



## The Optimal Time for Preventive Replacement of Solar Energy Device Which Consisting of Number (n) of Tubes

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### Abstract

The subject of this article is about the lifetime of solar devices consisting of number (n) of tubes. The solar energy devices, when they reach an aging stage, the tubes start to fail, the maintenance costs increase, and the production of the device decreases.

The costs of solar energy device maintenance companies increase as the device gets old. Therefore, maintenance companies are looking for the age to replace the device with a new device. In this article, we searched for the best age to replace device (in aging stage) with a new device.

By relying on strategy of balance the costs of tubes failure in unexpected time, we have determined the optimum time for preventive replacement of solar energy device. Then we studied the factors that influence the optimum time for preventive replacement of solar energy device.

**Keywords:** Preventive replacement, Evacuated-tube, Solar energy devices, Weibull distribution, Normal distribution.

### 1. Introduction

When devices get older, devices begin to deteriorating. Maintenance companies begin to maintain a device and make plans for this maintenance. Repeated failure is very expensive and sometimes it is possible to reach the total maintenance costs more than the price of the new device. One of the maintenance policies when device failures increase, is the preventive replacement of a device with a new device.

One of the solar energy devices is device consisting of number (n) of tubes. And when the device gets bigger, the tubes begin to fail. Each tube fails, we replace it with a new one. And with the aging of the device, the costs of replacing the tubes increase after

they fail, so one solution is to preventive replacement for device with a new device.

(Cox [1] and Barlow et al. [12]) have investigated the renewal theory, and Ross [6], carried out an investigation in stochastic processes. (Barlow et al. [11] and Blanchard et al. [14]) investigated preventive maintenance. (Bergman [13]) investigated the theory of replacement. (Nakagawa and Osaki [3]) has investigated the damage model, but (Esary et al. [2], Savits [4] and Gotlieb [5]) carried out an investigation in shock model.

(Qian et al. [7], Nakagawa and Ito [8], Satow and Osaki [9] and Sheu et al. [10]) investigated the optimum time of a preventive replacement to a system that was shocked. (Bahrami et al. [15])

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investigated the constant interval replacement model for preventive replacement. (Kuckelkom et al. [16]) has achieved in the method of manufacturing a solar energy device consisting of number of tubes.

We benefited from a strategy of balancing the costs of tubes failure in unexpected times, then we determined the time at which its expected cost per unit time would be minimal, by this we have set the optimal time for preventive replacement of solar energy device. We took advantage of weibull distribution and normal distribution in the numerical example. Then we studied the factors that influence the optimum time for preventive replacement of solar energy device.

## 2. Evacuated-tube collector

An evacuated-tube collector is device consisting of a number ( $n$ ) of similar tubes. An evacuated-tube collector are used to produce electricity, heat, light, hot water, and heating and cooling operations. As the system gets older, these tubes start to fail. When the tube fails, we replace it with a new tube. As the system of evacuated-tube collector ages, the failure of the tubes increases, and the cost of replacing them increases, and production of these systems becomes expensive. So aim is to find the best age in which to implement a preventive replacement of a system.

In Figure 1, we see a solar energy device made of tubes placed on the roof of a house to provide hot water.



Figure 1. System of evacuated-tube collector

### 2.1. Stages of the life of the device

The life of the part in the system begins with a decrease in the failure rate which is called the (DFR) phase, then the fixed failure rate stage called (CFR) comes and when the part starts in the system to erode, the failure rate increases, we call this stage (IFR). The last stage in the life of the machine is stage of deteriorate and erode, which costs of maintenance and production become high. In figure 2, see tubes failure rate, over time

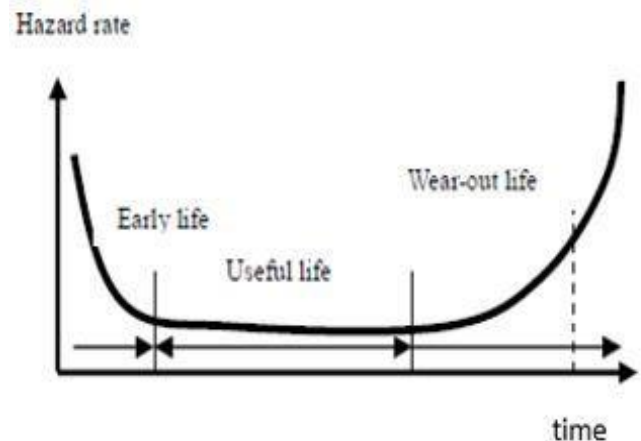


Figure 2. Curve of life of the device

One of the solutions that can be adopted in the final stage of device degradation is the policy of preventive replacement. When we reach the end of device life, failure costs and production costs are high. Therefore, choosing the best time to preventive replacement the device is the solution to eliminate the high costs of failure.

### 2.2. Costs

The solar energy device consists of number ( $n$ ) of tubes. When every tube fails, we will replace it with tube another one. As for when the device is at the end of its life, we replace solar energy device preventively. So there are two costs during the life of the device, the cost of replace the tube after it fails and the cost of preventive replacement for solar energy device. The costs are as follows:

$C_{RT}$  : The cost of replace the tube after it fails,

$C_{PR}$  : The cost of preventive replacement for solar energy device.

2.3. Model

The solar energy system is replaced preventively when it ages, this is because the tubes increases their failure and the cost of their replacement increases. The planned life of solar energy system replacement is  $T$ . So,  $T$  is the time of preventive replacement for solar energy device.

The life cycle of the device is from zero to the age in which the preventive replacement of the device is carried out. So, the life cycle of the device is  $(0, T)$ . During the life cycle of the machine, the tubes fail and we replace them. It is assumed that  $X$  is the time of the tube failed. We replace the tube in the event of a failure of the tube before the time of the preventive replacement of the device. And with distribution  $P(X \leq T) = F(T)$ .

2.4. Expected cost per unit time

To calculate the best time to preventive replacement for device, let's count Expected cost per unit time.

Expected cost per unit time  $C(T)$  is equal to

$$\frac{\text{Expected cost of cycle}}{\text{Time of cycle}}$$

The expected cost of cycle is the costs that we place on the device until the implementation of the preventive replacement of the device. Its fixed cost is the cost of a preventive replacement of a device that will be implemented at the end of device life cycle, Then, we add, the cost of replacing each tube after its failure multiplied by the probability of its failure. The device contains ( $n$ ) of the similar tubes so it became expected cost of cycle equal to:

$$C_{PR} + nC_{RT}P(X \leq T) = C_{PR} + nC_{RT}F(T)$$

As for the life cycle time of the device, it is the time planned  $T$  to preventive replacement for device.

So, expected cost per unit time  $C(T)$  is equal to

$$C(T) = \frac{C_{PR} + nC_{RT}F(T)}{T} \tag{1}$$

The expected cost per unit time will help us know the movement of the expected costs of the device as time progresses.

2.5. The optimal time to perform a preventive replacement of a device

Our aim is to find out when the expected cost per unit time is minimal, we implement preventive replacement for a device when its expected cost per unit time is minimal.

To get the minimum expected cost per unit time we need to follow the next step,

$$\frac{dC(T)}{dT} = 0 \quad \text{Note: } f(T) = \frac{dF(T)}{dT}$$

$$\frac{nC_{RT}f(T)T - C_{PR} - nC_{RT}F(T)}{T^2} = 0$$

$$nC_{RT}f(T)T - C_{PR} - nC_{RT}F(T) = 0$$

$$nC_{RT}f(T)T = C_{PR} + nC_{RT}F(T)$$

$$T = \frac{C_{PR} + nC_{RT}F(T)}{nC_{RT}f(T)}$$

$$T = \frac{\frac{C_{PR}}{nC_{RT}} + F(T)}{f(T)} \tag{2}$$

$T^*$  :is the optimal time for preventive replacement for device.

To obtain  $T^*$ , we have to solve the equation (2).

2.6. Numerical example

A factory for manufacturing solar energy systems. One of the devices that it manufactures is solar energy devices that consist of number ( $n$ ) of tubes. This factory performs maintenance policies for the devices, when the tube fails, replace it with a new tube. As the device ages, the tubes failure increases, its production decreases, and costs become high, so the solution is to preventive replacement for device.

2.6.1. Application

After observing the tubes in the devices, it became clear that the average time of the tube failure is 5 years. The solar energy device consists of a number  $n = 40$  of tubes, cost replace each tube after it fails

is  $C_{RT}=25\$$ . The cost of preventive replacement of device when it is in the old age stage is  $C_{PR}=500\$$ .

2.6.2. Normal distribution

We took first that the failure of the tubes occurs to the normal distribution, with  $\mu = 5$  years and  $\sigma = 6$  months. So,

$$F(t) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{t - \mu}{\sigma \sqrt{2}} \right) \right] \quad \text{With}$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$\text{And } f(t) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{t - \mu}{\sigma} \right)^2}$$

The aim of this article is to determine the optimal time for a preventive replacement of the solar energy device, for calculi the optimal time, we put all the values into the equation (2). After using equation (2), is obtained  $T^* = 4.25$  years. That is, when **4.25** years (4 years and 3 months) have passed, we have to preventive replacement of device.

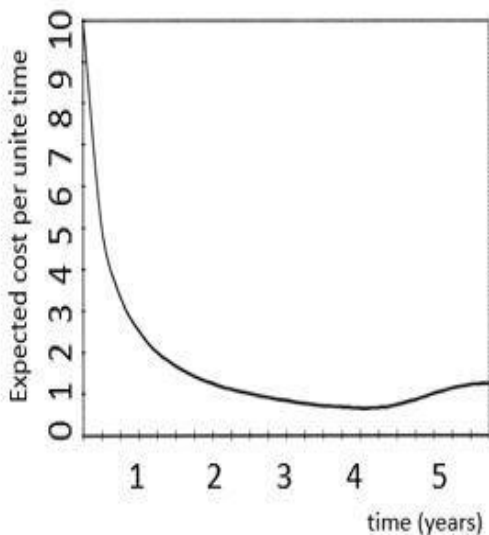


Figure 3. Diagram between expected cost per unit time and time

This means that if 4 years and 3 months have passed for solar energy system and we have not replaced it preventively with another device, tubes failure will

increase rapidly, and maintenance costs will be significant. So the best solution is when 4 years and 3 months have passed we implement a preventive replacement for the device.

2.6.3. Weibull distribution

Because the failure of the tubes increases as the age of the solar energy device ages, the device becomes in its aging stage at the end of its life. The weibull distribution with  $K$  (shape parameter) and  $\lambda$  (scale parameter), which is used in the case of old age of device.

So,

$$F(t) = 1 - e^{-\left(\frac{t}{\lambda}\right)^K} \quad \text{And}$$

$$f(t) = \frac{K}{\lambda} \left(\frac{t}{\lambda}\right)^{K-1} e^{-\left(\frac{t}{\lambda}\right)^K}$$

Tube failure time distribution is weibull distribution with scale parameter  $\lambda = 5$  years and shape parameter  $K = 1.7$ .

After using equation (2), is obtained  $T^* = 4.7$  That is, when **4.7 years** have passed, we have to preventive replacement of device.

3. Economic Results & Discussion

In the table 1 we change average time of tube failure, for see its effect on the time of preventive replacement of a device.

Table 1. The optimal time of preventive replacement for device when average time of failure changes.

$\lambda$	$T^*$
$\lambda = 5$ years	4.7 years
$\lambda = 6$ years	5.69 yers
$\lambda = 7$ years	6.63 years
$\lambda = 8$ years	7.58 years

Note when  $\lambda$  increases, the optimal time of preventive replacement for device increases. This means that the better the industrial specifications for the tubes, the average time of tube failure increases, which increases the optimal time of preventive replacement for device.

In table 2 we increased the cost of replacing the tube after it failed.

Table 2. The optimal time of preventive replacement for device when the cost of replacing the tube changes

$C_{RT}$	$T^*$
$C_{RT}=25\$$	4.7 years
$C_{RT}=35\$$	3.7 years
$C_{RT}=45\$$	3.1 years
$C_{RT}=50\$$	2.88 years

We notice when it becomes more expensive to replace the tube after it fails, the optimum time for the preventive replacement of device is decreases.

In Table 3 we increased the cost of the preventive replacement of device.

Table 3. The optimal time of preventive replacement for device when the cost of the preventive replacement for device changes

$C_{PR}$	$T^*$
$C_{PR}=400\$$	4 years
$C_{PR}=425\$$	4.2 years
$C_{PR}=450\$$	4.38years
$C_{PR}=500\$$	4.7 years

We observe when the cost of preventive replacement for device increases, the optimum time for the preventive replacement of device increases.

#### 4. Conclusions

In this article, we studied a solar energy device consisting of number (n) of tubes. When a device gets old, the tubes failure increases and the cost of replacing tubes increases. So we choose the best lifespan to preventive replacement of device.

By putting all the values in equation (2) we can get the optimum time for the preventive replacement of device. Preventive replacement for a device that would rid hardware maintenance companies of the costs of tubes failure in the aging phase of the device.

We concluded from the numerical example that some variables affect the optimum time for the preventive replacement of device. The industrial specifications for device is better (i.e. the greater the mean failure time of the tube), the optimal time of preventive replacement for device increases. But when the cost of replace the tube after failure increases, the optimal time of preventive replacement for device decreases. Also when the cost of preventive replacement for device increases, the optimal time of preventive replacement for device increases.

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#### Nomenclature

$C_{RT}$	The cost of replace the tube after it fails
$C_{PR}$	the cost of preventive replacement
$C(T)$	Expected cost per unit time
$K$	Shape parameter for weibull distribution
$T^*$	Optimal time for preventive replacement
$\lambda$	Scale parameter for weibull distribution

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