ICISE 2020
6th International Conference on Industrial and Systems Engineering
• 9 & 10 Sep. 2020 • Ferdowsi University of Mashhad
ششمین کنفرانس بین المللی مهندسی صنایع و سیستم‌ها
۱۹ و ۲۰ شهریور ۱۳۹۹ • دانشگاه فردوسی مشهد

Adjustable robust profit hub location problem using column and constraint generation algorithm

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Abstract

In this paper, adjustable robust optimization with polyhedral uncertainty set is used to deal with uncertain demands in profit hub location problem. Hub location problem seeks to find best location for establishing hub facilities, as well as allocating demands nodes to them. In the proposed model, location decisions are taken in the first stage without revealing of uncertainty, while allocation decision is taken in the second stage in presence of uncertain demands. Column and constraint generation algorithm is used to solve proposed model more efficiency. The adjustable robust model and column and constraint generation algorithm analyzed using well-known AP data set with different level of conservatism. Computational experiments show the superiority of column and constraint generation algorithm comparison with Benders decomposition algorithm in the number of iterations. Also the number of hub facilities and objective function (profit) are decreased with increasing in uncertainty budget.

Keywords: Adjustable robust, Hub location problem, Column and constraint generation algorithm, Benders decomposition algorithm, Uncertain demands.

1. Introduction

Hub location is an important problem in the location literature and widely used in transportation, telecommunications and other applications. This problem seeks to find the best location for establishing hub facilities and determine the optimal pathway for sending commodities. In the hub location problem, some facilities are considered as a connection point (hubs) between each non-hub nodes. Hence, commodities are transmitted through one or two hubs in this network. This may decrease total transportation costs because of existing of the economy of scale property.

The first model in hub location problem was proposed by [1] that introduced the first quadratic mathematical model for hub location problem. In this model, only one hub is used in each pathway, While in the model proposed by [2] the number of hubs between each origin destination nodes are determined based on hub establishment and routing costs. [3] proposed a two-stage stochastic programming model for multiple allocation hub location problem by considering uncertain demands and transportation cost. [4] introduced robust optimization for multi-objective capacitated p-hub location problem which demands and time required for process of commodity are uncertain. [5] considered a hub location problem with uncertain demands and hub establishment fixed cost, also robust-stochastic model presented to deal with uncertain demands and hub establishment fixed cost. Also they proposed a two-stage stochastic programming model, where demands are defined as uncertain parameters. [6] proposed robust optimization for single and multiple allocation hub location problems with uncertain demands. They introduced a nonlinear programming formulation that is transfer to mixed integer conic quadratic problem. [7] proposed a robust optimization for intermodal multiple allocation p-hub location problem. They used Benders decomposition algorithm for large-scale instances and examine the effects of considering uncertainty in the model. [8] presented robust optimization model for single and multiple allocation hub location problem with uncertain hub establishment fixed cost and capacity of hub facilities. They showed that the costs are increased when uncertainties are not considered in the model. [9] presented robust optimization for hub location problems with uncertain demands, hub establishment cost and inter hub flow discount factor. [10] introduced robust optimization for uncapacitated multiple allocation hub location problem with considering uncertain demands and transportation cost. The proposed model with considering uncertain demands and transportation cost together is solved using branch and cut algorithm. [11] proposed robust optimization for multiple allocation hub location problem with uncertain demands and hub establishment fixed cost. Also, Benders decomposition algorithm and hybrid heuristic approach are used for solving large scale problems.

In this paper, adjustable robust optimization is applied to uncapacitated multiple allocation profit hub location problem which demands are considered as uncertain parameters and captured by intervals. Also column and constraint generation algorithm is used to solve proposed model.

The remainder of this paper is organized as follows: Section 2, introduces the deterministic model of uncapacitated profit hub location problem, section 3 introduces the adjustable robust optimization applied to the proposed model, section 4 presents column and constraint generation algorithm. In section 5, computational experiments are done for analyzing the performance of column and constraint generation algorithm to solve the proposed model, finally section 6 concludes the paper and suggest future studies.

2. Multiple allocation profit hub location problem

In this section, the deterministic model of multiple allocation profit hub location problem is introduced. N and H are the set of nodes and potential hubs, respectively. w_{ij} is the demands that transferred from node $i \in N$ to node $j \in N$ and f_k is the hub establishment cost for potential hub $k \in H$. d_{ij} is the distance or transportation cost between node $i \in N$ and node $j \in N$. c_i denotes the setup cost for serving node $i \in N$, it is assumed that it will be possible to serve commodities originated (or with destination) at $i \in N$ without activating node $i \in N$ as a servicing node. That is, there is no need to incur in the setup cost c_i for serving node $i \in N$ if it becomes a hub. r_{ij} is the revenue that earned by transferring flows from node $i \in N$ to node $j \in N$. χ, α and δ are collection cost per unit, inter hub flow discount factor and transfer cost per unit, respectively. Transportation cost is calculated by $c_{ij}^{kl} = \chi d_{ik} + \alpha d_{kl} + \delta d_{lj}$ equation and for economic of scale property it is assumed that $\alpha \le \chi$ and $\alpha \le \delta$. z_k is a binary variable and is equal to 1 if and only if a hub is established at node $k \in H$. x_{ij}^{kl} is the fraction of flows originated from node $i \in N$ and destined to node $j \in N$ routed through hub $k \in H$ and $l \in H$. s_i is defined as a binary variable and equal to 1 if and only if node i is served (i.e. activated as a non-hub node). y_{kl} is defined as a binary variable and equal to 1 if and only if hub edge (arc $k \in H$ and $l \in H$) is activated.

The mathematical model of multiple allocation profit hub location problem that was proposed by [12] with a little change is as follows:

$$Max \sum_{i\in N}\sum_{j\in N}\sum_{k\in H}\sum_{l\in H} w_{ij}\,(R_{ij} - c_{ij}^{kl})x_{ij}^{kl} - \sum_{k\in H} f_k\, z_k - \sum_{i\in N} c_i\, s_i - \sum_{k\in H}\sum_{l\in H} r_{kl}\, y_{kl} \tag{1}$$

Subjected to:

$$s_k + z_k = 1 \qquad \forall k \in H \tag{2}$$

$$\sum_{k\in H}\sum_{l\in H} x_{ij}^{kl} = 1 \qquad \forall i \in N, j \in N \tag{3}$$

$$y_{kl} \le z_k \qquad \forall k \in H, l \in H, k \neq l \qquad (4)$$

$$\sum_{l \in H} x_{ij}^{kl} + \sum_{l \in H, l \neq k} x_{ij}^{lk} \le z_k \qquad \forall i \in N, j \in N, k \in H \qquad (5)$$

$$x_{ij}^{kl} + x_{ij}^{lk} \le y_{kl} \qquad \forall i \in N, j \in N, k \in H, l \in H \qquad (6)$$

$$z_k, s_i, y_{kl} \in \{0,1\} \qquad (7)$$

$$x_{ij}^{kl} \ge 0 \qquad (8)$$

The first term of the objective function represents the net profit for routing the commodities. The other terms represent the total setup costs of the hubs that are chosen, the non-hub nodes that are selected to be served, and the hub edges that are used, respectively. Constraints (2) represent that a node is selected as a hub or non-hub node. Constraints (3) ensures that demands are fully transmitted. Constraints (4) represent that the arc between two nodes can be selected if and only if two edges selected as hubs. Constraints (5) not allowed direct connection created between non hub nodes. According to the constraints (6), flows can be routed via hubs if and only if the arc between hub nodes is created. Constraints (7) and (8) are the standard integrality and non-negativity constraints.

3. Proposed adjustable robust profit hub location problem

In the previous model, its assumed that all parameters are known in the planning time. While, in the real world some parameters have uncertain nature. Adjustable robust optimization is used to cope with uncertain demands in proposed model. In adjustable robust optimization, decisions are divided into two separate states ([13]). The location decisions are taken in the first stage without revealing of uncertainty and allocation decisions are taken in the second stage in presence of uncertainties. Polyhedral uncertainty set is used in this paper and the level of conservatism is controlled by an uncertainty budget ([14]). Demands is assumed taken a value in interval of $[w_{ij}^n - w_{ij}^d, w_{ij}^n + w_{ij}^d]$. w_{ij}^n and w_{ij}^d are nominal and deviation values of demands, respectively. Γ_w is a parameter that denotes the level of conservatism. The mathematical model of the adjustable robust profit hub location problem with uncertain demands is as follows:

$$\max_{z,s,y} \left(-\sum_{k \in H} f_k\, z_k - \sum_{i \in N} c_i\, s_i - \sum_{k \in H} \sum_{l \in H} r_{kl}\, y_{kl} \right) + \max_{x \in \gamma(z,s,y,w)} \min_{w_{ij} \in W} \left(\sum_{i \in N} \sum_{j \in N} \sum_{k \in H} \sum_{l \in H} w_{ij} \,(R_{ij} - c_{ij}^{kl}) x_{ij}^{kl} \right) \qquad (9)$$

Subjected to:

(2), (4), (7)

Where $\gamma(z, s, y, w) = \{x: (3), (5), (6), (8)\}$. In this model, z_k, s_i and y_{kl} are the first stage decision variables, while second stage decision variable consist of x_{ij}^{kl}. First stage decision variables are maximized according to the worst-case of second stage decision variables.

The objective function of the proposed model is non-linear and have max max-min terms that makes problem hard to solve. Hence a decomposition algorithm should be used to solved proposed model. Column and constraint generation algorithm is applied to solve proposed model.

4. Column and constraint generation algorithm

Column and constrain generation algorithm was proposed by [15]. In this algorithm, model is divided into two separate problems named as master and sub problems. First stage and second stage decision variables exist in master and sub problems, respectively. The sub problem is as follows:

$$\underset{x\in\gamma(z,s,y,w)}{\text{Max}} \quad \underset{w_{ij}\in W}{\text{Min}} \left(\sum_{i\in N}\sum_{j\in N}\sum_{k\in H}\sum_{l\in H} w_{ij}\,(R_{ij}-c_{ij}^{kl})x_{ij}^{kl}\right) \tag{10}$$

Subjected to:

$$\sum_{k\in H}\sum_{l\in H} x_{ij}^{kl}=1 \qquad \forall i\in N, j\in N \tag{11}$$

$$\sum_{l\in H} x_{ij}^{kl}+\sum_{l\in H, l\neq k} x_{ij}^{lk}\le \bar{z}_k \qquad \forall i\in N, j\in N, k\in H \tag{12}$$

$$x_{ij}^{kl}+x_{ij}^{lk}\le \bar{y}_{kl} \qquad \forall i\in N, j\in N, k\in H, l\in H \tag{13}$$

$$x_{ij}^{kl}\ge 0 \tag{14}$$

With duality property, max-min term in the objective function (10) can transform max form to min form. q_{ij}, u_{ijk} and p_{ij}^{kl} are dual multiplier of constraints (11), (12) and (13) respectively. The dual of sub problem with considering uncertain demands interval is as follows:

$$Min \sum_{i\in N}\sum_{j\in N} q_{ij}+\sum_{i\in N}\sum_{j\in N}\sum_{k\in H}\bar{z}_k u_{ij}^{k}+\sum_{i\in N}\sum_{j\in N}\sum_{k\in H}\sum_{l\in H}\bar{y}_{kl}\,p_{ij}^{kl} \tag{15}$$

Subjected to:

$$q_{ij}+u_{ij}^{k}+u_{ij}^{l}+p_{ij}^{kl}+p_{ij}^{lk}\ge(\overline{w}_{ij}-\widehat{w}_{ij}\theta_{ij})(R_{ij}-c_{ij}^{kl}) \qquad \forall i,j,k,l \tag{16}$$

$$q_{ij}+u_{ij}^{k}\ge(\overline{w}_{ij}-\widehat{w}_{ij}\theta_{ij})(R_{ij}-(\chi d_{ik}+\delta d_{kj})) \qquad \forall i,j,k \tag{17}$$

$$\sum_{i\in N}\sum_{j\in N}\theta_{ij}\le\Gamma_w \tag{18}$$

$$0\le\theta_{ij}\le 1 \qquad \forall i,j \tag{19}$$

$$u_{ij}^{k}, p_{ij}^{kl}\ge 0 \tag{20}$$

Master problem with first stage decision variables is as follows:

$$Max\left(-\sum_{k\in H} f_k\, z_k-\sum_{i\in N} c_i\, s_i-\sum_{k\in H}\sum_{l\in H} r_{kl}\, y_{kl}\right)+\eta \tag{21}$$

Subjected to:

$$\eta\le(\overline{w}_{ij}-\widehat{w}_{ij}\theta_{ij}^{iter})(R_{ij}-c_{ij}^{kl})x_{ijkl}^{iter} \qquad \forall iter \tag{22}$$

$$\sum_{k\in H}\sum_{l\in H} x_{ijkl}^{iter} = 1 \qquad \forall i \in N, j \in N, iter \qquad (23)$$

$$\sum_{l\in H} x_{ijkl}^{iter} + \sum_{l\in H, l\neq k} x_{ijlk}^{iter} \leq z_k \qquad \forall i \in N, j \in N, k \in H, iter \qquad (24)$$

$$x_{ijkl}^{iter} + x_{ijlk}^{iter} \leq y_{kl} \qquad \forall i, j \in N, k, l \in H, iter \qquad (25)$$

$$x_{ijkl}^{iter} \geq 0 \qquad \forall i, j \in N, k, l \in H, iter \qquad (26)$$

$$z_k, s_i, y_{kl} \in \{0,1\} \qquad (27)$$

Constraints (22) is the optimality cut that is added in each iteration to the master problem. Also constraints (23) - (26) are added in each iteration to the master problem. In other words, in each iteration new constraints and variables are added to the master problem. Algorithm 1 shows the pseudo code of the column and constraints generation algorithm.

Algorithm 1: Column and constraint generation algorithm

Data: $LB = -\infty, UB = +\infty$

While $UB - LB \leq \varepsilon$

Step 1: Solve master problem

$$LB = \sum_{k\in H} f_k\, \bar{z}_k + \sum_{i\in N} c_i \bar{s}_i + \sum_{k\in H}\sum_{l\in H} r_{kl}\, \bar{y}_{kl} - \eta$$

Step 2: Solve dual of sub problem

$$UB = -\sum_{i\in N}\sum_{j\in N} \bar{q}_{ij} - \sum_{i\in N}\sum_{j\in N}\sum_{k\in H} \bar{z}_k\, \bar{u}_{ij}^{k} - \sum_{i\in N}\sum_{j\in N}\sum_{k\in H}\sum_{l\in H} \bar{y}_{kl}\, \bar{p}_{ij}^{kl} + \sum_{k\in H} f_k\, \bar{z}_k + \sum_{i\in N} c_i \bar{s}_i + \sum_{k\in H}\sum_{l\in H} r_{kl}\, \bar{y}_{kl}$$

Step 3: Add optimality cut and new constraints and variables to the master problem

End while

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5. Computational experiment

In this section, the performance of the proposed model and column and constraint generation algorithm is analyzed. The well-known set of instances such as the AP data set (Australian Post) is used in this paper for analyzing, that used widely in hub location problems. The proposed mathematical model is solved by using GAMS software, and run in an Intel Core i5 with 2.5 GHz CPU and 6 GB of RAM.

Figure 1 and table 1 shows the convergence trend for Benders decomposition and column and constraint generation algorithms, respectively, in a case of uncertainty budget equals to 0.1.

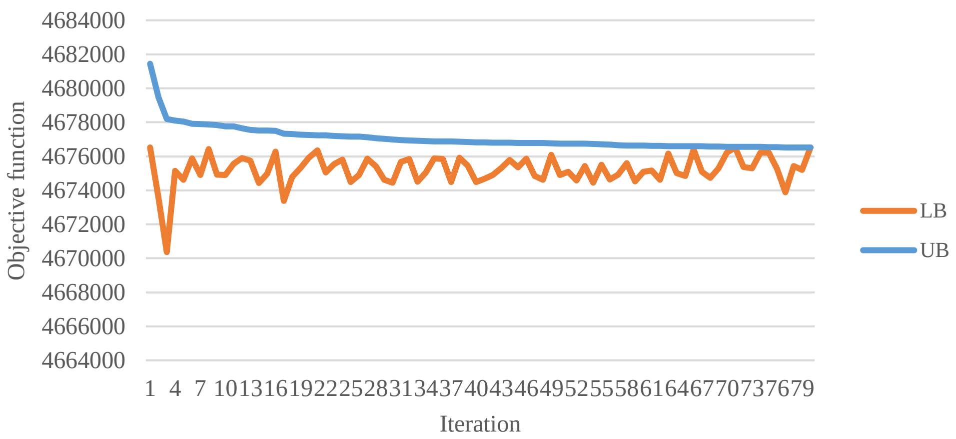

Figure 1. Convergence trend of Benders decomposition algorithm for AP 10-node instance, $\Gamma_w = 0.1$

Comparing figure 1 and table 1 confirms the rapid convergence of the column and constraint generation algorithm to solve the proposed model compared with Benders decomposition algorithm. In other words, column and constraint generation algorithm needs few iterations to converging optimal solution.

Table 1. Convergence of column and constraint generation algorithm for AP 10-node instance, $\Gamma_w = 0.1$

Iteration	LB	UB
1	4676522	7059836
2	4676522	4676522

The impacts of uncertainty budget (Γ_w) on the hub network configuration and its objective function is analyzed and reported in table 2 for AP 10-node instance. In case of high value for uncertainty budget, objective function (profit) decreased with less established hub facilities.

Table 2. Performance of model with different Γ_w for AP 10-node instance

Γ_w	# Iterations	CPU time (s)	OBJ	Hub configuration
0.1	2	2.328	4676520	3, 8, 10
0.2	2	2.296	3583400	1, 9, 10
0.3	2	2.251	2724420	9, 10
0.4	2	2.64	2015190	9, 10
0.5	2	1.97	1434530	9, 10
0.6	2	1.953	1010000	10
0.7	2	1.812	652138	10
0.8	2	2.421	350201	10
0.9	2	1.89	131532	10
1	2	1.204	-29762	10

6. Conclusion

In this paper, adjustable robust optimization is used to deal with uncertain demands in multiple allocation profit hub location problem. The location decisions (hub nodes, non-hub nodes and arcs) are taken in the first stage without revealing of uncertainties and allocation decisions are taken in the second stage in presence of uncertainty. Polyhedral uncertainty set is used in this paper and the level of conservatism is controlled by an uncertainty budget parameter. Column and constraint generation algorithm is used to solve proposed model more efficiently. Computational experiments showed that the column and constraint generation algorithm had less iterations comparing with Benders decomposition algorithm. Also number of hub facilities and objective function are decreased when uncertainty is high. It is interesting for future research to considering hub location problem with more applications. Also mathhuristic two-phase algorithm can used to solve such model in large scale instances.

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