	A	B	C	D	E	F	G	H	I	J	K
3598	3595	Foreign Policy Analysis	FOREIGN POL ANAL-US	1743-8586	562	1.329	1.289	1.542	0.419	0.001320	52.105
3599	3595	Journal of Cultural Economy	J CULT ECON-UK	1753-0350	493	1.329	1.110	1.349	0.200	0.001100	60.144
3600	3597	Journal of the Society for Social Work and Research	J SOC SOC WORK RES	2334-2315	375	1.328	1.148	1.454	0.379	0.000770	62.500
3601	3598	JOURNAL OF INSECT SCIENCE	J INSECT SCI	****-****	2,806	1.325	1.248	1.397	0.280	0.003660	53.960
3602	3599	PLANT SOIL AND ENVIRONMENT	PLANT SOIL ENVIRON	1214-1178	2,337	1.324	1.117	1.724	0.230	0.001620	52.198
3603	3600	JOURNAL OF HYMENOPTERA RESEARCH	J HYMENOPT RES	1070-9428	574	1.322	1.080	1.152	0.333	0.000960	52.970
3604	3601	Quality Engineering	QUAL ENG	0898-2112	935	1.320	1.097	1.377	0.209	0.000940	41.448
3605	3601	RADIOCHIMICA ACTA	RADIOCHIM ACTA	0033-8230	2,763	1.320	1.220	1.113	0.516	0.001480	43.676
3606	3603	BIOTECHNOLOGIE AGRONOMIE SOCIETE ET ENVIRONNEMENT	BIOTECHNOL AGRON SOC	1370-6233	656	1.319	1.319	1.238	0.261	0.000690	33.568
3607	3604	FAMILY RELATIONS	FAM RELAT	0197-6664	3,138	1.317	1.288	1.965	0.439	0.001920	56.177
	3604	JOURNAL OF MATHEMATICAL PHYSICS	J MATH PHYS	0022-2488	17,280	1.317	1.178	1.280	0.268	0.019010	51.818

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A new capability index for non-normal distributions based on linex loss function

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A new capability index for non-normal distributions based on linex loss function

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ABSTRACT
Capability indices are the most common tools in process capability analysis, measuring how much the product meets the costumer expectations. One type of capability indices are loss-based indices that consider the cost of the difference between the product characteristic and its target value. In this work, we apply an asymmetric loss function to construct a new loss-based capability index for non-normal processes allowing a more customized way of considering the costs in capability analysis. The performance of the proposed capability index is studied by simulation results. Three real examples are given to show the utilization of the proposed index.

KEYWORDS
asymmetric loss function; bootstrap; linex function; process capability analysis; process capability index

Introduction

In the capability analysis a process is assessed by certain statistical methods to determine whether it meets a set of specifications or requirements. Suppliers and manufacturers use these methods to assure costumers that their products are of high quality with the least amount of non-conformities. Capability indices are the most widely used tools among all available capability analysis methods. A process capability index measures how much the process fulfills the costumer expectations. Costumer expectations are usually numerical values within which the process is expected to operate.

The earliest process capability index was proposed by Kane (1986) as

$$C_p = \frac{USL - LSL}{6\sigma}, \quad [1.1]$$

where USL and LSL are the upper and lower specification limits respectively, and σ denotes the standard deviation of the process. In [1.1] the denominator shows the actual process capability while the numerator shows the consumer's quality requirements, and can be thought of as indicating the potential of the process to produce conforming product (Anis 2008). If a shift in the mean of the process happens, a large proportion of items might fall out of the specification limits while C_p is still high, meaning that the C_p index only relates the process spread to the specification limits and does not consider the possible shifts of the process mean away from the target value (Anis 2008).

Another capability index was introduced by Kane (1986) as

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\}, \quad [1.2]$$

where μ stands for the process mean and

$$k = \frac{\mu - M}{(USL - LSL)/2}, \quad [1.3]$$

with $M = (LSL + USL)/2$. The index k represents a measure of the distance that the process lies offcenter, and C_{pk} shows the reduction in process capability caused by the lack of centering (Anis 2008).

As can be seen, C_p and C_{pk} indices consider the reduction of variability as a criterion for quality improvement. They are not concerned with the quality loss, which is the consequence of failing to meet customer's requirements. Indeed, a wide range of capability indices concentrates only on determining the capability of processes without taking into account any loss perspective and loss functions. Loss functions are quite versatile, they are utilized in different fields of industrial engineering such as quality engineering, tolerances design and capability analysis (Abdolshah et al. 2011). In order to consider the quality loss, loss-based indices were developed.

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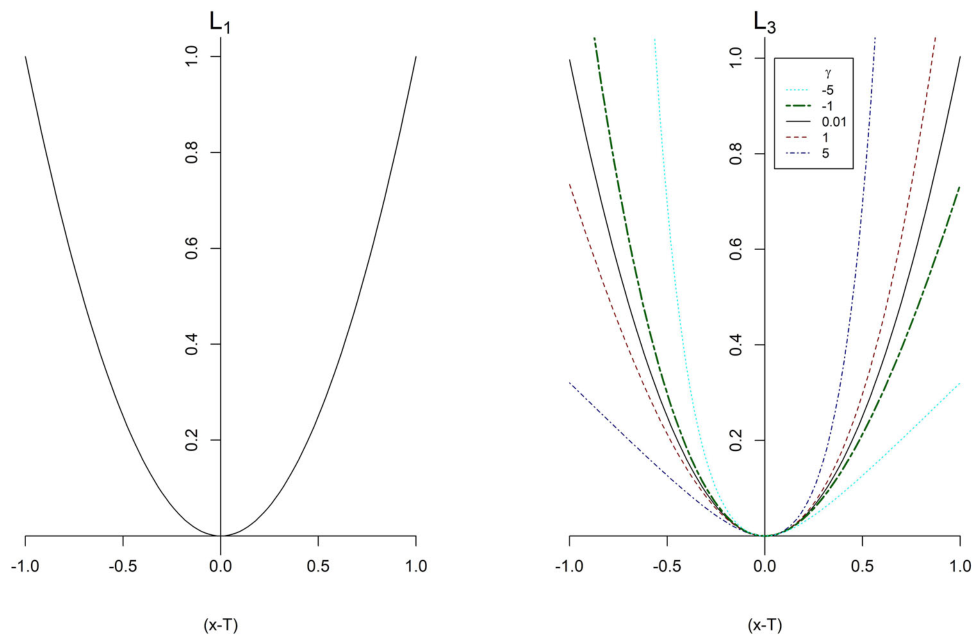

Figure 1. Plots of loss functions L_1 and L_3 for selected values of γ.

In this work, we introduce a new loss-based capability index which permits a more customized way of including costs in comparison to the previous indices available in literature. This index can be applied for non-normal processes. In the literature, so far, all the proposed loss-based indices use symmetric approach for counting the loss. That is, only the difference between the process mean and the costumer expectation is measured as the loss, while the direction of this difference has always been ignored in the measurements (e.g., Johnson (1992), Hsieh and Tong (2006) and Eslamipoor and Hosseini-Nasab (2016)). Some practitioners find it unfair to assign equal costs to the differences only based on their equal values, whereas the differences might have been located in different directions. Our proposed index considers the direction of the location process mean around the costumer expectation in addition to its distance.

The present article is structured as follows. In the following section, one of the most well-known loss-based capability indices is recalled and its features are discussed. The motivation for introducing the new index is also stated. Then, the new index and its estimator are proposed. Next, the performance of the proposed capability index is assessed by simulation studies. In addition, three real data examples are presented for illustrating the application of the proposed index. In the final section, the conclusions offered and suggestions for future studies are made.

C_{pm} and C_{pmk} indices

One of the important purposes in quality management is quantifying and controlling the losses stemmed from noncompliance with customer specifications. In order to evaluate these losses, C_{pm} index was introduced by Hsiang (1985) which is also known as Taguchi index. This index assumes the actual performance of process characteristics in relation to the target value and the specification limits as

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \quad [2.1]$$

where T is the target value of the quality characteristic. As seen, C_{pk} is derived from C_p by modifying the numerator, whereas C_{pm} is obtained by modifying the denominator (Anis 2008). By combining the two modifications, a new index C_{pmk} is obtained and was first presented by Pearn, Kotz, and Johnson (1992) as

$$C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}. \quad [2.2]$$

C_{pm} and C_{pmk} employ quadratic loss function

$$L_1(x) = (x - T)^2, \quad [2.3]$$

to represent the quality loss. Figure 1 shows the functional behavior of L_1 with respect to $(x - T)$, where x is the process location. L_1 rises with $|x - T|$, meaning that loss value increases as x goes farther from T. This is the basic feature of a loss function that considers the distance between the quality measure and the target value. However, the quadratic loss function only focuses on the distance and is not sensitive to its sign. Say $|x-T|=a>0$ for some x. Then the loss value is $L_1(a)$ regardless $x-T=a$ or $x-T=-a$. This could be a fault for measuring the capability of processes in which the location of x around T is important as well as its distance. For example in the average weight of antiviral medicine tablets, $x-T=-a$ could mean just not enough dose to cure the infection, whereas $x-T=a$ could mean overdosing with sever ramifications for the patient. So here we need a class of loss functions that are sensitive to the location of x around T. These functions are known as asymmetric loss functions. Some of mostly used asymmetric loss functions are double linear (LinLin), double quadratic (QuadQuad) and linear-exponential (linex) functions. Consider a loss function of the general form as

$$L_2(x-T)=2[\beta+(1-2\beta)I(x-T<0)]|x-T|^a, \qquad [2.4]$$

where $\beta\in(0,1)$ is a parameter representing the degree of asymmetry and $I(\cdot)$ is a unit indicator if $x-T<0$, and zero otherwise. For $a=1$ the function is the LinLin loss and for $a=2$ it is the QuadQuad loss. For $\beta<0.5$ L_2 assigns higher cost to negative differences ($x-T<0$); for $\beta>0.5$ higher cost is instead given to positive differences, and for $\beta=0.5$ L_2 penalizes symmetrically positive and negative differences. See Granger (1999) for a review on [2.4].

Linex loss function was originally introduced by Varian (1975) and Zellner (1986) as

$$L_3(x-T)=2\frac{e^{\gamma(x-T)}-\gamma(x-T)-1}{\gamma^2}, \qquad [2.5]$$

where γ is a constant. The parameter γ determines the degree of asymmetry. In Figure 1, L_3 is plotted versus γ. When $\gamma=1,5$ in this graph, L_3 is quite asymmetric with positive $(x-T)$ values being more costly than negatives. More generally when $\gamma>0$, L_3 rises almost exponentially for $x-T>0$ and almost linearly for $x-T<0$. The opposite happens for $\gamma<0$ (e.g., $\gamma=-1,-5$ in Figure 1); L_3 increases almost linearly for $x-T>0$ and almost exponentially for $x-T<0$. For small values of γ (e.g., $\gamma=.01$ in Figure 1), L_3 is almost symmetric and presents a very close curve to the quadratic loss function L_1. Indeed, we have

$$\lim_{\gamma\to 0}L_3(x-T)=(x-T)^2,$$

by twice the application of L'Hopital's rule (See Parsian and Kirmani (2002) for a discussion about key properties of (2.5)).

All the asymmetric loss functions introduced here have been widely used in different areas (e.g., economy, econometrics, forecast theory, etc) and can be considered as good choices in processes that the location of the quality measure is important. In this work, we select linex. In this function, the value of γ specifies whether positive or negative values of $(x-T)$ should cost more and if so, what will be the rate of increase in cost. Select a value for γ as follows: In case of having a process in which positive values of $(x-T)$ are more costly than negative values, select a positive γ. Choose a negative γ in case negative values of $(x-T)$ are more costly than positive values. In any case, choose greater values for $|\gamma|$ to include higher rates of increase in costs in the process. As seen in [2.5], γ can take any value over real numbers except zero. However we used values over $[-10,10]-\{0\}$ for the present work.

A new S'_{pmk} index

In this section we introduce a new capability index based on linex loss function. Before that, we recall S_{pmk} index suggested by Chen and Ding (2001) as

$$S_{pmk}=\frac{\Phi^{-1}\left(\frac{1+F(USL)-F(LSL)}{2}\right)}{3\sqrt{1+\left(\frac{\mu-T}{\sigma}\right)^2}}=\frac{\Phi^{-1}\left(1-\frac{P}{2}\right)}{3\sqrt{1+\left(\frac{\mu-T}{\sigma}\right)^2}}, \qquad [3.1]$$

where Φ^{-1} is the inverse of the standard normal distribution function, $F(x)$ is the cumulative distribution function (c.d.f) of the process and P is the percentage of nonconforming items. S_{pmk} is used for non-normal distributions and considers departures from the target value using the quadratic loss function. An estimator $\hat{S}_{pmk}$ of S_{pmk} is determined by replacing μ and σ^2 with their unbiased estimators $\bar{X}$ and S^2.

The new index, S'_{pmk}, requires the same assumptions and is defined as

$$S'_{pmk}=\frac{\Phi^{-1}\left(\frac{1+F(USL)-F(LSL)}{2}\right)}{3\sqrt{1+\frac{2}{\sigma^2}\frac{e^{\gamma(\mu-T)}-\gamma(\mu-T)-1}{\gamma^2}}}, \qquad [3.2]$$

where again an estimator $\hat{S}'_{pmk}$ of S'_{pmk} is determined by replacing μ and σ^2 with their unbiased estimators as

$$\hat{S}'_{pmk}=\frac{\Phi^{-1}\left(\frac{1+F(USL)-F(LSL)}{2}\right)}{3\sqrt{1+\frac{2}{S^2}\frac{e^{\gamma(\bar{X}-T)}-\gamma(\bar{X}-T)-1}{\gamma^2}}}. \qquad [3.3]$$

Note that the process must be under control and stable in order to use capability indices.

To construct confidence intervals, we will use the nonparametric bootstrap method of Efron (1982) as

Table 1. Simulation results for normal distribution with $\sigma = 1$, $T = 0$, $LSL = -5$ and $USL = 5$.

			n = 25			n = 50			n = 100			n = 150		
S_{pmk}, μ	γ	S'_{pmk}	$\hat{S}'_{pmk}$	CR(%)	MSE	$\hat{S}'_{pmk}$	CR(%)	MSE	$\hat{S}'_{pmk}$	CR(%)	MSE	$\hat{S}'_{pmk}$	CR(%)	MSE
0.4778	0.01	0.4791	0.4828	94.2	0.0047	0.4796	94.3	0.0023	0.4797	95.2	0.0011	0.4793	94.8	0.0007
−2	0.5	0.5380	0.5414	93.5	0.0048	0.5384	94.0	0.0023	0.5385	95.0	0.0011	0.5381	95.1	0.0007
	1	0.5908	0.5940	93.6	0.0049	0.5910	93.9	0.0024	0.5911	94.5	0.0011	0.5908	94.9	0.0008
	5	0.8146	0.8203	91.1	0.0077	0.8160	93.4	0.0037	0.8153	94.7	0.0018	0.8149	95.5	0.0012
	10	0.9095	0.9192	90.8	0.0112	0.9127	92.7	0.0053	0.9109	94.7	0.0025	0.9103	95.3	0.0016
0.9808	0.01	0.9816	0.9930	93.6	0.0262	0.9844	93.8	0.0125	0.9832	94.7	0.0059	0.9822	95.1	0.0039
−1	0.5	1.0191	1.0309	92.8	0.0246	1.0222	93.7	0.0117	1.0208	94.5	0.0055	1.0199	95.1	0.0036
	1	1.0528	1.0650	92.1	0.0235	1.0562	93.6	0.0111	1.0545	94.3	0.0052	1.0536	95.5	0.0034
	5	1.2070	1.2240	91.1	0.0242	1.2131	92.8	0.0113	1.2098	94.6	0.0053	1.2087	95.4	0.0034
	10	1.2769	1.2981	90.3	0.0285	1.2851	92.7	0.0132	1.2806	94.6	0.0063	1.2792	95.2	0.0041
0.9808	0.01	0.9800	0.9971	92.6	0.0264	0.9910	94.8	0.0127	0.9845	94.4	0.0064	0.9837	94.4	0.0043
1	0.5	0.9373	0.9548	93.2	0.0287	0.9488	95.0	0.0139	0.9421	94.8	0.0070	0.9412	94.6	0.0047
	1	0.8886	0.9069	93.2	0.0316	0.9008	94.5	0.0154	0.8937	94.8	0.0078	0.8927	95.2	0.0053
	5	0.3940	0.4431	93.8	0.0518	0.4237	94.8	0.0249	0.4079	95.2	0.0119	0.4041	95.2	0.0080
	10	0.0660	0.1113	93.9	0.0200	0.0890	94.8	0.0059	0.0766	95.3	0.0021	0.0733	94.9	0.0013
0.4778	0.01	0.4765	0.4829	93.7	0.0047	0.4810	95.0	0.0023	0.4784	95.1	0.0012	0.4780	95.4	0.0008
2	0.5	0.4113	0.4183	93.8	0.0046	0.4161	94.6	0.0023	0.4133	95.3	0.0011	0.4129	95.3	0.0008
	1	0.3417	0.3492	93.9	0.0045	0.3467	94.7	0.0022	0.3438	95.3	0.0011	0.3433	94.9	0.0007
	5	0.0255	0.0299	94.2	0.0003	0.0280	94.7	0.0001	0.0266	95.3	0.0001	0.0263	94.6	0.0000
	10	0.0003	0.0006	93.9	0.0000	0.0005	94.7	0.0000	0.0004	95.4	0.0000	0.0004	94.8	0.0000

the distribution of $\hat{S}'_{pmk}$ is unknown. This method is based on a resampling procedure for estimation purposes. The percentile method is explained in next and will be used for bootstrap confidence interval calculations.

Suppose that the quantity θ is of interest and there is an estimator $\hat{\theta}$ which is used for estimation of θ. We have a sample of observed data (main sample). B bootstrap samples are drawn from the main sample and the estimations $\hat{\theta}^*_1, \ldots, \hat{\theta}^*_B$ are calculated for each. A $100(1-\alpha)\%$ percentile confidence interval is obtained as

$$\left[\hat{\theta}^*_{\frac{\alpha}{2}\times B}, \hat{\theta}^*_{\left(1-\frac{\alpha}{2}\right)\times B}\right].$$

Hence a percentile bootstrap confidence interval for S'_{pmk} is calculated as

$$\left[\hat{S}'^*_{pmk,\frac{\alpha}{2}\times B}, \hat{S}'^*_{pmk,\left(1-\frac{\alpha}{2}\right)\times B}\right]. \quad [3.4]$$

Note that a process will be interpreted inadequate if $S'_{pmk} < 1$ and capable if $S'_{pmk} \geq 1$.

Performance evaluation

The performance of the suggested index is evaluated using simulation in R. The simulations will be performed for three probability distributions: normal, Poisson and Weibull with specified parameters. For each distribution, random samples of sizes 25, 50, 100 and 150 are generated. The number of simulation runs is set to 1000 and $\hat{S}'_{pmk}$ is calculated by replacing μ and σ^2 with mean and the variance of the generated samples in [3.2]. For comparison results, only positive values 0.01, 0.5, 1, 5 and 10 are considered for γ, because negative values will give similar results but just in a reverse manner. Finally, a 95 percent confidence interval for the coverage ratio is computed. As described in Alevizakos, Koukouvinos, and Castagliola (2019), a 95 percent lower limit of the stated nominal value for the coverage rate is obtained as $\left(0.95 - 1.96\sqrt{0.05 \times 0.95/1000} \times 100\%\right) = 93.64\%$. In the tables, the average of $\hat{S}'_{pmk}$ over the 1000 runs as well as the average mean of quadratic error (MSE) of estimations are reported.

The simulation results for the normal distribution are presented in Table 1. Although S'_{pmk} index is designed for non-normal distributions, it is worth to study its performance under the normal distribution. Data are simulated for $\mu = -2, -1, 1, 2$ and $\sigma = 1$. The target value T is set to 0 with $LSL = -5$ and $USL = 5$. The table is partitioned into four panels for different n values. The trend within each panel is similar to other panels, so we concentrate on the panel with $n = 25$ as an example. For negative values of $(\mu - T)$ i.e., $\mu = -2, -1$, S'_{pmk} increases as γ increases, while for positive $(\mu - T)$ i.e., $\mu = 1, 2$, the trend goes into reverse. Now let γ be fixed and μ change. For instance, let $\gamma = 1$. S'_{pmk} decreases from 0.5908 for $\mu = -2$ to 0.3417 for $\mu = 2$; while S_{pmk} remains the same 0.4778. Indeed, both indices behaved as expected: S_{pmk} considers an equal capability for the process when the mean locates at the same distance from T. Whereas S'_{pmk} detects the process less capable when the mean causes a positive $(x - T)$, even if it locates at the equal distance from T taken

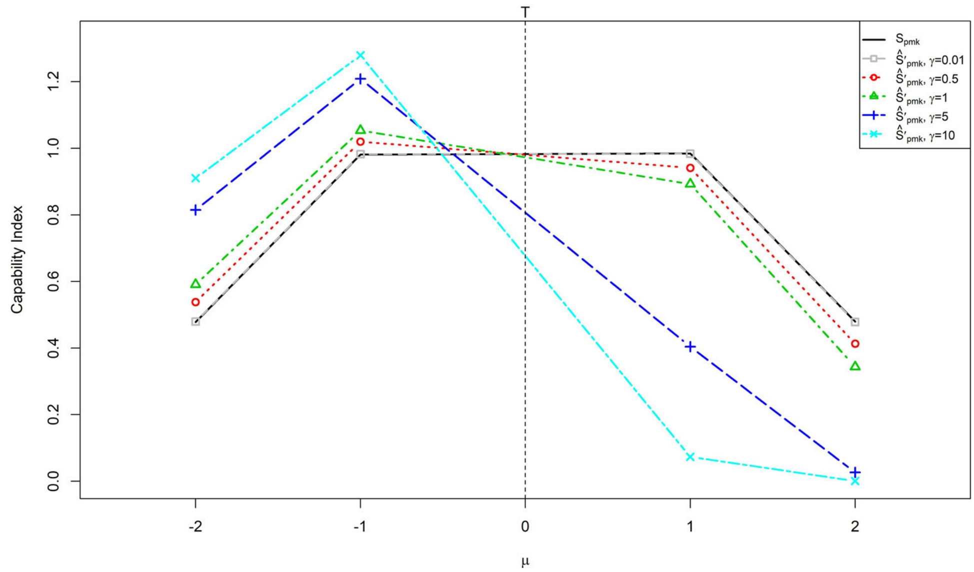

Figure 2. Simulation results for normal distribution when $n = 150$.

by the mean with a negative $(x - T)$. The same happens when μ goes from −1 to 1, but more strictly. When $\mu = -1, S'_{pmk} = 1.0528 > 1$ determines the process as capable, while $S_{pmk} = 0.9808$ shows incapability. For $\mu = 1$, $S'_{pmk} = 0.8886 < 1$ interprets the process incapable, so does S_{pmk}. Furthermore, *MSE* values are indicative of the good performance of $\hat{S}'_{pmk}$ in estimating S'_{pmk}, as all the values are less than 0.01 and decrease as the sample size n gets larger. The coverage ratio (*CR*) is greater than or close to the nominal value (93.64%) for all cases and improves with increasing n.

Figure 2 is presented to offer a better understanding of these results. In this figure, the indices estimations are plotted versus μ for $n = 150$. The curve of $\hat{S}'_{pmk}$ is very similar to S_{pmk} when $\gamma = 0.01$, but gets more and more different as γ increases. The values at the left of T are greater than the corresponding values at the right and with the same distance from T.

Next simulations are run for Poisson distribution as a discrete distribution. Mean values for Poisson data generation are as $\mu = 6, 7, 9, 10$. The same procedure of the normal simulations is used but with $LSL = 0$, $USL = 16$ and $T = 8$. The results are summarized in Table 2. Again only the panel with $n = 25$ is discussed as the trend is the same within all the four panels. The proposed index behaves as expected in this case, too. S'_{pmk} values for $\mu = 9, 10$ are smaller than the values for $\mu = 6, 7$, because they cause positive $(x - \mu)$. For $\mu = 6, 7, S'_{pmk}$ increases as γ gets larger, while for $\mu = 9, 10$ it decreases as γ rises. However, there is one main difference between these results and those obtained for normal distribution. S_{pmk} does not remain the same for equal $|x - T|$'s, because variance of the underlying distribution affects the indices in addition to the mean. In the normal distribution simulations, mean and variance were unrelated and μ changed while σ was fixed, whereas for Poisson distribution $\mu = \sigma$. The performance of $\hat{S}'_{pmk}$ is somehow weak for some cases in $n = 25$, 50 regarding coverage ratios and *MSE* values, but they give more satisfying results as n increases.

The last case of simulations is performed for Weibull distribution with $\mu = 2, 3, 5, 6$ and shape parameter 2, $LSL = 0$, $USL = 8$ and $T = 4$. The results are presented in Table 3. The simulation procedure is the same as the previous ones and the two indices behavior is the same as Poisson one. All *MSE* values are less than 0.01 except some for $n = 25$, but coverage ratios are all good regarding the nominal 93.64%, except when $\mu = 2, 3$ for $n = 25$. As the final point, take a closer look at results for $\mu = 2$ in Table 3. For $\gamma = 0.01, 0.5, 1$ the process is diagnosed incapable because $S'_{pmk} < 1$; whereas for $\gamma = 5, 10, S'_{pmk} > 1$ and the process is capable. This is a testament to the important role of γ in determining how much practitioners, in addition to the distance magnitude, care about the direction of the distance between μ and T. As a guideline on how to choose γ, Figure 1 (right panel) can be used. Practitioners can see how changing γ affects the loss function and choose which case is more suitable for their process.

Real data examples

In this section, three real data examples are used to demonstrate the application of our proposed index, one for a discrete and the other two for continuous processes.

Table 2. Simulation results for Poisson distribution with $T = 8$, $LSL = 0$ and $USL = 16$.

			n = 25			n = 50			n = 100			n = 150		
S_{pmk}, μ	γ	S'_{pmk}	$\hat{S}'_{pmk}$	CR(%)	MSE	$\hat{S}'_{pmk}$	CR(%)	MSE	$\hat{S}'_{pmk}$	CR(%)	MSE	$\hat{S}'_{pmk}$	CR(%)	MSE
0.9690	0.01	0.9703	0.9559	89.5	0.0058	0.9643	91.7	0.0027	0.9676	93.5	0.0013	0.9693	94.0	0.0009
6	0.5	1.0246	1.0115	88.7	0.0034	1.0193	92.7	0.0014	1.0224	94.3	0.0007	1.0237	94.9	0.0004
	1	1.0655	1.0536	88.3	0.0023	1.0607	92.5	0.0009	1.0637	94.4	0.0004	1.0645	94.6	0.0003
	5	1.1820	1.1755	93.0	0.0029	1.1793	94.9	0.0015	1.1815	94.7	0.0007	1.1813	95.0	0.0005
	10	1.2131	1.2091	94.1	0.0040	1.2114	94.8	0.0021	1.2131	95.1	0.0010	1.2126	95.0	0.0007
1.0297	0.01	1.0300	1.0080	78.2	0.0019	1.0189	86.4	0.0006	1.0247	89.4	0.0002	1.0266	91.2	0.0001
7	0.5	1.0394	1.0226	82.5	0.0014	1.0310	89.1	0.0004	1.0356	91.4	0.0002	1.0369	93.3	0.0001
	1	1.0472	1.0341	87.8	0.0014	1.0408	91.5	0.0005	1.0445	93.2	0.0002	1.0453	95.0	0.0001
	5	1.0765	1.0722	93.7	0.0032	1.0744	93.9	0.0017	1.0762	94.8	0.0008	1.0758	94.7	0.0005
	10	1.0870	1.0840	93.3	0.0045	1.0855	93.6	0.0023	1.0871	94.8	0.0011	1.0865	94.8	0.0007
0.8030	0.01	0.8029	0.7944	93.3	0.0111	0.7984	94.4	0.0060	0.8019	94.9	0.0030	0.8013	94.8	0.0020
9	0.5	0.7955	0.7829	93.3	0.0137	0.7887	94.4	0.0074	0.7934	95.0	0.0037	0.7931	94.7	0.0024
	1	0.7860	0.7673	93.1	0.0179	0.7756	94.4	0.0095	0.7822	95.0	0.0047	0.7823	94.7	0.0031
	5	0.5623	0.5265	93.1	0.0975	0.5361	94.2	0.0694	0.5514	95.0	0.0437	0.5497	94.7	0.0325
	10	0.1198	0.3035	92.7	0.1607	0.2605	94.1	0.1116	0.2189	94.7	0.0664	0.1893	94.7	0.0440
0.6229	0.01	0.6223	0.6174	92.9	0.0112	0.6231	94.6	0.0057	0.6237	95.6	0.0029	0.6217	95.7	0.0019
10	0.5	0.5873	0.5811	93.0	0.0151	0.5880	94.6	0.0079	0.5888	95.6	0.0041	0.5866	95.7	0.0026
	1	0.5378	0.5314	93.2	0.0213	0.5391	94.6	0.0115	0.5398	95.6	0.0060	0.5371	95.7	0.0039
	5	0.0554	0.1218	93.3	0.0335	0.0953	94.0	0.0139	0.0759	95.0	0.0045	0.0668	95.7	0.0022
	10	0.0007	0.0227	93.2	0.0095	0.0071	93.8	0.0009	0.0025	94.9	0.0000	0.0016	95.3	0.0000

Table 3 Simulation results for Weibull distribution with shape parameter 2, $T = 4$, $LSL = 0$ and $USL = 8$.

			n = 25			n = 50			n = 100			n = 150		
S_{pmk}, μ	γ	S'_{pmk}	$\hat{S}'_{pmk}$	CR(%)	MSE	$\hat{S}'_{pmk}$	CR(%)	MSE	$\hat{S}'_{pmk}$	CR(%)	MSE	$\hat{S}'_{pmk}$	CR(%)	MSE
0.7165	0.01	0.7183	0.7556	90.6	0.0117	0.7387	91.3	0.0058	0.7284	93.7	0.0024	0.7239	95.2	0.0015
2	0.5	0.8048	0.8462	90.5	0.0138	0.8274	91.7	0.0069	0.8158	93.2	0.0028	0.8110	95.2	0.0018
	1	0.8816	0.9270	90.1	0.0166	0.9064	91.5	0.0083	0.8935	93.7	0.0034	0.8883	94.9	0.0021
	5	1.2008	1.2688	89.7	0.0432	1.2385	91.3	0.0221	1.2180	93.5	0.0093	1.2107	93.0	0.0058
	10	1.3322	1.4148	89.4	0.0667	1.3782	91.7	0.0342	1.3530	93.0	0.0146	1.3443	93.6	0.0090
0.8145	0.01	0.8149	0.8461	90.8	0.0148	0.8274	92.8	0.0056	0.8205	95.0	0.0024	0.8177	94.9	0.0015
3	0.5	0.8325	0.8683	88.8	0.0160	0.8471	92.2	0.0060	0.8390	94.7	0.0025	0.8360	93.5	0.0016
	1	0.8475	0.8873	87.8	0.0175	0.8640	91.5	0.0065	0.8548	94.5	0.0027	0.8516	93.3	0.0017
	5	0.9086	0.9641	85.2	0.0288	0.9321	90.8	0.0105	0.9187	92.3	0.0046	0.9147	93.0	0.0028
	10	0.9325	0.9941	85.7	0.0354	0.9586	90.1	0.0127	0.9437	92.6	0.0056	0.9393	93.7	0.0035
0.4666	0.01	0.4665	0.4842	91.7	0.0092	0.4743	93.4	0.0041	0.4694	93.7	0.0020	0.4688	94.7	0.0013
5	0.5	0.4611	0.4772	91.9	0.0101	0.4680	93.5	0.0046	0.4635	94.0	0.0023	0.4631	94.8	0.0015
	1	0.4541	0.4680	92.0	0.0116	0.4598	93.5	0.0054	0.4558	94.1	0.0027	0.4558	95.1	0.0017
	5	0.3059	0.3299	92.8	0.0393	0.3180	94.3	0.0245	0.3097	94.6	0.0147	0.3107	94.6	0.0104
	10	0.0618	0.1872	93.0	0.0641	0.1452	94.4	0.0356	0.1088	94.7	0.0168	0.0958	94.6	0.0097
0.3250	0.01	0.3247	0.3364	92.8	0.0057	0.3302	94.1	0.0026	0.3267	94.6	0.0013	0.3265	94.6	0.0008
6	0.5	0.3063	0.3183	92.6	0.0069	0.3120	94.5	0.0033	0.3083	94.8	0.0016	0.3082	94.7	0.0010
	1	0.2802	0.2932	93.1	0.0087	0.2865	94.7	0.0043	0.2824	94.9	0.0021	0.2823	94.8	0.0014
	5	0.0287	0.0746	92.9	0.0132	0.0518	94.7	0.0044	0.0391	94.9	0.0013	0.0356	94.7	0.0006
	10	0.0004	0.0153	93.0	0.0037	0.0043	94.6	0.0006	0.0013	94.8	0.0000	0.0009	94.7	0.0000

Table 4. Data for example 1.

Sample Number	1	2	3	4	5	6	7	8	9	10
Number of Nonconforming Parts	10	15	31	18	24	12	23	15	8	8

Example 1

For the first example, we use data by Montgomery (2012). The data is shown in Table 4, where the number of nonconforming plastic parts in an injection molding process is collected.

We consider LSL, USL and T as 0, 30 and 15 respectively. The number of nonconforming parts that exceed T are more costly for the costumer, so we set $\gamma = 5$. Assuming that the process follows a Poisson distribution, we have $F(USL) = 0.9992$ and $F(LSL) = 0.0000$, where $\bar{x} = 16.4000$ and $s^2 = 58.0444$. By substituting these values in [3.3] we have

$$\hat{S}'_{pmk} = \frac{\Phi^{-1}\left(\frac{1+0.9992-0.0000}{2}\right)}{3\sqrt{1+\frac{2}{58.0444}\cdot\frac{e^{5(16.4-15)}-5(16.4-15)-1}{5^2}}} = 0.7042. \quad [5.1]$$

Table 5. Data for example 2.

59.984	59.981	59.981	60.003	59.982	60.005	60.004	59.983	59.981	59.980
60.000	59.998	59.982	59.983	59.981	59.982	59.999	60.001	59.982	59.988
59.995	59.998	59.982	59.983	59.981	59.994	60.002	59.988	59.980	59.982
59.982	59.983	59.981	59.986	59.987	60.001	59.982	60.003	60.001	59.984
59.985	59.979	59.987	59.990	59.998	59.984	59.989	59.999	59.985	60.003
60.004	60.001	60.000	59.982	59.981	59.984	59.998	59.983	59.999	59.987
59.991	59.992	59.992	59.983	59.981	59.996	59.997	60.000	60.000	59.991
60.002	60.001	59.990	59.987	59.982	60.006	59.981	59.982	59.984	59.985
60.003	60.004	59.992	59.991	59.986	59.992	59.991	59.981	59.998	59.985
60.001	59.980	59.993	59.984	59.981	59.984	59.988	59.999	60.000	60.001

As seen, $\hat{S}'_{pmk} < 1$, so the process of injection molding is not capable. However $\hat{S}_{pmk} = 1.0951 > 1$ determines the process as capable. The reason of disagreement between the two indices here could be interpreted as following. S_{pmk} only considers the difference between the process mean and the target, but does not find it large enough to interpret the process as incapable. Whereas S'_{pmk} also considers the direction of the difference and detect it deviant enough to describe the process as incapable. To choose which index to use, it is up to the researcher to decide whether or not the direction of difference is important enough to reject the capability of the process, despite the amount of difference is not as considerable.

Example 2

In this example, we use data given in Chen and Ding (2001). The data include diameters of the inner rings of bearings produced by a manufacturer (Table 5). Let d denote the diameter. The specifications of d are: $T = 60$ mm, $LSL = 59.981$ mm and $USL = 60.004$ mm. If d is outside of the specification limits, the bearing should be sent to repair. The repair method depends on whether d is below LSL or above USL. If it is below LSL, then the inner ring is thicker than required. The excess thickness will be simply reduced by some lathing. If d is above USL, a more complicated method should be applied to adjust the thickness. First, a piece of a certain material is welded to the inner ring, and then it is lathed to get the required thickness. The latter repair method costs more, so γ is set to 1.

As stated in Chen and Ding (2001), normality of data is not accepted. Thus $\hat{F}(LSL) = 4/100$ and $\hat{F}(USL) = 98/100$, where $\hat{F}$ is the empirical distribution function. Substituting these values in addition to $\bar{x} = 59.9903$ and $s^2 = 0.00007$ in [3.3], we have

$$\hat{S}'_{pmk} = \frac{\Phi^{-1}\left(\frac{1+0.98-0.04}{2}\right)}{3\sqrt{1 + \frac{2}{0.00007}\left(e^{(59.9903-60)} - (59.9903-60) - 1\right)}} = 0.4096. \quad [5.2]$$

Chen and Ding (2001) used S_{pmk} index for capability measurement and obtained $\hat{S}_{pmk} = 0.4092$. Although $\bar{x}$ and T are close, both of the indices specify the process as incapable. This could be because of the rather large sample deviation, s; because both of the indices are sensitive to the deviation. Beside, there is a small difference between $\hat{S}_{pmk}$ and $\hat{S}'_{pmk}$. Maybe the reason is the small difference between $\bar{x}$ and T here. Indeed no matter of what type, loss functions make more influence as the quality measure goes farther from the target value. Hence the difference between the two indices' performance appears more as the quality measure takes more distance from the target value. This was already approved in the simulation results.

Example 3

For this example, we use the data by Chopra et al. (2012). They studied the tablet production process for acyclovir 300 mg which is an antiviral medication prescribed to treat certain virus infections. The data was collected about various characteristics of the produced tablets. We consider the average weight of the tablets with specifications $T = 300$ mg, $LSL = 294$ mg and $USL = 306$ mg. The data is available in Chopra et al. (2012) with $\bar{x} = 299.822$ and $s^2 = 1.266$ calculated. The data follow normal distribution as stated in Chopra et al. (2012), so $F(LSL) = 0.000$ and $F(USL) = 1.000$. We set $\gamma = 10$, and we have

$$\hat{S}'_{pmk} = \frac{\Phi^{-1}\left(\frac{1+1.000-0.000}{2}\right)}{3\sqrt{1 + \frac{2}{1.266}\,\frac{e^{10(299.822-300)} - 0.7(299.822-300) - 1}{10^2}}} = 1.7448. \quad [5.3]$$

The process is capable as $\hat{S}'_{pmk} > 1$. The same is determined by $\hat{S}_{pmk} = 1.7363$. It seems that the close value of $\bar{x}$ to T and the rather small s make both of the indices determine the process as capable.

Conclusions

In this article a new index S'_{pmk} is introduced for capability analysis of non-normal processes. It is a loss-based capability index, considering the loss costs caused by the difference of the process mean μ from the target value T. S'_{pmk} has a new feature comparing to the former loss-based capability indices. It is not only sensitive to the distance between μ and T, but is also sensitive to the location of μ around T. This is possible by the asymmetry parameter γ. Setting γ to a desired value provides two options. First, it determines whether smaller or greater values of μ with respect to T cause more costs. Second, it specifies the increase rate of costs based on the distance between μ and T. According to its properties, S'_{pmk} is recommended to be used in complicated processes in which more customization of loss costs is required.

S'_{pmk} is constructed based on an asymmetric loss function linex. There are also some other asymmetric loss functions available (e.g., LinLin and QuadQuad loss functions) that might have interesting properties in application to capability indices. This can be investigated in further studies with comparison to S'_{pmk} performance in the present work. In addition, S'_{pmk} is defined by concerning S_{pmk} as the main idea. S_{pmk} is designed for non-normal processes, but many other capability indices are designed for more specified conditions. So another idea for further studies is to design new indices with asymmetric loss functions based on other types of loss-based capability indices e.g., L_e (Johnson 1992), PCI_θ (Hsieh and Tong 2006) and $C_p(u,v)(STB)$ (Eslamipoor and Hosseini-Nasab 2016). In this work we used S'_{pmk} only in capability analysis, but it can be applied to other fields of quality control such as process monitoring, acceptance sampling, etc.

Conflict of interest

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