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Study the Role of R^2 Term in Cosmological AdS-like Space by AdS/CFT Correspondence

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Abstract

It has been shown that AdS/CFT correspondence may be realized between some AdS-like cosmological space and CFT living on the boundary. By extending such works we studied the role of R^2 term in the corrected action to the thermodynamic quantities, particularly by adding a boundary counterterms to the gravitational action and considering an AdS-like cosmological space. With such HD (higher derivative) terms and metric we calculated the free energy, as a function of N^2T^4 times $3/2(1-1/N^2)$, that is comparable with perturbative result followed from boundary QFT. In fact the difference between the results obtained from AdS-like cosmological space and the results obtained from strong coupling limit of $\aleph = 4$ SCFT, by taking into account next to leading term in large N expansion, is just a 3/2 factor. Higher derivative term contributions also appear in entropy and energy via the redefinition of gravitation constant. We explicitly identify higher order counterterms effect in energy as a constant term namely E_0. This value can be interpreted as the Casimir energy, so that, there is a full and perfect match between QFT and gravitational action when both R^2 term and boundary counterterms are considered. So we can claim that our results have assured us that there is a good fit between AdS-like cosmological space and $\aleph = 4$ SU(N) super Yang-Mills quantum theory.

Keywords AdS/CFT correspondence · Cosmology · Black hole · Gravitation · Higher derivative (HD) terms · Thermodynamic quantities · Casimir energy

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1 Introduction

Black holes, apart from their intrinsic importance, provide a testing ground for quantum theory of gravitation, this is done by holographic principle [1]. Holographic principle is a relation between space-time geometry and the number of degrees of freedom. It is well known that there exists a conventional theory without gravity, living on the boundary of a space-time region, with one degree of freedom per Planck area, by which all bulk phenomena including quantum gravity can be described. Holographic principle is clearly essential for existence of such a theory. AdS is probably a necessary condition for existence of the kind of duality that has been found in this space-time. Anti-de Sitter space can be scaled into direct product of an infinite time axis with a unit spatial ball [2]. The conformal field theory on the boundary of AdS [3, 4] is an example of a dual theory.

Besides this, AdS/CFT correspondence (an excellent review can be seen in [5]) may provide us with new insights into strong coupling regions of SUSY QFTs. An example of this kind is given by strong coupling limit of $\aleph = 4$ super YM theory [6, 7]. Thermodynamics of $\aleph = 4$ super YM theory in relation with AdS/CFT correspondence has been discussed in numerous works [8, 9]. One of these thermodynamic relations implies that an entropy should be assigned to a black hole equal to one-fourth of its horizon area measured in Planck units [10, 11]. In a fundamental theory of quantum gravity this Bekenstein-Hawking entropy should have a statistical interpretation. It has been argued [12–14] that string theory provides such an interpretation.

This correspondence has also been discussed in some works both in different dimensions and different states. For instance a certain extremal 5-dimensional black hole was constructed in [15] so that its horizon area is non-vanishing. It was shown that logarithm of its ground state degeneracy, matchese Bekenstein-Hawking entropy. These results have been extended to a number of directions, for example in [16] it was generalized to rotating black holes. In [17] a similar 5-dimensional example was considered, and it was further shown that entropy of slightly non-extremal black holes also matches the Bekenstein-Hawking result. This allowed calculation of the temperature of Hawking radiation for a D-brane. Similar results were obtained in 6-dimensions for slightly non-extremal black strings [18]. The original 3-brane solution of type IIB supergravity was constructed in [19]. Another interesting problem is whether AdS/CFT correspondence could be realized as the one between some AdS-like cosmological space and CFT living on the boundary. In this field we can refer to a number of recent proposals for entropy bounds in cosmology [20–24]. Especially in [25] the test on proportionality of free energies for a d5 cosmological space (after AdS/CFT identification of parameters) and a $\aleph = 4$ SU(N) super Yang-Mills quantum theory was studied.

Another important discussion in AdS/CFT is supergravity. In fact, in AdS/CFT we have a superstring theory in one side and a gauge theory on the other side. But we know supergravity provides a low energy effective theory of the massless sector of superstring theory and can be used to study its low energy properties, so it is convenient to have a brief review of this theory in this paper. Supersymmetry is a symmetry between fermions and bosons belonging to supermultiplets and are related by supertransformations. Superalgebra in supersymmetry is formed by Supertransformations and spacetime transformations such as Poincare transformations. Various kinds of superalgebras in this theory are dependent upon spacetime dimension, spacetime symmetry and the number of supersymmetries.

Super Poincare algebra consists of generators of supertransformations (supercharges) Q_α, translation generators P_μ and Lorentz generators $M_{\mu\nu}$. If we have only one Majorana spinor supercharge we have $\aleph = 1$ super Poincare algebra. Components of the supercharge Q_α are fermionic generators and satisfy anticommutation relations. We can find possible supermultiplets (certain set of particle states with different spins) by studying irreducible representations of super Poincare algebra for one particle states. By these explanations of supersymmetry, we can now turn to supergravity.

Supergravity is a field theory which has a local supersymmetry and is a generalization of Einstein's general relativity. The Rarita-Schwinger field $\Psi_{\mu\nu}(x)$ is the gauge field of the local supersymmetry, represented by a particle with spin $3/2$ called a gravitino. Supergravity also has a local symmetry under the general coordinate transformation, whose gauge field is the gravitational field $e^a_\mu(x)$. The supergravity multiplet for $\aleph = 1$ super Poincare algebra consists of a pair of fields $(e^a_\mu(x), \Psi_{\mu\nu}(x))$. So, there exists a supergravity theory (called $\aleph = 1$ Poincare supergravity) which contains these two fields [26, 27].

If supergravity contains $\aleph$ Majorana spinor supercharges Q_i (i = 1, 2, . . . , $\aleph$) we have an $\aleph$ -extended supersymmetry [28]. Extended supergravities generically contain gravitational field, Rarita-Schwinger fields, vector fields, spinor fields and scalar fields. For example $\aleph$ = 2 Poincare supergravity [29] that is the simplest extended supergravity, has a gravitational field $e^a_\mu(x)$, two Majorana Rarita-Schwinger fields $\Psi^i_\mu(x)$ (i = 1, 2). Also a vector field $B_\mu(x)$ and its field equations have a Minkowski spacetime solution $e^a_\mu(x) = \delta^a_\mu(x)$, B_μ =0, $\Psi^i_\mu(x)$=0. Lagrangian of this type of supergravity is invariant up to total divergences under the general coordinate transformation, local Lorentz transformation, U(1) gauge transformation, $\aleph$ = 2 local supertransformation and a global U(2) symmetry [30]. The last symmetry is similar to the global U(1) symmetry in $\aleph$ = 1 supergravity. SU(2) in U(2)$\sim$ SU(2) $\times$ U(1) is a symmetry of the Lagrangian but U(1) is a symmetry of the field equations. We can also construct $\aleph$ = 2 supergravity with a cosmological term. In this theory the so called $\aleph$ = 2 Anti de Sitter Supergravity (because its field equations have an AdS spacetime solution) a cosmological term with a negative cosmological constant and a mass term of the Rarita-Schwinger fields are added to Lagrangian. Another modification is the covariant derivative on Rarita-Schwinger fields that contains a minimal coupling to the U(1) gauge field B_μ, which is not present in m = 0 supergravity. This Lagrangian is invariant up to total divergences under general coordinate transformation, the local Lorentz transformation, the U(1) gauge transformation and $\aleph$ = 2 local supertransformation. Global U(2) symmetry of the m = 0 theory is broken by the coupling to the gauge field. Since this theory has the minimal coupling to the gauge field, it is called $\aleph$ = 2 gauged supergravity. Similarly, $\aleph$ = 3, 4, 5, 6, 8 extended supergravities can be constructed [31].

As we explained briefly $\aleph$=4 SYM theory before, it is crucial to have an overview of $\aleph$=4 SYM with finite chemical potential in Minkowski space which is a dual picture of $\aleph$ = 2 AdS5 supergravity (STU model). We can obtain this theory by compacting the eleven dimensional supergravity in a three-fold Calabi-Yau [32]. In fact STU model that admits a chemical potential for $U(1)^3$ symmetry, is the special form of $\aleph$ = 2 supergravity in several dimensions. This model has three real scalar fields as well as an 8-charged (4 electric and 4 magnetic) non-extremal black hole. Of course there are other models with four-charged and three-charged black holes, but there are some differences between these two models. For example, the entropy vanishes for three charged model, corresponding to three different chemical potentials (except in the non-BPS case). To get a regular black hole, a four charged model is required. The situation is different in 5 dimensions, so that in this case there is no distinction between BPS and non-BPS branch. The most interesting configurations in 5 dimensions are three-charged models [33]. In fact a three-charged non-extremal black hole with a four - dimentional boundary which includes QCD is an important thermal background for this correspondence. Much work has been done in this area. For instance in Ref. [34] a non-extremal black hole with three equal charges was considered to extract a drag force for a moving quark at $\aleph = 2$ supergravity. As an extension to Refs. [35–37] the STU model with three different charges was considered in Ref. [38]. Also the quark-anti quark configuration and rotating quark-anti quark pair in STU background was studied in Ref . [39]. In Ref. [40] the drag force was calculated for a flat space, a three dimensional sphere and a pseudo-sphere by considering a non-extremal black hole with only one charge. In another work [41] a non-zero chemical potential was considered and electrical conductivity was calculated with a conclusion that conductivity behaves in a universal manner for gauge theories dual to R-charged black hole in d = 4. Also the STU model with a time-dependence that is dual to a boost-invariant expanding plasma was studied in Ref. [42]. In Ref. [35] a flat space is considered and the shear viscosity in the SYM theory dual to the STU model is computed. In Refs. [43, 44] the viscosity of gauge theory plasma with a chemical potential was calculated] by using a d=5 RN- AdS black hole. Some other works on thermodynamics for STU model are presented in [45–49], a good review of such works can be found in [39].

In present paper we just consider the gravity part of supergravity; we especially consider the higher derivative corrections in bulk gravity for several reasons as follows: any effective stringy gravity action besides the usual Einstein term (and a possible cosmological term) includes higher derivative terms of different order as loop corrections to string amplitudes [50]. On the other hand, from the point of view of AdS/CFT correspondence in any effective supergravity action, the R^2-terms give next-to-leading terms in large N expansion of boundary CFTs in the strong coupling limit [51, 52].

Another reason is, that higher derivative gravity is probably a good candidate for construction of realistic brane-world cosmologies. It has been shown that it is necessary to include the higher curvature terms for a better understanding of both the thermodynamic behavior and thermal phase structure. Of course for computing the thermodynamic quantities in any effective gravitational action including higher derivatives we should calculate the entropy. This fact that entropy formula is valid to any effective gravitational action including higher curvature interactions, was proposed for the first time in [53], and then was generalized in some works [54, 55]. The Gauss-bonnet (GB) invariant which is attributed to the low energy effective string action can be the suggestive combination of the higher derivative curvature term. It is worth mentioning that the resulting field equations contain no more than second derivatives of the metric tensor [56, 57], thus in this theory we don't have any ghost. This is also true for Randall-Sundrum type warped geometry [58, 59]. A natural tool to explore the AdS/non-CFT correspondence, is probably attainable by using the general higher derivative terms to the effective effect.

Another application of higher derivative corrections is to study the role of these corrections on quantities related to quark-gluon plasma (QGP) such as shear viscosity, drag force, jet-quenching and the ratio of shear viscosity η to entropy density s. In fact quark-gluon plasma which is composed of free quarks and gluons is a phase of quantum chromodynamics (QCD) which exists at high temperature or high density. One important hydrodynamical quantity of QGP is the shear viscosity which is related to entropy, specially the ratio of shear viscosity η to entropy density s has a universal value: $\eta/s = 1/4\pi$ [60–63]. However this universal value may be enhanced or reduced for several cases [64–66]. For example, in string theory, α corrections enhance this value, while probably higher derivative corrections will reduce it. In STU model the shear viscosity has been computed, also the effects of higher derivatives in five-dimensional gauged supergravity has been calculated for the ratio of shear viscosity to entropy [35, 37, 67, 68]. In another work, the ratio of shear viscosity to entropy density was obtained by using diffusion constant and applying higher derivative correction for three-charged black hole in STU model [39]. Other important properties of QGP are drag force and also jet-quenching parameter. Jet-quenching parameter is obtained by calculating the expectation value of a closed light-like Wilson loop and using the dipole approximation. In Refs [69, 70] effect of higher derivative corrections such as Gauss-Bonnet on drag force and jet-quenching parameter has been studied. Also, in Ref. [71] the effects of higher derivative correction and external electric field on jet-quenching parameter in STU model was studied. Considering the effect of higher derivative corrections for R-charged AdS5 black holes with three equal charges, thermodynamic quantities such as Hawking temperature and entropy are presented in [72]. The four derivative terms (up to linear orders) for a STU model was studied in [67]. In Refs. [61, 73] the effect of curvature squared corrections on the ratio of η/s was computed. More references and discussion in this field can be found in Refs. [39, 74, 75].

The main purpose of present paper is dealing with a question that, if higher derivative terms are considered how thermodynamics quantites will be changed in higher-dimensional AdS-like cosmological space. Before doing that, it is necessary to have a brief review of $AdS_5 \times S^5$ spacetime and relevant papers.

2 Action, Metric and Free Energy in $AdS_5 \times S^5$ Space

As we know D-branes are useful to establish the precise relation between gauge theory and curved spacetime. But superstring theory actually requires a 10 dimentional spacetime for consistency. We roughly divide spacetime made by D3-brane into $AdS_5 \times S^5$ near the origin with an approximately flat spacetime surrounding it. The $\aleph = 4$ SYM corresponds to supergravity on $AdS_5 \times S^5$ [76, 77]. These two descriptions of the brane namely, gauge theory and supergravity, are complementary to each other. Gauge theory description is valid when $g_s N_c \gg 1$, whereas supergravity description is valid when $g_s N_c \ll 1$. Black D3-brane is given by [19].

$$\begin{aligned} ds_{10}^2 &= Z^{-1/2}(-dt^2 + d\overrightarrow{x}^2) + Z^{-1/2}(dr^2 + r^2 d\Omega_5^2) \\ Z &= 1 + (L/r)^4 \quad , \quad L^4 \cong g_s N_c l_s^4 \end{aligned} \tag{1}$$

This metric is the zero-temperature solution of the D3-brane, or the extreme black hole solution, which corresponds to $\aleph = 4$ SYM at zero temperature. In [6] it was pointed out that near extremal black 3-brane of Hawking temperature T, may be used to study large N SYM theory heated up to the same temperature and entropy of SU(N) SYM theory which was identified with the Bekenstein-Hawking entropy of the geometry.

Since horizon of a near-extremal 3-brane is located far down its throat, the same answer for the Bekenstein-Hawking entropy is obtained if we replace 3-brane metric by throat approximation, $r \ll L$. The resulting metric [3, 78],

$$ds^2 = \frac{r^2}{L^2}\left[-\left(1 - \frac{r_0^4}{r^4}\right)dt^2 + dx^2 + dy^2 + dz^2\right] + \frac{L^2}{r^2}\left(1 - \frac{r_0^4}{r^4}\right)^{-1} dr^2 + L^2 d\Omega_5^2 \tag{2}$$

is a product of S^5 with a certain limit of the Schwarzschild black hole in AdS_5 [79]. The horizon is given by $r_h = r_0$.

AdS_5 part of (Euclidean) metric (2) has the following Ricci scalar and Ricci tensor:

$$R = \frac{-20}{L^2} \quad , \quad R_{mn} = \frac{-4}{L^2} g_{mn} \tag{3}$$

It is obvious that metric (2) is a solution of equation of motion defined by the following action that is a 10-dimentional gravity:

$$I = -\frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g_{10}} R_{10} \tag{4}$$

In AdS/CFT correspondence, since $Vol(S^5) = \pi^3 L^5$, we have the following identification:

$$N_c^2 = \frac{\pi^4 L^8}{2G_{10}} \tag{5}$$

After Wick-rotating the time variables by $t \to it$, the free energy F can be obtained from action I in (4). Following [80], one can identify free energy F of the theory with Euclidean gravitational action times temperature, i.e.

$$I = \beta F \tag{6}$$

In above relation the temperature is defined as:

$$\beta = \frac{1}{T} \tag{7}$$

We can find the corresponding Hawking temperature, by employing thermodynamic relation dE=TdS, or through the absence of a conical singularity in periodic Euclidean metric,

$$T = \frac{g'_{00}(r_h)}{4\pi} \tag{8}$$

$$T = \frac{1}{4\pi}\left[\frac{r^2}{L^2}\left(1 - \frac{r_0^4}{r^4}\right)\right]' |_{(r = r_h)} = \frac{1}{4\pi}\left[\frac{2r}{L^2} + \frac{2r_h^4}{L^2 r^3}\right] |_{(}\ r = r_h) = \frac{r_h}{\pi L^2} \tag{9}$$

$$r_h = \pi L^2 T \tag{10}$$

Using $\sqrt{-g_{10}} = r^3 L^2 \sin\theta_1^4 \sin\theta_2^3 \sin\theta_3^2 \sin\theta_4^1$, (3) and (4) and compactification of S^5 we arrive at the following form of the action:

$$I = -\frac{1}{16\pi G_{10}} \int_0^\beta dt \int d\Omega_5 \int d^3x \int_{r_h}^{r_{Max}} L^2 r^3 \left(\frac{-20}{L^2} + \frac{12}{L^2} + ...\right) dr \tag{11}$$

where dots stand for matter fields which appear by compactification. In the limit that such matter fields have a trivial behavior we have:

$$I = \frac{1}{16\pi G_{10}} \left(\frac{8}{L^2}\right) \int_0^\beta dt \int d\Omega_5 \int d^3x \int_{r_h}^{r_{Max}} L^2 r^3 dr = \frac{2}{16\pi G_{10}} \frac{V_3 \Omega_5}{T} (r_{Max}^4 - r_h^4) \tag{12}$$

here V_3 is the volume of 3d flat space. We also assume β has a period of 1/T . Using (6) we find:

$$F_{SAdS} = TI = \frac{2}{16\pi G_{10}} V_3 \Omega_5 (r_{Max}^4 - r_h^4) \tag{13}$$

The expression for F_{SAdS} and I contains the divergence originating from large r. In order to subtract the divergence, we regularize I in (1) by cutting off the integral at a large radius r_{Max} followed by action I, which must be subtracted off its zero temperature limit, I_{AdS}. If we continue above method for free energy in thermal AdS spacetime, we will have:

$$I = \frac{1}{16\pi G_{10}} \left(\frac{8}{L^2}\right) \int_0^{\beta_0} dt \int d\Omega_5 \int d^3x \int_0^{r_{Max}} L^2 r^3 dr = \frac{2}{16\pi G_{10}} \frac{V_3 \Omega_5}{T_0} r_{Max}^4 \quad (14)$$

We remind that care is needed for thermal AdS temperature. This requirement that Euclidean geometry does not have a conical singularity, determines the temperature of a black hole. The periodicity β_0 for thermal AdS spacetime is arbitrary, but we are interested in free energy difference, so we must match the thermal AdS temperature with that of SAdS. The relation is:

$$\beta f^{1/2} = \beta_0 f_0^{1/2} \quad (15)$$

where f and f_0 are g_{00} components for the SAdS black hole and the thermal AdS spacetime respectively. We have:

$$\frac{1}{T_0} = \frac{1}{T_0} \frac{(g_{00})^{1/2}_{SAdS}}{(g_{00})^{1/2}_{AdS}} \frac{1}{T} = \frac{1}{T}\left(1 - \frac{r_h^4}{2r_{Max}^4}\right) \quad (16)$$

Using (14) and (16), we find

$$I_{AdS} = \frac{2}{16\pi G_{10}} V_3 \Omega_5 \frac{1}{T}\left(1 - \frac{r_h^4}{2r_{Max}^4}\right)(r_{Max}^4) \quad (17)$$

By using (6) we have:

$$F_{AdS} = T I_{AdS} = \frac{2}{16\pi G_{10}} V_3 \Omega_5 \left(1 - \frac{r_h^4}{2r_{Max}^4}\right)(r_{Max}^4) \quad (18)$$

After subtracting I_{SAdS} from I_{AdS} and in the limit $r_{max} \rightarrow \infty$ we find the following expression for free energy,

$$F = -\frac{V_3 \Omega_5}{16\pi G_{10}} r_{Max}^4 \quad (19)$$

Using (10), and (19) and AdS/CFT dictionary (5), free energy F can be rewritten in the following form by putting L=1:

$$F = -\frac{V_3}{8} \pi^2 N^2 T^4 \quad (20)$$

We now compare the above results with those of $\aleph = 4$ SYM gauge theory. The corresponding free energy, as shown in refs [6, 7] is given by:

$$F = -\frac{V_3}{6} \pi^2 N^2 T^4 \quad (21)$$

which is different from AdS result by a factor 4/3, this difference may be regarded as due to only employing the leading approximation term.

3 Action, Metric and Free Energy in AdS-like Cosmological Space

Next question is the role of AdS/CFT correspondence in the situation that cosmological AdS-like space is considered. It is very similar to the above method. We can obtain a metric of a kind of cosmological model if inside horizon $r < r_h$, we replace t by r and r by t [25]:

$$ds^2 = -\frac{L^2}{t^2}\left(\frac{r_0^4}{t^4}-1\right)^{-1} dt^2 + \frac{t^2}{L^2}\left[\left(\frac{r_0^4}{t^4}-1\right) dr^2 + d\overrightarrow{x}^2\right] + L^2 d\Omega_5^2 \tag{22}$$

It is clear that what we have done is, exchanging the physical role of time and radial coordinates [81]. Since we have only exchanged the coordinates r and t, the metric will satisfy Einstein equations. In this situation, the singularity of black hole corresponds to t=0, so there is a curvature singularity. The singularity at t=0 might be regarded as a big bang.

One can consider an analogue $\widetilde{F}$ of free energy in metric (22). $\widetilde{F}$ would be gained by substituting the metric given in (22) into gravitational action ($\widetilde{I}$) as in (6) after Wick-rotation t→ it. Since r is the time coordinate t in black hole metric (2), r could have a period of $1/T$ after the Wick-rotation. As (11) the on-shell bulk action is:

$$\widetilde{I} = \frac{1}{16\pi G_{10}} \int_0^\beta dr \int d\Omega_5 \int d^3x \int_0^{r_h} L^2 t^3 \left(-\frac{8}{L^2}\right) dt = -\frac{1}{8\pi G_{10}} \beta V_3 \Omega_5 r_h^4 \tag{23}$$

Using (10) we find

$$\widetilde{I} = -\frac{1}{8\pi G_{10}} \frac{V_3 \Omega_5}{T} (\pi L^2 T)^4 \tag{24}$$

Then we find the following expression for free energy $\widetilde{F}$:

$$\widetilde{F} = T\widetilde{I} = -\frac{1}{8\pi G_{10}} V_3 \Omega_5 (\pi L^2 T)^4 \tag{25}$$

Taking (6) the above relation can be rewritten as:

$$\widetilde{F} = -\frac{V_3}{4} \pi^2 N^2 T^4 \tag{26}$$

which is different from the CFT result by a factor $3/2$, this difference again may be regarded as due to only employing the leading approximation term. We can obtain other thermodynamic quantities like entropy ($\widetilde{S}$) and energy ($\widetilde{E}$) from $\widetilde{F}$:

$$\widetilde{S} = -\partial_T \widetilde{F} = \frac{1}{2\pi G_{10}} V_3 \Omega_5 \pi^4 L^8 T^3 \tag{27}$$

$$\widetilde{E} = \widetilde{F} + T\widetilde{S} = \frac{3}{8\pi G_{10}} V_3 \Omega_5 (\pi L^2 T)^4 \tag{28}$$

4 The Role of R^2-term and Boundary Terms in Casimir Energy and in Some Thermodynamic Quantities

As mentioned above, the difference in results between cosmological AdS-like space and CFT may be regarded as due to only employing the leading approximation method. So it will be interesting to regard higher derivatives in action. Moreover higher derivatives are also interesting, because any effective stringy gravity includes higher derivative terms of different order which from the point of view of AdS/CFT correspondence gives next-to-leading terms in large N expansion [82, 83]. So, higher derivative gravity may serve as quite a good candidate for construction of realistic brane-world cosmologies [84, 85].

The presence of higher derivative corrections to the supergravity action will change both the thermodynamics of anti-de Sitter black holes and cosmological AdS-like space. We consider these corrections on gravity side of the correspondence. It was already shown [86] that such a bulk theory may correctly reproduce trace anomaly of ℵ = 4 super YM theory. We will check such HD AdS/CFT conjecture by considering Gibbons-Hawking and the counterterm action. We will also check such HD AdS/CFT conjecture for cosmological AdS-like space. Before doing this, we should mention that for thermodynamic properties it is necessary to add the surface term to the Einstein-Hilbert action. In fact the full gravitational action in n+1 spacetime dimensions has three contributions as:

$$I_E = I_{bulk} + I_{GT} + I_{CT} \tag{29}$$

here, I_{bulk}, I_{GT}, and I_{CT} are called bulk action, Gibbons-Hawking action, and counterterm action, respectively. In the following we will calculate all these three terms. The general bulk action of n+1-dimensional R^2 gravity is given by:

$$I_{bulk} = \int d^{n+1}x\sqrt{-g_{n+1}}\left(\frac{1}{\kappa^2}R - \Lambda - \frac{1}{2}(\partial\varphi)^2 + \alpha^1 R^2 + \alpha^2 R^3 + \alpha^3 R^4 + ...\right) \tag{30}$$

where dots stand for terms depending on antisymmetric tensor field strengths, derivatives of dilaton and other matter fields. We will focus on $\alpha^1 R^2$ string correction to the supergravity action. In the limit that other matter fields have a trivial behaviour with a constant dilation φ , we will have:

$$I_{bulk} = \int d^{n+1}x\sqrt{-g_{n+1}}\left(\frac{1}{\kappa^2}R - \Lambda + \alpha^1 R^2\right) \tag{31}$$

If we put $a = \alpha^1$ and follow the method used in [87, 88] we can define metric ansatz as the following:

$$ds^2 = -\left(k - \frac{\mu}{r^{n-2}} + \frac{r^2}{L^2}\right)dt^2 + \left(k - \frac{\mu}{r^{n-2}} + \frac{r^2}{L^2}\right)^{-1} dr^2 + r^2 d\Omega^2_{n-1} \tag{32}$$

where $d\Omega^2_{n-1}$ is the unit metric on S^{n-1} , μ is the parameter corresponding to the mass of black hole and k is a constant (which could be 1, 0 and -1).

In this case, the curvature tensors become

$$R = -\frac{n(n+1)}{L^2} \quad , \quad R_{mn} = -\frac{n}{L^2}g_{mn} \tag{33}$$

So the curvatures are covariantly constant. Then in equations of motion following the bulk action, the terms containing covariant derivatives of the curvatures vanish which is leading to:

$$0 = -\frac{1}{2}g_{\alpha\beta}\left(\frac{1}{\kappa^2}R - \Lambda + aR^2\right) + 2aRR_{\alpha\beta} + \frac{1}{\kappa^2}R_{\alpha\beta} \tag{34}$$

By substituting (33) into (34), we find

$$0 = \frac{n^2(n+1)(n-3)a}{L^4} - \frac{n(n-1)}{\kappa^2 L^2} - \Lambda \tag{35}$$

As before the temperature is given by:

$$T = \frac{g'_{00}(r_+)}{4\pi} = \frac{nr_+^2 + k(n-2)L^2}{4\pi L^2 r_+} \tag{36}$$

Here, r_+ is the root of $g_{00} = 0$.

From above equation, r_+ can be rewritten in terms of T as

$$r_+ = \frac{4\pi L^2 T \sqrt{16\pi^2 L^4 T^2 - 4nk(n-2)L^2}}{2n} \tag{37}$$

In (37), the plus sign corresponds to k=-1 or k= 0 and the minus sign corresponds to k= 1. Substituting (35) into (31) one gets

$$I_{bulk,k,n+1} = -\frac{\beta\Omega_{k,n-1}}{L^2}\left(\frac{1}{\kappa^2} - \frac{2n(n+1)a}{L^2}\right)\left(r_+^n - L^2 k r_+^{n-2}\right) \tag{38}$$

As there is a boundary, we need to add a surface term to the action, which is called the Gibbons-Hawking surface term

$$I_{GT} = -\frac{2}{\kappa^2}\int d^n x\sqrt{-\gamma}K \tag{39}$$

and is evaluated at $r = \infty$. Here, $\gamma_{\mu\nu}$ is the (n + 1)-dimensional metric at the surface, and K is the trace of the extrinsic curvature of the surface as described below. If we change $u := r_0/r$, the (n +1)-dimensional metric $\gamma_{\mu\nu}$ is given by decomposing the metric as

$$ds^2_{n+2} = g_{uu}du^2 + \gamma_{\mu\nu}x_\mu x_\nu \tag{40}$$

K and the unit normal to u= (constant) surface, n^M, that is pointing in the direction of decreasing u or increasing r are given by:

$$g_{MN}n^M n^N = 1 \rightarrow n^u = -\frac{1}{\sqrt{g_{uu}}} \quad , \quad K = n^u\frac{\partial_u\sqrt{\gamma}}{\sqrt{\gamma}} \tag{41}$$

The second surface term is the counterterm action that may be arranged as an expansion in powers of the boundary curvature and its derivatives:

$$\begin{aligned} I_{ct} &= \frac{2}{\kappa^2}\int d^n x\sqrt{-\gamma}K \quad \left[\frac{(n-1)}{L} - \frac{L^2}{2(n-2)}R + \frac{L^3}{2(n-4)(n-2)^2}\right. \\ &\qquad \left.\left(R_{\mu\nu}R^{\mu\nu} - \frac{n}{4(n-1)}\right)R^2 + ...\right] \end{aligned} \tag{42}$$

where R and $R_{\mu\nu}$ are the Ricci scalar and Ricci tensor for the boundary metric, respectively. Calculating (39) and (42), using (38) and substituting into (29) we have:

$$\begin{aligned} I_{E,k,n+1} &= -\frac{\beta\Omega_{k,n-1}}{L^2}\left(\frac{1}{\kappa^2} - \frac{2n(n+1)a}{L^2}\right) \\ &\quad \times\left(r_+^n - L^2 k r_+^{n-2} + L^2 k\delta_{n,2} - \frac{3}{4}L^4k^2\delta_{n,4} + \frac{5}{8}L^6 k\delta_{n,6} - ...\right) \end{aligned} \tag{43}$$

$$\begin{aligned} F_{E,k,n+1} &= T I_{E,k,n+1} = -\frac{\Omega_{k,n-1}}{L^2}\left(\frac{1}{\kappa^2} - \frac{2n(n+1)a}{L^2}\right) \\ &\quad \times\left(r_+^n - L^2 k r_+^{n-2} + L^2 k\delta_{n,2} - \frac{3}{4}L^4k^2\delta_{n,4} + \frac{5}{8}L^6 k\delta_{n,6} - ...\right) \end{aligned} \tag{44}$$

$$S = \frac{A}{4\widetilde{G}_{n+1}} = \frac{\Omega_{n-1}r_+^{n-1}}{4\widetilde{G}_{n+1}} = 4\pi\Omega_{n-1}\left(\frac{1}{\kappa^2} - \frac{2n(n+1)a}{L^2}\right)r_+^{n-1} \tag{45}$$

$$E = F + TS \tag{46}$$

$= \frac{(n-1)\Omega_{k,n-1}}{L^2}(\frac{1}{\kappa^2} - \frac{2n(n+1)a}{L^2})(r_+^n + L^2 k r_+^{n-2}) + \frac{\Omega_{k,n-1}}{L^2}(\frac{1}{\kappa^2} - \frac{2n(n+1)a}{L^2})(-L^2 k\delta_{n,2} + \frac{3}{4}L^4k^2\delta_{n,4} - \frac{5}{8}L^6 k\delta_{n,6} + ...)$

Our relations show that the first law of thermodynamics (dE=TdS) still remains valid. This fact can be checked by deriving relations (45) and (46) from r_+ and using relation (36). We will also explain at the end of section 5 the correction to entropy that has appeared as a redefinition of gravitation constant as $\left(\frac{1}{\kappa^2} \longrightarrow \frac{1}{\kappa^2} - \frac{2n(n+1)a}{L^2}\right)$.

It is interesting to evaluate energy for n=4 and k=1, namely

$$E = 2\pi^2 \left(\frac{1}{16\pi G_5} - \frac{40a}{L^2}\right)\left(3\mu + \frac{3}{4}L^2\right) \tag{47}$$

The energy has a constant term, so by using the AdS/CFT dictionary, $L^3/G_5 = 2N_c^2/\pi$ the constant term will be written as

$$E = 2\pi^2 \left(\frac{1}{16\pi G_5} - \frac{40a}{L^2}\right)\left(\frac{3}{4}L^2\right) = \frac{3N_c^2}{16L} - 60\pi^2 a \tag{48}$$

This can be interpreted as the Casimir energy [89] that coincides with that calculated for ℵ = 4 SYM. In fact the calculation of Casimir energy from QFT gives a non-zero result, which in this case is given by:

$$E_{Casmir} = \frac{3N_c^2}{16L} - \frac{3}{16L} \tag{49}$$

By choosing $a = 3/(960L\pi^2)$ in (48), we see a very good consistency between the two results obtained from different methods. We will use this in next section.

5 The Role of R^2 Term to Thermodynamics of AdS/CFT Correspondence on Cosmological Level

The question that arises is, the role of adding R^2 term to action in AdS/CFT correspondence when cosmological AdS-like space is considered. Similar to the previous sections we can obtain a metric of a kind of the cosmological model if inside the horizon $r < r_h$, we change the role of r as t and the role of t as r:

$$ds^2 = -\left(k - \frac{\mu}{t^{n-2}} + \frac{t^2}{L^2}\right)dr^2 + \left(k - \frac{\mu}{t^{n-2}} + \frac{t^2}{L^2}\right)^{-1} dt^2 + t^2 d\Omega_{n-1}^2 \tag{50}$$

That is, we have exchanged the physical role of time and radial coordinates. Substituting (35) into (31) one gets

$$\begin{aligned}\widetilde{I} &= \int d^{n+1}x\sqrt{-g_{n+1}}\left(\frac{1}{\kappa^2}R - \Lambda + a^1 R^2\right) = -\int_0^\beta dr \int d\Omega_{k,n-1} \\ &\quad \int_0^{r_+} t^{n-1}\left(\frac{8}{L^2\kappa^2} - \frac{16n(n+1)a}{L^4}\right)dt \\ &= -\frac{\beta\Omega_{k,n-1}}{nL^2}\left(\frac{8}{\kappa^2} - \frac{16n(n+1)a}{L^2}\right)r_+^n \end{aligned} \tag{51}$$

By inserting (37) into above relation we find

$$\widetilde{F} = -\frac{\Omega_{k,n-1}}{nL^2}\left(\frac{8}{\kappa^2}-\frac{16n(n+1)a}{L^2}\right)\left(\frac{4\pi L^2 T \pm \sqrt{16\pi^2 L^4 T^2 - 4nk(n-2)L^2}}{2n}\right)^n \quad (52)$$

$$\begin{aligned}\widetilde{S} &= -\frac{d\widetilde{F}}{dT} \quad (53)\\ &= \frac{\Omega_{k,n-1}}{L^2}\left(\frac{8}{\kappa^2}-\frac{16n(n+1)a}{L^2}\right)\left(\frac{4\pi L^2 T \pm \sqrt{16\pi^2 L^4 T^2 - 4nk(n-2)L^2}}{2n}\right)^{n-1}\\ &\times\left(\frac{4\pi L^2}{2n} \pm \frac{16\pi^2 L^4 T}{2n\sqrt{16\pi^2 L^4 T^2 - 4nk(n-2)L^2}}\right)\\ \widetilde{E} &= \widetilde{F} + T\widetilde{S} \quad (54)\\ &= \frac{\Omega_{k,n-1}}{L^2}\left(\frac{8}{\kappa^2}-\frac{16n(n+1)a}{L^2}\right)\left(\frac{4\pi L^2 T \pm \sqrt{16\pi^2 L^4 T^2 - 4nk(n-2)L^2}}{2n}\right)^{n}\\ &\times\left(-\frac{1}{n} \pm \frac{4\pi L^2 T}{\sqrt{16\pi^2 L^4 T^2 - 4nk(n-2)L^2}}\right)\end{aligned}$$

If we evaluate above results for n=4, k=0 and L=1 we find

$$r_+ = \pi L^2 T \quad (55)$$

$$\widetilde{I} = -\left(\frac{1}{\kappa^2} - \frac{40a}{L^2}\right)\frac{2\beta V_3}{L^2} r_+^4 \quad (56)$$

$$\widetilde{S} = \left(\frac{1}{\kappa^2} - \frac{40a}{L^2}\right) 8\pi V_3 r_+^3 \quad (57)$$

$$\widetilde{F} = -\left(\frac{1}{\kappa^2} - \frac{40a}{L^2}\right)\frac{2V_3}{L^2} r_+^4 \quad (58)$$

$$\widetilde{E} = \left(\frac{1}{\kappa^2} - \frac{40a}{L^2}\right)\frac{6V_3}{L^2} r_+^4 \quad (59)$$

By using $N^2 = (\pi/2)(L^3/G_5)$, $a = 3/(960L\pi^2)$ and putting L=1 we have:

$$\widetilde{S} = \left(1 - \frac{1}{N^2}\right)\pi^2 V_3 N^2 T^3 \quad (60)$$

$$\widetilde{F} = -\frac{3}{2}\left(1 - \frac{1}{N^2}\right)\frac{\pi^2 V_3 N^2 T^4}{6} \quad (61)$$

$$\widetilde{E} = \frac{3}{4}\left(1 - \frac{1}{N^2}\right)\pi^2 V_3 N^2 T^4 \quad (62)$$

We now compare the above results with those of ℵ = 4 super Yang-Mills theory at least on the level of free energy. From the perturbative QFT, free energy is given by:

$$F = -(1 - \frac{1}{N^2})\frac{\pi^2 V_3 N^2 T^4}{6} \quad (63)$$

which is different from the cosmological AdS-like space result by a factor 3/2. Thus, our results show that there is a cosmological AdS/CFT correspondence between cosmological

AdS-like space and ℵ = 4 SU(N) super Yang-Mills. It indicates that we can calculate QFT quantities starting from cosmological background.

At this stage there are two points worth mentioning: The first point to comment on, is the first law of thermodynamics, in fact once again our relations show that the first law of thermodynamics (dE=TdS) still remains valid. This fact can be checked by deriving relations (57) and (59) from r_+ and also by using relation (55). The second point is concerned with our correction to entropy (relation (57)). This fact that entropy is not directly related to the horizon area in higher-derivative gravity has been already noted in some previous works. We can see a logarithmic correction due to quantum modification in some works [90–96]. However, according to Wald' entropy formula of black holes [97], it is quite difficult to produce such a logarithmic term in black hole entropy in some effective local theory of gravity even with higher derivative curvature terms. In Ref. [98], quantum corrections to the entropy area relation of order $e^{-\kappa/\hbar}$ was studied. In Ref. [99] a charged dilatonic black holes with the Yang-Mills and Gauss-Bonnet curvature corrections was considered and as is usual in higher derivative theories, it does not coincide with the area of the event horizon. In fact it is a positive definite function of M as $S = 4\pi M^2(1 + \alpha\frac{2-q}{2M^2} + \alpha^2\frac{73-45q^2}{120M^4})$. Corrections to four dimensioanl non-extremal black hole solutions of string theory with fundamental string momentum and winding charges are studied in Ref. [100] in the presence of higher derivative corrections of the form λR^2. It was found that the Bekenstein-Hawking entropy is increased by an overall factor $(1 + \frac{2\lambda}{r_h^2})$, where r_h is the radius of the black hole.

In Ref. [101] a general class of black holes in ℵ=2 supergravity that contains solutions of type-IIA string theory compactified on a general Calabi-Yau 3-fold with higher derivative corrections was considered. The entropy of these black holes no longer satisfies the Bekenstein-Hawking area formula but includes the correction due to higher derivative as $S = \frac{A}{2} = \frac{A}{4} + \frac{A}{4}$. That means the correction to the entropy is of the same order as the Bekenstein-Hawking entropy. The case for spherically symmetric horizons in Lanczos-Lovelock lagrangian was considered in Ref. [102], it was shown that in principle, the corrections to the entropy and the energy coming from the higher order Lanczos-Lovelock terms do not need to preserve the structure of first law of thermodynamics. The expression for entropy is in the form of $S = \frac{A_{D-2}}{4}a^{D-2}[1 + (D-2)(D-3)\frac{2\alpha}{a^2}]$ where α is regarded as the inverse string tension and a is horizon for this spacetime. Leading order entropy correction arising from R^2 terms for the supersymmetric black ring and the 5D BMPV (the charged rotating black hole in 5D) spinning black hole, was studied in Ref. [103]. The correction to the entropy is $\Delta S = \frac{\pi}{6}A\ c_2.Y(\frac{1}{Q} - \frac{A^2}{4Q^4})$. Here Y_A, are scalar components of vector multiplets and c_{2A} the components of the second Chern class of CY_3 which is also defined as $A = \sqrt{Q^3 - J^2}$.

In Ref. [104] it was shown that for a stationary black hole including higher derivative corrections $(R + \lambda R_{\mu\upsilon\rho\varepsilon}R^{\mu\upsilon\rho\varepsilon})$ the entropy is equal to a quarter of the horizon area in units of the effective gravitational coupling, so that $G_{eff} = G_N(1 - 4\lambda R^{trtr})^{-1}$, in this case the entropy is $S = \frac{A_H}{4G_{eff}} = \frac{A_H}{4G_N}(1 - 4\lambda R^{rtrt})$. Our resuls also show that R^2 term contribution appears in entropy via the redefinition of gravitation constant as $(\frac{1}{\kappa^2} \longrightarrow \frac{1}{\kappa^2} - \frac{2n(n+1)a}{L^2})$. It should be noted that in our analysis there is no relation for entropy with logarithmic corrections. Some other corrections to entropy at higher derivatives can be found in Refs. [105, 106].

6 Conclusions

AdS/CFT correspondence on cosmological level has been studied in some previous works. Without HD terms the possibility of such a cosmological AdS/CFT duality has been found in ref [25]. We tried in this paper, to understand the role of HD terms in cosmological AdS/CFT correspondence from a different point of view. We used AdS Einstein gravity with R^2 term and added a Gibbon-Hawking surface term as well as a boundary counterterm to the gravitational action and considered an AdS-like cosmological space.

Then we calculated some thermodynamic quantities such as entropy, energy and free energy. To check the validity of our results, we compared our calculations with perturbative results obtained from boundary QFT. We know from the viewpoint of AdS/CFT correspondence, that higher derivative terms in gravity side correspond to the 1/N corrections in large N limit of gauge theory side. For instance, the free energy obtained from R^2 term and metric as a function of N^2T^4 times $3/2(1-1/N^2)$, is similar to the result obtained from perturbative boundary QFT . In fact by taking into account the next to leading term in large N expansion, the difference between results obtained from AdS-like cosmological space and the results obtained from strong coupling limit of $\aleph = 4$ SCFT, is just a factor 3/2.

Calculation of energy and entropy showed that R^2 term contribution also appears in these quantities via the redefinition of gravitation constant as $(\frac{1}{\kappa^2} \longrightarrow \frac{1}{\kappa^2} - \frac{2n(n+1)a}{L^2})$. This can be checked in our paper in some thermodynamic quantities like entropy (relations (45), (53)) and energy (relations (46), (54)).

We explicitly identify higher order counterterms effect in the energy as a constant term namely E_0. This value can be interpreted as the Casimir energy, so that, there is a full and perfect match between the QFT result and gravitational action when R^2 term and boundary counterterms are considered. Then our results indicate that cosmological AdS/CFT correspondence (with a corrected action) could be valid in the next-to-leading order of the 1/N.

In this area, we suggest that behavior of thermodynamic quantities and validity of cosmological AdS/CFT correspondence be checked when a non-constant dilaton term is added to gravitational action.

For more work one can focus on $R_{\mu\upsilon}R^{\mu\upsilon}$ and $R_{\mu\upsilon\rho\varepsilon}R^{\mu\upsilon\rho\varepsilon}$ correction terms in gravity action and study the validity of cosmological AdS/CFT correspondence.

Finally it is also interesting to find the correction imposed on the ratio of shear viscosity to the entropy density for this new correspondence.

Declarations

Conflict of Interests The author declares that there is no conflict of interest.

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