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ORIGINAL RESEARCH PAPER

Mixed supply function-Cournot equilibrium model of futures and day-ahead electricity markets

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Abstract

Futures contract is one of the useful financial derivatives for hedging the market players against the risks of undesirable price fluctuations in the power systems. Market players are allowed to trade electric energy in both futures and day-ahead electricity markets. The behaviour of market players in each market affects the prices and power transactions of the other market. In this study, the mutual impacts of the futures and day-ahead electricity markets are studied using a mixed supply function-Cournot equilibrium model. The day-ahead electricity market is modelled by Cournot method and futures market negotiations are modelled by the supply function method. The proposed model considers the strategic behaviour of consumers in the futures market, mark-to-marketing (MTM) settlement in the futures market, dynamics of market players' behaviour during the trading period, and transmission system constraints. An upgraded risk management method is also introduced and evaluated. The proposed model is applied to a test system and the impacts of different parameters on the results are discussed.

1 | INTRODUCTION

In the wholesale level of today's power systems, both supply and demand sides are managed by private sectors that are trying to benefit from trading electric energy. The supply side of the system mainly includes the producers that own some power generation units. Consumers are usually large industrial loads, retailers, or distribution companies. Previously, the day-ahead electricity market was the only trading choice in restructured power systems. But financial issues like risk management concerns and experiences like the California crisis led to attracting more attention to utilising financial derivatives in the power systems. The futures market is one of the main trading options in the financial markets. Nowadays, a considerable amount of electric energy is traded in the futures market. A futures contract in the power system is a standardised contract between two parties in which both parties agree to trade a particular amount of power at a predetermined price and a specified date in the future [1]. Using the futures contracts, producers and consumers can hedge themselves against the risks of undesirable day-ahead electricity price fluctuations. Futures have special characteristics. First, futures are standardised contracts that make them easy to trade among market players. Second, the ability of trading contracts allows creating new parties in the market and increases the level of competition in the market. Third, futures contracts are traded in a legal futures market by daily mark-to-marketing (MTM) settlement, which makes them more reliable to exercise and reduces the risk of contract failure.

Futures and day-ahead electricity markets are executed simultaneously. On the one hand, the predicted day-ahead market price is usually used as a guide for market players in the futures market to decide about their contract prices. On the other hand, signing futures by different market players changes their behaviour in the day-ahead electricity market, transfers a volume of power from the day-ahead market to the futures market, and consequently affects the market prices [2]. So there are mutual impacts between the futures and the day-ahead markets. Hence, if the market operators or regulators want to study the future of the power system in different situations, they need to consider these mutual impacts between futures and day-ahead markets in their studies.

The problem of mutual interaction between different types of contracts and electricity markets has been widely studied in the literature from different viewpoints. Studies in [3–8] solve the problem from the viewpoint of market players. In [3], two separate mixed integer linear models for a consumer and a generation company for optimal power allocation between forward contracts and the day-ahead market are introduced. In [4], a mathematical model for supplying a load by self-production, purchasing from the day-ahead electricity market, and weekly and monthly forward contracts is proposed. Dynamic programming is applied in [3, 4] to solve the problem. The authors in [5] solve the profit maximisation problem of a wind hydro-pump storage unit that can sell its output power through forward contracts and the day-ahead electricity market. In [6], the decision-making problem of a distribution company for participation in the bilateral contracts and the day-ahead market is formulated as a stochastic model. In [7], a bidding strategy method for a price-taker producer in the day-ahead electricity market considering the optimal involvement in weekly forward contracts is introduced. The authors in [8] apply a multi-stage mixed integer stochastic model for determining the optimal trading strategy of a risk-averse producer in the day-ahead electricity market and forward and option contracts. In [9–15], the problem from the viewpoint of the market operators is solved. In [9], a mathematical model for optimal operation of an electricity market with day-ahead and reserve markets and bilateral contracts is introduced. Prices of the contracts are assumed to be predetermined. Market players' bids are equal to their marginal costs and necessary reserve power is assumed to be certain, known, and constant. In [10], a Nash equilibrium model for a power system with only forward contracts market is proposed. According to this method, each producer submits his/her marginal cost to each consumer for obtaining the volume of forward contracts. The day-ahead electricity market has not been taken into account in [10]. In [11], a Cournot–Nash equilibrium model is proposed for joint day-ahead and forward markets. Cournot game is used for both day-ahead and forward markets, strategic gaming of consumers in forward contracts are not considered, risk management is not modelled and all demand is aggregated in a single load. The supply function equilibrium of an electricity market with bilateral contracts is investigated in [12]. Impacts of electricity market prices on the volume of forward contracts, the generation capacity of producers, and gaming of the consumers in the contract negotiation process are not considered in the proposed method. In [13], an iterative algorithm for determining the optimal price adjustment of forward contracts in a power system with fossil fuel power generators in a transmission network, suppliers as intermediaries, and consumers with flexible and inflexible loads and renewable resources in a distribution network is proposed. The volumes of forward contracts are assumed to be known and constant. In [14], the forward contracts negotiation equilibrium problem in an electricity market parallel with the day-ahead market is solved. Impacts of the forward contracts on the day-ahead market are not considered in the proposed model. In [15], a supply function Nash equilibrium model for an electricity market parallel with a forward market is introduced. Mutual impacts of the day-ahead market and forward contracts and strategic behaviour of consumers in the contract negotiations process are considered in the proposed model, but the transmission system constraints are ignored.

In order to highlight the gap in the literature, all the reviewed studies are evaluated based on the following factors, which refer to the main characteristics of the joint operation of day-ahead and futures markets: (1) Considering the strategic behaviour of both producers and consumers in the contract negotiation process, (2) considering the risk management preferences of the market players, (3) including the transmission system constraints into the model, (4) modelling the trading period in the futures market, (5) considering mutual impacts of day-ahead and futures markets, (6) formulating the MTM settlement process, and (7) considering both price and volume of contracts as decision variables. Evaluation results are presented in Table 1. As shown in Table 1, none of the reviewed papers consider all the introduced indices for modelling the behaviour of market players in the day-ahead and futures market. All the market player viewpoint studies do not consider the mutual impacts of contracts and the day-ahead market and strategic behaviour of the contract parties in the contract negotiations. In market operator viewpoint studies, only references [11] and [15] consider the mutual impacts of contracts and the day-ahead market. However, reference [11] ignores the consumers' strategic behaviour in the trading period and risk-averse behaviour of market players, and both [11] and [15] do not model the impacts of transmission system congestion on the results. More importantly, proposed models for contracts in [11] and [15] and also other system operator viewpoint studies do not model the futures market with the ability to update the producers' and consumers' contracts in different days of trading period and MTM settlement. In this study, a mixed supply function-Cournot Nash equilibrium model is proposed to study the behaviour of market players in both futures and day-ahead markets from the viewpoint of the market operator. The proposed method considers all introduced indices in Table 1 for a joint futures and day-ahead market study in the modelling procedure. The main contributions of this study are as follows:

1. Proposing a mixed supply function-Cournot equilibrium model for the joint day-ahead and futures markets considering their mutual impacts.
2. Modelling the futures market considering the possibility of updating the market players' contracts on different days of the trading period and also MTM settlement.
3. Considering the strategic behaviour of consumers alongside the producers in the futures market.
4. Upgrading the proposed risk management method in [15] such that it can present a more realistic vision about the concerns of market players during the trading period.
5. Studying the impacts of transmission system congestion on the Nash equilibrium of the joint futures and day-ahead markets.

The rest of the paper is organised as follows. In Section 2, problem description and the assumptions are presented. Uncertainties are modelled in Section 3. Proposed risk management method is explained in Section 4. Problem is formulated in Section 5. In Section 6, simulation results are presented. Finally, conclusions are made in Section 7.

TABLE 1 Comparing the reviewed studies according to the introduced indices

Indices	Reference number [3, 5],	[4, 7, 8]	[6]	[9, 10]	[11]	[12]	[13]	[14]	[15]
Strategic behaviour of consumers and producers in contract negotiations							√	√	√
Risk management preferences of market players	√	√	√					√	√
Considering grid equations			√			√	√		
Updating contracts in the trading period		√	√						
Considering mutual impacts of day-ahead market and futures contracts					√				√
Mark to marketing settlement									
Considering both price and power of contracts as decision variables								√	√

2 | PROBLEM DESCRIPTION AND ASSUMPTIONS

2.1 | Problem description

An oligopoly power system with some fossil fuel power producers and large-scale consumers, with an imperfect competition model, are considered. It is assumed that the futures and day-ahead electricity markets are executed simultaneously in the system and all producers and consumers are allowed to trade energy in both markets.

In this study, impacts on parallel execution of day-ahead and futures markets on the behaviour of market players, the share of each market from dispatched power, and markets' prices are studied. To this end, optimal gaming of the market players in futures and day-ahead markets should be known. For this purpose, it is assumed that the whole system has reached its Nash equilibrium point. Hence, the main goal of this study is to propose a model that represents the optimal behaviour of producers and consumers in the Nash equilibrium of joint day-ahead and futures markets.

For the sake of simplicity in presenting the problem, it is assumed that the power delivery period is a specific hour on a specific day in future. It can be easily extended to different hours of different days.

2.2 | Problem timeline and actions of market players

There are two main time periods in the problem, trading and delivery periods. Futures market contracts are concluded in the trading period and the day-ahead market is run on the delivery period. The trading period represents a period that can consist of several days, months or years. Market players participate in the futures market at each day of the trading period. The trading period ends before running the day-ahead market, that is, one day before the delivery period. The delivery period is referred to as the time period that the contract powers are settled and scheduled powers of the day-ahead market are traded.

On each day of the trading period, market players participate in the futures market considering their agreed contracts in previous days of the trading period, updated forecasts of uncertain parameters over the delivery period, the impacts of their behaviours in each market on the other market, and their risk management preferences. The phrase risk management preferences mean the strategy that a market player chooses to confront with the risk of losing money in the markets. A market player that seeks a totally stable revenue, trades most of his/her power in futures market with fixed prices, while a market player that wants to benefit from probable desirable price fluctuations, trades a larger share of his/her power in the day-ahead market.

The goal of producers and consumers from participation in futures market at each day of the trading period is to update the total quantity and average price of all their contracts in the futures markets by involving in new contracts with new prices and quantities.

Market players should notice that each of their actions in the futures market in a trading day affects both futures and day-ahead markets prices and quantities in overall. So, they should have an estimation of the impacts of their actions in the futures market on the day-ahead market prices and quantities and the effects of changing the day-ahead market prices on the futures market prices and quantities.

After finishing the trading period, market players can participate in the day-ahead market for the understudy delivery period. In the day-ahead market, market players should consider their agreed futures contracts in the trading period and the latest updates in the forecasted parameters and behave such that their total profit in the aggregation of the futures and day-ahead market maximises.

2.3 | Power system structure

The power system is divided into some areas. Each area is modelled by a large-scale consumer and some producers. Hence, the same indices are used to refer to consumers and areas. Areas are connected to each other by transmission lines with predetermined power transmission capacities. Producer i and consumer j are modelled by an affine marginal cost function $a_i + b_i^e Q_i$ and marginal utility function $c_j - d_j^e Q_j$, respectively [2].

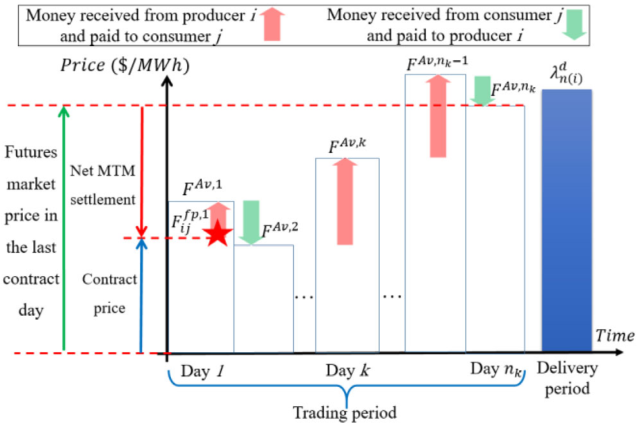

FIGURE 1 Settlement procedure in the futures market

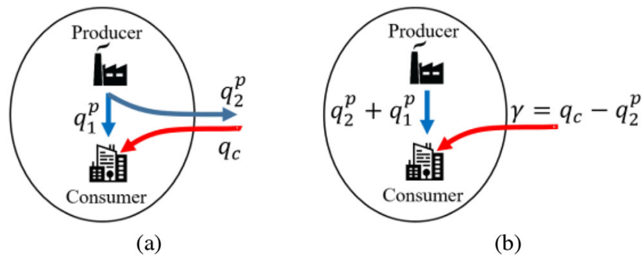

FIGURE 2 Comparing (a) the Poolco model with (b) equivalent bilateral and arbitrager model

2.4 | Futures market description

At each day of the trading period, market players can decide about the quantity and price of new contracts for different delivery periods. This study focuses on one specific delivery period. On each day of the trading period, market players are allowed to participate in the futures market, submit bids, and contract with other parties for the understudy delivery period. At the end of each trading day, a price is assigned to the market as the futures market price or daily settlement price of that day. This price is determined by computing the average price of some of the last contracts on that day. In this study, the focus is on the price and quantity of contracts that are concluded at each day and the sequence of contracts are not considered. Hence, as a simplification, the average price of all contract at each trading day is calculated as the futures market price on that day.

Futures market price is different at each day of the trading period, and at the end of the trading period, all contracts are settled by the futures market price in the last day of the trading period. Now, while the futures market price in the last day of the trading period is assigned to the contracts, they should be settled by the price that is agreed during the trading period. MTM settlement is used to solve this issue. According to the MTM settlement, at each day of the trading period, the difference between the contract price and futures market price is exchanged between the contract parties such that when the contract is settled by futures market price in the delivery period, the net price of the contract is equal to the agreed price. This process is depicted in Figure 1. The following example is presented to explain the MTM settlement process in more detail. In Figure 1, assume that on trading day 1, a contract is concluded with the price of $F_{ij}^{fp,1}$ between producer i and consumer j. $F^{Av,1}$ is assumed to be the daily settlement price of the day *1*. The producer i pays $F^{Av,1} - F_{ij}^{fp,1}$ and the consumer j receives $F^{Av,1} - F_{ij}^{fp,1}$. Then, price $F^{Av,1}$ is assigned to the contract on the trading day 1. Now, noting that the assigned price to the contract of the trading day 2 is $F^{Av,2}$, producer i pays $F^{Av,2} - F^{Av,1}$, the consumer j receives $F^{Av,2} - F^{Av,1}$, and then, $F^{Av,2}$ is assigned to the contract on the trading day 1. This process is repeated for all trading days and all other contracts during the trading period. Hence, the price of the contracts in the delivery period will be equal to the settlement price of the last day of the trading period. The differences between this futures market price and the actual contract prices are settled during the trading period. This settlement is called MTM settlement and is depicted in Figure 1. Since the MTM settlement is performed before the delivery period, the interest rate of the exchanged money should be considered in the final profit calculation.

Futures market negotiations among producers and consumers are modelled by the supply function method, which will be discussed in detail later. In the delivery period, market players are obligated to exercise and settle the contracts.

2.5 | Day-ahead electricity market operation

The day-ahead electricity market is executed after the trading period and closing the futures market. So the quantities and prices of the contracts are known and constant in the day-ahead electricity market. In the day-ahead electricity market, producers and consumers trade the remained quantity of their generation capacity and required demand, respectively. Producers behave strategically in the day-ahead market. However, consumers are price-takers in the day-ahead electricity market to supply all their loads anyhow.

Cournot method is applied to model the day-ahead electricity market. Using Cournot leads to a more straightforward model than the supply function model especially when transmission system constraints are considered and also gives a more realistic vision than Bertrand model [16].

The proposed equilibrium model by Hobbs [17] is upgraded in this study to model the day-ahead electricity market operation. The proposed Poolco model in [17] is inspired by a bilateral model, upgraded by considering arbitragers in the system. In detail, reference [17] shows that a Poolco model for an electricity market is equivalent with a bilateral model in which each producer sells its output power only to the consumer that is located in its area. Then, arbitragers buy power from consumers in low price areas and sell it to high price areas until the price difference between every two areas becomes equal to the related transmission price. This equality is illustrated for a single area in Figure 2. In Figure 2, q_1^p and q_2^p are amounts of scheduled power of the producer in the day-ahead market that are consumed by the consumer located in the same area of the producer and consumed by other consumers in other areas, respectively. q^c is the total consumed power of the consumer in the day-ahead market, and variable γ is the power that is transmitted to the area by arbitragers. Using variable γ turns the bilateral model into an equivalent Poolco model.

In [17], it is also assumed that all produced energy passes through a virtual hub node. Transmission System Operator (TSO) charges producers a congestion-based wheeling fee W_n $\$/MWh$ for transmitting power from the hub node to area n, and power scheduling is performed by maximising the total revenue of the TSO considering the transmission system constraints.

This proposed Poolco model is upgraded as follows to be applied to the proposed method in this study:

1. An affine marginal cost function is considered for producers instead of a fixed marginal cost function in [17],
2. impacts of futures market contracts on the day-ahead market are considered in the upgraded model,
3. strategic gaming of consumers in the futures market is modelled, and
4. uncertainties are involved in the model.

3 | UNCERTAINTY MODELLING

In this study, 'uncertainty' and 'risk' refer to events that can be predicted by a specified error. At first, in this section, the uncertainty modelling process is described, and then in Section 4, the proposed risk management method is introduced.

On each day of the trading period, market players are confronted with uncertainties about the delivery period. In this study, uncertainties in the demand of consumers are considered as the main source of the uncertainty in the system. As mentioned before, consumers are modelled by an affine function $c_j - d_j^e Q_j$. It is assumed that the uncertainty in the demand of each consumer is applied to the intercepts of demand functions. Another point is that at each trading day k, the uncertainty scenarios about the demand in the delivery period are different. So in order to show the dependency of intercept of each consumer to the uncertainty and updating the uncertainty scenarios in each trading day k, the marginal utility function of consumer j is reformulated as $c_{j,s}^k - d_j^e Q_j$.

Set S_k is used to refer to the uncertainty scenarios in the trading day k. Since most consumers are affected by the same economic or social factors, the correlation between the uncertainties of consumers assumed to be equal to one.

A scenario tree is assigned to generate the values of uncertainty scenarios, that is, $c_{j,s,}^k$ as shown in Figure 3. In this study, for the sake of simplicity in the presentation, some sample days in the entire trading period, that is, days 1, x, y, x, and w are chosen and the simulations are performed for these days. Each day represent a stage in the scenario tree in Figure 3. K stages of the scenario tree are assigned to $n_k = K - 1$ trading days and one delivery period. At each trading day k, each path in the scenario tree that starts from a specific node at stage k and ends at stage K represents an uncertainty scenario for that specific node and trading day. The value of c_j at stage K in that path represents the demand at delivery period in that uncertainty scenario, that is, $c_{j,s}^k$. Paths A and B in Figure 3 depict two sample uncertainty scenarios for trading day 1. Assuming that two branches are extracted from each node, the number of uncertainty scenarios in trading day 1 is $2^{(K-1)}$. In the second stage, which represents the day x of the trading period, the accuracy of forecasts about the delivery period increases and one of the nodes $S_{2,1}$ or $S_{2,2}$ in Figure 3 happens. Based on the fact in which branch happens in stage 2, the set of uncertainty scenarios of the delivery period in the trading day 2 changes (green and yellow areas in Figure 3). So the set of uncertainty scenarios at each trading day are different and the market players can update their contracts based on the updated scenarios.

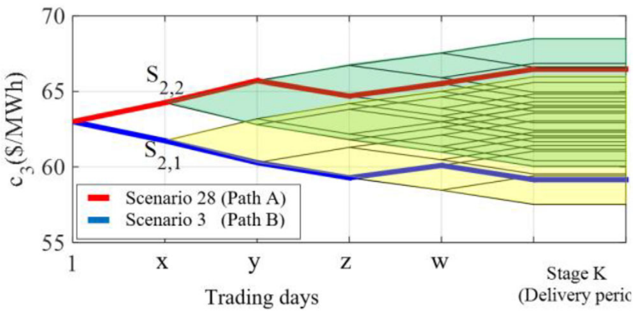

FIGURE 3 Scenario tree for modelling the uncertainty in the trading period

The procedure of updating uncertainty scenarios and consequently updating the contracts in the trading period continues until the delivery period. The normal distribution is used to assign probabilities to the branches of each node at the scenario tree.

4 | INTRODUCING THE PROPOSED RISK MANAGEMENT METHOD

In [15], a risk management method called 'concern scenario' method is proposed. According to this method, probability of each scenario at each node of the scenario tree is replaced with a value that represents the amount of concern of each producer or consumer of that scenario. For instance, since producers (consumers) are more worried about decreasing (increasing) the market price in the delivery period, they put more attention to the scenarios in which the total demand and market prices decrease (increases). So greater values are assigned to these scenarios, compared to other scenarios, in that node for producers (consumers). The proposed risk management method has been compared with the CVaR method in [15] and its efficiency is evaluated. However, the proposed concern scenario method in [15] considers only the concerns of the market players and neglects the probability of occurring in each scenario. While a scenario may cause huge concerns for a market player, its accruing probability may be much small that it reduces the impact of that scenario on the results. In this study, both concerns of

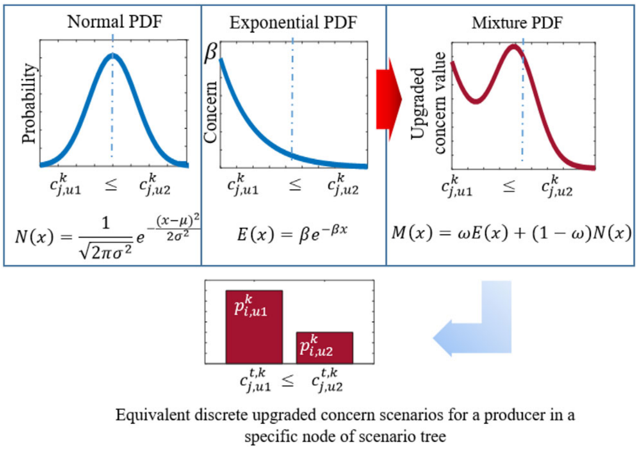

FIGURE 4 Mixture distribution of aggregated normal and exponential distributions

market players and probabilities of each scenario are considered in the modelling. To this end, first, normal distribution scenarios of demand at each node of the scenario tree for the delivery period are generated. This normal distribution is considered for all market players. Then, the exponential distribution function is used to generate the concern scenarios for each market player in that node of the scenario tree. Since the concern of each market player is different, the exponential distribution that is assigned to each market player is also different. The exponential distribution is used for modelling the concerns for two reasons. First, using a distribution function for modelling the concern values leads to the values that are comparable with normal distribution values. Second, compared to the proposed beta PDF in [15], for modelling the concerns, the exponential distribution is a monotonic function that gives a more realistic vision about the growing concerns of the market player by increasing or decreasing the market prices. Figure 4 indicates the normal and exponential distributions for a specific producer. Assuming that the demand scenarios at each node for the delivery period are sorted increasingly, based on Figure 4, applying the exponential distribution means that the concerns decrease by increasing the demand or the market price, which makes it suitable for producers. In order to adapt it to consumers, first, the values of concerns for different scenarios are calculated. Then, the first value is assigned to the last scenario, the second value to the one before the last, and so on. As illustrated in Figure 4, greater values for parameter β in the exponential distribution for each producer (consumer) means higher concern of that producer (consumer) for scenarios that lead to decreasing (increasing) the demand and consequently market prices. Hence, parameter β can be used as an index for showing the number of concerns of each market player. Assigning a greater value for β for a producer or consumer means higher concern of that market player about the delivery period.

Now, for each producer and consumer at each node of the scenario tree, we have two values for each scenario, probability of occurring and concern value. A mixture distribution formula is used to create a single PDF for each producer and consumer at each node. Mixture distribution is the weighted sum of the values of both PDFs at each scenario. Finally, two discrete scenarios are extracted from the resulted mixture distribution for two branches extracted from each node. Figure 4 represents the process of creating mixed probability-concern scenarios at each node of the scenario tree. Resulted values are called mixed probability-concern values and should be calculated for all nodes.

Now, in order to calculate the expected profit of the market players, we need to calculate the mixed probability-concern value of each uncertainty scenario, that is, each path from a specific node in scenario tree to the stage K. The mixed probability-concern value of each path is calculated by multiplying the mixed probability-concern values of branches in that path. So on each trading day k, market players are confronted with some uncertainty scenarios about the delivery period, which are called delivery period scenarios. The values of these scenarios are the intercepts of consumers' marginal utility functions at delivery period and the probability of each scenario is calculated by multiplying the values of mixed probability-concern values in the path of reaching to that value. Indices $\rho^{P,k}_{i,s}$ and $\rho^{C,k}_{j,s}$ are used as the mixed probability-concern values of each delivery period scenario on trading day k for producers and consumers, respectively.

5 | PROBLEM FORMULATION

In this section, the proposed formulation for the joint operation of day-ahead and futures markets is presented. Two sets of formulations should be presented for two different conditions: (1) An arbitrary day in trading period, that is, $k \neq K$, and (2) delivery period, that is, $k = K$. At first, the problem is formulated for the trading period. Then, the formulation is revised to adapt to condition 2.

5.1 | Problem formulation of the trading period

On each day of the trading period, producers and consumers participate in the futures market considering contracted quantities on the previous trading days and all possible scenarios for the demand in the delivery period. Four sets of formulations should be considered to model the problem: Futures market, day-ahead market, producers' profit, and consumers' profit formulations. In the end, the procedure of finding the equilibrium of the model is explained.

5.1.1 | Futures market formulation for trading day k

The supply function method is used to model the futures market. Each producer submits an affine bid function to each consumer. The slope of this bid function is equal to the slope of the marginal cost function, and its intercept, that is, α^{c}_{ij} is the decision-making variable of the producer i. So bid function of producer i to consumer j in stage k of scenario tree is represented as $F_{ij}^{c,k} = \alpha_{ij}^{c} + b_i^e Q_{ij}^{c,k}$. Similarly, the bid function of consumer j to producer i is represented as $F_{ji}^{c,k} = \in_{ji}^{c} - d_j^e Q_{ji}^{c,k}$ in which $\in_{ji}^{c}$ is the consumer's decision-making variable. Calculating the intersection of bid functions of producer and consumer in each contract yields the contract price and quantity of each contract on trading day k as follows:

$$Q_{ji}^{c,k} = Q_{ij}^{c,k} = \frac{\in_{ji}^{c,k} - \alpha_{ij}^{c,k}}{b_i^e + d_j^e} [0, 1] \tag{1}$$

$$F_{ji}^{c,k} = F_{ij}^{c,k} = \frac{b_i^e \in_{ji}^{c,k} + d_j^c \alpha_{ij}^{c,k}}{b_i^e + d_j^e} \tag{2}$$

5.1.2 | Day-ahead market estimation formulation for trading day k

When market players participate only in the futures market in the trading period, they must consider the impacts of their behaviour in the futures market on the day-ahead market. Hence, the day-ahead market must also be modelled in the trading period. Cournot method is used to model the day-ahead electricity market. As mentioned before, at each trading day k, some delivery period scenarios are generated. The power system should be modelled for each scenario. So the TSO optimisation problem should be solved for each scenario separately. According to the proposed method in [17] and explanations in Section 2.5, the optimisation problem of the TSO at trading day k for scenario s of the delivery period is as below:

$$\max \sum_{j\in A} W_{j,s}^{k} \left(\gamma_{j,s}^{k} + \sum_{r=1}^{k} \left(\sum_{i\in P} Q_{ji}^{c,r} - \sum_{i\in P_j} \sum_{j\in C} Q_{ij}^{c,r} \right) \right) \tag{3}$$

such that

$$\sum_{m\in A} V_j V_m B_{jm} \left(\delta_{j,s}^{k} - \delta_{m,s}^{k} \right) = -\left(\gamma_{j,s}^{k} + \sum_{r=1}^{k} \left(\sum_{i\in P} Q_{ji}^{c,r} - \sum_{i\in P_j} \sum_{j\in C} Q_{ij}^{c,r} \right) \right) \left(\psi_{j,s}^{k} \right) \forall j \in C \tag{4}$$

$$V_j V_m B_{jm} \left(\delta_{j,s}^{k} - \delta_{m,s}^{k} \right) \le \bar{T}_{jm}, \left(\bar{\mu}_{jm,s}^{L,k} \right) \forall (j, m) \in L \tag{5}$$

$$V_j V_m B_j m (\delta_{j,s}^{k} - \delta_{m,s}^{k}) \ge \underline{T}_{j,m}, (\underline{\mu}_{jm,s}^{L,k}) \forall (j, m) \in L \tag{6}$$

The objective function in Equation (3) represents the revenue of the TSO from transferring power to each area in the delivery scenario s of the delivery period. Term $\gamma_{j,s}^{k}$ is the injected power to area j in the day-ahead electricity market from TSO's viewpoint. DC load flow equations are used to model the transmission system. Constraint (4) represents the Kirchhoff's Current Law (KCL) in all areas. Constraints (5) and (6) are the power transmission constraints of the grid's lines. It should be noted that the TSO optimisation problem in Equations (3) to (6) is solved for the day-ahead market. Hence, $Q_{ji}^{c,k}$ and $Q_{ij}^{c,k}$ are assumed to be constant in this optimisation. The decision-making variables in the TSO optimisation problems are $\gamma_{j,s}^{k}$ and $\delta_{j,s}^{k} \forall j \in A$.

5.1.3 | Producers' profit formulation

On each trading day k, producers should decide about the volume of their contracts on that day. To this end, they should consider the impacts of their actions on their revenue from the day-ahead market and try to bid such that their aggregated profit from both futures and day-ahead markets is maximised. Hence, the profit optimisation problem of producer i for participation in the futures market on trading day k can be formulated as below:

$$\max E\left(\pi_i^{k}\right) = \sum_{s\in S_k} \rho_{i,s}^{P,k} \left[\lambda_{n(i),s}^{k} Q_{i,s}^{p,k} + \sum_{j\in C} R_{ij}^{P,k} \right. \tag{7}$$

$$\left(- a_i \left(Q_{i,s}^{p,k} + \sum_{r=1}^{k} \sum_{j\in C} Q_{ij}^{c,r} \right) \right. \tag{7}$$

$$\left. -\frac{1}{2} b_i^e \left(Q_{i,s}^{p,k} + \sum_{r=1}^{k} \sum_{j\in C} Q_{ij}^{c,r} \right)^2 \right] \tag{7}$$

such that

$$Q_{i,s}^{p,k} + \sum_{r=1}^{k} \sum_{j\in C} Q_{ij}^{c,r} \le \bar{Q}_i \left(\bar{\mu}_{i,s}^{k} \right) \quad \forall s \in S_k \tag{8}$$

$$\lambda_{n,s}^{k} = c_{n,s}^{k} - d_n^e \left(\sum_{i\in P_n} Q_{i,s}^{p,k} + \sum_{r=1}^{k} \sum_{i\in P} Q_{ni}^{c,k} + x_{i,n,s}^{k} \right) \quad \forall n \in C, s \in S_k \tag{9}$$

$$\lambda_{n,s}^{k} = \lambda_{hub,s}^{k} + W_{n,s}^{k} \left(\mu_{i,n,s}^{hub,k} \right) \quad \forall s \in S_k \tag{10}$$

$$\sum_{n\in C} x_{i,n,s}^{k} = 0, \left(\mu_{i,s}^{x,k} \right) \quad \forall s \in S_k \tag{11}$$

$$Q_{i,s}^{p,k} \ge 0 \quad \forall s \in S_k \tag{12}$$

Decision-making variables are the bids in the futures market on trading day k, that is, $\alpha_{ij}^{c,k}$ $\forall j \in C$, and bid in the day-ahead market at each scenario of the delivery period, that is, $Q_{i,s}^{p,k}$. $\alpha_{ij}^{c,k}$ $\forall j \in C$ appear in variables $Q_{ij}^{c,r}$ and $F_{ij}^{c,k}$ as formulated in Equations (1) and (2). Set S_k includes all delivery period scenarios at node k, which according to Figure 3 contains $2^{(K-k)}$ scenarios for each consumer's demand. $Q_{ij}^{c,r}$, $F_{ij}^{c,r}$ and $F^{Av,r}$ for $r = 1, \dots, k-1$ are determined in previous trading days, and hence are constant on trading day k. $F^{Av,k}$ is the average of contract prices on trading day k that is known as the settlement price of trading day k. The first term in the objective function in Equation (7) represents the revenue from the day-ahead market. $R_{ij}^{P,k}$ is the total revenue of producer i from contracting with consumer j in the futures market considering MTM settlement till trading day k, which according to the explanations in Section 2.4 is formulated as below:

$$R_{ij}^{p,k} = F^{Av,k} \sum_{r=1}^{k} Q_{ij}^{c,r} + (1+IR)^{(365-D_k)} Q_{ij}^{c,k} \left(F_{ij}^{c,k} - F^{Av,k} \right) + (1+IR)^{(365-D_k)} \left(F^{Av,k-1} - F^{Av,k} \right) \sum_{r=1}^{k-1} Q_{ij}^{c,r} \tag{13}$$

The first term in Equation (13) assigns the daily settlement price of the trading day k to all contracts between producer i and consumer j in all trading days $r = 1, \dots, k$, the second term in Equation (13) represents the revenue from MTM settlement of contract between producer i and consumer j on trading day k, and the third term in Equation (13) is the revenue from MTM settlement of contracts between producer i and consumer j that is agreed in the previous days of the trading period, that is, $r = 1, \dots, k-1$. The rest of the objective function is the total operation cost of the producer in the aggregation of both markets.

Constraint (8) is the generation capacity of producer i. Constraint (9) represents the market price of the area that the producer is located in. Constraint (10) is the relationship between the hub node price and the area price. Constraint (11) indicates that the sum of injected power to the areas from the viewpoint of producer i at delivery period scenario s is equal to zero. Constraint (12) guarantees the positivity of generated power by producer i.

The results of this optimisation problem are considered as the contract prices and quantities of producer i on the trading day k.

5.1.4 | Consumers' profit formulation

The profit of each consumer is calculated by subtracting the utility of electricity for that consumer from the total payment through the futures contracts and the day-ahead market. As mentioned before, the total consumed energy by each consumer in the day-ahead market is equal to the generated power of the producers located in that area plus injected power from the hub node. So the expected profit of each consumer j is formulated as below:

$$\max E\left(U_j^k\right) \tag{14}$$

$$= \sum_{s \in S_k} \rho_{j,s}^{C,k} \Bigg[-\lambda_{j,s}^k \left(\sum_{i \in P_j} Q_{i,s}^{p,k} + z_{j,j,s}^k \right) - \sum_{i \in P} R_{ji}^{C,k} + c_{j,s}^k \left(\sum_{i \in P_j} Q_{i,s}^{p,k} + z_{j,j,s}^k + \sum_{r=1}^{k} \sum_{i \in P} Q_{ji}^{c,r} \right) - \frac{1}{2} d_j^e \left(\sum_{i \in P_j} Q_{i,s}^{p,k} + z_{j,j,s}^k + \sum_{r=1}^{k} \sum_{i \in P} Q_{ji}^{c,r} \right)^2 \Bigg] \tag{14}$$

such that

$$\lambda_{n,s}^k = c_{n,s}^k - d_n^e \left(\sum_{i \in P_n} Q_{i,s}^{p,k} + \sum_{r=1}^{k} \sum_{i \in P} Q_{ji}^{c,r} + z_{j,n,s}^k \right) \quad \forall n \in C, s \in S_k \tag{15}$$

$$\lambda_{n,s}^k = \lambda_{hub,s}^k + W_{n,s}^k \left(\mu_{j,n,s}^{hub,k} \right) \quad \forall s \in S_k \tag{16}$$

$$\sum_{n \in C} z_{j,n,s}^k = 0, (\mu_{j,s}^{x,k}) \quad \forall s \in S_k \tag{17}$$

Decision-making variables of consumer j are the bids in the futures market, that is, $\in_{ji}^{c,k}$ $\forall i \in P$. $\in_{ji}^{c,k}$ $\forall i \in P$ appear in variables $Q_{ji}^{c,r}$ and $F_{ji}^{c,k}$ as formulated in Equations (1) and (2). The first term in the objective function in Equation (14) is the payment in the day-ahead market. $R_{ij}^{C,k}$ is the total payment of consumer j for contracting with producer i in the futures market considering MTM settlement till trading day k, which according to the explanations in Section 2.4 is formulated as below:

$$R_{ji}^{C,k} = F^{Av,k} \sum_{r=1}^{k} Q_{ji}^{c,r} + (1+IR)^{(365-D_k)} Q_{ji}^{c,k} \left(F_{ji}^{c,k} - F^{Av,k} \right) + (1+IR)^{(365-D_k)} \left(F^{Av,k-1} - F^{Av,k} \right) \sum_{r=1}^{k-1} Q_{ji}^{c,r} \tag{18}$$

The rest of Equation (14) represents the utility of consumer j for consuming electric energy. Different terms of Equation (18) are defined similarly to the terms of Equation (13).

Constraints (15) to (17) for consumers are the same as Constraints (9) to (11) for producers.

5.1.5 | Obtaining the Nash equilibrium of the model

According to [19], the proposed formulation in this study fits in the field of conjectural Counrot and Nash equilibrium methods. In order to find the equilibrium of this model for the trading day k, prices and quantities of futures contracts in Equations

(3) to (18) are replaced with their equivalents in Equations (1) and (2). Since the producers and consumers optimisation problems are in the form of quadratic programming method and hence, are convex, similar to the proposed solution methods in [17] and [18], Karush–Kuhn–Tucker (KKT) conditions of all optimisation problems, that is, Equations (3) to (6) for TSO operation, Equations (7) to (13) for all producers and Equations (14) to (18) for all consumers are written. Moreover, the following market-clearing conditions must be satisfied to ensure that injected power to each node from the viewpoint of the TSO, producers, and consumers are the same:

$$\gamma_{n,s}^{k} = x_{i,n,s}^{k} = z_{j,n,s}^{k} \quad \forall\, n \in C, i \in P, j \in C,\ s \in S_k \tag{19}$$

Applying equality in Equation (19) causes Equations (9) and (15) to be similar for all market players. Hence, these equations can be turned into one equation. A similar process is followed for Equations (11) and (17). Now, by solving the remaining equalities and inequalities constraints obtained from KKT conditions of all optimisations, the Nash equilibrium of the model will be found. More details about writing the KKT conditions of the proposed method are presented in the Appendix.

5.2 | Problem formulation for the delivery period

In the delivery period, market players can only participate in the day-ahead electricity market. Hence, the futures market and variables Equations (1) and (2) are fixed and known in the formulations. Moreover, consumers are price takers in the day-ahead market. Hence, the optimisation problem in Equations (14) to (18) is also omitted. Total contracted power in the futures market on all trading days must be considered as constants in the formulation. So optimal gaming of the market players can be found by solving the KKT optimal conditions of Equations (3) to (6) and (7) to (13) and considering the contracted prices and quantities as constants in the formulation.

6 | CASE STUDY

The proposed method is applied to a test system with three areas and parameters that are shown in Figure 5 [18]. The nominal voltage of all areas is the same. Symbol ‘P’ represents the producers and symbol ‘C’ represents the consumers. Parameters of producers and slope of marginal utility functions of consumers are presented in Tables 2 and 3 [14]. The daily interest rate is 0.04. It is assumed that the delivery period is one hour a day and the trading period is one year. For the sake of simplicity, it is assumed that producers and consumers only participate in the futures market on trading days 1, x, y, z, and w. The number of days between each trading day and the first trading day is presented in Table 4. Since five trading days are considered, 32 scenarios are generated for each consumer in the delivery period as shown in Figure 6. Figure 2 shows the scenario tree generated for consumer 2 and highlights the paths of two different delivery scenarios that are marked in Figure 5 in more detail. Similar normal distributions are considered for demand uncertainty in each node of the scenario tree. Concerns of producers and consumers are changed by changing the parameter β in exponential distribution formulation. Values of parameter β for different market players are presented in Table 5. Based on Table 5 and according to explanations in Section 4, producer 2 and consumer 2 have the most concerns about the future. Parameter ω in mixture PDF is assumed to be equal to 0.5. PATH Solver in GAMS software is used to calculate the Nash equilibrium of the proposed method. The system configuration for running the program is a dual-core CPU, 2.66 GHz, and 4 GB RAM. The runtime of the program for each trajectory on the scenario tree is about 3 min.

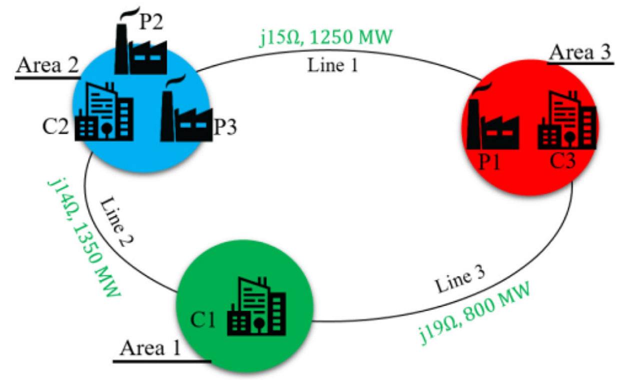

FIGURE 5 Proposed test system structure

TABLE 2 Producers’ cost function and generation capacity information

Number of producer	Marginal cost functions coefficients: a_i (\$/MWh)	b_i(\$/MW^{2}h)	Generation capacity (GW)
1	20	0.017	3
2	5.6	0.026	4
3	16	0.007	3.5

TABLE 3 Slop of marginal utility functions of consumers information

	C1	C2	C3
d_j(\$/MW^{2}h)	0.010	0.003	0.005

TABLE 4 Number of the days between the first trading day and trading day k

	$k=1$	$k=x$	$k=y$	$k=z$	$k=w$
D_k	0	80	160	250	330

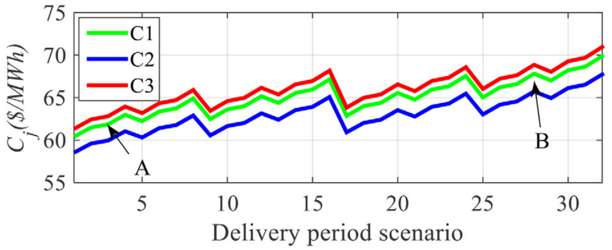

FIGURE 6 Intercepts of marginal utility functions of consumers in different scenarios

TABLE 5 Parameter β for different market players

Producers			Consumers		
1	2	3	1	2	3
0.3	0.2	0.4	0.15	0.2	0.1

6.1 | Simulation results

On each day of the trading period, one of the branches of the scenario tree happens. Therefore, on each day of the trading period, some scenarios of the trading period are omitted and we face the remained scenarios. So, in the end, for each realisation of the system in the delivery period, we will be faced with a trajectory in the scenario tree, like blue and red paths in Figure 3 that indicates what actually happened during the trading period. So simulations should be performed for each trajectory separately. In this subsection, first, simulation results are presented for the above-average demand trajectory 28 and then the results are compared with the under-average demand trajectory 3. The trajectories 28 and 3 in the scenario tree are depicted in Figure 3 as trajectories A and B, respectively. Simulation results for trajectory 28 are presented in Figures 7–9. Figure 7 indicates the cumulative contracted powers in the considered five trading days and scheduled power in the day-ahead market for each producer and consumer. Simulation results show that about 77% of the total demand is traded through futures. Seventy percent of the contracts belong to the first two trading days.

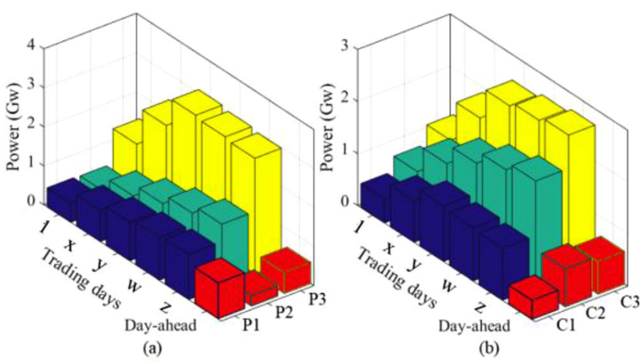

FIGURE 7 Total power contracts on different trading days and day-ahead scheduled powers for (a) producers, and (b) consumers at scenario 28

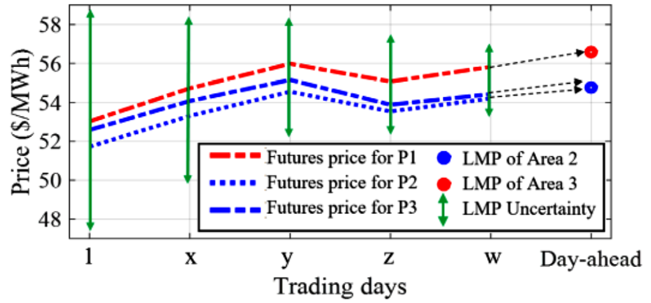

FIGURE 8 Average contract price of each producer with different consumers in different trading days, and locational marginal price (LMP) of areas 2 and 3 in the delivery period

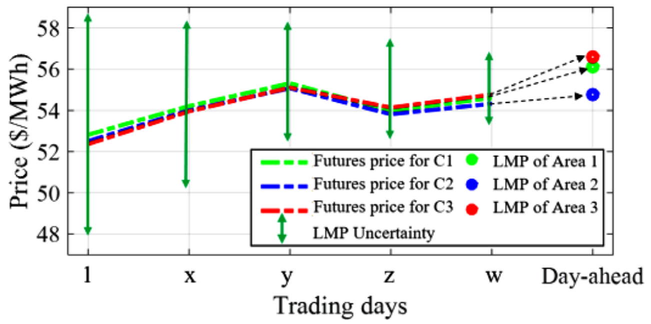

FIGURE 9 Average contract price of each consumer with different produces in different trading days and LMP of areas in the delivery period

P1 sells more share of his/her generation capacity than other producers in the day-ahead market. This happens because in trajectory 28, congestion in transmission lines 1 and 2 increase the market power of P1 in the system, and hence P1 benefits from selling power with greater prices in the day-ahead market. This fact is illustrated in Figures 8 and 9 where locational marginal price (LMP) of area 3 that includes P1 has the highest value compared to the other areas. Increasing the price of area 3 forces the C3 to buy his/her required demand from the futures market. This causes the share of futures from traded power of C3 be more than other consumers, whereas his/her concern about the future is lower than other consumers' concerns. P3, which has the most concern about the delivery period, tries to sell power in the futures market as much as possible.

Figures 8 and 9 indicate that the futures contract prices usually place between the upper and lower bounds of the day-ahead market prices uncertainty and try to converge to the day-ahead market prices at the delivery period. Transmission lines congestion increases the competition between P2 and P3 and decreases the market price in area 2. This increases the share of futures of P2 and P3. Decreasing the day-ahead market price of area 2 also encourages the C2 to buy more power through the day-ahead market and decreases his/her share of futures contracts as shown in Figure 6(b).

Simulation results of trajectory 28 are compared with trajectory 3 in which the total demand is lower than most of

TABLE 6 Comparing the results of delivery period scenarios 5 and 28

		Scenario 28	Scenario 3
Day-ahead market prices ($/MWh)	Area 1	56.9	49.09
	Area 2	54.75	48.26
	Area 3	56.58	50.19
Futures market price ($/MWh)		55.36	50.26
Total scheduled day-ahead power (MW)		1716	778
Total contracted power (MW)		5724	6095

the trajectories in Table 6. According to Table 6, by reducing the demand, market prices in different areas and futures contract prices decrease as it is expected. Final futures market price, which according to the definition of the MTM settlement process is the futures market price in the last day of the trading period, obtain close to the day-ahead market prices in both cases. In trajectory 28, the futures market prices are slightly less than the average day-ahead market price in different areas, whereas in trajectory 3, the futures market prices are slightly more than the average day-ahead market price in different areas. Reducing the demand and market prices make the producers reluctant for participating in the day-ahead market, and hence less power is traded in the day-ahead market and more power is traded in the futures market compared to trajectory 28.

6.2 | Impacts of the MTM process on the results

MTM settlement has almost similar impacts on all producers and all consumers. So in order to avoid presenting repetitive information, the simulation results are presented only for the arbitrary producer P3 and consumer C2. Figure 10 illustrates the financial interactions of P3 and C2 during the trading period and their net profit from MTM for trajectories 3 and 28. In order to calculate the net profit of market players from MTM, first, revenue from the MTM settlement during the trading period is computed considering the interest rates. This revenue is equal to the last two terms of Equations (13) and (18) for producers and consumers, respectively. Let us call it Re1. Then, the revenue from the difference between the futures market price on the last trading day and contract price is calculated for each contract. Let us call it Re2. Finally, the net profit from MTM is obtained by computing Re1 – Re2. As shown in Figure 10, based on the variation of futures market prices in the trading period, both producers and consumers may gain or lose money by MTM. In trading days that futures market price reduces, consumers pay and producers receive money and vice versa. Since futures market price is mainly decreasing in trajectory 3, producers benefit from receiving a part of their contract prices in advance and their profit increases. In trajectory 28, the futures market price is mainly increasing. Hence, consumers receive money in the trading period while they have to give it back in the delivery period. In this condition, consumers benefit from the interest rate and the time value of the money. However, even though the traded money due to the MTM settlement on each trading day may reach 10% of market players' profit, the net profit of the market players from MTM settlement is not considerable. In fact, as mentioned before, the goal of MTM settlement is mainly reducing the possibility of the contract default by each of the contract parties not gaining profit.

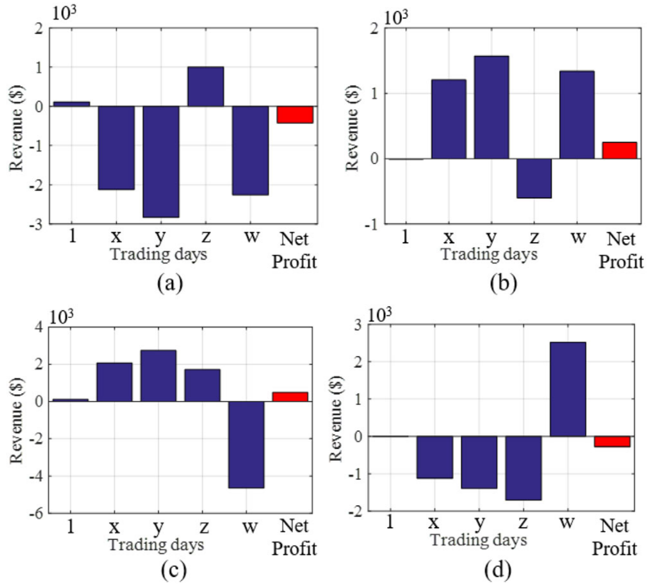

FIGURE 10 Comparing the revenue of market players from mark-to-marketing in the trading period for (a) P3 in scenario 28, (b) C2 in scenario 28, (c) P3 in scenario 3, and (d) C2 in scenario 3,

TABLE 7 Comparing the average market prices with and without futures

	Average day-ahead market prices ($/MWh)		
	Area 1	Area 2	Area 3
Without futures	52.79	51.98	53.77
With futures	52.51	51.54	53.50

6.3 | Impact of futures contracts on the day-ahead market prices

Table 7 compares the average day-ahead market prices in different areas with and without the futures market. Simulation of the case in which the futures market is not considered in the system is performed similarly to Section 5 assuming that $Q_{ij}^{c,k} = Q_{ji}^{c,k} = 0 \; \forall \; i \in P, \; j \in C$. As shown in Table 7, considering that the futures market decreases the day-ahead market prices slightly up to 0.8%, this difference increases up to 2% when uncertainty increases by about 100%.

TABLE 8 Simulation results with and without considering the concern of C1

		Consumers		
		C1	C2	C3
	With C1 concerns	1023	2154	2814
Contract quantities (MW)	Without C1 concerns	925	2190	2925
	With C1 concerns	9279	16,052	18,798
Average profit ($)	Without C1 concerns	9467	15,827	18,810

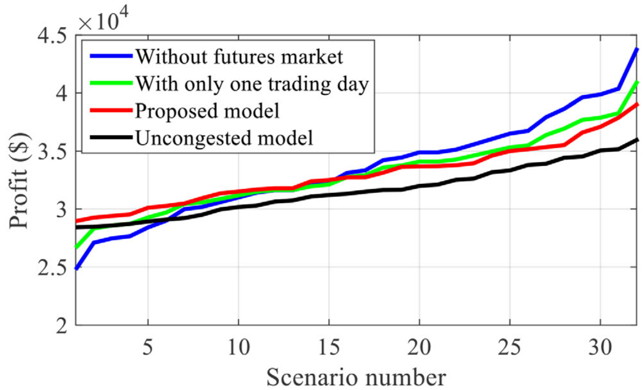

FIGURE 11 Comparing the variation of profit of P1 over all delivery period scenarios in different cases

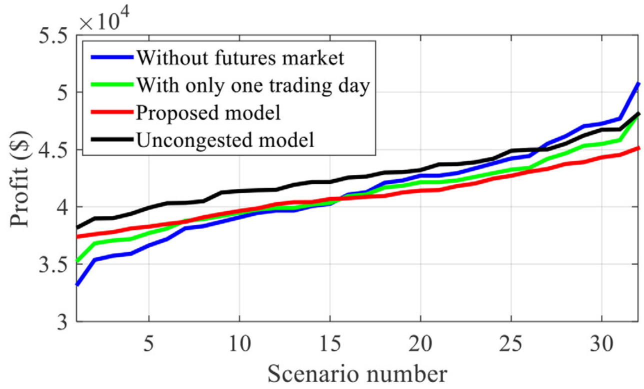

FIGURE 12 Comparing the variation of profit of P2 over all delivery period scenarios in different cases

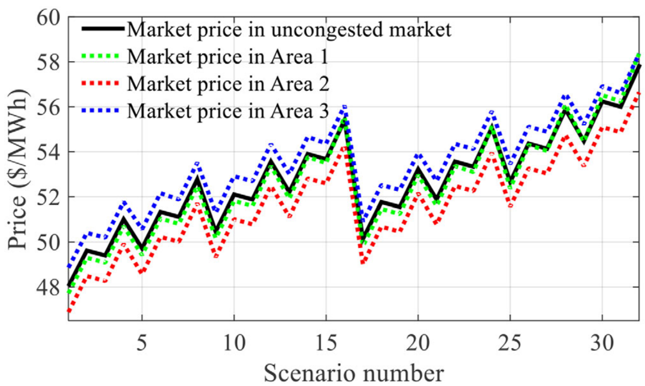

FIGURE 13 Comparing the variation of areas' market prices over all delivery period scenarios in congested and uncongested models

6.4 | Impact of concerns of market players on the contracted quantities and profit of consumers

The average total quantities of contracted powers and profits for consumers over all the delivery period scenarios are compared in Table 8 in two cases, considering the concerns of C1 and ignoring the concerns of C1. As indicated in Table 8, by removing the concern of C1, his/her total contracted powers in the futures market decreases by about 16%. In this situation, the average profit of C1 increases by 1.7% because of ignoring his/her concerns in the decision-making process. Other consumers are slightly affected by reducing the activity of C1 in the futures market. Similar results are extracted when the concerns of each producer are ignored.

6.5 | Impact of the futures on the profit of the market players

The main goal of financial deliveries is reducing the risk of undesirables profit fluctuations and uncertainty about the future. In this subsection, the effects of futures on the profit of the arbitrary market players P1 and P2 are studied. Simulation results are presented for four different cases in Figures 11 and 12. In order to show the impact of futures contracts on the amount of variation in the profit of market players clearly, the profits in different scenarios are sorted and presented in Figures 11 and 12. As shown in Figures 11 and 12, utilising futures decreases the variations of the profit of P1 and P2 by about 47% and 56%, respectively. Considering the risk management preferences of P1 and P2 reduces their profit in some scenarios. In fact, risk management tools always provide a compromise between profit and risk. As a producer is more risk-averse and tries to reduce the risk of losing money in the low market price scenarios, his/her chance for gaining high profits in high market price scenarios also decreases. P2 is more risk-averse than P1, and hence his/her concern is mainly on reducing the profit variation, and this leads to reduce his/her profit due to risk management preferences more than reducing the profit of P1. Comparing the results of the proposed method with the case that there is only one trading day shows that both P1 and P2 use the possibility of trading in the days after the first trading day to contract more power and reduce their profit variations.

The transmission system constraint of the understudy test system provides market power for P2 and leads to increasing the profit of P2 by increasing the electricity price in area 3 as shown in Figure 13. Removing the congestion has decreased the market price in area 3 and increased the market price in area 2, which leads to decreasing the profit of P1 and increasing the profit of P2.

7 | CONCLUSION

Futures contracts are known as one of the popular risk management derivatives for trading electric energy. Simultaneous execution of futures and day-ahead markets causes mutual interactions between these two markets. This study proposes a Nash equilibrium of the joint futures and day-ahead markets to study these mutual interactions. Supply function and Cournot methods are applied to model the market players in futures and day-ahead markets, respectively. A new risk management method is also applied to the model. Simulation results show that each concern of market players about the delivery period increase the share of contracted power from all traded power increases. Transmission system congestion affects the behaviour of market players in the system. Congestion in an area forces the consumers to contract more power to hedge themselves against the risk of buying power with high prices in the day-ahead market. Producers that have market power prefer to trade more power in the day-ahead market. MTM benefits the producers when contract prices decrease during the trading period and benefit the consumers when contract prices increase during the trading period. Using futures contracts reduces the profit variations of market players up to 56% but reduces their profit in some scenarios as a result of compromising between risk and benefit. Considering the futures alongside with day-ahead market reduces the day-ahead market prices up to 0.8% for the proposed test system. As the delivery period uncertainty increases, the impact of the futures market on reducing the day-ahead market prices increases.

Nomenclature

A. Indices

- i power system producers
- j & n power system consumers and areas
- $n(i)$ area n that includes producer i
- s delivery period scenarios
- k trading days

B. Sets

- P set of producers
- C set of consumers and areas (one consumer in each area)
- P_j set of producers at area j
- S_k set of delivery period scenarios estimated on trading day k

C. Constants

- a_i intercept of marginal cost function of producer i
- b_i^e slope of marginal cost function of producer i in the day-ahead and futures market
- $c_{j,s}^k$ intercept of marginal utility function of consumer j at delivery period scenario s estimated on trading day k
- d_j^e slope of marginal utility function of consumer j in the day-ahead and futures market
- $\bar{Q}_i$ maximum output power of the producer i
- $\rho_{i,s}^{P,k}$ mixed probability-concern values of producer i for delivery period scenario s estimated on trading day k
- $\rho_{j,s}^{C,k}$ mixed probability-concern values of consumer j for delivery period scenario s estimated on trading day k
- $\bar{T}_{nm}/\underline{T}_{nm}$ upper/lower bounds for transmitted power in the line between area n and m
- B_{nm} susceptance of the transmission line between area n and m
- V_n nominal voltage of the transmission system at area n
- IR daily interest rate
- D_k number of the days between the first trading day and trading day k
- ω weighting factor of mixture PDF

D. Variables

- α_{ij}^c intercept of the bid function of producer i to consumer j in the futures market
- $\in_{ji}^c$ intercept of offer function of consumer j to producer i in the futures contract market
- $Q_{i,s}^{p,k}$ optimal power bid of producer i estimated on trading day k for scenario s of the delivery period
- $Q_{ij}^{c,k}/F_{ij}^{c,k}$ contract power/price of producer i with consumer j on trading day k of the futures market
- $Q_{ji}^{c,k}/F_{ji}^{c,k}$ contract power/price of consumer j with producer i on trading day k of the futures market
- $\lambda_{j,s}^k$ area j market price at delivery period scenario s estimated on trading day k
- $\lambda_{hub,s}^k$ hub node price at delivery period scenario s estimated on trading day k
- $W_{j,s}^k$ congestion-based wheeling fee for transmitting power from hub node to area j at delivery period scenario s estimated on trading day k
- $\gamma_{j,s}^k$ injected power of the day-ahead market from the hub node to area j at uncertainty scenario s estimated on trading day k from the TSO viewpoint
- $x_{i,j,s}^k$ injected power of the day-ahead market from the hub node to area j at uncertainty scenario s estimated on trading day k from the viewpoint of producer i
- $z_{j,n,s}^k$ injected power of the day-ahead market from the hub node to area n at uncertainty scenario s estimated on trading day k from the viewpoint of consumer j
- $\delta_{n,s}^k$ phase angel of area n at delivery period scenario s estimated on trading day k
- $R_{ij}^{P,k}$ total revenue of producer i from contract with consumer j in the futures market on trading day k
- $R_{ij}^{C,k}$ total cost of consumer j from contract with producer i in the futures market on trading day k
- $F^{Av,k}$ futures market price on trading day k
- $\psi_{j,s}^k$ Lagrangian multiplier related to the constraint (4)
- $\bar{\mu}_{jm,s}^{L,k}$ Lagrangian multiplier related to the constraint (5)
- $\underline{\mu}_{jm,s}^{L,k}$ Lagrangian multiplier related to the constraint (6)

$\bar{\mu}_{i,s}^{k}$ Lagrangian multiplier related to the constraint (8)
$\mu_{i,n,s}^{bub,k}$ Lagrangian multiplier related to the constraint (10)
$\mu_{i,n,s}^{x,k}$ Lagrangian multiplier related to the constraint (11)
$\mu_{j,n,s}^{bub,k}$ Lagrangian multiplier related to the constraint (16)
$\mu_{j,n,s}^{z,k}$ Lagrangian multiplier related to the constraint (17)

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APPENDIX

In this appendix, the process of finding the Nash equilibrium of the system is presented in more detail. In order to find the Nash equilibrium of the system, first KKT optimally condition of producers, consumers and TSO optimisations are derived. The KKT optimally condition of TSO at trading day k for scenario s of the delivery period is as below:

$$\frac{\partial L\,(TSO)}{\partial \gamma_{j,s}^{k}} = -W_{j,s}^{k} + \psi_{j,s}^{k} = 0 \quad \forall j \in C, s \in S_k \tag{20}$$

$$\frac{\partial L\,(TSO)}{\partial \delta_{j,s}^{k}} = B^{T}{}_{j}\psi_{s}^{td} + E^{T}{}_{n}\left(\bar{\mu}_{s}^{L,k} - \underline{\mu}_{s}^{L,k}\right) = 0 \quad \forall j \in C, s \in S_k \tag{21}$$

$$\sum_{m\in A} V_j V_m B_{jm}\left(\delta_{j,s}^{k} - \delta_{m,s}^{k}\right) = -\left(\gamma_{j,s}^{k} + \sum_{r=1}^{k}\left(\sum_{i\in P} Q_{ji}^{c,r} - \sum_{i\in P_j}\sum_{j\in C} Q_{ij}^{c,r}\right)\right) \quad \forall j \in C, s \in S_k \tag{22}$$

$$V_j V_m B_{jm}\left(\delta_{j,s}^{k} - \delta_{m,s}^{k}\right) \le \bar{T}_{jm}, \perp \bar{\mu}_{jm,s}^{L,k} \ge 0 \quad \forall (j,m) \in L, s \in S_k \tag{23}$$

$$V_j V_m B_{jm}\left(\delta_{j,s}^{k} - \delta_{m,s}^{k}\right) \ge \underline{T}_{jm}, \perp \underline{\mu}_{jm,s}^{L,k} \ge 0 \quad \forall (j,m) \in L, s \in S_k \tag{24}$$

In Equation (2), $B(n,m) = Img(Y_{BUS}(n,m))$, where Y_{BUS} is the admittance matrix of the grid. $B^{T}{}_{n}$ refer to row n of the transpose of B. E is an $|L| \times |C|$matrix in which if line l is connected to nodes n and j and the power is transmitted from (to) node n then $E(l,n) = V_n V_j B_{nj}$ ($E(l,n) = -V_n V_j B_{nj}$), else $E(l,n) = 0$. Vectors ψ_{s}^{td}, $\bar{\mu}_{s}^{L,k}$,$\underline{\mu}_{s}^{L,k}$ include variables $\psi_{j,s}^{k} \forall j \in C$, $\bar{\mu}_{jm,s}^{L,k} \forall (j,m) \in L$, and $\forall (j,m) \in L\, \mu\_jm, s^{L,k}$, respectively. The KKT optimally condition of producer i in trading day k is written as below:

$$\frac{\partial L\left(\pi_i^k\right)}{\partial Q_{i,s}^{p,k}} = \rho_{i,s}^{p,k}\left[-\lambda_{n(i),s}^k + d_{n(i)}^e Q_{i,s}^{p,k} + a_i + b_i^e\left(Q_{i,s}^{p,k} + \sum_{r=1}^{k}\sum_{j\in C} Q_{ij}^{p,k}\right)\right] + \bar{\mu}_{i,s}^k - d_{n(i)}^e \mu_{i,n(i),s}^{hub,k} \geq 0 \perp Q_{i,s}^{p,k} \geq 0 \quad \forall i \in P, s \in S_k \tag{25}$$

$$\frac{\partial L\left(\pi_i^k\right)}{\partial \alpha_{i,j}^{f,k}} = \sum_{s\in S_k}\rho_{i,s}^{p,k}\left[-d_j^e Q_{i,s}^{A,p,k} - \left((1+IR)^{365-k} - 1\right), F^{Av,k}\left(-(1+IR)^{365-k} d_j^e Q_{ij}^{p,k} + (1+IR)^{365-k} F_{ij}^{p,k}\left(+\left[(1+IR)^{365-k} - 1\right], \frac{d_j^e}{M}\left(\sum_{r=1}^{k}\sum_{l\in C} Q_{il}^{p,k}\right) - a_i - b_i^e\left(Q_{i,s}^{p,k} + \sum_{r=1}^{k}\sum_{i\in P} Q_{n,i}^{c,k}\right)^2\right]\right.\right. - \sum_{s\in S_k}\bar{\mu}_{i,s}^k + \sum_{s\in S_k} d_j \mu_{i,n(i),s}^{hub,k} \quad \forall i \in P, j \in C, j = n(i) \tag{26}$$

$$\frac{\partial L\left(\pi_i^k\right)}{\partial \alpha_{i,j}^{f,k}} = \sum_{s\in S_k}\rho_{i,s}^{p,k}\left[-\left((1+IR)^{365-k} - 1\right), F^{Av,k}\left(-(1+IR)^{365-k} d_j^e Q_{ij}^{p,k} + (1+IR)^{365-k} F_{ij}^{p,k}\left(+\left[(1+IR)^{365-k} - 1\right], \frac{d_j^e}{M}\left(\sum_{r=1}^{k}\sum_{l\in C} Q_{il}^{p,k}\right), -a_i - b_i^e\left(Q_{i,s}^{p,k} + \sum_{r=1}^{k}\sum_{i\in P} Q_{n,i}^{c,k}\right)^2\right]\right.\right. - \sum_{s\in S_k}\bar{\mu}_{i,s}^k + \sum_{s\in S_k} d_j^e \mu_{i,n(i),s}^{hub,k} \quad \forall i \in P, j \in C, j \neq n(i) \tag{27}$$

$$\frac{\partial L\left(\pi_i^k\right)}{\partial x_{i,n(i),s}^k} = \rho_{i,s}^{p,k} d_{n(i)}^e Q_{i,s}^{p,k} - d_{n(i)}^e \mu_{i,n(i),s}^{hub,k} + \mu_{i,n(i)s}^{x,k} = 0 \quad \forall i \in P, s \in S_k \tag{28}$$

$$\frac{\partial L\left(\pi_i^k\right)}{\partial x_{i,m,s}^k} = -d_m^e \mu_{i,n(i),s}^{hub,k} + \mu_{im,s}^{x,k} = 0 \quad \forall i \in P, s \in S_k,\ m \neq n(i) \tag{29}$$

$$Q_{i,s}^{p,k} + \sum_{r=1}^{k}\sum_{i\in P} Q_{n,i}^{fc,k} \leq \bar{Q}_i \perp \bar{\mu}_{i,s}^k \geq 0 \quad \forall i \in P, s \in S_k \tag{30}$$

$$c_n - d_n^e\left(\sum_{i\in P(n)} Q_{i,s}^{p,k} + \sum_{r=1}^{k}\sum_{i\in P} Q_{n,i}^{c,r} + x_{i,n,s}^k\right) - \lambda_{hub,s}^k - W_{n,s}^k = 0 \quad \left(\mu_{i,n,s}^{hub,k}\right) \quad \forall n \in C, s \in S_k \tag{31}$$

$$\sum_{n\in C} x_{i,n,s}^k = 0 \quad \left(\mu_{i,s}^{x,k}\right) \quad \forall i \in P, s \in S_k \tag{32}$$

where $M = |P| \times |C|$. The KKT optimally condition of consumer j in trading day k is written as below:

$$\frac{\partial L\left(U_j^k\right)}{\partial \in_{ji}^{f,k}} = \sum_{s\in S_k}\rho_{j,s}^{C,k}\left[-d_j^e Q_{j,s}^{c,k} - \left((1+IR)^{365-k} - 1\right), F^{Av,k}\left(+(1+IR)^{365-k} b_i^e Q_{ij}^{c,k} + (1+IR)^{365-k} F_{ij}^{c,k}\left(-\left[(1+IR)^{365-k} - 1\right], \frac{b_i^e}{M}\left(\sum_{r=1}^{k}\sum_{m\in P} Q_{mj}^{c,k}\right) - c_j + d_j^e\left(Q_{j,s}^{c,k} + \sum_{r=1}^{k}\sum_{m\in P} Q_{mj}^{c,k}\right)\right]\right.\right. - \sum_{s\in S_k} d_j^e \mu_{j,n,s}^{hub,k} = 0 \quad \forall i \in P, j \in C \tag{33}$$

$$\frac{\partial L\left(U_j^k\right)}{\partial z_{jn(j),s}^k} = -d_j^e \rho_{j,s}^{C,k} Q_{j,s}^{c,k} - d_j^e \mu_{j,n,s}^{hub,k} + \mu_{j,s}^{z,k} = 0 \quad \forall j \in C, s \in S_k \tag{34}$$

$$\frac{\partial L\left(U_j^k\right)}{\partial z_{jm,s}^k} = -d_m^e \mu_{j,m,s}^{hub,k} + \mu_{jm,s}^{z,k} = 0 \quad \forall j \in C, s \in S_k,\ m \neq n(i) \tag{35}$$

$$Q_{j,s}^{c,k} = \sum_{i\in P(n(j))} Q_{i,s}^{p,k} + Q_{n(j),s}^{m,k} + z_{jn,s}^k \quad \forall s \in S_k \tag{36}$$

$$c_n - d_n^e\left(\sum_{i\in P(n)} Q_{i,s}^{p,k} + \sum_{r=1}^{k}\sum_{i\in P} Q_{ji}^{c,r} + z_{j,n,s}^k\right) - \lambda_{hub,s}^k - W_{n,s}^k = 0 \left(\mu_{j,n,s}^{hub,k}\right) \quad \forall n \in C, s \in S_k \quad \forall n \in C, s \in S_k \tag{37}$$

$$\sum_{n\in C} z_{j,ns}^k = 0, \left(\mu_{j,s}^{z,k}\right) \quad \forall j \in C, s \in S_k \tag{38}$$

Considering Equation (19), Equations (31) and (37) are similar for all market players. Hence, these equations can be turned into one equation. A similar process is followed for Equations (38) and (32). Now, by solving the remained equalities and inequalities constraints obtained from KKT conditions of all optimisations, the Nash equilibrium of the model will be found.