12th International Congress on Civil Engineering, 12-14 July 2021
Ferdowsi University of Mashhad, Mashhad, Iran

Determination of the Effective Moment of Inertia for RC Beams strengthened with FRP Sheets Using Multi-Expression Programming

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Abstract

Large deflections in reinforced concrete (RC) beams under different loadings can cause cracking of walls and damage to nonstructural components. The long-term deflection of RC beams is one of the design issues that must be accurately controlled by designers to satisfy serviceability requirements. Flexural strengthening is one of the ways to deal with this phenomenon of excessive deflection in RC beams. Nowadays, using Fiber-Reinforced Polymers (FRP) is a common method for strengthening and rehabilitation of RC structural members. In recent years, many researches have been performed on the deflection and effective moment of inertia of RC beams strengthened with FRP bars, but investigations on the effective moment of inertia of RC beams reinforced with FRP sheets by EBR method are relatively limited and the need for further studies in this regard is felt. The current design codes use simple and approximate methods to calculate deflection of these structural elements, but to prevent their unexpected behavior under service and ultimate load, more precise relationships are needed. Estimation of deflections under the service load level requires the effective moment of inertia of the beams. Various studies have been conducted to determine the relationships for the effective moment of inertia of RC beams strengthened with FRP sheet, but the results showed that the accuracy of the existing relationships is not acceptable. In this research, using an evolutionary algorithm called multi-expression programming (MEP) and a database of experimental results collected from literature, a relation for the effective moment of inertia of such beams has been proposed. Comparison of the accuracy of the propose model with other existing relations proves higher accuracy of the suggested model

Keywords: RC beam, Effective moment of inertia, Multi-Expression Programming, FRP, Deflection

1. INTRODUCTION

Various factor such as errors in the design, failure of materials and construction errors, may cause problems in the performance of RC structures. In addition, changes in building codes or change in the use can necessitate a re-evaluation of the existing of structures, and in some cases the structures might need to be rebuilt or strengthen. Nowadays, the strengthening and retrofitting methods are among the interesting topics that have attracted many researchers. Using FRP sheets is one of most applicable strengthening methods for RC reinforcing structural members, due to positive mechanical properties of this system, such as high stiffness, high weight to resistance ratio and the proper resistance against creep and fatigue [1-7]. Beams are one of the most important structural members that undergo various deformations due to the applied loads and addition of the FRP sheet reinforcement to their bottom face is a practical method to increase their bending capacity and ductility and thus improve their performance level. This approach was first used for strengthening of bridge girders by installation of steel sheets with epoxy resin [8]. Widespread application of composites materials due to their suitable properties compared to steel sheets, resulted in many experimental and numerical studies on the possible applications of composite materials for strengthening RC structures.

For instance, Saadatmanesh and Ehsani [9] performed experimental study on the rectangular and T-shaped concrete beams strengthened with GFRP sheets, and observed that the FRP strengthening improves the flexural strength, especially for the beams with lower steel reinforcement ratio. Philip et al. [10] examined the influence of three parameters of FRP sheet type, size and the installation methods on the behavior of RC beams. Sharif et al. [11] performed research on RC beams strengthened with FRP sheets having different thicknesses and different bonding methods. Chajes et al. [12] conducted four-point bending tests on RC beams strengthened with Aramid, Glass and Graphite FRP sheets and estimated their bending strength. They found that addition of external FRP sheets can improve flexural strength and stiffness of RC beams about 40 to 60%. Aboutaha et al. [13[studied ductility of CFRP strengthened RC beams and reported that the FRP debonding is the major failure type of the system, which significantly affect ductility and strength of strengthened beams. Ashour et al. [14] tested 16 sample RC beams with externally bonded FRP sheet and found that the addition of FRP sheet improvs flexural capacity, while reducing ductility of the sample beams. Maalej and Leong [15] investigated the effect of beam depth on the performance FRP strengthened RC beams, and found that there is no significant relationship between the beam depth and its ductility. Alzaid et al. [16] proposed an analytical model for predicting the flexural capacity of FRP strengthened RC beams. This model is developed based on considering different failure modes by estimating the strains in concrete, steel rebar and FRP sheet.

In spite of improving flexural capacity of RC beams, the brittle of RC beams strengthened with FRP sheets is a challenge, which can be partly solved by utilization of sheets from different types (hybrid combination) [17-19]. In addition to improving flexural strength of RC beams, FRP strengthening should results in appropriate performance of structural members under the service loads. This cannot be achieved by simply increasing strength of members, because for example a member which is designed based on the ultimate strength approach may experience large deflections under service loads. These large deflections in turn can cause damage to non-structural components or the corrosion of internal reinforcing steel bars may increase due to the formation of deep cracks in the beam. Therefore, it is necessary to control the deflection of FRP reinforced RC beams in addition to the strength controls. One of the important parameters form the serviceability standpoint is the short-term deflection. The short-term deflection of normal RC beams is a function of moment of inertia. ACI318-05 propose to utilize the effective moment of inertia after cracking for this purpose, which is derived from the relation proposed by Branson et al. [20]. However, this relation is applicable for RC beams with FRP rebar and ACI440-2R-17 has no suggestion for calculation of RC beams strengthened with external FRP sheets [21]. In the mentioned standard it is only stated that the deflection of beams must comply with the service load requirements of ACI318. The effect of FRP sheets on the deflection of strengthened beams can be computed using the equivalent section analysis method. Moreover, it should be noted that the Italian and European standards also do not proposed any relationship for calculation of de the effective moment of inertia of RC beams with externally bonded FRP sheets, just proposed to utilize the integral of moment-curvature diagrams in order to compute deflections [22, 23]. Therefore, Habibi and Tavakkolizadeh [24] utilized the approach proposed by Hall and Ghali [25] to modify the Branson relation [26] for RC beams with externally bonded FRP. However, they proposed some modified relations for predicting the effective moment of inertia of FRP strengthened RC beams, but evaluations showed that accuracy of these relation can be further improved. Therefore, the objective of the present study is to utilize a database of experimental results attained from four-point bending tests on FRP strengthened RC beams and an evolutionary algorithm called Multi-Expression Programming to propose new and more accurate predictive relations for the effective moment of inertia.

2. THE BASIC RELATIONS

As mentioned in the previous section, the experimental results from four-point bending tests are used in the present study. The maximum deflection of a simple RC beam with external FRP sheets is derived from the following relation:

$$\delta_{\max} = \frac{PL_a}{48E_c I}\left(3L^2 - 4{L_a}^2\right) \qquad (1)$$

In this equation, L is the beam span, P is the total applied load which is divided to two $\frac{P}{2}$ loads at a distance of L_a from the supports. E_c and I are the concrete elastic modulus and moment of inertia of the beam, which should be replaced with the effective moment of inertia after cracking, I_e As mentioned previously. The objective of the present study is to propose an accurate relationship for the effective moment of inertia of

FRP strengthened RC beams using the experimental data reported by various researchers and a variant of genetic programming, called MEP. Calculation of the effective moment of inertia requires knowledge of the cracking moment and the moment of inertia of the equivalent cracked section. The former is derived from Eq. (2), in which h and $f_c^{'}$ are depth of the beam section and the compressive strength of concrete, respectively. I_g is the moment of inertia of the gross section.

$$M_{cr} = 1.2\sqrt{f_c^{'}}\,\frac{I_g}{h}\,. \tag{2}$$

$$I_g = \frac{bh^3}{12}\,. \tag{3}$$

b in Eq. (3) is the cross-section width of beam.

Fig. 1 demonstrates the transformed equivalent cracked section of the beam in which, determines the neutral axis location and A_s ,d , b_p and t_p are cross-section area of the tensile steel rebar, effective depth of beam, and width and thickness of the reinforcing FRP sheet. The moment of inertia of cracked section is derived from Eqs. (4) to (6).

$$I_{cr} = \frac{E_c b_c X^3}{12E_p} + \frac{E_c X b_c}{E_p}\left(\frac{X}{2}\right)^2 + \frac{A_s E_s}{E_p}\left(d - X\right)^2 + b_p t_p\left(h - X\right)^2. \tag{4}$$

$$X = \frac{-B + \left(B^2 + 4AC\right)^{0.5}}{2A}\,. \tag{5}$$

$$A = \frac{E_c b_c}{2E_p} \qquad B = \frac{A_s E_s}{E_p} + b_p t_p \qquad C = \frac{A_s d E_s}{E_p} + hb_p t_p\,. \tag{6}$$

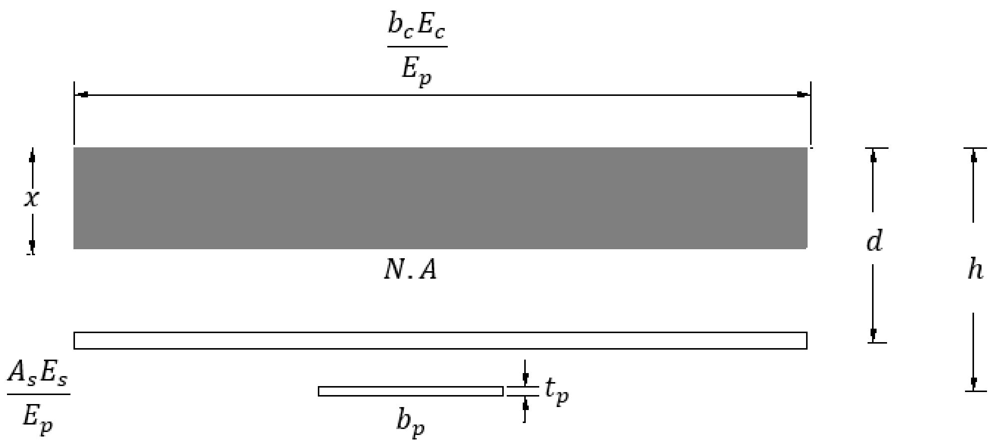

Fig. 1- The transformed equivalent cracked beam section

In the previous equations, $\frac{E_s}{E_p}$ is the moduli ratio of steel to FRP. Using the above-mentioned relations, the value of effective moment of inertia is computed for a set of experimental tests results, which are presented in the next section. Then the computed values are used as input of MEP to derive the new predictive relation.

3. EXPERIMENTAL DATABASE

Various studies have been performed on the deflection behavior of RC beams with externally bonded FRP. In the present study, a series of the experimental data from four-point bending tests are collected, which are presented in Table 1 It is attempted to select studies such that to cover a common variation ranges of all effective parameters. Table 1 only gives summary of values of the effective parameters in the selected experimental tests.

Table 1- Summary of the selected experimental results

Ref	b (mm)	h (mm)	No. of tension rebar	Diameter of tension rebar (mm)	f_{co} MPa)	E_f M Pa)	f_f (MPa)	L_a (mm)	Diameter of stirrup (mm)	Type of rebar
Ahmed et al.[27]	125	225	2	8	41/42	240000	3500	500	6	Carbon
Rahimi, and Hutchinson[28]	200	150	2	10/16	50	127000 /36000	1074/1 532	750	6	Carbon/ Glass
Dong et al.[29]	152	305	2	10	38.2	73100	960	990	6	Carbon
Alagusundara moorthy et al.[30]	230	380	2	25	31	138000 /22800 0	490/20 68	1830	9	Carbon
Pham and Al-Mahaidi[31]	140	260	2/3	12	47.7/53.7	213500	3900	700	10	Carbon
Almusallam, and Al-Salloum[32]	150	200	3	10	35.5/36.5	30000	600	875	8	Glass
Gao et al.[33]	150	200	2	10	47.8/62.1	235000	4200	500	8	Carbon
Esfahani et al.[34]	150	200	2	12/16/20	23.8/24.1/ 25.2	237000	2845	600	8	Carbon

HE PROPOSED MODEL

Multi-Expression Programming (MEP) is a variant of genetic programming which has the ability to program multiple solutions within a same chromosome [35]. In the simplest form of MEP, chromosomes are linear strings. As mentioned, the main advantage of MEP is its ability in coding multiple responses within a chromosome, which provide the opportunity to search wider spaces of the search space. The algorithm commences with production of a random population, Then, couples of the parents are selected based on binary evaluation process, and the next generation is produced by combination of these parents or mutation and replacing the worsts of the current generation with the best produced children. This process is repeated until reaching the best expression or reaching the predefined number of generations [35]. Recently, MEP attracted various researchers in the field of civil and structural engineering, especially as a computational tool for performing nonlinear programming and proposition of predictive mathematical models in the form of functions of multiple variables [36-38].

The experimental data collected from the literature (see Table 1) are used as inputs of MEP algorithm, and also for its verification. 70% of the experimental results were randomly selected for derivation of the relation. After performing multiple analysis using different sets of algorithm parameters, the following Eq. is derived for the effective moment of inertia of RC beams with externally bonded FRP sheet:

$$I_e = \left(\frac{Mcr}{Ma}+1\right)\left(\frac{2I_{cr}}{\left(\frac{\rho}{\rho_b}\right)^2}\right)+\left(\frac{\rho}{\rho_b}\right)I_{cr}+\frac{2Icr}{\left(\frac{\rho}{\rho_b}\right)}. \tag{7}$$

The algorithm parameters that result in this equation are given in Table 2. The remaining 30% of experimental data were used for the accuracy evaluation of the proposed relation

Table 2- The MEP parameters used in derivation of the proposed model

Code length	20
Probability of operators	0.4
Probability of variables	0.5
Probability of constants	0.1
Number of constants	4
Crossover probability	0.95
Mutation probability	0.015
Tournament size	2
Number of runs	40
Number of generations	100
Number subpopulations	60
Subpopulation size	100

The accuracy of the proposed method is also compared with three relations suggested by Habibi and Tavakkolizadeh, which are given by Eq. (8) to (10) [24]:

$$I_e = I_g \left(\frac{Mcr}{Ma}\right)^2 + 2.6 I_{cr} \left(1 - \left(\frac{Mcr}{Ma}\right)^{0.25}\right). \tag{8}$$

$$.\, I_e = \frac{0.93 I_g I_{cr}}{I_{cr} + 1.25\left(\frac{Mcr}{Ma}\right)^{1.8} \left(I_g - I_{cr}\right)}. \tag{9}$$

$$I_e = I_g \left(\frac{Mcr}{Ma}\right)^{2.36} + 2.07 I_{cr} \left(1 - \left(\frac{Mcr}{Ma}\right)^{1.32}\right). \tag{10}$$

Three different error measures, namely Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Integral Absolute Error (IAE) are used for the performance assessment. These error measures are computed as follows:

- Root Mean Square Error:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{m} \left(\frac{X_{the} - X_{exp}}{X_{exp}}\right)^2}{n}}. \tag{11}$$

- Mean Absolute Error:

$$MAE = \frac{\sum_{i=1}^{m} \left|\frac{X_{the} - X_{exp}}{X_{exp}}\right|}{n}. \tag{12}$$

- Integral Absolute Error:

$$IAE = \frac{\left(\sum_{i=1}^{m} \sqrt{\left|\left(X_{the} - X_{exp}\right)^2\right|}\right)}{\sum_{i=1}^{n} X_{exp}}. \tag{13}$$

Here, X is a general quantity, and X_{the} and X_{exp} are the theoretical and experimental estimations of this quantity, respectively. $\bar{X}_{the}$. and $\bar{X}_{exp}$ are the mean values of the theoretical and experimental results, respectively, and m is the number of experiments. Obviously, lower values for these measures indicate higher accuracy of the model. The attained results are presented in Table 3. Based on the attained results, it is evident that proposed relationship is considerably more accurate than the other available formulae.

Table 3- Results of the accuracy evaluation of the relations for the effective moment of inertia

Model	RMSE	MAE	IAE	R^2
Proposed Equation	6.64	53.58	0.424	0.77
Habibi and Tavakkolizadeh ,1st eq	75.66	51.96	0.577	0.85
Habibi and Tavakkolizadeh ,2nd eq	71.59	147.89	0.586	0.49
Habibi and Tavakkolizadeh ,3rd eq	30.73	175.301	1.779	0.76

5. CONCLUSION

Due to importance of the deflection form the serviceability standpoint, various studies have been performed on the deflection behavior of FRP strengthened RC beams. Despite the extensive research in this field, there is limited reliable relations for prediction of the deflection of RC beams strengthened with externally bonded FRP sheet. A necessary parameter for calculation of the FRP strengthened RC beams is the effective moment of inertia. Therefore, using a database of experimental results and Multi-Expression Programming, a new predictive model was proposed for the effective moment of inertia. Three different error measures were used to compare accuracy of the proposed model with some of the existing limited relations. Based on the attained results, the suggested model has a remarkably higher accuracy in prediction of the effective moment of inertia for RC beams with externally bonded FRP sheets, and can improve accuracy of the deflection predictions for these beams.

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