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Systematic study on α-decay half-lives: a new dependency of effective sharp radius on α-decay energy

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Abstract Within the framework of the proximity formalism, we present a systematic study to analyze the effects of the α-decay energy through the effective sharp radius parameter on the α-decay half-lives of 227 nuclei in the range $61 \leq Z \leq 99$. Wentzel-Kramers-Brillouin (WKB) calculations with the proximity potential Zhang 2013 are carried out to obtain the theoretical values of the α-decay half-lives. In this work, we introduce a new Q_α-dependent (QD) form of the effective sharp radius which significantly reduces the standard deviation of estimated half-lives using the Zhang 2013 model in comparison with the corresponding experimental data in our selected mass range. We evaluate the validity of this simple formula using the Geiger-Nuttall (G-N) plots and semi-empirical formulas. The modified form of the Zhang 2013 model is also found to work well in α-decay studies of superheavy nuclei (SHN) with $Z = 117 - 120$. Our results reveal that the calculated half-lives for the use of new proposed form of the effective sharp radius in the proximity potential can reproduce the closed-shell effects at neutron magic number $N = 126$ and $N = 184$.

1 Introduction

Alpha-decay is the nuclear decay process whereby the parent nucleus emits an alpha particle. Experimentally, new elements can be identified by the observation of α-decay chain from unknown parent nucleus to a known nuclide [1–8]. In this situation, it can be said that α-decay of nuclei is one of the efficient frameworks to identify new chemical elements or new nuclides. Also, for many years the α-decay process from heavy and superheavy nuclei is considered as an important tool for investigating the nuclear structure such as the ground-state half-life, the nuclear incompressibility, the nuclear spin and parity, the shell effects, and the nuclear interaction [9–15]. Theoretically, α-radioactivity can be treated as a simple quantum tunneling phenomenon through a potential barrier between α cluster and the daughter nucleus. Hence, one can use the Wentzel-Kramers-Brillouin (WKB) approximation method to estimate the penetration probability of the emitted α particle. It is well known that the selection of an appropriate model to determine the interacting potential between α-particle and daughter nucleus plays an important role in calculating the α-decay half-lives within the framework of the WKB method. Up to now, various successful theoretical models, both phenomenologically and microscopically, towards the description of the α radioactivity process have been developed such as the cluster model [16–18] the liquid-drop-model [19–22], the density dependent M3Y effective interaction [13,14,23,24], the unified model for α decay [25,26], and the generalized liquid-drop-model [27–30]. Another trustworthy theoretical formalism for estimating the α+core interaction potential is that adopted by Blocki *et al.* and is based on the proximity-force theorem [31]. It is well known that the nuclear proximity potential can be described as the product of two factors; one is a geometrical factor depending upon the mean curvature of the interacting surfaces and the other is a universal function depending upon the separation distance. Note should be taken that, this function is independent of the masses of colliding nuclei. Various versions of nuclear proximity potentials are available in the literature [32–34]. With the passage of time, many attempts have been performed to systematically evaluate the validity of the different versions of proximity potentials in reproducing the α-decay half lives of the heavy and superheavy nuclei [35–38]. The main objective of these studies is to select a suitable nuclear potential for α-decay studies. Analyzing the role of various physical effects such as the

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temperature of parent nucleus in the α-particle and heavier-cluster decays of radioactive nuclei within the framework of the proximity formalism has also received much attention in recent years [39–41]. It would be desirable to know that the temperature dependence can be applied to the various parts of this formalism like the radius parameter R_i, surface thickness parameter b and nuclear surface tension coefficient γ. For example, Daei-Ataollah et al. [39] introduced a modified temperature-dependent (TD) surface energy coefficient in the proximity potential based on the thermal properties of liquids and hot nuclei. By including the proposed temperature dependence in the proximity potential model denoted as Dutt 2011, they analyzed the thermal effects of 344 parent nuclei on the α-decay half-lives. It is found that the TD form of the theoretical model that they used to compute the nuclear part of the potential is in better agreement with experimental data of the α-decay half-lives than the temperature-independent one. Additionally, the obtained results confirm the closed-shell effects around $N = 126$. In recent years, several theoretical attempts [42,43] have also been made for analyzing the thermal effects on the α- and cluster-decay processes using the TD form of the effective sharp radius proposed by Guet and coworkers [44] based on the semi-classical extended Thomas-Fermi density vibrational method in calculating the fission barriers of the nuclei ^{208}Pb and ^{240}Pu. The proposed form allows us to incorporate the temperature dependence in the effective radii of α-particle and daughter nucleus as follows,

$$R_{1(2)}(T) = R_{1(2)}(T = 0)(1 + 0.0007T^2), \qquad (1)$$

where the T-IND form of $R_{1(2)}(T = 0)$ can be calculated by using the existing equations in the various versions of the proximity potential formalisms. Besides, the temperature T (in MeV) is correlated to the kinetic energy of the emitted α particle $E_{\text{kin}} = (A_d/A_p)Q_\alpha$ and the α radioactivity energy Q_α through a semi-empirical statistical relation [45,46]. During recent years, a rich knowledge has been provided over the influence of the temperature dependence of the effective radius R_i on the studies of α-decay process of heavy and superheavy nuclei [42,43,47].

In 2013, Ni and co-workers [48] performed the first attempt to study the nuclear charge radii of heavy and superheavy nuclei from the experimental α-decay data. They introduced a simple empirical formula for the nuclear radius as a function of the logarithm of α emission half-lives and the released energy Q_α of the emitted α particle. However, researches on the extraction of the radii of heavy and superheavy nuclei from the characteristics of α-decay process are rare. Hence, in the present work, we attempt to present a new procedure for introducing a clear Q_α dependence of the effective sharp radius in the proximity formalism using the precise measurements of the α-decay half-lives. In fact, this procedure considers the dependence of the effective sharp radius on the α-decay energy as a modification term in the geometric configuration of the dinuclear system (involving the α particle and the residual core nucleus) and thus in the calculations of the α-nucleus potential. To this purpose, we perform a systematic study over a large number of α-decay processes involving 227 parent nuclei with $Z = 61 - 99$. For calculating the effective α-core nuclear interactions, we use the static nucleus-nucleus potential introduced by Zhang et al. in Ref. [49]. We note that the theoretical values of the α-decay half-lives for different parent nuclei are calculated in the framework of the semiclassical WKB approximation. The prime motivations behind the present attempt can be summarized as follows. (i) We would like to introduce a new Q_α dependence of the effective sharp radius $R_i(Q_\alpha)$ by minimizing the root-mean-square (rms) deviations $\sigma = \sqrt{\sum_{i=1}^{n}(\log_{10}(T_{1/2}^{calc(i)}/T_{1/2}^{exp(i)}))^2/n}$, where n represents the number of parent nuclei considered in the fitting procedure, between the logarithmic values of the theoretical and experimental half-lives for the presently studied α-decay processes. (ii) The precision of the proposed formula must be tested for α-decay half-lives. In order to access this aim, we use it for Zhang 2013 proximity potential to calculate the theoretical values of $\log_{10}T_{1/2}$ in comparison with the corresponding experimental data and also some semi-empirical formulas available in the literature. (iii) The validity of the new analytical formula $R_i(Q_\alpha)$ will be examined using the Geiger-Nuttall α-decay law. Additionally, it will be interesting to see whether the proximity potential Zhang 2013 supplemented with this formula can reproduce the shell closure effects on α transitions. (iv) In the present study, we are interested in estimating the α-decay half-lives of superheavy nuclei with a proton number of $117 \leq Z \leq 120$ and a neutron number of $162 \leq N \leq 200$ by using the modified version of the Zhang 2013 potential.

This article is organized in the following way. Section 2 gives the relevant details of the theoretical frameworks used to calculate the interaction potential and α-decay half-life. Section 3 is devoted to the calculation results and the discussion. We summarized the main conclusions of the present study in Sect. 4.

2 Theoretical framework

It is well known that the interaction potential between two reacting nuclei is an essential ingredient that can reflect the basic characteristics of a theoretical model. In theories of α radioactivity in heavy and superheavy elements, the α-nucleus potential can be determined through either a phenomenological or a microscopic approach. Note that this potential is the sum of three parts; the attractive short range nuclear potential V_N, the repulsive Coulomb potential V_C, and the centrifugal parts V_l due to angular momentum,

$$V_{\text{tot}}(r) = V_N(r) + V_C(r) + V_l(r) = V_N(r) + \frac{Z_1 Z_2 e^2}{r} + \frac{\hbar^2 l(l+1)}{2\mu r^2}, \quad (2)$$

where r defines the separation distance between the centers of mass of the α-particle (with a charge of Z_1) and daughter nucleus (with a charge of Z_2). Further, in this relation $\mu = \frac{m_1 m_2}{m_1+m_2}$ and l are the reduced mass of the α-daughter nucleus system and the orbital angular momentum carried away by the emitted α-particle, respectively.

2.1 Nuclear interaction potential

In the present study, the nuclear part of the α+daughter potential is obtained using a new version of the proximity potential introduced by Zhang et al. [49]. According to this model, the nuclear contribution of the interaction potential between two colliding nuclei can be written as,

$$V_N(r) = 4\pi b\gamma \overline{R}\Phi\left(\frac{r - R_1 - R_2}{b}\right) \text{ MeV}, \quad (3)$$

where the mean radius of the nuclear reaction system $\overline{R}$ can be calculated by,

$$\overline{R} = \frac{R_1 R_2}{R_1 + R_2} \text{fm}. \quad (4)$$

Here, R_1 and R_2 are the radii of α particle and daughter nucleus, respectively. They have the following form,

$$R_{1(2)} = 1.28 A_{1(2)}^{1/3} - 0.76 + 0.8 A_{1(2)}^{-1/3} \text{fm}. \quad (5)$$

In Eq. (3), γ and $\Phi(\frac{r-R_1-R_2}{b})$ are the nuclear surface tension coefficient and the universal function, respectively. They can be calculated by Eqs. (4) and (17) of Ref. [49]. Further, the parameter b represents the width of the nuclear surface and is taken to be 1 fm.

2.2 Alpha-decay half-life

In the present study, the α-decay half-life $T_{1/2}$ for the selected parent nucleus can be calculated by using the following relation,

$$T_{1/2} = \frac{h \ln 2}{2 E_\upsilon P_0 P_\alpha} \quad (6)$$

where P_0 is the α preformation factor, which was obtained as 0.43, 0.35 and 0.18 for even-even, odd-A and odd-odd parent nuclei, respectively [53]. In addition, the zero-point vibration energy E_υ can be obtained by the following laws,

$$\begin{aligned} &\text{even}(Z)\text{-even}(N) \text{ parent nuclei} : E_\upsilon = 0.1045 Q_\alpha \\ &\text{odd}(Z)\text{-even}(N) \text{ parent nuclei} : E_\upsilon = 0.0962 Q_\alpha \\ &\text{even}(Z)\text{-odd}(N) \text{ parent nuclei} : E_\upsilon = 0.0907 Q_\alpha \\ &\text{odd}(Z)\text{-odd}(N) \text{ parent nuclei} : E_\upsilon = 0.0767 Q_\alpha. \end{aligned} \quad (7)$$

In above formulas, Q_α is the released energy of the emitted α-particle. It is well known that the value of the α-radioactivity energy must be positive for the α-decay to be possible. Accordingly, the decay energy Q_α can be calculated using the mass excess values as follows

$$Q_\alpha = \Delta M_p - (\Delta M_\alpha + \Delta M_d), \quad (8)$$

where ΔM_p, ΔM_d and ΔM_α are the mass excesses of the parent, daughter and alpha nuclei, respectively. Note that the experimental data of mass excess are taken from the latest evaluated atomic mass table AME2016 [54,55]. Within the framework of the WKB approximation, one can calculate the penetration probability P_α through the Coulomb barrier between the α-particle and the daughter nucleus as,

$$P_\alpha = \exp\left[-\frac{2}{\hbar}\int_{R_a}^{R_b} \sqrt{2\mu(V_{\text{tot}}(r) - Q_\alpha)}dr\right] \quad (9)$$

where R_a and R_b refer to two physical turning points determined from the following condition,

$$V_{\text{tot}}(R_a) = Q_\alpha = V_{\text{tot}}(R_b). \quad (10)$$

3 Results and discussion

It is the purpose of the present study to systematically analyze the effect of the Q_α-dependence of the effective sharp radius on the α-decay half-lives of 227 α emitters with $Z = 61-99$. In Fig. 1, we display the variation of the calculated values of Q_α as a function of neutron number for isotopes with $Z = 84 - 91$, including $^{187-218}$Po, $^{197-215}$At, $^{193-214}$Rn, $^{199-216}$Fr, $^{202-224}$Ra, $^{207-225}$Ac, $^{210-230}$Th and $^{212-231}$Pa elements. From this figure it can be seen that for each of the selected elements within the range of $Z = 84 - 91$, the Q_α-values almost linearly decrease with increasing the neutron number in the vicinity of $N = 126$. A sudden drop can be seen at $N = 126$ which is due to the neutron closed shell effect. Another point to note in Fig. 1 is that the shell effects become stronger when the proton number approaches from $Z = 91$ to $Z = 84$ and in fact comes close to the proton magic number $Z = 82$. As pointed before, we use the proximity potential proposed by Zhang and co-workers in 2013 to calculate the nuclear potential between the emitted α particle

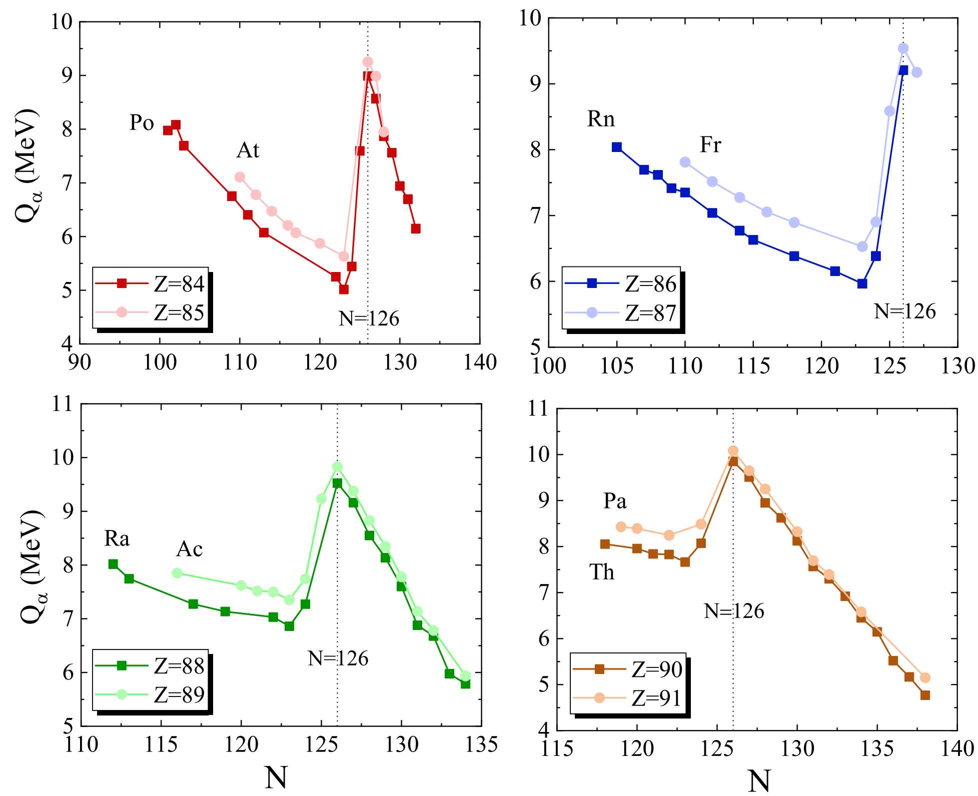

Fig. 1 Variation of the calculated values of the α-decay energy Q_α as a function of the neutron number of the daughter nuclei for different isotopes with $Z = 84 - 91$

and the daughter nucleus. The proximity model denoted as Zhang 2013 is one of the latest versions of the proximity formalism introduced by analyzing different α-decay process of natural α-emitters [49]. Notice that, in the present study, the Q_α-dependence of the proximity formalism is taken into account through an arbitrary function $f(Q_\alpha)$ applied to the radius parameters $R_{1(2)}$, Eq. (5), as follows

$$R_i(Q_\alpha) = R_i \times f(Q_\alpha). \tag{11}$$

We use this dependence in the proximity potential Zhang 2013 and then minimize the rms deviation σ between logarithmic values of α-radioactivity half-lives of theoretical and experimental data to estimate the $f(Q_\alpha)$ function for our selected mass range. We display in Fig. 2 the extracted values of the $R_i(Q_\alpha)/R_i$ ratio as a function of the α-decay energies. It is clearly seen that the function $f(Q_\alpha)$ follows a linear decreasing trend with increasing the Q_α-values from $Q_\alpha = 0.940$ to 10.084 MeV. The observed linear behavior can be parameterized using the following relation,

$$\frac{R_i(Q_\alpha)}{R_i} = f(Q_\alpha) = 0.96207 - 0.00376 Q_\alpha. \tag{12}$$

Since the accuracy of the obtained formula must be carefully verified, so we investigate its validity for identifying the shell closure effects. To reach this goal, in Fig. 3, we plot the variation of the mean radius $\overline{R}$ of the α-nucleus system with

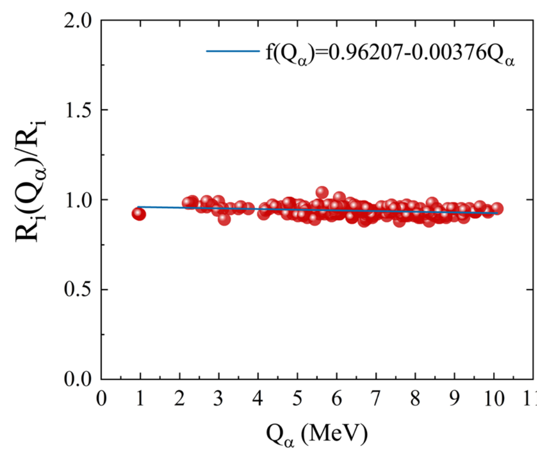

Fig. 2 The Q_α-dependence of the $R_i(Q_\alpha)/R_i$ ratio based on the proximity potential Zhang 2013 for the selected α-decay processes with $61 \le Z \le 99$. The present fitted function $f(Q_\alpha)$, Eq. (12), corresponds to the solid line

and without the Q_α-dependent (QD) term against the neutron number of daughter nuclei for the isotopic chains with $Z = 84-91$. It must be noted that the calculations of the QD sharp effective radius $R_i(Q_\alpha)$ have been performed using Eq. (12). In Fig. 3 (upper panel), by increasing the atomic number Z from 84 to 91, we see that the Q_α-independent (Q-IND) values of $\overline{R}$ exhibit an increasing linear trend by increasing the neutron number N. In Fig. 3 (lower panel), we can see that the calculated values of $\overline{R}(Q_\alpha)$ decrease sharply at $N = 126$

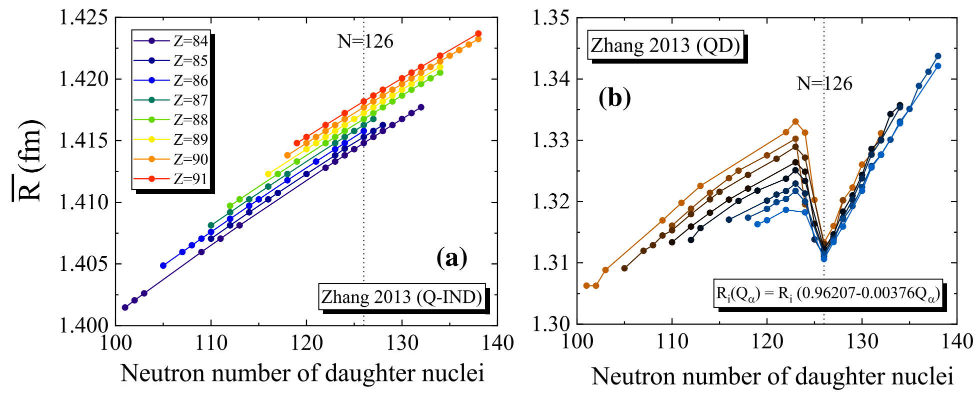

Fig. 3 Calculated values of the mean radius of the nuclear reaction system $\overline{R}$ with (panel **b**) and without (panel **a**) the Q_α-dependence of the effective radius versus the neutron number of daughter nuclei. Notice that panel **b** shows the new results applying QD relation given by Eq. (12)

by considering the Q_α dependency of the radius parameter using the obtained formula (12). When $N > 126$, the QD values of the mean radius increase with increasing the neutron number. These phenomena reflect the strong shell closure effects at neutron magic number $N = 126$. By including the present Q_α dependence in Zhang 2013 proximity model, one can obtain the strength of the total emitted α-daughter nucleus interaction potential. Figure 4 shows the radial distribution of this potential for two α-decays ^{212}Po and ^{237}Cm, as two examples. To achieve further understanding, the results of this modified proximity potential are compared with those obtained by the original proximity potential Zhang 2013 (Q-IND). Remarkably, the short-dotted line in each panel denotes the α-radioactivity energy Q_α. It is clear from Fig. 4 that when we take into account the Q_α-dependence of the proximity formalism through the presently suggested formula (12), we find that the height and width of the total interaction potential shift toward the higher values. It is shown that for Q-IND and QD versions of the Zhang 2013 proximity potential the values of the outer turning point r_{out} are identical, whereas the values of inner turning point r_{in} are different. Under these conditions, one can expect that the Q_α-dependence applied to the interaction potential affects the barrier penetration probability P_α and thus the calculated α-decay half-lives $T_{1/2}^{\text{cal}}$. The variation of the decimal logarithm of the penetration probability of α particles from tunneling through the potential barrier based on the Q-IND and QD forms of the Zhang 2013 model are displayed in Fig. 5 (left panel), which includes the results of even-Z parent nuclei such as $^{187-218}$Po ($Z = 84$), $^{193-214}$Rn ($Z = 86$), $^{202-224}$Ra ($Z = 88$) and $^{210-230}$Th ($Z = 90$) isotopes, and in Fig. 5 (right panel), which includes the results of odd-Z parent nuclei such as $^{197-215}$At ($Z = 85$), $^{199-216}$Fr ($Z = 87$), $^{207-225}$Ac ($Z = 89$) and $^{212-231}$Pa ($Z = 91$) isotopes. Note that the values of P_α are obtained from the one-dimensional WKB approximation. Our results indicate that the barrier penetration probability logarithm decreases by imposing the QD sharp radius parameter (12) in the formalism of the Zhang 2013 model. In addition, as seen in Fig. 5, the calculations of $\log_{10}(P_\alpha)$ strongly indicate the effect of the neutron closed-shell at $N = 126$. For further insight, the logarithmic values of calculated half-lives with and without taking into account the effects of Q_α dependency in comparison with the corresponding experimental data are shown in Fig. 6. The obtained results for even- and odd-Z parent nuclei are plotted in left and right panels, respectively. From Fig. 6, one can conclude that the calculated α-decay half-lives

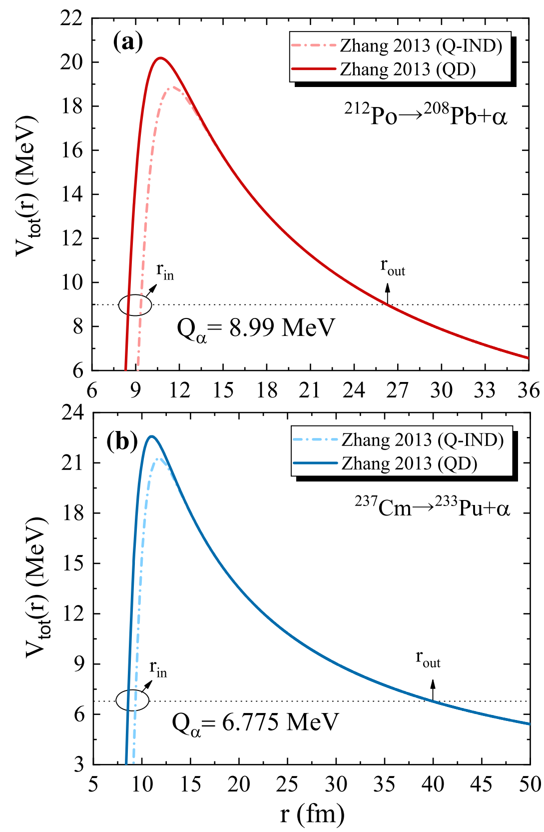

Fig. 4 Total interaction potential between α-particle and daughter nucleus for α-decay of **a** ^{212}Po and **b** ^{237}Cm parent nuclei based on the proximity models Zhang 2013 (Q-IND) and Zhang 2013 (QD) employed in the present work

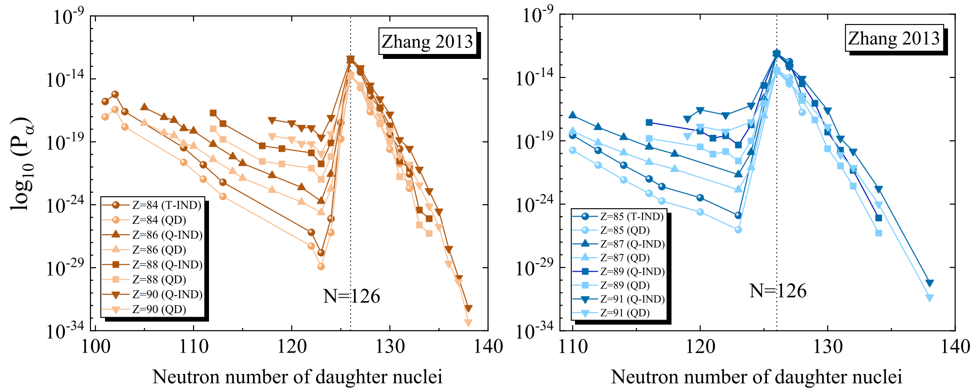

Fig. 5 Variation of the decimal logarithm of the WKB barrier penetration probability of the α-particle as a function of the neutron number of daughter nuclei using the Q-IND and QD forms of the potential Zhang 2013 for even-Z isotopes (left panel) and odd-Z isotopes (right panel)

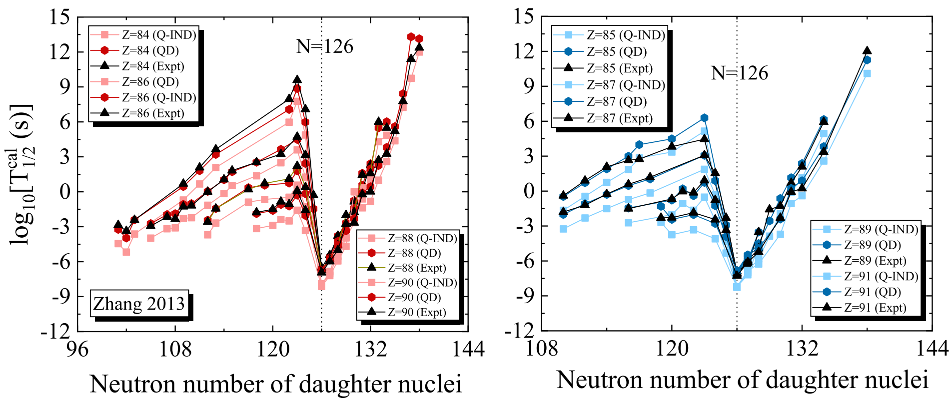

Fig. 6 Same as Fig. 5 but for the decimal logarithm of the half-lives. The corresponding experimental data are also shown for comparison

increase by considering Q_α-dependence in the radius parameter. In addition, it is shown that such dependence enables us to improve the agreement between the theoretical and experimental data of $\log_{10}(T_{1/2})$ for the considered α-decay modes. The strong shell effect of the well-known neutron magic number $N = 126$ can also be seen in the half-life calculations.

The effect of inclusion of Q_α-dependence on the theoretical half-lives of all 227 α-emitters is displayed in Fig. 7. We plot the logarithmic differences between the calculated and experimental values of α-decay half-lives as a function of the mass number of parent nuclei using the original and modified forms of the proximity potential Zhang 2013. From this figure, it is evident that the present Q_α-dependence improves the description of the available experimental half-lives in the selected mass range. In order to intuitively analyze their deviations, the standard rms deviations σ of the calculated alpha radioactivity half-life values resulting from the Zhang 2013 (Q-IND) and Zhang 2013 (QD) proximity potentials in comparison with the experimental data are tabulated in Table 1. On analyzing the table, it is found that the present modified proximity type potential enables us to obtain the more accurate results of alpha radioactivity half-lives. The present Q_α-dependence in the proximity model Zhang 2013 with the least standard deviation ($\sigma = 0.444$) can be adopted as an appropriate nuclear potential to study the α-decays. Table 1. The standard deviations between logarithmic values of alpha radioactivity half-lives of calculations and experimental data based on the different nuclear proximity potentials. The standard deviations for different semi-empirical formulas we used in comparison with the experimental data are also shown.

3.1 Comparison with the semi-empirical formula

It is important to investigate the validity of the Zhang 2013 proximity potential supplemented with the new proposed QD form of the sharp effective radius parameter $R_i(Q_\alpha)$ in comparison with some semi-empirical formulas available in the literature. In the past few decades, various efforts have been made to develop the semi-empirical formulas for the calculation of the α-decay half-lives. It is well known that almost all the suggested formulas depend on the mass number A, the atomic number Z and the released energy of the emitted α particles Q_α. Further, the authors tried to determine the adjustable parameters of these formulas by fitting to the experimental α-decay half-lives.

- **Royer formula 2000 (Royer-00)**

In 2000, on the basis of the liquid droplet model consisting of the proximity effects, Royer [19] proposed a new semi-empirical formula for α-decay half-life which is dependent

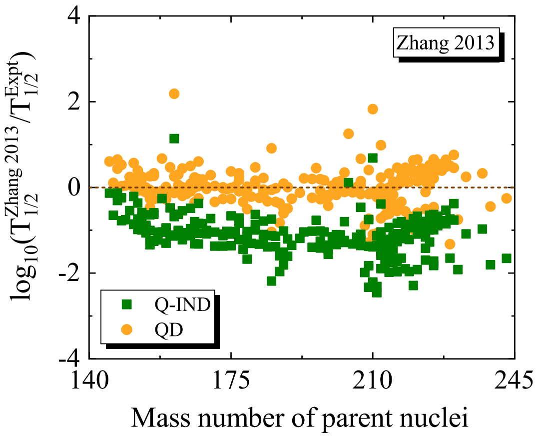

Fig. 7 The logarithmic differences between the theoretical and experimental values of the alpha half-lives as a function of the mass number of the parent nuclei using the Q-IND and QD forms of the proximity potential Zhang 2013

Table 1 The standard deviations between logarithmic values of alpha radioactivity half-lives of calculations and experimental data based on the different nuclear proximity potentials

Proximity potential	Standard deviation
Zhang 2013 (Q-IND)	1.211
Zhang 2013 (QD)-present	0.444
Empirical formula	**Standard deviation**
Royer-00	0.552
Mod-Royer-10	0.572
Royer-AP-17	0.688
DK-09(1)	0.590
DK-09(2)	0.625
Dong-10	0.432
Wong-15	0.410

The standard deviations for different semi-empirical formulas we used in comparison with the experimental data are also shown

on the atomic mass number, the charge number of the parent nucleus and the released energy Q_α as follows,

$$\log_{10} T_{1/2} = aZQ_\alpha^{-1/2} + bA^{1/6}Z^{1/2} + c, \tag{13}$$

where the adjustable parameters a, b and c depend on the parity of the parent nucleus combination and they are different in even–even, even–odd, odd–even and odd–odd nucleus [19]. In that study, the alpha emission half-lives have been deduced from the WKB semiclassical approximation without a preformation factor, which implicitly supposes that the α-decay is a very asymmetric fission process. The obtained results for a recent data set of 373 alpha radioactivities revealed that the RMS deviation between the theoretical and the experimental values of the decimal logarithm of the half-lives is 0.63. While, this deviation is only 0.35 for the even–even nuclei, see Ref. [19] for details. We labeled this formula as "Royer-00" formula.

- **Modified Royer formula 2010 (Mod-Royer-10)**

In 2010, Royer [56] presented a modified analytical formula depending on the angular momentum of alpha particle to determine the α-decay half-lives $\log_{10} T_{1/2}$(s) as

$$\log_{10} T_{1/2} = aZQ^{-1/2} + bA^{1/6}Z^{1/2} + c + d\frac{ANZ[l(l+1)]^{1/4}}{Q_\alpha} + eA\left[1-(-1)^l\right]. \tag{14}$$

We note that the constant coefficients of this relation have been adjusted on a recent experimental data set of 344 alpha transitions. It is shown that the difference between the experimental and theoretical data is relatively weak for most nuclei. However, the difference is significant for some specific nuclei such as $^{113}_{53}$I, $^{149}_{64}$Gd, $^{206}_{85}$At, and $^{218}_{91}$Pa. The reason for this difference may be attributed to the uncertainty of the extracted experimental data on these nuclei. In the present study, this modified version is marked as "Mod-Royer-10" formula.

- **Akrawy and Poenaru formula 2017 (Royer-AP-17)**

In 2017, Akrawy and Poenaru [57] introduced a new semi-empirical formula for calculations of α-decay half-lives based on the Royer formula (13). They fixed the coefficients a, b and c of this relationship by fitting the calculated values of $\log_{10} T_{1/2}$ to the corresponding experimental data of 356 α transitions. The extracted values of these coefficients for even-even, even-odd, odd-even and odd-odd nuclei have been presented in Ref. [57]. In that study, the results of three well-known relationships semFIS, ASAF and UNIV have been also used for comparison. These results indicate that, despite its simplicity, the new suggested formula behaves quite well for reproducing the experimental data. This is labeled as "Royer-AP-17" formula.

- **Denisov and Khudenko formula 2009 (DK-09)**

In 2009, Denisov and Khudenko [58] formulated the empirical formula for α-decay half-lives by taking into account the electron screening effect as

$$\log_{10} T_{1/2} = aZQ_\alpha^{-1/2} + bA^{1/6}Z^{1/2}\mu^{-1} + c + d\frac{A^{1/6}[l(l+1)]^{1/2}}{Q_\alpha} + e\left[(-1)^l - 1\right], \tag{15}$$

here $\mu = [A/A-4)]^{1/6}$. To find the constants of this relation, they fitted the calculated α-decay half-lives to the welldefined experimental data for alpha transitions of 344 parent nuclei. In above relation, the constants (a, b, c, d, e) are extracted as (1.6088, –1.1549, –26.1721, 0, 0) for even–even nuclei, (1.6910, –1.0726, –30.2365, 0.7198, –0.6965) for even–odd nuclei, (1.6925, –1.0853, –30.0842, 0.2453, –0.6406) for odd–even and (1.6577, –0.9874, –30.8222, 0.5893, –0.2914) for odd–odd nuclei, respectively [58]. We labeled this formula as "DK-09(1)". Next, the authors modified these fitting constants as reported in Ref. [59]. In the present study, this modified version is called "DK-09(2)".

- **Dong formula 2010 (Dong-10)**

In 2010, Dong et al. [60] extended the Royes's formulas by taking account of the contribution of centrifugal barrier within the generalized liquid drop model (GLDM). The half-lives have been calculated for α radioactivity nuclei in the ground-states and isomeric states. We marked the suggested formula as "Dong-10" and it is given as follows,

$$\log_{10}T_{1/2} = a + bA^{1/6}Z^{1/2} + cZQ_{\alpha}^{-1/2} + \frac{l(l+1)}{\sqrt{(A-4)(Z-2)A^{-2/3}}}. \tag{16}$$

It must be noted that the fitting parameters (a, b, c) are reported in Ref. [60]. One can find that the calculated favored α-decay half-lives agree well with the corresponding experimental data. Such agreement confirms the reliability of GLDM in the decay studies of isomeric states.

- **Wong formula 2015 (Wong-15)**

In 2015, Wong et al. [61] introduced an improved empirical formula for evaluating recent α-decay half-lives of 341 nuclei [62] by considering the hindrance effect resulting from the change of the ground state spins and parities of parent and daughter nuclei together with a new correction factor for nuclei near the shell closures.

$$\log_{10}T_{1/2} = a + bA^{1/6}Z^{1/2} + cZQ_{\alpha}^{-1/2} + \frac{d^{1-(-1)^l}\, l(l+1)}{\sqrt{(A-4)(Z-2)A^{-2/3}}} + S, \tag{17}$$

where the four parameters a, b, c and d have been obtained by fitting the experimental data. In addition, the last term S is a phenomenological correction factor [61]. The obtained results reveal that the precision in the suggested formula is higher than that in the previous analytical formula for the α-decay half-lives such as those introduced by Refs. [19,60]. Note that the authors also employed the Q_{α}-values derived from different nuclear mass models to predict α-decay with the suggested formula. In the present work, this formula is called as "Wong-15".

According to Ref. [63], one can find out that the above seven semi-empirical formulas can reproduce the experimental data reasonably well. For a better understanding, the standard deviations σ between logarithmic values of theoretical and experimental values of alpha radioactivity half-lives using the selected semi-empirical formulas are summarized in Table 1. The obtained values of σ reveal that the Wong-15 and Dong-10 formulas can be suitable choices to study the α-decay processes considered in the present work compared to other semi-empirical formulas. In fact, the Wong and Dong formulas along with modified form for the proximity potential Zhang 2013 possess small standard deviations ($\sigma < 0.445$).

3.2 Validity of the Geiger-Nuttall law in α radioactivity

The first significant correlation in α-decay systematics has been introduced by Geiger and Nuttall [64,65]. What needs to be emphasized is that the Geiger-Nuttall (G-N) α-decay law reflects the linear correlations between the logarithm of the half-lives of alpha radioactive decay processes and the energy of the emitted α-particles Q_{α} as follows,

$$\log T_{1/2} = aQ_{\alpha}^{-1/2} + b, \tag{18}$$

where a and b are constants. We are now in a position to probe the validity of the G-N law for the calculated values of α-decay half-lives within the framework of the present proximity potential. In order to assess this purpose, in Fig. 8, we display the G-N plots for the values of $\log_{10}(T_{1/2})$ calculated by the Zhang 2013 model with and without the Q_{α}-dependence of the effective radius parameter (12) as a function of the total energy of the α-decay process $Q_{\alpha}^{-1/2}$ (in MeV$^{-1/2}$) for Ra, Rn, Th and Ac isotopes. We would like to point out that the calculated values of $\log_{10}(T_{1/2})$ for all isotopes are found to be linear with different slopes and intercepts which are tabulated in Table 2.

It is quite evident from the tabulated results that the imposing of the effects of the Q_{α} dependency on the proximity formalism leads to increase the logarithmic values of the calculated half-lives. Besides, the linearity of the G-N plots reveals the validity of the presently obtained formula for the radius parameter $R_i(Q_{\alpha})$.

3.3 Prediction of α-decay half-lives for superheavy elements

The synthesis and identification of superheavy nuclei (SHN), elements with $Z > 104$ away from the valley of stability, and study of their decay properties are one of the main topics of interest in the field of nuclear physics for both theoreticians and experimentalists. It must be emphasized that α-decay is the dominant decay mode for such elements. From the

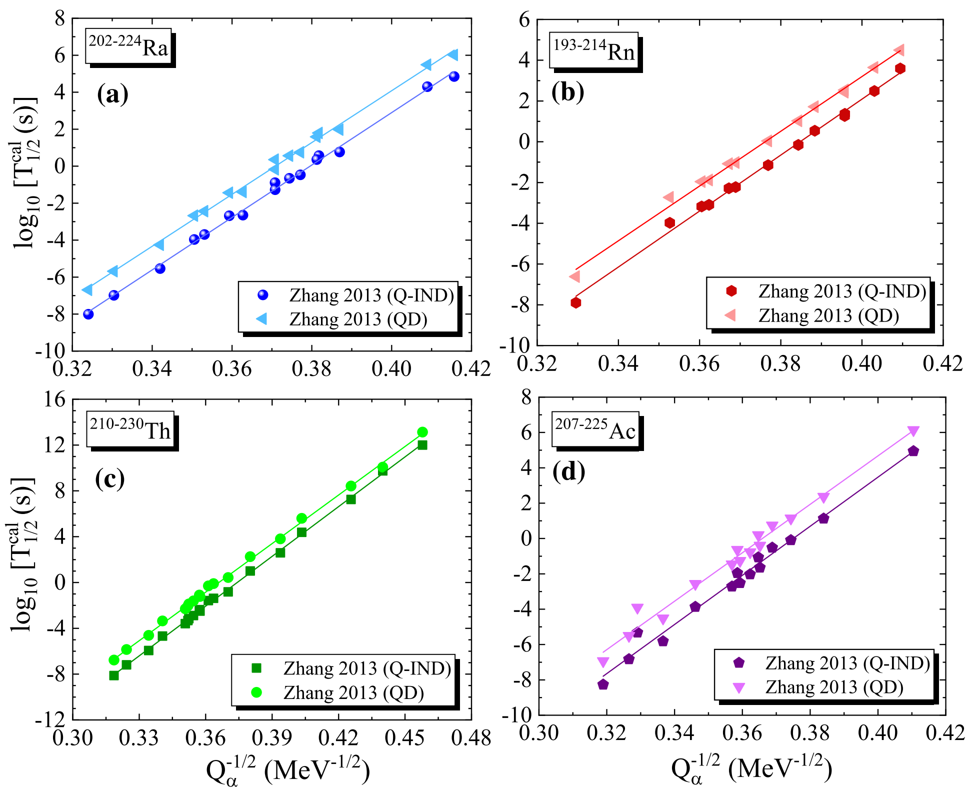

Fig. 8 The Geiger-Nuttall law for the emission of α-particle from **a** $^{202-224}$Ra, **b** $^{193-214}$Rn, **c** $^{210-230}$Th, and **d** $^{207-225}$Ac parent nuclei between $\log_{10}(T_{1/2})$ and $Q_\alpha^{-1/2}$ using the proximity potentials Zhang 2013 (Q-IND) and Zhang 2013 (QD)

Table 2 The slopes a and intercepts b of the G-N plots for α-decay from different isotopes based on the Zhang 2013 (Q-IND) and Zhang 2013 (QD) models

Parent nuclei	Zhang 2013 (Q-IND)		Zhang 2013 (QD)	
	a (MeV$^{1/2}$s)	b (s)	a (MeV$^{1/2}$s)	b (s)
$^{202-222}$Ra	141.82	–53.83	138.32	–51.36
$^{193-214}$Rn	137.32	–52.83	133.65	–50.27
$^{210-230}$Th	142.05	–52.98	141.24	–51.68
$^{207-225}$Ac	139.01	–52.12	135.31	–49.56

experimental point of view, the formation of SHN is very challenging. However, it has been well recognized that the fusion reactions (including hot [1] and cold [66] types) can be used for synthesizing of superheavy elements. For example, during recent years the neutron-rich isotopes from element $Z = 112$ to $Z = 118$ were produced in the fusion-evaporation reactions induced by ^{48}Ca beam on different actinide targets [1,4,67–71]. Recently, many experimentalists are trying to synthesize the new superheavy elements such as $Z = 119$, $Z = 120$ and so on. Another interesting attempt in α-decay studies is the prediction of the α half-lives of unknown isotopes in superheavy region which are not synthesized yet. We note that the features of predicted α-decay energy and half-lives for even-even nuclei and odd-A parent nuclei with Z = 117, 118, 119, and 120 based on the improved empirical formula by Deng et al. [72] reflect the strong shell effects at $N = 184$. Such phenomena suggest that the next neutron magic number after $N = 126$ is $N = 184$. In addition, the results of Ref. [72] revel that the predictions of α-decay half-lives by this improved formula can be considered as references for experimental data of new SHN such as the elements with $Z = 117 - 120$.

Here and in the following we intend to analyze the variation trend of Q-IND and QD forms of the mean radius $\overline{R}$ as a function of the neuron number of daughter nuclei for even-even nuclei and odd-A nuclei with $Z = 117 - 120$ using Eqs. (5) and (12), respectively. The results are shown in Fig. 9. By incorporating the present Q_α-dependence of the sharp effective radius (12), we can see that the predicted values of the mean radius of the nuclear reaction system reflect the strong shell effects at neutron magic number $N = 184$. In order to test the present modified form of the Zhang 2013 model for the calculation of the SHN, in the second step, we have calculated the α-decay half-lives of 45 elements with $Z = 105 - 116$ whose experimental data are available. In Fig. 10, we display logarithms of the ratio between theoretical α-decay half-lives calculated with the Q-IND and QD forms of the Zhang 2013 model and experimental ones as a function of the mass number of the parent nuclei. From this figure, we can find that the incorporation of temperature effects, using Eq. (12), improves the results of the Zhang 2013 model. We note that the values of the standard deviations σ are fully consistent with these results. In fact, we

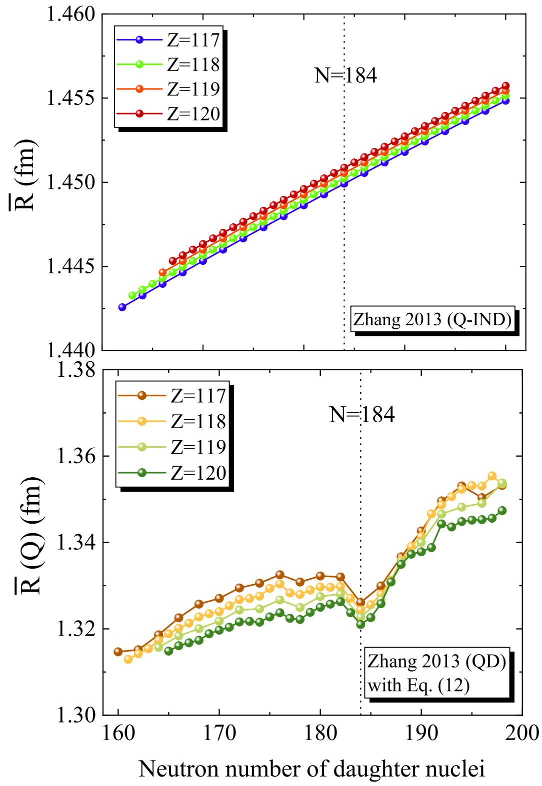

Fig. 9 Same as Fig. 3 but for superheavy nuclei with $Z = 117 - 120$

give the following values of rms standard deviations for 45 superheavy elements, $\sigma = 1.645$ and $\sigma = 0.697$ with Q-IND version of Zhang 2013 model and its QD form obtained by Eq. (12). In order to test the quality of the present modified form of the Zhang 2013 model for predicting the α-decay half-lives of the SHN, we have calculated the half-lives of 110 elements with $Z = 117 - 120$. In Fig. 11, the calculated α-decay half-lives are compared with the results of the Q-IND model and also those obtained by the empirical formula presented in Ref. [72].

We can see from this figure that predictions by the Zhang 2013 (QD) model are consistent with those obtained by the formula suggested by the Deng and coworkers. Further, the α-decay half-lives using our modified model have a valley around $N = 184$ which imply the closed-shell effects.

4 Summary

The α-decay half-lives of 227 nuclei with $Z = 61 - 99$ have been systematically evaluated using the proximity potential Zhang 2013 in order to investigate the role of Q_α-dependence of the nucleus-nucleus potential through the effective sharp radius $R_{1(2)}$ in the α-decay process. The theoretical values of the α-decay half-lives are calculated using the framework

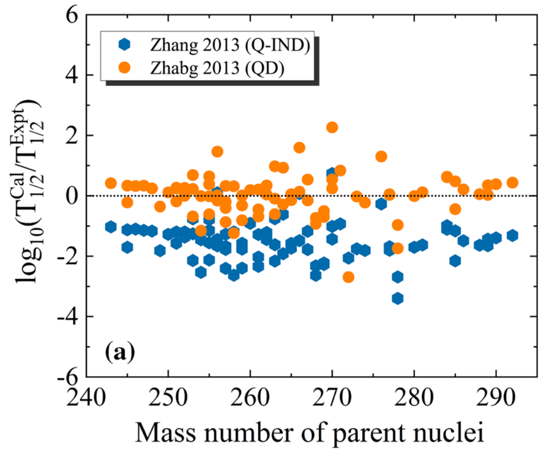

Fig. 10 The logarithmic differences between experimental α-decay half-lives and calculations versus the mass number of the parent nuclei for 45 superheavy elements with $Z = 105 - 116$. The calculations are based on the Q-IND and QD forms of the proximity potential Zhang 2013

of one-dimensional WKB approximation. The main conclusions of the present paper can be summarized as follows.

- A new QD form for the effective sharp radius $R_{1(2)}$ can be obtained if one consider the consistency between the logarithmic values of theoretical and experimental data of the alpha radioactivity half-lives. The results of the proposed model reveal that both height and width of the total interaction potential between the daughter nucleus and the α-particle increase by incorporating the present Q_α-dependence of the effective radius, Eq. (12), in the proximity potential Zhang 2013. While this dependency will give us possibility to reduce the penetration probability of emitted α-particle through the potential barrier.
- In this study, we explore the validity of the suggested form of the effective sharp radius $R_{1(2)}$ by calculating the α-decay half-lives of all 227 parent nuclei in comparison with the corresponding experimental data. The results show that the α-decay half-lives calculated by using Zhang 2013 (QD) model ($\sigma = 0.444$) are in better agreement with available experimental data than its original Q-IND version ($\sigma = 1.211$). In addition, the computational results based on the QD form of the Zhang 2013 model confirm the shell closure effects at neutron magic number $N = 126$ on the effective sharp radius, penetration probability and α-decay half-life.
- We test the quality of the suggested QD form of the effective radius using the G-N law for different cases of alpha radioactivity between the calculated values of $\log_{10}(T_{1/2})$ and $Q_\alpha^{-1/2}$. It is shown that the logarithmic values of α-decay half-lives calculated by the modified proximity potential Zhang 2013 are linearly dependent on the inverse-square of the released energy Q_α. This result can

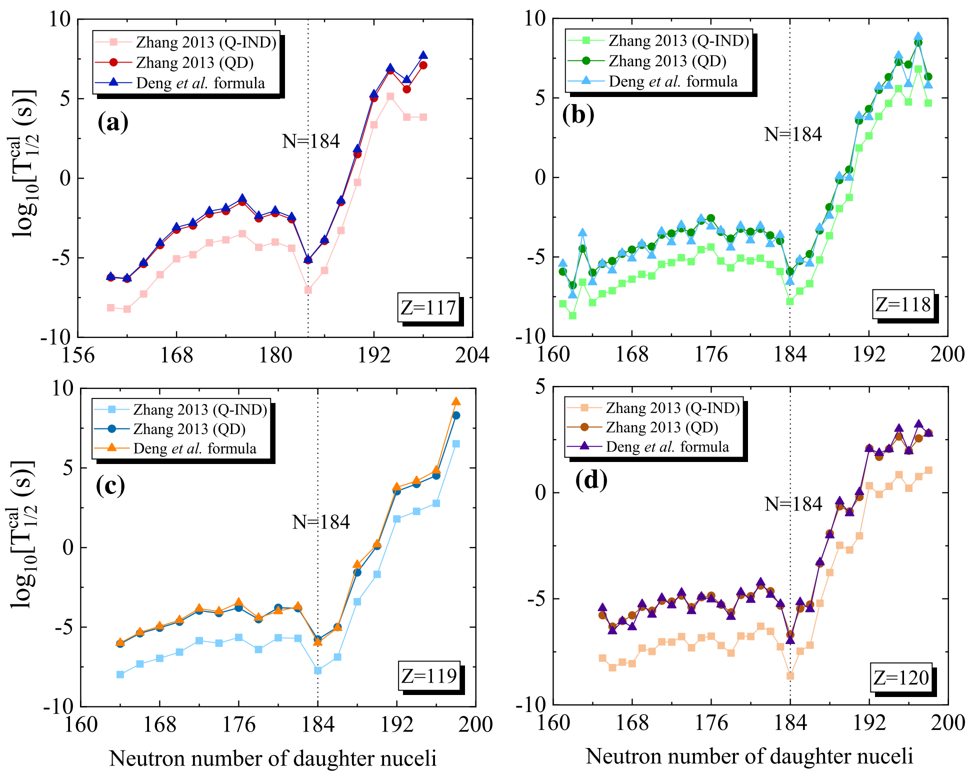

Fig. 11 The predicted α-decay half-lives of even-even nuclei and odd-A nuclei with $Z = 117 - 120$ based on the Q-IND and QD forms of the proximity potential Zhang 2013 as a function of the neutron number of daughter nuclei. The predictions calculated by Deng et al. formula [72] are also shown for comparison

demonstrate that the calculations are reliable. In order to further examine the ability of the simple analytical formula (12), we have compared the standard deviation σ of the Zhang 2013 (QD) model with those obtained by the seven different semi-empirical formulas. On the basis of the presented results for the selected mass range, one can find that Wong-15 ($\sigma = 0.410$) and Dong-10 ($\sigma = 0.432$) formulas are more precise than other five empirical formulas.

- In the present study, a comparison of the predicted alpha radioactivity half-lives of superheavy nuclei in region $117 \leq Z \leq 120$ has been performed using Zhang 2013 (QD) model and empirical formula presented in Ref. [72]. The obtained results reveal that the predictions made by the modified form of the proximity potential are consistent with counterparts by the formula developed by Deng et al. [72] for even-even nuclei and odd-A nuclei with $Z = 117, 118, 119$, and 120.

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