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Admissibility analysis for autonomous underwater vehicle based on descriptor model with time-varying delay

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Abstract

This article investigates an enhanced optimal robust time-delay stabilizer for an autonomous underwater vehicle in the descriptor model. Time-delay, model uncertainty, and actuator saturation constraint are some practical challenges in autonomous underwater vehicle controller design. In this regard, an appropriate autonomous underwater vehicle descriptor model is obtained, and sufficient stabilization conditions are determined in the terms of linear matrix inequality. The obtained criterion guarantees the system to be regular, impulse-free, and stable. Meanwhile, the delay-dependent and rate-dependent conditions are taken into account. Furthermore, uncertainty and time-delay are time-variant. This method includes a tuning factor for practical design aspects and tradeoff among desired requirements. Also, as an essential general requirement in non-linear systems, the maximal estimate of the attraction domain is proposed as an optimization problem. Numerical examples and simulations illustrate that the proposed methods are effective and useful in less conservative results. The technique can be generalized and applied to the most conventional autonomous underwater vehicles.

Keywords

Autonomous underwater vehicle, descriptor systems, linear matrix inequality, actuator saturation

1. Introduction

Today, many marine investigators have focused on AUVs all over the world. AUVs receive widespread attention in many applications such as search and rescue, photometric survey, and environmental monitoring (Lei 2020). The controller should be able to deal with fundamental problems like time-delay, actuator saturation, uncertainty, and disturbances. In the past decade, different methods have been developed for AUV control ((Khodayari and Balochian, 2016) and (Khodayari and Balochian, 2015)). Nevertheless, they have ignored the impact of time-delays. Nowadays, many AUV's missions include regions as the nominal target rather than a particular route. Therefore, paying attention to familiarity with the bound of attraction region is very significant and should be considered in the design procedures. In the rest of this article, we hint at some main AUV stabilization problems.

Time-delay impact: In most industrial problems, natural phenomenon, and circuit systems, time-delay can impose a severe limitation on the controller design and even results in instability. Recognizing time-delay stability is reasonably hard because of its infinite dimension (Li et al., 2019). Generally, time-delay is divided into two main bifurcations: the delay-independent and delay-dependent. The delay-dependent system is less conservative than the delay-independent one. So, more investigations have been devoted to it (Han et al., 2017). In the AUV structure, there are some non-homogeneous primary delay sources such as delay of Doppler Velocity Log (DVL), Medium Access Delay (MAC), actuator delays, and thruster delay. One of the essential delay origins is transmission data delay between the control center and AUV in remote mode (Kim and Yoo,

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2018). From another aspect, because of the AUV trip, the distance between AUV and the control station is time-varying, inducing time-varying communication delays.

Actuator saturation constraint: on the other hand, most dynamic systems encounter saturation constraints of their actuators, adding an extra restriction on the AUV control analysis. Inspired by this, various techniques have been presented to handle saturation (Yan et al., 2018). However, the Domain of Attraction (DOA) has not been explored in these articles. Generally, the problem of actuator's saturation constraints can be solved using some primary techniques such as poly-topic solution or anti-windup techniques.

Descriptor (singular) systems: over the past decades, much attention has been devoted to exploring descriptor systems. Descriptor systems that are known by other names, such as singular systems or semi-state systems, need admissibility in addition to stability. It means that we need stability, regularity, impulse free, and causality (Chen et al., 2017). Descriptor time-delay systems are time-delay differential equations that tighten with algebraic equations. Hence, investigating these systems is much more complicated than standard state-space systems (Zhou et al., 2019). In singular time-delay system, due to its infinite dynamic modes and non-dynamic modes, the uniqueness and existence of system solutions may not be guaranteed, and it may have unexpected impulsive treatment.

Parameter uncertainty: AUV modeling is complicated due to the hydrodynamic coefficients estimation and parameter coupling. Besides, these parameters vary in different velocities, missions, and environments. Therefore, the theoretical methods are difficult to apply directly to the AUV control problem. In practice, sometimes, the gap between the practice and theory rises to 100%. The stability problem of the singular time-delay system has been explored by a useful theorem in El Haouti et al. (2020) with less conservative result; however, some practical vital issues like uncertainty and actuator saturation have not been considered simultaneously.

Different requirements concerning DOA are significant in dynamic systems. In most of the practical systems, a lower bound on DOA is an open issue. Most of the time, these estimations are conservative and need more considerations. Therefore, reasonable estimation on DOA becomes vital. For nonlinear time-delay systems, DOA has been studied rarely despite their significant role in engineering systems. DOA in nonlinear time-delay systems has been studied in Scholl et al. (2020); however, it includes only conventional state-space and does not cover singular time-delay systems.

Here, we present two stages to solve the problem; the first one transforms singular system into an equivalent neutral one under a reasonable assumption. In the second stage, delay-dependent stability criterion is proposed for stability in terms of LMI (based on the Lyapunov–Krasovskii Functional (LKF)) to tackle singularity, time-delay, and compensation of actuator saturation impact with time-variable parameter uncertainty.

Almost AUV control problems are categorized into two main branches. The first issue is tracking and the second one is stabilization. Depending on AUV missions, it can include some nominal points that also are called as trim points, so stabilization in each trim point is very vital in this subject. To explore the stabilization methods, based on Lyapunov theory, we can refer to Makhlouf (2018), and in a more complex statement with mixed time-delay, it is better to study Naifar et al. (2020). In Naifar et al. (2020), quasiuniform stability for fractional-order systems has been developed and can be enumerated as the new useful method.

Based on our knowledge, the AUV stabilization in the uncertain descriptor time-delay model by desired design parameters in the presence of uncertainty and actuator constraints has not been addressed adequately yet and rare works have dealt with this. The present result can be used in trial stability of singular time-delay systems, retarded systems, and neutral ones with desired tuning parameters that would be useful in desired practical requirements.

The rest of this article is organized as follows: In Section 2, useful notations are presented. In Section 3, the problem statement with some helpful definitions and lemmas is expressed. In Section 4, numerical examples and comparison of the proposed method with previous techniques are given. In Section 5, AUV general equations in descriptor form in-depth channel are provided, and some simulation results for a REMUS100 (Remote Environmental Monitoring Units) AUV in-depth channel are given to verify the effectiveness of the proposed algorithms. Finally, the article is concluded in Section 6.

2. Notations

Symbol	Explanation
T	Matrix transposition
$P > 0$	P is symmetric and positive definite matrix
$\mathrm{sym}\{\cdot\}$	$\mathrm{sym}\{X\} = X + X^T$
$\mathrm{col}\{\ldots\}$	Column vector
$\mathrm{diag}\{\ldots\}$	Block diagonal matrix
$\bar{\lambda}(P)$	Maximal eigenvalue of matrix P
$co\{\cdot\}$	The convex hull of a set
ρ	Spectral radius of the matrix

3. Problem statement and preliminaries

In this section, some useful definitions and lemmas are presented, a new theorem is proved, and another helpful theorem is introduced.

3.1. Definitions, theorems, and lemmas

In this article, we consider the following singular time-delay systems subject to norm-bounded uncertainties as follows

$$\begin{cases} E\dot{x}(t) = (A_0 + \Delta A_0)x(t) + (A_d + \Delta A_d)x(t-h(t)) \\ \qquad\qquad + B\,\mathrm{sat}(u(t)) \\ x(t) = \phi(t) \quad t \in [-h,0] \quad 0 \le h(t) \le h_m \quad \dot{h}(t) \le d_1 < 1 \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input emitted from the designed controller, and E, A_0, and $A_d \in R^{n\times n}$ are known real constant matrices. We assume *rank* $E = r \le n$. $h_m > 0$ is as maximum discrete time-delay. ΔA_0 and ΔA_d represent norm-bounded uncertainty of the system's main matrices. The saturation function is defined with the following equations

$$\begin{aligned} &\mathrm{sat}(u(t)) = [\mathrm{sat}(u_1(t)), \mathrm{sat}(u_2(t)), \ldots \mathrm{sat}(u_m(t))]^T \\ &\text{where } \mathrm{sat}(u_i(t)) = \mathrm{sign}(u_i(t)) \min\{|u_i|, \overline{u}_i\} \ \text{ with } \ \overline{u}_i > 0 \end{aligned} \tag{2}$$

The purpose of this article is to establish a delay-dependent admissibility criterion, which can generate acceptable bounds on a time-delay as large as possible. Inspired by the neutral system, we try to transform system (1) into an equivalent neutral form under specified limitations. Before transformation, it is necessary to express some useful definitions and lemmas.

Definition 1 (Li and Lin, 2017): The pair *(E,A$_0$)* is regular if *det (SE-A$_0$)* is not identically zero; the pair *(E,A$_0$)* is impulse free if *deg (det (SE-A$_0$)) = rank (E).*

Definition 2 (Dai, 1989): The singular time-delay system is regular and impulse free if the pair *(E,A$_0$)* is regular and impulse free. The singular time-delay system is admissible if it would be regular, impulse free, and stable.

Lemma 1 (Fridman, 2014): If the pair *(E,A$_0$)* is regular and impulse free, then the solution to the singular time-delay system exists, and it is impulse free and unique on [0, ∞).

Lemma 2 (Schur complement) (Fridman, 2014): Given the matrices *A, B*, and *C*, the following holds

$$M = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix} \ge 0 \Leftrightarrow C \ge 0 \quad \text{and} \quad A - BC^{-1}B^T \ge 0 \tag{3}$$

Assumption 1: Assuming that the pair *(E,A$_0$)* is regular, impulse free, and all eigenvalues of $\widehat{C}(t)$ are inside the unit circle (Schur-Cohn), that is

$$\rho\left(\widehat{C}(t)\right) = \max\{|(1-d_1)\rho(C)|, ||(1-d_2)\rho(C)||\} < 1 \tag{4}$$

Lemma 3 (Dai, 1989): Assume that in equation (1) the pair *(E,A$_0$)* is regular and impulse free, then there exist two invertible matrices $M, N \in R^{n\times n}$ such that

$$\mathrm{MEN} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = \overline{E}, \quad MA_0N = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix} = \overline{A} \tag{5}$$

Let

$$MA_dN = \overline{A}_d = \begin{bmatrix} A_{d1} & A_{d2} \\ A_{d3} & A_{d4} \end{bmatrix}, \quad N^{-1}x(t) = \mu(t) = \begin{bmatrix} \mu_1(t) \\ \mu_2(t) \end{bmatrix} \tag{6}$$

where the partitions are compatible with the structure of $\overline{E}$. Then, the system given by equation (1) would be equivalent to equation (7)

$$\overline{E}\dot{\mu}(t) = \overline{A}\mu(t) + \overline{A}_d\mu(t-h) \tag{7}$$

Equation (7) is equal to equations (8) and (9)

$$\dot{\mu}_1(t) = A_1\mu_1(t) + A_{d1}\mu_1(t-h(t)) + A_{d2}\mu_2(t-h(t)) \tag{8}$$

$$0 = \mu_2(t) + A_{d3}\mu_1(t-h(t)) + A_{d4}\mu_2(t-h(t)) \tag{9}$$

By differentiating equation (9), we have

$$\begin{aligned} \dot{\mu}_2(t) &+ \left(1-\dot{h}(t)\right)A_{d3}\dot{\mu}_1(t-h(t)) \\ &+ \left(1-\dot{h}(t)\right)A_{d4}\dot{\mu}_2(t-h(t)) = 0 \end{aligned} \tag{10}$$

Using equations (8)–(10), we have the following equations

$$\begin{aligned} \begin{bmatrix} \dot{\mu}_1(t) \\ \dot{\mu}_2(t) \end{bmatrix} &= \begin{bmatrix} A_1\mu_1(t) + A_{d1}\mu_1(t-h(t)) + A_{d2}\mu_2(t-h(t)) \\ -\mu_2(t) - A_{d3}\mu_1(t-h(t)) - A_{d4}\mu_2(t-h(t)) \end{bmatrix} \\ &+ \left(1-\dot{h}(t)\right)\begin{bmatrix} 0 & 0 \\ -A_{d3} & -A_{d4} \end{bmatrix}\begin{bmatrix} \dot{\mu}_1(t-h(t)) \\ \dot{\mu}_2(t-h(t)) \end{bmatrix} \end{aligned} \tag{11}$$

Let

$$\begin{aligned} &\widehat{A} = \begin{bmatrix} A_1 & 0 \\ 0 & -I_{n-r} \end{bmatrix}, \quad \widehat{A}_d = \begin{bmatrix} A_{d1} & A_{d2} \\ -A_{d3} & -A_{d4} \end{bmatrix}, \\ &C = \begin{bmatrix} 0 & 0 \\ -A_{d3} & -A_{d4} \end{bmatrix} \end{aligned} \tag{12}$$

Moreover, define

$$\widehat{C}(t) = \left(1 - \dot{h}(t)\right)C \tag{13}$$

Therefore, equation (11) is equivalent to the following neutral system based on equation (14)

$$\begin{aligned} &\dot{\mu}(t) - \widehat{C}(t)\dot{\mu}(t-h(t)) = \widehat{A}\mu(t) + \widehat{A}_d\mu(t-h(t)), \\ &\mu(t) = \varphi(t) \quad t\in[-h,0] \end{aligned} \tag{14}$$

The transformation might introduce new dynamics, leading to some conservative results, which should be compensated in a complementary way. Hence, the asymptotical stability of equation (14) will guarantee the admissibility of equation (1) and vice versa. Here, different choice of M and N in equation (5) does not affect Assumption 1. Now, we investigate the stability of the system given in equation (15) instead of exploring the admissibility of the system given in equation (1). The main problem changes to equation (15) with additive norm-bounded uncertainty

$$\begin{aligned} &\dot{x}(t) - \left(\widehat{C} + \Delta\widehat{C}(t)\right)\dot{x}(t-\tau(t)) = \left(\widehat{A} + \Delta\widehat{A}\right)x(t) \\ &\quad + \left(\widehat{A}_d + \Delta\widehat{A}_d(t)\right)x(t-h(t)) + B\mathrm{sat}(u(t)) \\ &x(t) = \varphi(t) \quad t\in[-h,0] \end{aligned} \tag{15}$$

In equation (15), $\tau(t)$ and $h(t)$ are known, bounded function of time, and continuously differentiable with their respective rates of change bounded as follows

$$\begin{aligned} &0 < h(t) \le h_m, \quad 0 \le \tau(t) \le \infty, \quad \dot{h}(t) \le d_1, \quad \dot{\tau}(t) \le d_2, \quad h_m > 0, \\ &d_1 < 1 \quad \text{and} \quad d_2 < 1 \end{aligned} \tag{16}$$

Generally, we suppose two independent values of delay and get $\tau(t)$ instead of $h(t)$ in terms of $(\widehat{C} + \Delta\widehat{C}(t))\,\dot{x}(t-\tau(t))$. Besides, we assume that uncertainty is denoted as follows

$$[\Delta A_0(t)\Delta A_d(t)\Delta \mathrm{C}(t)] = DF(t) = [E_0 \quad E_1 \quad E_2] \tag{17}$$

where D,E_0,E_1, and E_2 are known constant real matrices and $F(t)\in R^{i\times j}$ (i and j are integers) is an unknown real time-varying matrix function of uncertain parameters with Lebesgue-measurable elements such that $F^T(t)\,F(t)\le I$. In the following, we introduce a theorem that covers solving equation (14). Then, we use a more sophisticated theorem with a tuning factor. As mentioned before, here, the main strategy to tackle the actuators' saturation constraint is the poly-topic method (Gomes da Silva and Tarbouriech, 2005).

Denoting the i$^{\text{th}}$ row of K by k_i, we define a polyhedron

$$\Gamma(k,\overline{u}) = \{x\in R^n : |k_i x| \le \overline{u}_i, \quad i = 1,\ldots m\} \tag{18}$$

Let χ be the set of all diagonal matrices in $R^{m\times m}$ with diagonal elements that are either 1 or 0. For example, if $m = 2$, then

$$\begin{aligned} \chi &= \{D_1,D_2,D_3,D_4\} \\ &= \left\{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\right\} \end{aligned}$$

There are 2^m elements D_i in χ, and for every $i = 1,\ldots 2^m$, $D_i^- \triangleq I_m - D_i$ is also an element in χ. The main purpose of the poly-topic method is to embed *sat* $(Kx(t),\overline{u})$ within a convex hull of a group of linear feedbacks. Given two gain matrices $K,H\in R^{m\times n}$, the matrix set $\{D_iK + D_i^-H, i = 1,\ldots,m\}$ is formed by choosing some rows of K and the rest from H: finally, according to the lemma introduced in Li and Lin (2013), the saturation function is expressed as follows

$$(Kx(t),\overline{u}) \in co\{D_iK + D_i^-H, \quad i = 1,\ldots,m\} \tag{19}$$

It is guaranteed for all $x\in R^n$ that $|h_i(x)|\le\overline{u}$, $i = 1,\ldots,m$.

We consider the following control law obtained from algorithm output

$$u(t) = Kx(t) \tag{20}$$

According to equation (20), the closed-loop system of equation (15) is as follows

$$\dot{x}(t) - \overline{C}\dot{x}(t-\tau(t)) = \sum_{j=1}^{2^m}\lambda_j\widehat{A}_jx(t) + \overline{A}_1x(t-h(t)) \tag{21}$$

$$\begin{aligned} &\widehat{A}_j = B\left(D_jK + D_j^-H\right) + \overline{A}_0, \quad \sum_{j=1}^{2^m}\lambda_j = 1 \quad \text{and} \quad \lambda_j\ge 0 \\ &\text{such} \quad \overline{A}_0 = A_0 + DF(t)E_0, \quad \overline{A}_1 = A_1 + DF(t)E_1 \\ &\text{and} \quad \overline{C} = C + DF(t)E_2 \end{aligned} \tag{22}$$

Lemma 4 (Cao et al., 1998): For any $x,y\in R^n$ and a matrix $G>0$ with compatible dimensions, the following inequality holds

$$2x^Ty \le x^TGx + y^TG^{-1}y \tag{23}$$

Theorem 1: suppose that we have symmetric positive definite matrices $P = P^T>0, Q = Q^T>0, R = R^T>0$, and $W = W^T>0$ and other matrices M_j and $L_j j = 1,2$ with appropriate dimensions. If the following LMI holds, then asymptotic stability of the system in equation (15) would be guaranteed

$$\begin{bmatrix} \phi_{11(i)} & * & * & * & * \\ \phi_{21(i)} & \phi_{22} & * & * & * \\ \phi_{31(i)} & \phi_{32} & \phi_{33} & * & * \\ \overline{C}^T L_1^T & \overline{C}^T L_2^T & 0 & -(1-d_2)W & * \\ h_m M_1^T & 0 & h_m M_2^T & 0 & -h_m R \end{bmatrix} < 0, \quad i = 1,\ldots,2^m \tag{24}$$

$$\phi_{11(i)} = Q + M_1 + M_1^T + L_1\widehat{A}_i + \widehat{A}_i^T L_1^T, \quad \phi_{21(i)} = -L_1^T + L_2\widehat{A}_i + P, \quad \phi_{22} = h_m R + W - L_2 - L_2^T$$
$$\phi_{31} = \overline{A}_1^T L_1^T - M_1^T + M_2, \quad \phi_{32} = \overline{A}_1^T L_2^T, \quad \phi_{33} = -(1-d_1)Q - M_2 - M_2^T$$

Proof: Here, we propose a LKF candidate (Fridman, 2014)

$$V(t) = x^T(t)Px(t) + \int_{-h_m}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta + \int_{t-\tau(t)}^{t} \dot{x}^T(s)W\dot{x}(s)ds + \int_{t-h(t)}^{t} \dot{x}^T(s)Q\dot{x}(s)ds \tag{25}$$

Due to $P = P^T > 0, W = W^T > 0, Q = Q^T > 0$, and $R = R^T > 0$, it is clear that LKF condition is met. $(u(|\phi(0)|) \le V(t,\phi) \le v(\|\phi\|_C))$, such that $\|\phi\|_C = \max_{s\in[-h,0]}|\phi(s)|$.

Now, it should be proved that the derivative of this LKF along the system trajectory is not positive

$$\dot{V}(t) = 2x^T(t)P\dot{x}(t) - (1-\dot{\tau}(t))\dot{x}^T(t-\tau(t))W\dot{x}(t-\tau(t)) + \dot{x}^T(t)W\dot{x}(t) + \dot{x}^T(t)Q\dot{x} - \left(1-\dot{h}(t)\right)x^T(t-h(t)) Q\dot{x}(t-h(t)) + h_m\dot{x}^T(t)R\dot{x}(t) - \int_{t-h_m}^{t} \dot{x}^T(s)R\dot{x}(s)ds \tag{26}$$

From equation (16), it is evident that $h_m \ge h(t)$ and it was assumed that $R > 0$; hence

$$\int_{t-h_m}^{t} \dot{x}^T(s)R\dot{x}(s)ds \ge \int_{t-h(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds \Rightarrow - \int_{t-h_m}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le -\int_{t-h(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds \tag{27}$$

Now, the Leibniz–Newton formula provides

$$x(t-h(t)) = x(t) - \int_{t-h(t)}^{t} \dot{x}(s)ds \Rightarrow 0 = x(t) - \int_{t-h(t)}^{t} \dot{x}(s)ds - x(t-h(t)) \tag{28}$$

With an appropriate dimension of slack matrices, $M_j(j = 1,2)$, equation (29) is established

$$2\left[x^T(t)M_1 + x^T(t-h(t))M_2\right] \times \underbrace{\left[x(t) - \int_{t-h(t)}^{t} \dot{x}(s)ds - x(t-h(t))\right]}_{\triangleq 0} = 0 \tag{29}$$

Introducing the augmented matrix $\zeta(t) = \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}$, we have a different form of equation (29)

$$0 = 2\zeta^T(t)\begin{bmatrix} M_1 & -M_1 \\ M_2 & -M_2 \end{bmatrix}\zeta(t) - 2\zeta^T(t)\begin{bmatrix} M_1 \\ M_2 \end{bmatrix}\int_{t-h(t)}^{t} \dot{x}(s)ds \tag{30}$$

According to Lemma 4 and equation (30), we have

$$-2\zeta^T(t)\begin{bmatrix} M_1 \\ M_2 \end{bmatrix}\int_{t-h(t)}^{t} \dot{x}(s)ds \le h(t)\zeta^T(t)\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} R^{-1}\begin{bmatrix} M_1^T & M_2^T \end{bmatrix}\zeta(t) + \int_{t-h(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds \tag{31}$$

It means

$$\int_{t-h(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le h(t)\zeta^T(t)\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} R^{-1}\begin{bmatrix} M_1^T & M_2^T \end{bmatrix}\zeta(t) - 2\zeta^T(t)\begin{bmatrix} M_1 \\ M_2 \end{bmatrix}\int_{t-h(t)}^{t} \dot{x}(s)ds \tag{32}$$

Now, we consider other slack variables L_1, L_2. For any $L_j(j = 1,2) \ne 0$, we have equation (33)

$$2\left[x^T(t)L_2 + \dot{x}^T L_2\right] \times \underbrace{\left[\sum_{j=1}^{2^m} \lambda_j\widehat{A}_j x(t) + \overline{A}_1 x(t-h(t)) - \dot{x}(t) + \overline{C}\dot{x}(t-\tau(t))\right]}_{\triangleq 0 \text{ closed loop}(21)} = 0 \tag{33}$$

We add equation (33) on the right hand of equation (26) ($\dot{V}(t)$) and substitute equation (32) into equation (26) using equation (27). The derivative of equation (26) will change, resulting in the following

$$
\begin{aligned}
\dot{V}(t) \le\, & 2x^T(t)P\dot{x}(t) + h_m\dot{x}^T(t)R\dot{x}(t) - (1-d_1)x^T(t-h(t)) \\
& Qx(t-h(t)) + x^T(t)Qx(t) - (1-d_2)\dot{x}^T(t-\tau(t)) \\
& W\dot{x}(t-\tau(t)) + \dot{x}^T(t)W\dot{x}(t) + 2\left[x^T(t)L_1 + \dot{x}^T(t)L_2\right] \\
& \times \left[\sum_{j=1}^{2^m}\lambda_j\widehat{A}_j x(t) + \overline{A}_1 x(t-h(t)) - \dot{x}(t) + \overline{C}\dot{x}(t-\tau(t))\right] \\
& + 2\zeta^T(t)\begin{bmatrix} M_1 & -M_1 \\ M_2 & -M_2 \end{bmatrix}\zeta(t) \\
& + h_m\zeta^T(t)\begin{bmatrix} M_1 \\ M_2 \end{bmatrix}R^{-1}\begin{bmatrix} M_1^T & M_2^T \end{bmatrix}\zeta(t)
\end{aligned}
\tag{34}
$$

way and a tradeoff between different desires. One of these concepts refers to better estimation of the attraction region. Thus, another useful theorem is introduced in the following. Based on our knowledge, in some missions, DOA is one of the most significant performance indexes for system assessment.

Theorem 2 (Fezazi et al., 2016): If there exist symmetric positive definite matrices $\overline{P}, \overline{Q}, \overline{R}$, appropriately sized matrices $X, \overline{Y}_1, \overline{Y}_2, M, U$, a diagonal matrix S of appropriate dimension, positive scalars $\varepsilon, \beta, \delta$, and a real scalar α satisfying conditions (38)–(40)

$$
\psi = \begin{bmatrix}
\Omega_{11} + \varepsilon DD^T & * & * & * & * & * & * \\
\Omega_{21} + \sigma\varepsilon DD^T & \Omega_{22} + \sigma^2\varepsilon DD^T & * & * & * & * & * \\
\Omega_{31} & \Omega_{32} & \Omega_{33} & * & * & * & * \\
\Omega_{41} & \Omega_{42} & 0 & \Omega_{44} & * & * & * \\
\Omega_{51} & \Omega_{52} & 0 & 0 & \Omega_{55} & * & * \\
\Omega_{61} & \Omega_{62} & 0 & 0 & 0 & \Omega_{66} & * \\
E_0X^T & 0 & E_1X^T & 0 & E_2X^T & 0 & -\varepsilon I
\end{bmatrix} < 0 \tag{38}
$$

Hence, if inequality of equation (35) holds

$$
h_m\xi^T(t)\begin{bmatrix} M_1 \\ M_2 \end{bmatrix}R^{-1}\begin{bmatrix} M_1^T & M_2^T \end{bmatrix}\xi(t) + \xi^T\psi\xi^T < 0 \tag{35}
$$

Such that

$$
\xi^T(t) = \left[x^T(t)\dot{x}^T(t)x^T(t-h(t))\dot{x}(t-\tau(t))\right] \tag{36}
$$

$$
\psi = \begin{bmatrix}
\phi_{11(j)} & * & * & * \\
\phi_{21(j)} & \phi_{22} & * & * \\
\phi_{31(j)} & \phi_{32} & \phi_{33} & * \\
\overline{C}^TL_1^T & \overline{C}^TL_2^T & 0 & -(1-d_2)W
\end{bmatrix} \tag{37}
$$

Then, by applying Lemma 2 to equation (35), we obtain equation (24), and the proof is complete.

Theorem 1 handles time-delay neutral systems with uncertainty in parameters and input actuators saturations. However, in some practical problems, we need to qualify some other functional requirements. Moreover, we require a tuning factor to cope with the issue in a better and desired

$$
\begin{aligned}
&\Omega_{11} = AX^T + XA^T + BU + U^TB^T + Y_1^T + Y_1 + \overline{Q}, \\
&\Omega_{21} = -X + \alpha AX^T + \alpha BU + \overline{P} + \overline{Y}_2 \\
&\Omega_{22} = -\alpha X - \alpha X^T + h_m\overline{R} + \overline{W}, \quad \Omega_{31} = XA_d^T - \overline{Y}_1^T, \\
&\Omega_{32} = \alpha XA_d^T - \overline{Y}_2^T, \quad \Omega_{33} = -(1-d)\overline{Q} \\
&\Omega_{41} = -\overline{Y}_1^T, \quad \Omega_{42} = -\overline{Y}_2^T, \quad \Omega_{44} = -\frac{\overline{R}}{h_m}, \quad \Omega_{51} = XC^T, \\
&\Omega_{52} = \alpha XC^T, \quad \Omega_{55} = -(1-d)\overline{W} \\
&\Omega_{61} = M - SB^T, \quad \Omega_{62} = -\alpha SB^T, \quad \Omega_{66} = -2S^T
\end{aligned}
$$

$$
\begin{bmatrix} \overline{P} & * \\ U_i - M_i & \beta u_{0i}^2 \end{bmatrix} \ge 0 \quad i = 1,\ldots,m \tag{39}
$$

$$
\begin{aligned}
&\left(\overline{\lambda}\left(X^{-1}\overline{P} \quad X^{-T}\right) + h_m\overline{\lambda}\left(X^{-1}\overline{Q}X^{-T}\right)\right)\|\phi(\theta)\|_c^2 \\
&+ \left(\frac{h_m^2}{2}\overline{\lambda}\left(X^{-1}\overline{R}X^{-T}\right) + h_m\overline{\lambda}\left(X^{-1}\overline{W}X^{-T}\right)\right)\|\dot{\phi}(\theta)\|_c^2 \le \beta^{-1}
\end{aligned}
\tag{40}
$$

Then the system of equation (15) with control law $U = KX^{-T}$ is robustly stable, and the trajectory remains within the region of attraction. K is the output result of equations (38)–(40).

3.2. Optimization problem

DOA optimization can be carried out by putting the conditions on the maximal eigenvalues $X_1^{-1}\overline{P}X_1^{-T}$, $X_1^{-1}\overline{Q}X_1^{-T}$, $X_1^{-1}\overline{R}X_1^{-T}$, and $X_1^{-1}\overline{W}X_1^{-T}$. In other words, with positive scalars $\sigma_1 ... \sigma_4$, we have the following terms in LMI form

$$\begin{bmatrix} \sigma_1 I & X_1^{-1} \\ X_1^{-T} & \overline{P}^{-1} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \sigma_2 I & X_1^{-1} \\ X_1^{-T} & \overline{Q}^{-1} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \sigma_3 I & X_1^{-1} \\ X_1^{-T} & \overline{R}^{-1} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \sigma_4 I & X_1^{-1} \\ X_1^{-T} & \overline{W}^{-1} \end{bmatrix} \geq 0 \tag{41}$$

If the following LMI (42) holds, equation (41) will meet

$$\left[\sigma_1 + h_m \sigma_2 + \frac{h_m^2}{2}\sigma_3 + h_m \sigma_4\right]\delta^2 \leq \beta^{-1} \tag{42}$$

Where

$$\delta^2 = \max\left(\|\phi(\theta)\|_C^2, \|\dot{\phi}(\theta)\|_C^2\right) \tag{43}$$

Now, we make an optimization problem as follows:

Minimize Trace $(\overline{P}P^{-1} + \overline{Q}Q^{-1} + \overline{W}W^{-1} + \overline{R}R^{-1} + (X + X^T)(X^{-1} + X^{-T}))$.

Subject to $\sigma_{i=1...4} > 0, \delta > 0, \beta > 0$, and LMI (38)–(41)

$$\begin{bmatrix} X + X^T & * \\ I & X^{-1} + X^{-T} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \overline{Q} & * \\ I & Q^{-1} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \overline{P} & * \\ I & P^{-1} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \overline{W} & * \\ I & W^{-1} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \overline{R} & * \\ I & R^{-1} \end{bmatrix} \geq 0 \tag{44}$$

The new LMIs are solved by the complementarity algorithm (Peng et al., 2007) using the following procedure:

1. By determining, h_m, β, and constant initial value $\alpha = \alpha_0$, and choosing a sufficiently large initial δ such that there exists a feasible set of variable matrices, set $\alpha = \alpha_0$ and $\delta = \delta_0$.
2. Find a set of feasible matrices that satisfies $(\overline{P}, \overline{Q}, \overline{R}, \overline{W}, X, P^{-1}, Q^{-1}, R^{-1}, W^{-1}\sigma_{i=1,...,4})_0$.
3. Solve the following LMI minimization problem.

Minimize Trace

$$\left(\overline{P}P^{-1} + \overline{Q}Q^{-1} + \overline{W}W^{-1} + \overline{R}R^{-1} + \left(X + X^T\right)\left(X^{-1} + X^{-T}\right)\right) + \ldots + \overline{P}_0 P^{-1} + \overline{Q}_0 Q^{-1} + \overline{W}_0 W^{-1} + \overline{R}_0 R^{-1} + \left(X_0 + X^T\right)\left(X_0^{-1} + X_0^{-T}\right) \tag{45}$$

Subject to LMIs (44).

4. Substitute the new matrix variables from Step 3 into equation (44). If the result is feasible, then set $\alpha = \alpha_0$, $\delta = \delta_0$, and return to Step 3. If not, set the new matrices to be

$(\overline{P}, \overline{Q}, \overline{R}, \overline{W}, X, P^{-1}, Q^{-1}, R^{-1}, W^{-1}\sigma_{i=1,...,4})$ and return to Step 3. Superiority and numerical examples of this method are depicted in Section 4.

4. Illustrative examples

In this part, at first, the performance of proposed theorems is investigated by some examples in the literature. In the second part, the performance of the proposed method is explored on the AUV model in descriptor form.

4.1. Numerical examples

Example 1: Consider the singular system by equation (1) with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_0 = \begin{bmatrix} 0.5 & 0 \\ -1 & -1 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Delta A_0 = \Delta A_d = d_1 = B = 0$$

According to Definition 1, it is easily derived that the pair *(E,A*$_0$*)* is regular and impulse free. Let invertible matrices be as follows:

$$N = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix}, M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{such that}$$

$$\text{MEN} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \text{MAN} = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \quad MA_dN = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$$

Besides,

$$\widehat{A} = \begin{bmatrix} 0.5 & 0 \\ 0 & -1 \end{bmatrix}, \widehat{A}_d = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \widehat{C} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

The maximum value of h is given in Table 1.

Hence, our result (1.128s) is very close to the analytical bound $h_{\max}$ = 1.2092 (Fridman, 2002). In comparison with Zhi et al. (2018), that has been reached 1.208s, this remark should be mentioned that in Zhi et al. (2018), maximization of DOA and uncertainty constraints have not been

Table 1. Results and conditions of example 1.

Upper bound of h, s	Analytical bound (Fridman, 2002)	α
1.128	$h_{\max}$= 1.2092s	2.015

considered, so it is logical and reasonable that the result of Zhi et al. (2018) led to less conservatism compared to that of our method, even in the tiny gap.

Example 2 (Gomes da Silva et al., 2005): Consider the neutral time-delay system in equation (15) where

$$\overline{u} = 15,\ h = 1s,\ d_1 = 0.1\widehat{A} = \begin{bmatrix} 1 & 1.5 \\ 0.3 & -2 \end{bmatrix},$$

$$\widehat{A}_d = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix},\quad \widehat{C} = \begin{bmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix},\quad B = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$$

According to Figure 1, the maximal estimate of DOA with $\varepsilon = 0.2$ is 108.5288 indicating the performance of our method with respect to the traditional ones. The gain of the state feedback controller is $K = [-0.1388 \quad -0.0302]$.

Next, consider the system in Example 2 in retarded form ($\varepsilon = 0$) (Tarbouriech and Gomes da Silva, 2000) with some available results given in Table 3. If we set $C = 0$, the system reduces to a retarded type time-delay system. Tarbouriech and Gomes da Silva (2000) and Fridman et al. (2003) are based on the poly-topic approach (like us), whereas Gomes da Silva et al. (2011) are based on the sector

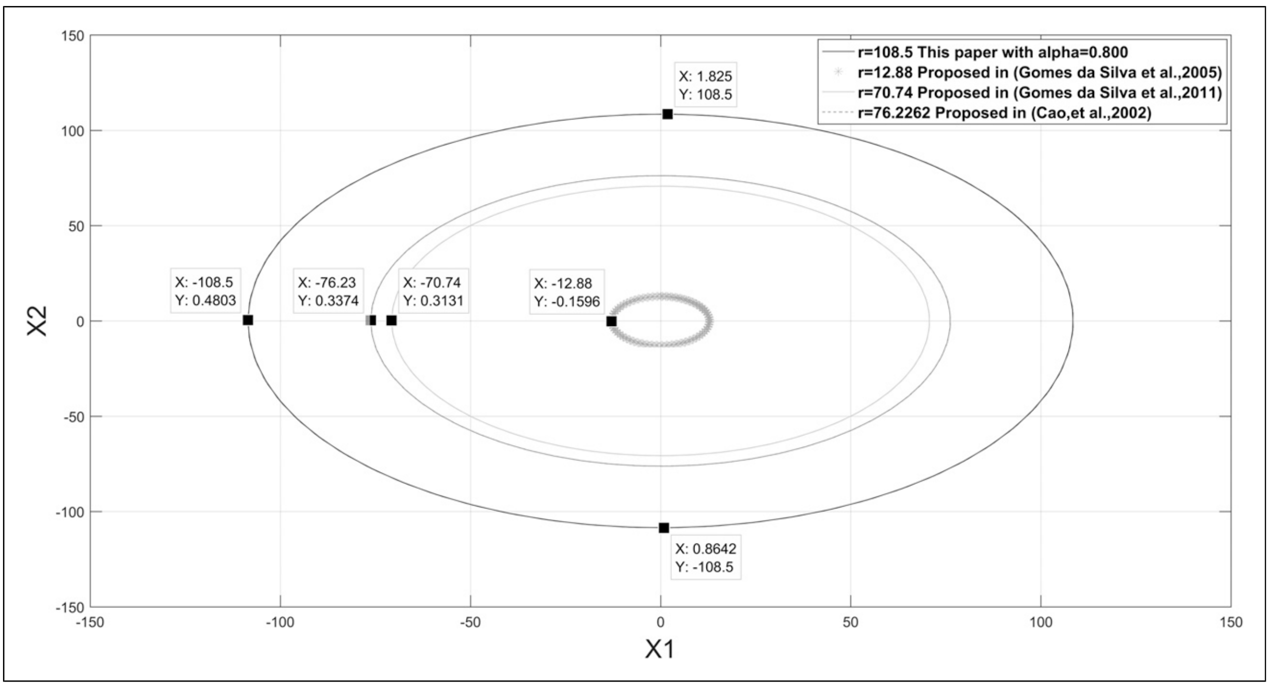

Figure 1. Comparing the results of Example 2 ($\varepsilon = 0.2$) with other literature in the neutral form.

Table 2. Domain of attraction of example 2 with $\varepsilon = 0.2$ (neutral model).

Methods	Max radius	K
(Gomes da Silva et al., 2005)	12.8800	[–0.2780–0.1390]
(Gomes da Silva et al., 2011)	70.7400	[0.1325 0.0153]
(Chen et al., 2015)	76.2262	[–0.2359–0.0453]
Proposed method	108.5288	[–0.1330–0.0246] $\alpha = 0.800001$

Table 3. Domain of Attraction of Example 2 with $\varepsilon = 0$ (retarded model).

Methods	Max radius	K
(Cao et al., 2002)	67.0618	Not reported
(Fridman et al., 2003)	79.4300	[–7.9130 0.7323]
(Gomes da Silva et al., 2011)	83.5500	[–0.1950 0.0649]
(Chen et al., 2015)	84.6074	[–0.2223–0.0246]
Proposed method	114.1750	[–0.1300–0.0206] $\alpha = 0.800001$

bounding approach. Table 2 and Table 3 give the present results.

As the last example, from Dey et al. (2014) we have the following problem statement

$$\widehat{A} = \begin{bmatrix} 0.5 & -1 \\ 0.5 & -0.5 \end{bmatrix}, \quad \widehat{A}_d = \begin{bmatrix} 0.6 & 0.4 \\ 0 & -0.5 \end{bmatrix}, \quad \widehat{C} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad d_1 = d_2 = 0 \quad \text{and} \quad \overline{u}_i = 5$$

The maximum tolerability of $h_{\max}$ in the literature is 3.310s, and we reached 3.3138s with $\alpha = 709$. A comparative result with other traditional methods is given in Table 4.

Remark 3: Note that, in optimization problem, weighting parameters α should be chosen such that a suitable situation is obtained. Generally, selecting such a setting is an ad-hoc way and might vary based on system specifications and structure. For other way solutions, one may use numerical or intelligent methods, such as the genetic algorithm, to get maximized the DOA in addition to the mentioned optimization problem. The proposed method produces less conservative results in both neutral and retarded systems for the upper bound of time-delay and estimating the maximum DOA, simultaneously.

Table 4. The maximum and permitted upper bound ($h_{\max}$).

Methods	$h_{\max}$(s)	K
(Cao et al., 2002)	0.350	Not reported
(Fridman et al., 2003)	1.854	[–25.8809–4.9315]
(Zhang et al., 2008)	2.248	Not reported
(Chen et al., 2015)	1.854	[–2.2346 0.0580]
(Dey et al., 2014)	3.310	[–605.9023–401.8906]
Proposed method	3.3138	[–0.1296–0.0169] $\alpha = 709$

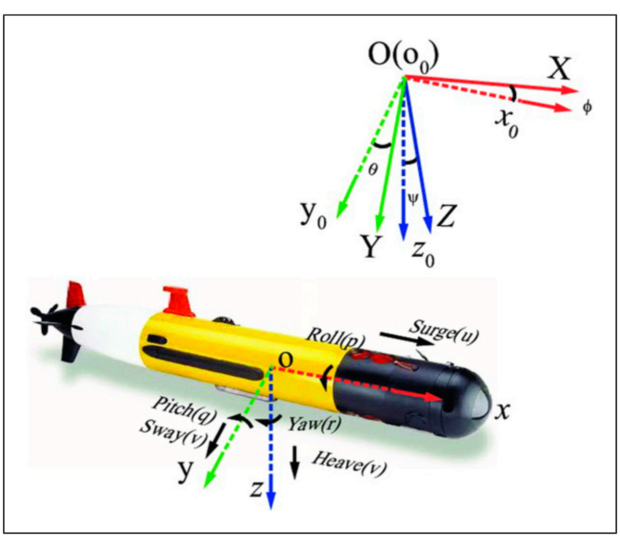

Figure 2. The reference frame of REMUS100 and its position and orientation (Wu et al., 2018).

5. AUV descriptor (singular) model

According to Figure 2, the AUV is determined uniquely by 6-DOF. Notably, for using the AUV, two coordinate systems are implemented for convenience. In Figure 2, both coordinate systems inertial reference frames (IRF) and body reference frame (BRF) are depicted. Assuming that the center of AUV coincides with the origin of BRF.

REMUS100 consists of four actuators and control fins. We explored AUV modeling in the state-space form in our previous works, and the reader can refer to Khodayari and Balochian (2016) and Khodayari and Balochian (2015) for more details. Here, we only focus on the descriptor model.

Assumption 2: The AUV is symmetric about three planes. In this literature, the environmental assumptions and vehicle/dynamics assumptions are according to Khodayari and Balochian (2016) and references in it, in SI units. Motion dynamics in the longitudinal and rotational vectors are described as follows

$$\begin{aligned}
\eta_1 &= [x \quad y \quad z]^T && \text{Positions vector} \\
\nu_1 &= [u \quad v \quad w]^T && \text{Linear velocities vector} \\
\tau_1 &= [X \quad Y \quad Z]^T && \text{External forces vector} \\
\eta_2 &= [\phi \quad \theta \quad \psi]^T && \text{Euler angles vector} \\
\nu_2 &= [p \quad q \quad r]^T && \text{Angular velocities vector} \\
\tau_2 &= [K \quad M \quad N]^T && \text{External moments vector}
\end{aligned}$$

To reach a singular model, we introduced augmented states as follows. Here, ξ,ξ_r,ξ_e are depth, desired depth, and depth error, respectively. We can attain to the following descriptor model:

Heave

$$\begin{aligned}
& m\left[\dot{\omega} - uq + vp - z_g\left(p^2 + q^2\right) + x_g(rp - \dot{q}) + y_g(rp + \dot{p})\right] \\
&= 0.5\,\rho L^4\left(Z_{\dot{q}}\dot{q} + Z_{pp}p^2 + Z_{q|q|}q|q| + Z_{rp}rp\right) \\
&\quad + 0.5\,\rho L^3\left(Z_{\dot{\omega}}\dot{\omega} + Z_{vp}vp + Z_{\omega|q|}\omega|q| + Z_{uq}uq\right) \\
&\quad + 0.5\,\rho L^2\left(Z_{uu}u^2 + Z_{uw}uw + Z_{\omega|\omega|}\omega|\omega|\right) \\
&\quad + 0.5\,\rho L^2 Z_{\delta_S}u^2\delta_S + (W - B)\cos\theta\cos\phi
\end{aligned} \tag{46}$$

Trim (pitch)

$$\begin{aligned}
& I_y\dot{q} + (I_x - I_z)rp + m\left[z_g(\dot{u} + \omega q - vr) - x_g(\dot{\omega} + vp - uq)\right] \\
&= 0.5\,\rho L^5\left[M_{\dot{q}}\dot{q} + M_{pp}p^2 + M_{rr}r^2 + M_{rp}rp + M_{qq}q|q|\right] \\
&\quad + 0.5\,\rho L^4\left[M_{\dot{\omega}}\dot{\omega} + M_{vr}vr + M_{vp}vp + M_{uq}uq + M_{|w|q}|\omega|q\right] \\
&\quad + 0.5\,\rho L^3\left[M_{uu}u^2 + M_{u\omega}u\omega + M_{\omega|\omega|}\omega|\omega|\right] \\
&\quad + 0.5\,\rho L^3 M_{\delta_S}u^2\delta_s + Z_B B\sin(\theta)
\end{aligned} \tag{47}$$

$$\dot{\xi} = \frac{d(\xi_r - \xi)}{dt} = -\dot{\xi} \tag{48}$$

$$\dot{\theta} = q\cos\theta - r\sin\phi \tag{49}$$

$$\dot{\xi} = -u\sin\theta + v\cos\theta\sin\phi + \omega\cos\theta\cos\phi \tag{50}$$

The definitions of hydrodynamic coefficients and their values are according to Khodayari and Balochian (2016) and references in it. Using the Maclaurin expansion of the trigonometric terms with first-order approximation and linearization around trim point ($\theta = \omega = q = \zeta_e \cong 0\, u_0 = 1.54\, m/s$), we have

$$E\dot{x}^*(t) = A_S x^*(t) + B_S u(t) + f^* \tag{51}$$

f^* is the error in simplification and linearization, and here, *u (t)* is the stern plane δ_s of the AUV. Finally, using state vector $x(t) = [\zeta_e \quad \omega \quad q \quad \theta]^T$ and input δ_s, the system is achieved as follows. By considering values of REMUS100 parameters and also considering $u_0 = 1.54\, m/s$, we have the following

$$E = \begin{bmatrix} & 0 & 0 & 0 \\ 0 & 65.98 & 1.93 & 0 \\ 0 & 1.93 & 8.33 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_s = \begin{bmatrix} 0 & -1 & 0 & 1.54 \\ 0 & -44.04 & -8.04 & 0 \\ 0 & 36.96 & -3.08 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B_s = \begin{bmatrix} 0 \\ -14.585 \\ -14.585 \\ 0 \end{bmatrix} \tag{52}$$

As we can see in equation (52), rank$(E) = 4$. Nevertheless, if we have some parameter uncertainty or the linear system in different equilibrium points or a total change of shaping or some reasons that have been discussed before, we could have a singularity in the system. For example, here E_{11} or E_{44} are close to zero and could cause singularity. According to Definitions 1 and 2, the determined AUV system matrix is regular and impulse free. The exact value of A_d is out of this study scope. Based on Lemma 2 and considering proper matrices M and N, the newly transferred matrices are

$$\widehat{A} = \begin{bmatrix} -1.9589 & 2.9991 & -1.1485 & 2.8776 \\ 0.1171 & -0.3628 & 1.0685 & 0.0115 \\ -0.1809 & 0.0520 & -0.2069 & 0.4908 \\ -1.907 & 1.888 & -1.2651 & 1.3833 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ -14.585 \\ -14.585 \\ 0 \end{bmatrix}$$

$$\widehat{A}_d = \begin{bmatrix} -0.5 & 0.2 & -0.39 & 0.1 \\ 0.5 & -0.2 & 0.68 & 0.3 \\ -0.054 & -0.001 & 0.02 & 0.0 \\ -0.015 & 0.01 & 0.001 & 0.03 \end{bmatrix},$$

$$\widehat{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.054 & -0.001 & 0.02 & 0.0 \\ -0.015 & 0.01 & 0.001 & 0.03 \end{bmatrix},$$

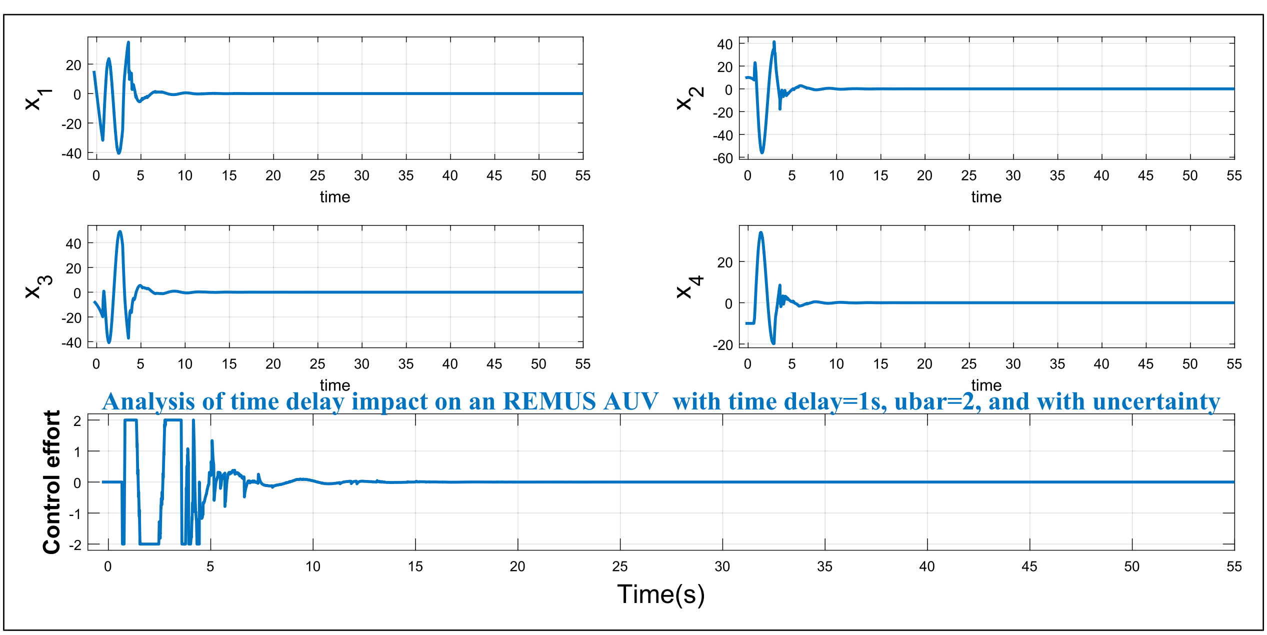

Figure 3. States situations and control effort for *h=1s* by *k*= [0.3726 0.4518 0.4184 0.9331].

$$A_d = \begin{bmatrix} -0.3 & -16.5 & -1.7 & -0.15 \\ -0.24 & -21.28 & -1.7 & -0.3 \\ -1.5 & -10.5 & -9.8 & -1.47 \\ 0.86 & 5.7 & 5.1 & 0.78 \end{bmatrix} \quad (53)$$

5.1. AUV simulations

According to the model in equation (53) in-depth channel and coupling parameter of w,q, and θ, the AUV simulation treatment in different situations is according to the following figures. Here, the state is $x(t) = [\zeta_e \quad \omega \quad q \quad \theta]^T$. For the first scenario, we investigate the effect of delay on AUV stability. In the following simulations, to make the time-variant uncertainty, $F(t) = \cos(t)$ was used, and a different initial continuous differentiable function is proposed for each state. These initial arbitrary functions are considered as $\sin(t)$, $\cos(t)$, $\exp(t)$, and ones for states 1 to states 4, respectively.

AUV Example 1: This example investigates the impact of the length of time-delay on stabilization. The main

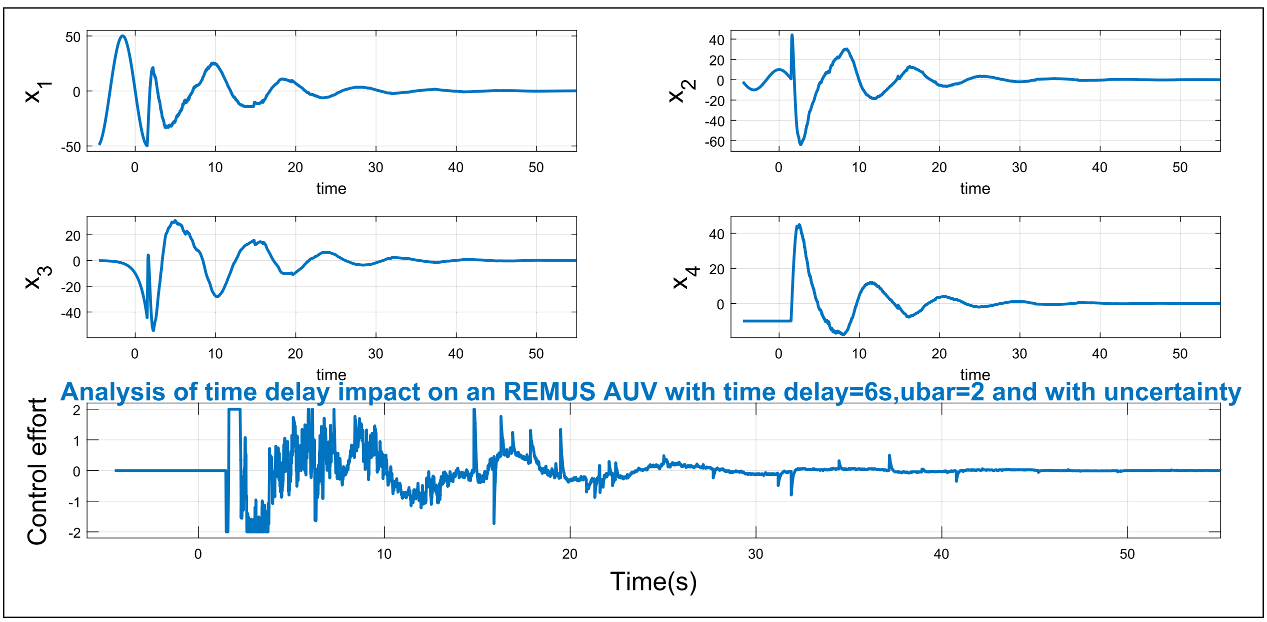

Figure 4. States situations and control effort for *h=6s by k=* [0.5987 0.2588 0.6618 0.8813].

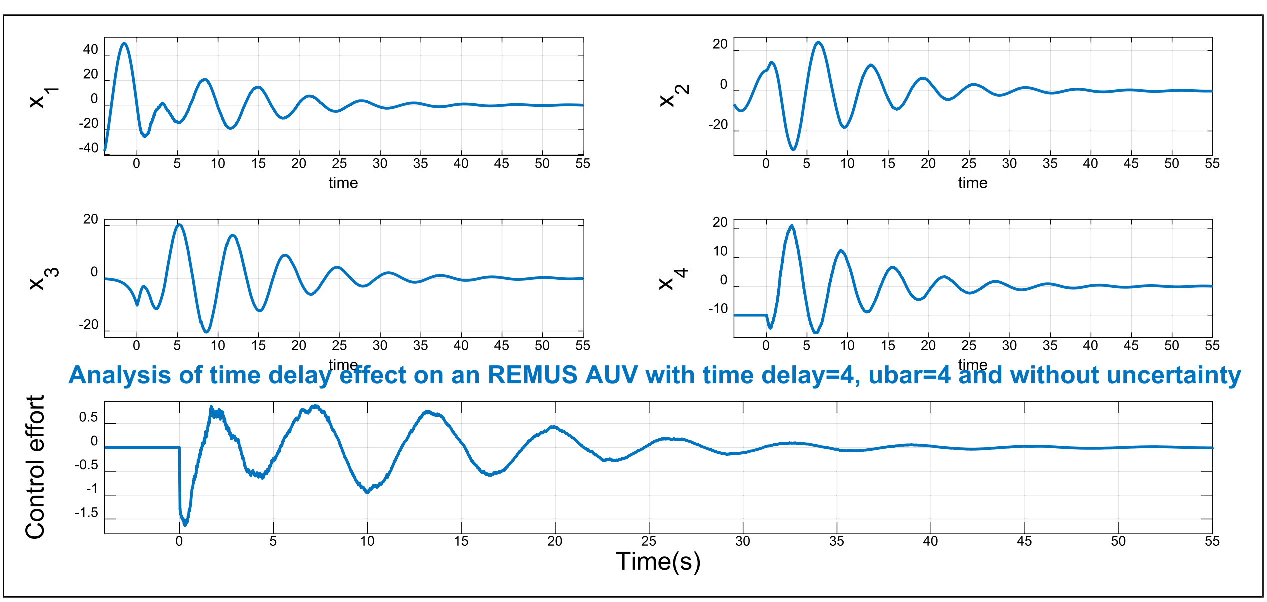

Figure 5. States situations on h = 4*s* without uncertainty by *k=* [-0.0115 0.1874 0.0420 0.2751].

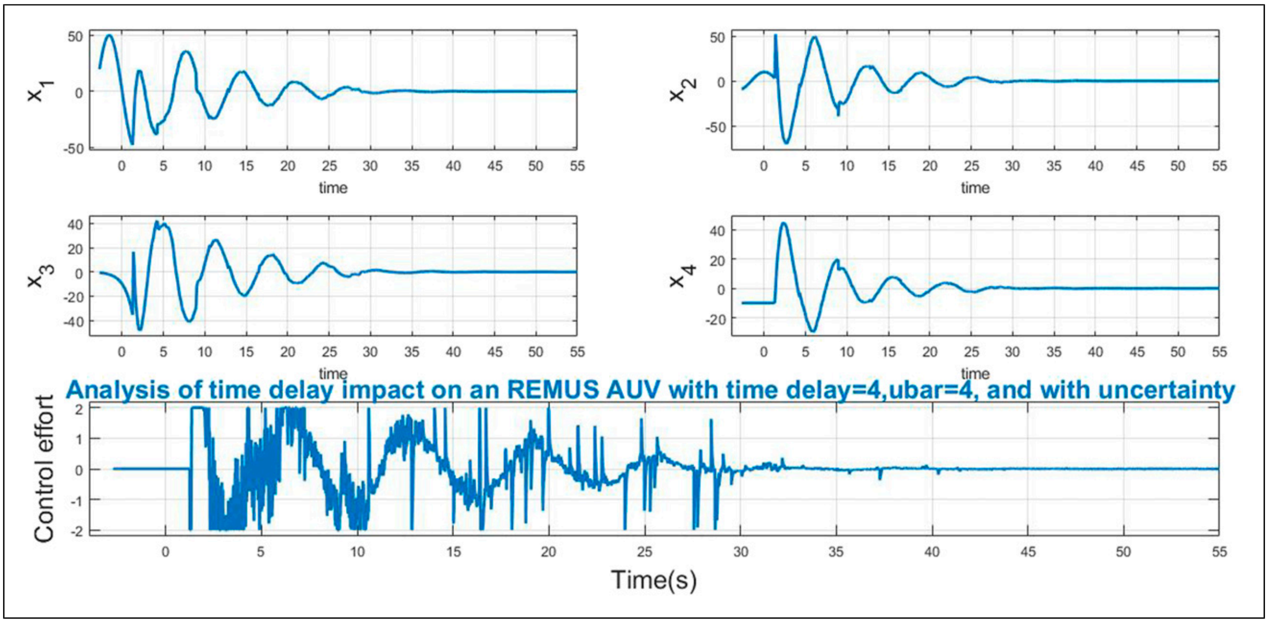

Figure 6. States situations on h = $4s$ with system uncertainty by k= [1.5514 0.4654 1.6111 1.7771].

matrices are selected according to equation (53), and other parameters are as follows

$$\bar{u}_i = 2,\ d_1 = d_2 = 0, \alpha = 10, D = [0.4 \quad 0.2 \quad 0 \quad 0.2];$$
$$E0 = [1 \quad 0 \quad 1 \quad 1]; E1 = [0.5 \quad 0.6 \quad 0.3 \quad 1];$$
$$E2 = [1 \quad 0 \quad 2 \quad 1]$$

As we can see in Figures 3 and 4, the control efforts are 0.1247 and 0.2783, respectively, in the Integral Absolute Error (IAE) performance index unit. It shows that in longer time-delay, there is more control effort. By increasing time-delay, some parameters like rise time, undershoot and overshoot increase, and the system encounter more control effort. However, the amplitude of u *(t)* remains in the range of saturation bound ($\bar{u}_i = 2$), and all states are stabilized.

AUV Example 2: This example investigates the effect of system uncertainty. Consider this initial situation: $\bar{u}_i = 2, d_1 = d_2 = 0, \alpha = 10, h = 4\,\mathrm{s}$.

$D_2 = [0 \ 0 \ 0 \ 0]; E0 = [0 \ 0 \ 0 \ 0]; E1 = [0 \ 0 \ 0 \ 0]; E2 = [0 \ 0 \ 0 \ 0]$ without uncertainty.

$D_1 = [0.4 \quad 0.2 \quad 0 \quad 0.2]; \quad E0 = [1 \quad 0 \quad 1 \quad 1]; E1 = [0.5 \quad 0.6 \quad 0.3 \quad 1]; E2 = [1 \quad 0.6 \quad 2 \quad 1]$ with uncertainty.

According to Figures 5 and 6, the system is stabilized in both situations in the range of saturation domain. Control efforts in stages 1 and 2 are 0.1024 and 1.4800 in the unit, respectively. It is logical that in the system with uncertainty in parameters, we encounter more control effort and worse stabilization. From a practical point of view, besides some items like overshoot and undershoot, we should pay attention to the maximum frequency on actuators. Depending on the type of actuators (electrical (almost up to 1~7Htz), hydraulic, or pneumatic), this frequency should be proportionate. According to our practical desired, all of the simulations can be done by different values of α finding other consequences in a tradeoff between our desires.

6. Conclusion

This article addresses the admissibility problem of descriptor AUV systems with time-delay via the neutral model transformation. A non-linear AUV model was obtained in the descriptor model. According to introduced theorems and lemmas, some delay-dependent and rate-delay-dependent stability criteria were presented. Also, the Domain of Attraction optimization was developed to improve the estimation of the region of attraction. The effectiveness and performance of the proposed method was demonstrated by numerical examples in the literature. In this method, a tuning factor has been considered for practical trials and making desirable situations. This method covers both retarded and neutral systems. Constraints such as input saturation of actuators, parameter uncertainty, time-varying rate of delay, and significant constraints of descriptor systems for admissibility were considered simultaneously besides DOA maximization. Results show that the presented approach is promising for autonomous manipulations and represents an essential passage toward developing a higher level of autonomy for intervention AUVs. In the future work in this field, less conservative conditions can be investigated by adding admissible disturbances to the problem statement. Also, this method can be extended to the discrete time-delay system and as a complementary activity; the issue can be investigated in switched systems.

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