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USING DIFFERENTIAL TRANSFORM METHOD (DTM) FOR SOLVING SOME FAMOUS PARTIAL DIFFERENTIAL EQUATIONS (PDE)

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Abstract. In this paper, we propose a reliable algorithm to develop exact and approximate solutions for the linear and nonlinear famous partial differential equations. We begin by showing how the DTM applies to the linear and nonlinear parts of any PDE and then give one example to illustrate the sufficiency of this method for solving such PDE's. This method can easily be applied to many linear and nonlinear problems and is capable of reducing the size of computational work. Exact solutions can also be achieved by the known forms of the series solution. It is an useful tool for analytical solutions.

1. Introduction and Preliminaries

Differential transform method (DTM) is a semi-numerical-analytic-technique that formalizes the Taylor series in a totally different manner. It was first introduced by Zhou in a study about electrical circuits [5], and Chen developed DTM to approximate the PDE's [3]. Using this technique, the given differential equation and related initial conditions are transformed into a recurrence equation that finally leads to the solution of a system of algebraic equations as coefficients of a power series solution . There is no need for linearization or perturbations, also large computational work and round-off errors are avoided.

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In the last few years, this method has been used to solve effectively, easily and accurately a large class of linear and non-linear partial differential equations.[2] The aim of this paper is to extend the differential transformation method to solve some nonlinear PDE's, e.g, the one-dimensional Boussinesq-like B(m,n) equation. The accuracy of the numerical results will be compared with that of the analytical ones. Note that we have computed the numerical results by MATLAB 7.0 programming.

2. Main results

With reference to the articles [1, 2, 3], in this section we introduce the basic definitions of the two-dimensional differential transformation method

Definition 2.1. If $w(x,t)$ is an analytic and continuously differentiable with respect to time t in the domain of interest, then the differential transform $W(k,h)$ of $w(x,t)$ at (x_0,t_0) is defined by:

$$W(k,h)=\frac{1}{k!h!}[\frac{\partial^{k+h}}{\partial x^k.\partial t^h}w(x,t)]_{x=x_0,t=t_0} \tag{2.1}$$

where the spectrum function $W(k,h)$ is the transformed function, which is also called the T-function for short.

Definition 2.2. The differential inverse transform of $W(k,h)$ is defined as follows

$$w(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)(x-x_0)^k.(t-t_0)^h \tag{2.2}$$

Combining Eqs.(2.1) and (2.2), and assuming $x_0=t_0=0$, we obtain

$$w(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}[\frac{\partial^{k+h}}{\partial x^k.\partial t^h}w(x,t)]_{x=x_0,t=t_0}x^k.t^h=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)x^k.t^h \tag{2.3}$$

In real applications, the function $w(x,t)$ is represented by a finite series of Eq.(2.3) which can be written as:

$$w(x,t)=\sum_{k=0}^{n}\sum_{h=0}^{m}W(k,h)x^k.t^h+R_{n,m}(x,t) \tag{2.4}$$

Eq.(2.3) implies that, $R_{n,m}(x,t)=\sum_{k=n+1}^{\infty}\sum_{h=m+1}^{\infty}W(k,h)x^k.t^h$, which is negligibly small. Usually, choosing the values of n,m are decided by convergency of the series coefficients. From the above definitions, it can be found that the concept of the two-dimensional differential transform is derived from the two-dimensional Taylor series expansion. With Eqs.(2.2) and (2.3), the fundamental mathematical operations performed by the two-dimensional differential transform can be readily obtained and are listed in Table 1.

TABLE 1. Operations of the two-dimensional differential transform

Original function $w(x,t)$	Transformed function $W(k,h)$
$u(x,t) \pm v(x,t)$	$U(k,h) \pm V(k,h)$
$\lambda u(x,t)$	$\lambda U(k,h)$
$x^m.t^n$	$\delta(k-m,h-n) = \begin{cases} 1, & k=m, h=n; \\ 0, & \text{o.w} \end{cases}$
$\frac{\partial^{r+s}}{\partial x^r \partial t^s} u(x,t)$	$\frac{(k+r)!(h+s)!}{k!h1} U(k+r,h+s)$
$u(x,t).v(x,t)$	$\sum_{r=0}^{k} \sum_{s=0}^{h} U(r,h-s)V(k-r,s)$
$u(x,t).v(x,t).q(x,t)$	$\sum_{r=0}^{k} \sum_{t=0}^{k-r} \sum_{s=0}^{h} \sum_{p=0}^{h-s} U(r,h-s-p)V(t,s)Q(k-r-t,p)$

By using the same theory as for the two-dimensional differential transform, one can reach the three-dimensional case.

Example 2.3. Consider the B(2,2) equation with the initial conditions

$$u_{tt} - (u^2)_{xx} - (u^2)_{xxxx} = 0 \tag{2.5}$$

$$u(x,0) = \frac{4}{3} sin^2(\frac{x}{4}) \tag{2.6}$$

$$u_t(x,0) = \frac{1}{3} sin(\frac{x}{2}) \tag{2.7}$$

To solve this initial value problem, we take two-dimensional differential transform from both sides of Eq.(2.5) to obtain

(2.8)

$$(h+1)(h+2)U(k,h+2) + \sum_{r=0}^{k} \sum_{s=0}^{h} [-2.(r+1)U(r+1,h-s)(k-r+1)U(k-r+1,s)$$

$$-2.U(r,h-s)(k-r+1)k-r+2)U(k-r+2,s) - 8.(r+1)U(r+1,h-s)(k-r+1)(k-r+2)(k-r+3)U(k-r+3,s) - 6.(r+1)(r+2)U(r+2,h-s)(k-r+1)(k-r+2)U(k-r+2,s) - 2.U(r,h-s)(k-r+1)k-r+2)(k-r+3)(k-r+4)U(k-r+4,s)] = 0$$

From the initial condition (2.6) we get

$$u(x,0) = \sum_{k=0}^{\infty} U(k,0)x^k = \frac{4}{3} sin^2(\frac{x}{4}) = \frac{4}{3} \sum_{k=0}^{\infty} \frac{(sin^2(\frac{x}{4}))^{(k)}_{(0)}}{k!}.x^k \tag{2.9}$$

and Eq.(2.7) yields

$$u_t(x,0)=\sum_{k=0}^{\infty}U(k,1)x^k=\frac{1}{3}sin(\frac{x}{2})=\frac{1}{3}\sum_{k=0}^{\infty}\frac{(sin(\frac{x}{2}))^{(k)}_{(0)}}{k!}.x^k \tag{2.10}$$

Substituting (2.9) and (2.10) into (2.8), all values of $U(k,h)$ for $k,h=0,1,2,...$ can be obtained, after substituted the obtained results, into Eq. (2.3), implies that

(2.11)

$$u(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U(k,h)x^kt^h=\frac{1}{12}(x^2+t^2)-\frac{1}{4^3.9}t^4+\frac{1}{6}x.t-\frac{2}{4^2.18}x.t^3+\frac{2}{4^6.9}x.t^5$$
$$-\tfrac{1}{4^2.6}x^2.t^2+\tfrac{5}{4^5.36}x^2.t^4-\tfrac{1}{4^2.9}x^3.t+\tfrac{5}{4^4.108}x^3.t^3+...$$

This is the same as the Taylor expansion of the exact solution [4].

$$u(x,t)=\frac{4}{3}sin^2(\frac{x+t}{4}) \tag{2.12}$$

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