V.F.: 'A multiobjective placement of [16] switching devices in distribution networks incorporating distributed energy resources', Electr. Power Syst. Res., 2016, 130, (8), pp. 34-45
- Alam, A., Alam, M.N., Pant, V., et al.: 'Placement of protective devices in [17] distribution system considering uncertainties in loads, temporary and permanent failure rates and repair rates', IET Gener. Transm. Distrib., 2018, 12, (7), pp. 1474–1485
- Silva, L.G.W., Pereira, R.A.F., Abbad, J.R., et al.: 'Optimised placement of [18] control and protective devices in electric distribution systems through reactive tabu search algorithm', *Electr. Power Syst. Res.*, 2008, **78**, (3), pp. 372–381 Izadi, M., Safdarian, A., Moeini-Aghtaie, M., *et al.*: 'Optimal placement of
- [19] protective and controlling devices in electric power distribution systems: a MIP model', IEEE Access, 2019, 7, pp. 122827–122837
- Abiri-Jahromi, A., Fotuhi-Firuzabad, M., Parvania, M., et al.: 'Optimized [20] sectionalizing switch placement strategy in distribution systems', *IEEE Trans. Power Deliv.*, 2012, **27**, (1), pp. 362–370 Lei, S., Wang, J., Hou, Y.: 'Remote-controlled switch allocation enabling
- [21] prompt restoration of distribution systems', IEEE Trans. Power Syst., 2018, **33**, (3), pp. 3129–3142
- [22] Izadi, M., Safdarian, A.: 'Financial risk constrained remote controlled switch deployment in distribution networks', IET Gener. Transm. Distrib., 2018, 12, (7), pp. 1547-1553
- Aghaei, J., Baharvandi, A., Rabiee, A., et al.: 'Probabilistic PMU placement [23] in electric power networks: an MILP-based multiobjective model', IEEE Trans. Ind. Inf., 2015, 11, (2), pp. 332-341
- Bernardon, D.P., Sperandio, M., Garcia, V.J., et al.: 'AHP decision-making [24] algorithm to allocate remotely controlled switches in distribution networks' *IEEE Trans. Power Deliv.*, 2011, **26**, (3), pp. 1884–1892 Mardani, A., Jusoh, A., Zavadskas, E.K.: 'Fuzzy multiple criteria decision-
- [25] making techniques and applications-two decades review from 1994 to 2014', Expert Syst. Appl., 2015, 42, (8), pp. 4126-4148
- Ahmed, F., Kilic, K.: 'Fuzzy analytic hierarchy process: a performance analysis of various algorithms', *Fuzzy Sets Syst.*, 2019, **362**, pp. 110–128 Ghaderi, A., Ginn III, H.L., Mohammadpour, H.A.: 'High impedance fault [26]
- [27] detection: a review', Electr. Power Syst. Res., 2017, 143, pp. 376-388

- [28] Kubler, S., Robert, J., Derigent, A., et al.: 'A state-of the-art survey & testbed of fuzzy AHP (FAHP) applications', Expert Syst. Appl., 2016, 65, pp. 398-422
- Mi, X., Wu, X., Tang, M., et al.: 'Hesitant fuzzy linguistic analytic [29] hierarchical process with prioritization, consistency checking, and inconsistency repairing', *IEEE. Access.*, 2019, **7**, pp. 44135–44149
- [30] Chang, D.-Y.: 'Applications of the extent analysis method on fuzzy AHP', Eur. J. Oper. Res., 1996, 95, (3), pp. 649-655
- [31] Jaganathan, S., Erinjeri, J.J., Ker, J.-i.: 'Fuzzy analytic hierarchy process based group decision support system to select and evaluate new manufacturing technologies', Int. J. Adv. Manuf. Technol., 2007, 32, (11-12), pp. 1253-1262
- Javanbarg, M.B., Scawthorn, C., Kiyono, J., et al.: 'Fuzzy AHP-based [32] multicriteriadecision making systems using particle swarm optimization', Expert Syst. Appl., 2012, **39**, (1), pp. 960–966 Saaty, T.L.: 'The analytic hierarchy process: planning, priority setting,
- [33] resources allocation' (McGraw-Hill, New York, NY, 1980)
- Goyal, R.K., Kaushal, S.: 'Deriving crisp and consistent priorities for fuzzy [34] AHP-based multicriteria systems using non-linear constrained optimization', Fuzzy Optim. Decis. Mak., 2018, 17, (2), pp. 195-209
- [35] Zadeh, L.A.: 'The concept of a linguistic variable and its application to approximate reasoning-III¹, *Inf. Sci.*, 1975, **9**, (1), pp. 43–80 Wang, L., Chu, J., Wu, J.: 'Selection of optimum maintenance strategies
- [36] based on a fuzzy analytic hierarchy process', Int. J. Prod. Econ., 2007, 107, (1), pp. 151-163
- Mikhailov, L.: 'Deriving priorities from fuzzy pairwise comparison judgements', *Fuzzy Sets Syst.*, 2003, **134**, (3), pp. 365–385 [37]
- Juneja, M., Nagar, S.K.: 'Particle swarm optimization algorithm and its parameters: a review'. 2016 Int. Conf. Control, Computing, Communication and Materials (ICCCCM), Allahbad, India, 2016, pp. 1–5 [38]
- Dehghanian, P., Fotuhi-Firuzabad, M., Bagheri-Shouraki, S., et al.: 'Critical [39] component identification in reliability centered asset management of power distribution systems via fuzzy AHP', IEEE Syst. J., 2012, 6, (4), pp. 593-602
- Ranga, C., Chandel, A. K., Chandel, R.: 'Condition assessment of power [40] transformers based on multi-attributes using fuzzy logic', IET Gener. Transm. *Distrib.*, 2017, **11**, (8), pp. 983–990 Kumar, S., Tyagi, B., Kumar, V., *et al.*: 'Incremental PMU placement
- [41] considering reliability of power system network using analytical hierarchical process', IET Gener. Transm. Distrib., 2018, 12, (16), pp. 3900-3909
- [42] Cacuci, D.G., Ionescu-Bujor, M., Navon, I.M.: 'Sensitivity and uncertainty analysis' (CRC Press, Taylor & Francis Group, 2003)