$O(9,9)$ symmetry of NS-NS couplings at order α'^3

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Recently, imposing the $O(1,1)$ symmetry on the circle reduction of the classical effective action of string theory, we have found all NS-NS couplings of type II superstring theories at order α'^3. In this paper we use the cosmological reduction on the couplings and show that, up to one-dimensional field redefinitions and total derivative terms, they are invariant under the $O(9,9)$ transformations.

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I. INTRODUCTION

A theory of gravity in a spacetime manifold with/without a boundary that is consistent with the rules of quantum mechanics is string theory. This theory includes the finite number of massless modes and the tower of infinite number of massive modes of the string excitations. At low energies, however, the massive modes are integrated out to produce an effective theory that includes only the massless fields. The effective action has a double expansions. The genus expansion includes the classical tree level, i.e., $\mathbf{S}_{\rm eff}+\partial\mathbf{S}_{\rm eff}$, a tower of quantum loop-level corrections, and the stringy expansion that is an expansion in terms of higher derivative couplings at each loop level. The classical effective action has the following higher derivative or α' expansion:

$$\begin{aligned}\mathbf{S}_{\rm eff}&=\sum_{m=0}^{\infty}\alpha'^m\mathbf{S}_m=\mathbf{S}_0+\alpha'\mathbf{S}_1+\alpha'^2\mathbf{S}_2+\alpha'^3\mathbf{S}_3+\cdots,\\ \partial\mathbf{S}_{\rm eff}&=\sum_{m=0}^{\infty}\alpha'^m\partial\mathbf{S}_m=\partial\mathbf{S}_0+\alpha'\partial\mathbf{S}_1\\ &\quad+\alpha'^2\partial\mathbf{S}_2+\alpha'^3\partial\mathbf{S}_3+\cdots.\end{aligned}\tag{1}$$

The leading order bulk action $\mathbf{S}_0$ includes the Hilbert-Einstein term and the boundary action $\partial\mathbf{S}_0$ includes the Hawking-Gibbons term [1,2]. These actions and their appropriate higher derivative extensions should be found by specific techniques in the string theory. Since the effective action includes the couplings at all orders of derivative, one complication in finding the couplings in the string theory is the freedom of the field redefinitions that include the higher derivatives of fields [3]. As a result, the effective action in the string theory can appear in many different equivalent schemes.

One of the most exciting discoveries in perturbative string theory is T duality [4,5] that appears when one compactifies theory on a torus, e.g., the compactification of the full bosonic string theory on tours T^d is invariant under $O(d,d,Z)$ transformations. After integrating out the massive modes, however, the T duality should appears as symmetry in the effective actions. It has been shown in [6,7] that the dimensional reduction of the classical effective actions of the bosonic and heterotic string theories on a torus T^d are in fact invariant under $O(d,d,R)$ transformations.

When one reduces the effective action on a circle, the invariance of the reduced action under the Z_2 subgroup of the $O(1,1,R)$ group constrains greatly the couplings in the effective action. In fact there is only one T-dual multiplet in the effective action of the bosonic string theory, and there are two T-dual multiplets in the effective action of type II superstring theories at the leading order of α', one for NS-NS couplings and one for R-R couplings [8]. By the T-dual multiplet we means the set of couplings in the effective action which are related into each others under the Z_2 transformations after reducing them on the circle. The Z_2 transformations or T-duality transformations are the Buscher rules [9,10] and some higher derivative corrections at each order of α' that depend on the scheme that one uses for the gauge invariant couplings in the effective action at that order of α' [11,12]. The corrected transformations, however, should satisfy the Z_2 symmetry. There is no scheme for the higher-derivative couplings in the original action in which the T-duality transformations are the standard Buscher rules [12].

Since the T-duality transformations have higher derivative corrections, the other T-dual multiplets include couplings at all orders of α'. In fact it has been observed in [13]

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that the couplings in the effective action of the bosonic string theory at order α' are related by the T-duality transformations to the couplings at order α'^2. They belong to one T-dual multiplet. If one extends the calculations in [13] to the order α'^3, then one would find that the couplings at order α' and α'^2 are related to some of the couplings at order α'^3. They belong to the same T-dual multiplet. However, there are couplings at this order that are not connected to the couplings at order α', α'^2 by the T-duality transformations. They belong to another T-dual multiplet. In other words, if one finds the couplings in the bosonic string theory at order α'^3 by S-matrix method, one would find they have two factors a_1, a_2, i.e., $\mathbf{S}_3 = a_1\mathbf{S}_3^1 + a_2\mathbf{S}_3^2$. One factor should be the same as the one appears in the couplings at order α', α'^2, and another one is proportional to $\zeta(3)$. These two factors should appear in the T-duality transformations as well. At the order α'^4, the couplings in the bosonic theory should have three factors a_1, a_2, a_3, i.e., $\mathbf{S}_4 = a_1\mathbf{S}_4^1 + a_2\mathbf{S}_4^2 + a_3\mathbf{S}_4^3$. One is the same as the factor in α', α'^2, one is proportional to $\zeta(3)$, and another is an independent factor. Similar patterns should appear for the higher orders of α'. Schematically, the bulk action (1) has the following expansion in terms of the T-dual multiplets:

$$\mathbf{S}_{\rm eff} = \sum_{n=0}^{\infty} a_n T_n = a_0 T_0 + a_1 T_1 + a_2 T_2 + a_3 T_3 + \cdots, \tag{2}$$

where $a_0, a_1, \cdots$ are some coefficients that cannot be fixed by the T duality. They should be fixed by the S-matrix calculations, e.g., $a_0 = 1$, $a_1 = 1$, $a_3 = \zeta(3)$. In the type II superstring theory, $a_1 = 0$, and there are T-dual multiplets that include the R-R couplings. The T-dual mutiplets in the bosonic string theory are

$$\begin{aligned}
T_0 &= \mathbf{S}_0, \\
T_1 &= \alpha'\mathbf{S}_1 + \alpha'^2\mathbf{S}_2 + \alpha'^3\mathbf{S}_3^1 + \alpha'^4\mathbf{S}_4^1 + \alpha'^5\mathbf{S}_5^1 + \cdots, \\
T_2 &= \alpha'^3\mathbf{S}_3^2 + \alpha'^4\mathbf{S}_4^2 + \alpha'^5\mathbf{S}_5^2 + \alpha'^6\mathbf{S}_6^2 + \alpha'^7\mathbf{S}_7^2 + \cdots, \\
T_3 &= \alpha'^4\mathbf{S}_4^3 + \alpha'^5\mathbf{S}_5^3 + \alpha'^6\mathbf{S}_6^3 + \alpha'^7\mathbf{S}_7^3 + \alpha'^8\mathbf{S}_8^3 + \cdots, \\
&\vdots
\end{aligned} \tag{3}$$

There are no parameters in these multiplets. If the spacetime manifold has no boundary in which the total derivative terms can be ignored, then each multiplet should be invariant under the T-duality transformations after reducing it on the circle. In principle, this constraint may fix all couplings in each T-dual multiplet. The couplings in the multiplet T_1 at orders α' and α'^2 in a particular scheme have been found in [12,13]. The couplings in the multiplet T_2 at order α'^3 have been also found in [14,15].

When the spacetime has a boundary, however, one should keep the total derivative terms before and after reduction and use the Stokes's theorem to transfer them to the boundary. They dictate that the invariance under the T-duality transformations requires some couplings on the boundary as well [16]. Hence, the bulk T-dual multiplets should be accompanied with appropriate boundary couplings to be fully invariant under the T duality. Schematically, the boundary action (1) should have the following expansion in terms of the boundary T-dual multiplets:

$$\begin{aligned}
\partial\mathbf{S}_{\rm eff} &= \sum_{n=0}^{\infty} a_n \partial T_n = a_0\partial T_0 + a_1\partial T_1 \\
&\quad + a_2\partial T_2 + a_3\partial T_3 + \cdots,
\end{aligned} \tag{4}$$

where $a_0, a_1, \cdots$ are the same coefficients that appear in the bulk T-dual multiplets (2). The boundary mutiplets are

$$\begin{aligned}
\partial T_0 &= \partial\mathbf{S}_0, \\
\partial T_1 &= \alpha'\partial\mathbf{S}_1 + \alpha'^2\partial\mathbf{S}_2 + \alpha'^3\partial\mathbf{S}_3^1 + \alpha'^4\partial\mathbf{S}_4^1 + \alpha'^5\partial\mathbf{S}_5^1 + \cdots, \\
\partial T_2 &= \alpha'^3\partial\mathbf{S}_3^2 + \alpha'^4\partial\mathbf{S}_4^2 + \alpha'^5\partial\mathbf{S}_5^2 + \alpha'^6\partial\mathbf{S}_6^2 + \alpha'^7\partial\mathbf{S}_7^2 + \cdots, \\
\partial T_3 &= \alpha'^4\partial\mathbf{S}_4^3 + \alpha'^5\partial\mathbf{S}_5^3 + \alpha'^6\partial\mathbf{S}_6^3 + \alpha'^7\partial\mathbf{S}_7^3 + \alpha'^8\partial\mathbf{S}_8^3 + \cdots, \\
&\vdots
\end{aligned} \tag{5}$$

The combination of bulk and boundary multiplets, i.e., $T_i + \partial T_i$, are then invariant under the T-duality transformations. In other words, neither the bulk multiplets nor the boundary multiplets are invariant separately under the T-duality transformations. Their anomalies cancel each other. There are, however, boundary couplings that are invariant under the T-duality transformations without anomaly. Some of them are related to the anomalous boundary multiplets by imposing the principle of the least action in the presence of the boundary with appropriate boundary values for the massless fields. Using these constraints, the boundary coupling in the multiplet ∂T_0 has been found in [16,17]. The boundary couplings in the multiplet ∂T_1 at order α' in a particular scheme have been found in [17].

When one uses the cosmological reduction on the classical effective action, the resulting one-dimensional effective action should have $O(d,d,R)$ symmetry [6,7]. This symmetry has been first observed for the leading order bulk couplings in [18–20] and for the couplings at order α' in a specific scheme in [21]. The T-duality transformations or $O(d,d,R)$ transformations in this case also receive higher derivative corrections. The corrected transformations satisfy the $O(d,d,R)$ symmetry [21]. In this case also there is no scheme for the original couplings in which the T-duality transformation are the standard $O(d,d,R)$ transformations of the leading order. Unlike the circle reduction, some of the couplings in the original action disappear upon the reduction [22]. Hence, this symmetry is not appropriate for fixing the couplings in the original action. However, this symmetry is useful for classifying the couplings in the one-dimensional effective action at all orders of α' [22,23].

Using the most general corrections for the T-duality transformations, including the transformations for the lapse function, and using integration by part, it has been shown in [22,23] that the cosmological reduction of the bulk action (1) at order α' and higher can be written in a scheme in which only the first time derivative of the generalized metric $\mathcal{S}$ appears. Trace of odd number of the first derivative of $\mathcal{S}$ is zero. It has been shown in [23] that the couplings that include $\mathrm{tr}(\dot{\mathcal{S}}^2)$ can be removed by the lapse function transformation. Then the one-dimensional bulk action can be written in a specific scheme as the following expansion [22,23]:

$$\begin{aligned}\mathbf{S}_{\mathrm{eff}}^c = \mathbf{S}_0^c + \int dt e^{-\Phi}(\alpha' c_{2,0}\mathrm{tr}(\dot{\mathcal{S}}^4) + \alpha'^2 c_{3,0}\mathrm{tr}(\dot{\mathcal{S}}^6)\\ + \alpha'^3[c_{4,0}\mathrm{tr}(\dot{\mathcal{S}}^8) + c_{4,1}(\mathrm{tr}(\dot{\mathcal{S}}^4))^2]\\ + \alpha'^4[c_{5,0}\mathrm{tr}(\dot{\mathcal{S}}^{10}) + c_{5,1}\mathrm{tr}(\dot{\mathcal{S}}^6)\mathrm{tr}(\dot{\mathcal{S}}^4)] + \cdots),\end{aligned} \tag{6}$$

where the coefficient $c_{m,n}$ depends on the specific theory; e.g., $c_{2,0}$ is nonzero for the bosonic string theory, whereas this number is zero for the superstring theory.

To find the cosmological reduction of the corresponding boundary action in (1), one has to take into account the one-dimensional total derivative terms and the T-duality transformations or the field redefinitions that have been used in [22,23]. However, it has been observed in [17] that if one adds the total derivative term resulting from the cosmological reduction of the leading order action to the boundary by using the Stokes's theorem, it cancels the cosmological reduction of the Hawking-Gibbons term, i.e., $\mathbf{S}_0^c = 0$. Since the cosmological reduction of the boundary term at the leading order of α' is zero, we expect it should be zero at all higher orders of α' as well, i.e.,

$$\partial\mathbf{S}_{\mathrm{eff}}^c = 0. \tag{7}$$

This may be used for conforming the boundary couplings in the effective action (1) at each order of α' that can be found by the Z_2 symmetry. Since the cosmological reduction of the leading order action is zero, then for studying the cosmological reduction of the boundary action at order α', one does not need the field redefinitions used in the bulk action. It has been shown in [17] that if one adds the total derivative terms at order α' to the reduction of the boundary couplings at order α', they become zero, which is consistent with the above conjecture.

The cosmological bulk action (6) may also be used for confirming the bulk couplings in the effective action (1) that can be found by the Z_2 symmetry. The NS-NS couplings in the effective action of type II superstring theory at order α'^3 have been found in [14,15] by imposing the Z_2 symmetry on the effective action. In fact it has been shown in [24] that there are 872 independent couplings at this order. The Z_2 symmetry is imposed on these couplings in a particular scheme. Interestingly, all parameters are fixed up to an overall factor in [14]. In that scheme there are 445 nonzero couplings that include derivatives of dilaton. A field redefinition has been used in [15] to write them in terms of 251 couplings in which the dilaton appears only as the overall factor $e^{-2\phi}$. In this paper we are going to show that the cosmological reduction of the couplings in [14] or in [15] can be written in the form of the cosmological action (6) at order α'^3. The gravitational sector of these couplings which have been found a long time ago by the S matrix and sigma model calculations [25–28], has been shown in [29] that satisfies the $O(9,9)$ symmetry.

The outline of the paper is as follows: in Sec. II, we review the observation that the cosmological reduction of the leading order bulk and boundary actions are invariant under the $O(d,d)$ transformations. In Sec. III, we first reduce the bulk NS-NS couplings at order α'^3 that have been found in [15] to find its corresponding one-dimensional bulk action. We add to it all one-dimensional total derivative terms and all possible field redefinitions with arbitrary coefficients to write the action in a scheme which has all the arbitrary parameters of the field redefinitions and the total derivative terms. We then impose the constraint on the parameters that the cosmological action has no derivative of the one-dimensional dilaton, no second and higher derivatives on metric and the B field, and we impose the constraint that the couplings involving the first derivative of metric which are not consistent with the $O(9,9)$ symmetry to be zero. Moreover, we impose the condition that the terms that have contribution to the $O(9,9)$-invariant structure $\mathrm{tr}(\dot{\mathcal{S}}^2)$ to be zero. We have found that in fact there is a solution for the parameters for such conditions. After imposing the resulting relations between the parameters, we find the action in the scheme that can be written explicitly as the α'^3 order terms of (6).

II. COSMOLOGICAL REDUCTION AT THE LEADING ORDER

In this section, we review the cosmological reduction of the leading order bulk and boundary actions. These actions are given as

$$\begin{aligned}\mathbf{S}_0 + \partial\mathbf{S}_0 = -\frac{2}{\kappa^2}\bigg[\int d^D x\sqrt{-G}e^{-2\phi}\left(R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H^2\right)\\ + 2\int d^{D-1}\sigma\sqrt{|g|}e^{-2\phi}K\bigg],\end{aligned} \tag{8}$$

where G is determinant of the bulk metric $G_{\mu\nu}$ and the boundary is specified by the functions $x^\mu = x^\mu(\sigma^{\tilde{\mu}})$. In the boundary action, g is determinant of the induced metric on the boundary

$$g_{\tilde{\mu}\tilde{\nu}} = \frac{\partial x^\mu}{\partial \sigma^{\tilde{\mu}}}\frac{\partial x^\nu}{\partial \sigma^{\tilde{\nu}}} G_{\mu\nu}. \tag{9}$$

The extrinsic curvature of boundary, i.e., $K_{\mu\nu}$, is defined as $K_{\mu\nu} = \nabla_\mu n_\nu - n_\mu a_\mu$ where n^μ is the unite vector orthogonal to the boundary that is outward going (inward going) if the boundary is spacelike (timelike), and $a_\nu = n^\rho \nabla_\rho n_\nu$ is acceleration. It satisfies the relation $n^\mu a_\mu = 0$. The extrinsic curvature is symmetric and satisfies $n^\mu K_{\mu\nu} = 0$.

When fields depend only on time, using the gauge symmetries it is possible to write the metric, B field, and dilaton as

$$G_{\mu\nu} = \begin{pmatrix} -n^2(t) & 0 \\ 0 & G_{ij}(t) \end{pmatrix}, \qquad B_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & B_{ij}(t) \end{pmatrix},$$
$$2\phi = \Phi + \frac{1}{2}\log\det(G_{ij}), \tag{10}$$

where the lapse function $n(t)$ can also be fixed to $n = 1$. The cosmological reduction of the bulk action then becomes

$$\mathbf{S}_0^c = -\frac{2}{\kappa}\int dt e^{-\Phi}\bigg[\frac{1}{4}\dot{B}_{ij}\dot{B}^{ij} - \frac{3}{4}\dot{G}_{ij}\dot{G}^{ij} - G^{ij}\dot{G}_{ij}\dot{\Phi} - \dot{\Phi}^2 + G^{ij}\ddot{G}_{ij}\bigg], \tag{11}$$

where $\dot{G}^{ij} \equiv G^{ik}G^{il}\dot{G}_{kl}$. Using the following total derivative term

$$\int dt \frac{d}{dt}[e^{-\Phi}G^{ij}\dot{G}_{ij}] = \int dt e^{-\Phi}[-G^{ij}\dot{G}_{ij}\dot{\Phi} - \dot{G}^{ij}\dot{G}_{ij} + G^{ij}\ddot{G}_{ij}] \tag{12}$$

one can write $\mathbf{S}_0^c$ as

$$\mathbf{S}_0^c = -\frac{2}{\kappa^2}\int dt e^{-\Phi}\bigg[\frac{1}{4}\dot{B}_{ij}\dot{B}^{ij} + \frac{1}{4}\dot{G}_{ij}\dot{G}^{ij} - \dot{\Phi}^2\bigg] - \frac{2}{\kappa}\int dt\frac{d}{dt}[e^{-\Phi}G^{ij}\dot{G}_{ij}]. \tag{13}$$

The total derivative term can be transferred to the boundary by using the Stokes's theorem.

On the other hand, the cosmological boundary is specified by $x^i = \sigma^i$, and $x^0 = t$ is independent of σ^i. Hence, $\sqrt{|g|}e^{-2\phi} = e^{-\Phi}$. The unit vector to the boundary is fixed, i.e., $\dot{n} = 0$, and the reduction of the trace of the extrinsic curvature becomes

$$K^c = \frac{1}{2}G^{ij}\dot{G}_{ij}. \tag{14}$$

Therefore, the reduction of the boundary term is canceled with the total derivative term in the bulk action, i.e.,

$$\mathbf{S}_0^c = -\frac{2}{\kappa^2}\int dt e^{-\Phi}\bigg[\frac{1}{4}\dot{B}_{ij}\dot{B}^{ij} + \frac{1}{4}\dot{G}_{ij}\dot{G}^{ij} - \dot{\Phi}^2\bigg],$$
$$\partial\mathbf{S}_0^c = 0. \tag{15}$$

Similar cancellation between the reduction of boundary action and the total derivative terms in the bulk action has been observed for the couplings at order α' in [17].

Using the generalized metric $\mathcal{S}$

$$\mathcal{S} \equiv \eta\begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix}, \tag{16}$$

where η is the metric of the $O(d,d)$ group which in the nondiagonal form is

$$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{17}$$

one can write the bulk action as

$$\mathbf{S}_0^c = -\frac{2}{\kappa^2}\int dt e^{-\Phi}\bigg[-\frac{1}{8}\mathrm{tr}(\dot{\mathcal{S}}^2) - \dot{\Phi}^2\bigg], \tag{18}$$

which is invariant under the global $O(d,d,R)$ transformations because the one-dimensional dilaton is invariant and the generalized metric transforms as

$$\mathcal{S} \to \Omega^T \mathcal{S}\Omega, \tag{19}$$

where Ω belongs to the $O(d,d,R)$ group, i.e., $\Omega^T\eta\Omega = \eta$. Note that $\mathrm{tr}(\dot{\mathcal{S}}) = 0$. Hence the reduction of the extrinsic curvature (14) can not be written in $O(9,9)$ invariant form. So it was necessary that this term was canceled with the total derivative term in the bulk action. In other words, there is no way to write the boundary action in $O(d,d)$ invariant form unless it is zero.

III. COSMOLOGICAL REDUCTION AT ORDER α'^3

The NS-NS couplings in the bulk effective action of type II superstring theory at order α'^3 have been found in [14,15] by imposing the Z_2 symmetry on the minimal gauge invariant couplings. In the particular scheme used in [15], the dilaton appears only as the overall factor $e^{-2\phi}$, and the metric and B field appear in the Riemann curvature, H and the first covariant derivative of H, i.e.,

$$\mathbf{S}_3 = -\frac{2c}{\kappa^2}\int d^{10}x\sqrt{-g}[e^{-2\phi}L_3(G,B) + \cdots], \tag{20}$$

where dots represent the R-R and fermion fields in which we are not interested, and c is an overall factor that cannot be fixed by the T-duality constraint. The gravitational sector is

$$L_3(G) = 2R_{\alpha}{}^{\epsilon}{}_{\gamma}{}^{\varepsilon}R^{\alpha\beta\gamma\delta}R_{\beta}{}^{\mu}{}_{\epsilon}{}^{\zeta}R_{\delta\zeta\varepsilon\mu} + R_{\alpha\beta}{}^{\epsilon\varepsilon}R^{\alpha\beta\gamma\delta}R_{\gamma}{}^{\mu}{}_{\epsilon}{}^{\zeta}R_{\delta\zeta\varepsilon\mu}. \tag{21}$$

The couplings in this sector are exactly the couplings that have been found by the S matrix and sigma-model calculations [25–28] provided that one chooses the overall parameter to be $c = -\zeta(3)/2^6$. There are 249 couplings that involve H. They appear in eight structures. There are two couplings with structure H^8, i.e.,

$$\begin{aligned} L_3^{H^8} = &\frac{1}{48}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta}{}^{\varepsilon\mu}H_{\gamma}{}^{\zeta\eta}H_{\delta\varepsilon}{}^{\theta}H_{\epsilon\zeta}{}^{\iota}H_{\theta\iota\kappa}H_{\mu\eta}{}^{\kappa} \\ &-\frac{9}{128}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta}{}^{\varepsilon\mu}H_{\gamma}{}^{\zeta\eta}H_{\delta\varepsilon}{}^{\theta}H_{\epsilon\zeta}{}^{\iota}H_{\eta\theta\kappa}H_{\mu\iota}{}^{\kappa}. \end{aligned} \tag{22}$$

There is one coupling with structure RH^6, i.e.,

$$L_3^{RH^6} = \frac{9}{8}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta}{}^{\varepsilon\mu}H_{\gamma}{}^{\zeta\eta}H_{\delta\varepsilon}{}^{\theta}H_{\epsilon\zeta}{}^{\iota}R_{\mu\iota\eta\theta}. \tag{23}$$

There are seven couplings with structure R^2H^4, i.e.,

$$L_3^{R^2H^4} = \frac{7}{2}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta}{}^{\varepsilon\mu}H_{\delta}{}^{\zeta\eta}R_{\gamma}{}^{\theta}{}_{\varepsilon\zeta}R_{\epsilon\theta\mu\eta} + \cdots, \tag{24}$$

where dots refer to the other six couplings in this structure. There are 22 couplings with structure R^3H^2, i.e.,

$$L_3^{R^3H^2} = -\frac{15}{2}H^{\alpha\beta\gamma}H^{\delta\epsilon\varepsilon}R_{\alpha\delta}{}^{\mu\zeta}R_{\beta\mu\epsilon}{}^{\eta}R_{\gamma\eta\varepsilon\zeta} + \cdots. \tag{25}$$

There are 77 couplings with the structure $(\nabla H)^2H^4$, i.e.,

$$L_3^{(\partial H)^2H^4} = \frac{5}{8}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta}{}^{\varepsilon\mu}H_{\delta\varepsilon}{}^{\zeta}\nabla_{\epsilon}H_{\gamma}{}^{\eta\theta}\nabla_{\zeta}H_{\mu\eta\theta} + \cdots. \tag{26}$$

There are 106 couplings with the structure $R(\nabla H)^2H^2$, i.e.,

$$L_3^{R(\partial H)^2H^2} = \frac{457}{48}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_{\varepsilon\zeta\mu\eta}\nabla_{\delta}H_{\beta}{}^{\varepsilon\mu}\nabla_{\epsilon}H_{\gamma}{}^{\zeta\eta} + \cdots. \tag{27}$$

There are 22 couplings with the structure $R^2(\nabla H)^2$, i.e.,

$$L_3^{R^2(\partial H)^2} = -\frac{5}{24}R_{\epsilon\mu\varepsilon\zeta}R^{\epsilon\varepsilon\mu\zeta}\nabla_{\delta}H_{\alpha\beta\gamma}\nabla^{\delta}H^{\alpha\beta\gamma} + \cdots. \tag{28}$$

And finally, there are 12 couplings with the structure $(\nabla H)^4$:

$$L_3^{(\partial H)^4} = \frac{1}{8}\nabla^{\delta}H^{\alpha\beta\gamma}\nabla_{\epsilon}H_{\gamma}{}^{\mu\zeta}\nabla_{\varepsilon}H_{\delta\mu\zeta}\nabla^{\varepsilon}H_{\alpha\beta}{}^{\epsilon} + \cdots. \tag{29}$$

We refer the interested readers to [15] for the explicit form of all couplings.

To find the cosmological reduction of these couplings we first find the cosmological reduction of the Riemann curvature, H and ∇H. They are

$$\begin{aligned} &R_{ijkl} = -\frac{1}{4}\dot{G}_{il}\dot{G}_{jk} + \frac{1}{4}\dot{G}_{ik}\dot{G}_{jl}, \qquad R_{i0jk} = 0, \qquad R_{i0j0} = \frac{1}{4}\dot{G}_{ik}\dot{G}^{k}{}_{j} - \frac{1}{2}\ddot{G}_{ij}, \\ &H_{ijk} = 0; \qquad H_{ij0} = \dot{B}_{ij}, \qquad \nabla_0 H_{ijk} = 0, \qquad \nabla_k H_{ij0} = 0, \\ &\nabla_l H_{ijk} = -\frac{1}{2}\dot{B}_{jk}\dot{G}_{il} + \frac{1}{2}\dot{B}_{ik}\dot{G}_{jl} - \frac{1}{2}\dot{B}_{ij}\dot{G}_{kl}, \qquad \nabla_0 H_{ij0} = -\frac{1}{2}\dot{B}_{j}{}^{k}\dot{B}_{ik} - \frac{1}{2}\dot{B}_{i}{}^{k}\dot{G}_{jk} + \ddot{B}_{ij}. \end{aligned} \tag{30}$$

Using the above reductions, one finds the following reduction for the Lagrangians $L_3^{H^8}$:

$$L_3^{H^8} = 13(\mathrm{Tr}(M^4))^2/64 + 61\mathrm{Tr}(M^8)/128, \tag{31}$$

where the 9×9 matrices $M = G^{-1}\dot{B}$ and $L = G^{-1}\dot{G}$. The reduction of the Lagrangian $L_3(G)$ is the following

$$L_3(G) = (\mathrm{Tr}(L^4))^2/64 + 5\mathrm{Tr}(L^8)/128 + \cdots, \tag{32}$$

where dots represent terms that have $\ddot{G}$, $\mathrm{Tr}(L^2)$, $\mathrm{Tr}(L)$ or $\mathrm{Tr}(L^3)$. The reduction of all other Lagrangians are

$$
\begin{aligned}
L_3^{RH^6} &= 9\mathrm{Tr}(L^2M^6)/16 + 9\mathrm{Tr}(LM^2LM^4)/16 - 9\mathrm{Tr}(LM^3LM^3)/32 + 9\mathrm{Tr}(L^2M^2)\mathrm{Tr}(M^4)/16 + \cdots,\\
L_3^{R^2H^4} &= 19\mathrm{Tr}(L^2M^2)^2/64 + 45\mathrm{Tr}(L^2M^2L^2M^2)/64 - 15\mathrm{Tr}(L^2M^2LMLM)/16\\
&\quad + 17\mathrm{Tr}(L^2M^3L^2M)/32 + 7\mathrm{Tr}(L^2MLM^2LM)/16 - 5\mathrm{Tr}(L^3M^2LM^2)/32\\
&\quad - 7\mathrm{Tr}(L^3M^3LM)/16 + 17\mathrm{Tr}(L^4M^4)/32 - 11\mathrm{Tr}(L^2M^2)\mathrm{Tr}(LMLM)/32 + \mathrm{Tr}(L^4)\mathrm{Tr}(M^4)/128 + \cdots,\\
L_3^{R^3H^2} &= -81\mathrm{Tr}(L^3ML^3M)/64 - 89\mathrm{Tr}(L^2M^2)\mathrm{Tr}(L^4)/128 + 147\mathrm{Tr}(L^4ML^2M)/64\\
&\quad - 99\mathrm{Tr}(L^5MLM)/64 + 23\mathrm{Tr}(L^6M^2)/32 + 75\mathrm{Tr}(L^4)\mathrm{Tr}(LMLM)/256 + \cdots,\\
L_3^{(\partial H)^2H^4} &= 2257\mathrm{Tr}(L^2M^6)/192 - 509\mathrm{Tr}(LM^2LM^4)/64 + 91\mathrm{Tr}(LMLM)\mathrm{Tr}(M^4)/96\\
&\quad + 169\mathrm{Tr}(LM^5LM)/192 + 33\mathrm{Tr}(L^2M^2)\mathrm{Tr}(M^4)/64 - 389\mathrm{Tr}(LM^3LM^3)/192 + \cdots,\\
L_3^{R(\partial H)^2H^2} &= 3(\mathrm{Tr}(L^2M^2))^2/8 - 23\mathrm{Tr}(LMLMLMLM)/48 + 1969\mathrm{Tr}(L^2M^2LMLM)/192\\
&\quad + 933\mathrm{Tr}(L^2M^3L^2M)/64 - 473\mathrm{Tr}(L^2MLM^2LM)/96 - 1757\mathrm{Tr}(L^3M^2LM^2)/192\\
&\quad - 17\mathrm{Tr}(L^3M^3LM)/2 + 1391\mathrm{Tr}(L^4M^4)/192 - 1385\mathrm{Tr}(L^2M^2)\mathrm{Tr}(LMLM)/384\\
&\quad - 229(\mathrm{Tr}(LMLM))^2/96 + 383\mathrm{Tr}(L^2M^2L^2M^2)/96 - 311\mathrm{Tr}(L^4)\mathrm{Tr}(M^4)/128 + \cdots,\\
L_3^{R^2(\partial H)^2} &= 17\mathrm{Tr}(L^3ML^3M)/16 - \mathrm{Tr}(L^2M^2)\mathrm{Tr}(L^4)/16 + 29\mathrm{Tr}(L^4ML^2M)/16\\
&\quad + 57\mathrm{Tr}(L^5MLM)/64 + 3\mathrm{Tr}(L^6M^2)/4 + 27\mathrm{Tr}(L^4)\mathrm{Tr}(LMLM)/256 + \cdots,\\
L_3^{(\partial H)^4} &= 3(\mathrm{Tr}(L^2M^2))^2/64 + 3\mathrm{Tr}(L^2M^2L^2M^2)/64 - 113\mathrm{Tr}(LMLMLMLM)/192\\
&\quad + 163\mathrm{Tr}(L^2M^3L^2M)/48 - 865\mathrm{Tr}(L^2MLM^2LM)/96 - 1247\mathrm{Tr}(L^3M^2LM^2)/192\\
&\quad - 41\mathrm{Tr}(L^3M^3LM)/64 + 2383\mathrm{Tr}(L^4M^4)/192 + 1439\mathrm{Tr}(L^2M^2)\mathrm{Tr}(LMLM)/384\\
&\quad + 113(\mathrm{Tr}(LMLM))^2/96 - 2485\mathrm{Tr}(L^2M^2LMLM)/192 - 277\mathrm{Tr}(L^4)\mathrm{Tr}(M^4)/128 + \cdots, \qquad (33)
\end{aligned}
$$

where dots represent terms which have $\ddot{G}$, $\ddot{B}$, $\mathrm{Tr}(L^2)$, $\mathrm{Tr}(M^2)$, $\mathrm{Tr}(L)$, or $\mathrm{Tr}(L^3)$. As it has been argued in [22], using the field redefinitions and total derivative terms, the couplings involving these structures can be converted to the other couplings which have no such structures. Before showing how this works, let us add all the above reductions to find the cosmological reduction of the Lagrangian $L_3(G,B)$, i.e.,

$$
\begin{aligned}
L_3(G,B) &= 23(\mathrm{Tr}(L^2M^2))^2/32 + 455\mathrm{Tr}(L^2M^2L^2M^2)/96 - 29\mathrm{Tr}(L^2M^2LMLM)/8\\
&\quad + 3553\mathrm{Tr}(L^2M^3L^2M)/192 + 2365\mathrm{Tr}(L^2M^6)/192 - 27\mathrm{Tr}(L^2MLM^2LM)/2\\
&\quad - 1517\mathrm{Tr}(L^3M^2LM^2)/96 - 613\mathrm{Tr}(L^3M^3LM)/64 - 13\mathrm{Tr}(L^3ML^3M)/64\\
&\quad - 97\mathrm{Tr}(L^2M^2)\mathrm{Tr}(L^4)/128 + (\mathrm{Tr}(L^4))^2/64 + 323\mathrm{Tr}(L^4M^4)/16\\
&\quad + 263\mathrm{Tr}(L^4ML^2M)/64 - 21\mathrm{Tr}(L^5MLM)/32 + 47\mathrm{Tr}(L^6M^2)/32 + 5\mathrm{Tr}(L^8)/128\\
&\quad - 473\mathrm{Tr}(LM^2LM^4)/64 - 443\mathrm{Tr}(LM^3LM^3)/192 + 169\mathrm{Tr}(LM^5LM)/192\\
&\quad - 13\mathrm{Tr}(L^2M^2)\mathrm{Tr}(LMLM)/64 + 51\mathrm{Tr}(L^4)\mathrm{Tr}(LMLM)/128 - 29(\mathrm{Tr}(LMLM))^2/24\\
&\quad - 205\mathrm{Tr}(LMLMLMLM)/192 + 69\mathrm{Tr}(L^2M^2)\mathrm{Tr}(M^4)/64 - 587\mathrm{Tr}(L^4)\mathrm{Tr}(M^4)/128\\
&\quad + 91\mathrm{Tr}(LMLM)\mathrm{Tr}(M^4)/96 + 13(\mathrm{Tr}(M^4))^2/64 + 61\mathrm{Tr}(M^8)/128 + \cdots, \qquad (34)
\end{aligned}
$$

which has 28 structures that have no $\ddot{G}$, $\ddot{B}$, $\mathrm{Tr}(L^2)$, $\mathrm{Tr}(M^2)$, $\mathrm{Tr}(L)$, or $\mathrm{Tr}(L^3)$. The cosmological reduction of the effective action (20) then is

$$
\mathbf{S}_3^c = -\frac{2c}{\kappa^2}\int dt[e^{-\Phi}L_3(G,B) + \cdots], \qquad (35)
$$

where $L_3(G,B)$ is given in (34).

To remove the total derivatives terms and field redefinition freedom from (35), we add all total derivative terms at order α'^3 and all field redefinitions with arbitrary coefficients to (35). We add the following total derivative terms:

$$-\frac{2c}{\kappa^2}\int dte^{-\Phi}\mathcal{J}_3 \equiv -\frac{2c}{\kappa^2}\int dt\frac{d}{dt}(e^{-\Phi}\mathcal{I}_3), \qquad (36)$$

where $\mathcal{I}_3$ is all possible terms at the seven-derivative level with even parity that are constructed from $\dot{\Phi}, \dot{B}, \dot{G}$, $\ddot{\Phi}, \ddot{B}, \ddot{G}, \ldots$. Using the package "XACT" [30], one finds there are 2288 such terms, i.e.,

$$\mathcal{I}_3 = j_1\dot{B}_i{}^j\dot{B}_j{}^k\dot{B}_k{}^l\dot{G}_l{}^m\dot{B}_m{}^n\dot{B}_n{}^p\dot{B}_p{}^i + \cdots, \qquad (37)$$

where the coefficients $J_1, \ldots, J_{2288}$ are 2288 arbitrary parameters.

One can change the field variables in (10) as

$$\begin{aligned} G_{ij} &\to G_{ij} + \alpha'^3\delta G_{ij}^{(3)}, \\ B_{ij} &\to B_{ij} + \alpha'^3\delta B_{ij}^{(3)}, \\ \Phi &\to \Phi + \alpha'^3\delta\Phi^{(3)}, \\ n &\to n + \alpha'^3\delta n^{(3)}, \end{aligned} \qquad (38)$$

where the matrices $\delta G_{ij}^{(3)}$, $\delta B_{ij}^{(3)}$ and $\delta\Phi^{(3)}, \delta n^{(3)}$ are all possible terms at the six-derivative level constructed from $\dot{\Phi}$, $\dot{B}$, $\dot{G}$, $\ddot{\Phi}$, $\ddot{B}$, $\ddot{G}$, …. The perturbations $\delta G_{ij}^{(3)}$, $\delta\Phi^{(3)}$, $\delta n^{(3)}$ contain even-parity terms and $\delta B_{ij}^{(3)}$ contains odd-parity terms, i.e.,

$$\begin{aligned} \delta n^{(3)} &= n_1\dot{B}_i{}^j\dot{B}_j{}^k\dot{B}_k{}^l\dot{B}_l{}^m\dot{B}_m{}^n\dot{B}_n{}^i + \cdots, \\ \delta\Phi^{(3)} &= e_1\dot{B}_i{}^j\dot{B}_j{}^k\dot{B}_k{}^l\dot{B}_l{}^m\dot{B}_m{}^n\dot{B}_n{}^i + \cdots, \\ \delta G_{ij}^{(3)} &= d_1\dot{B}_i{}^k\dot{B}_k{}^l\dot{B}_l{}^m\dot{B}_m{}^n\dot{B}_n{}^p\dot{B}_{pj} + \cdots, \\ \delta B_{ij}^{(3)} &= f_1\dot{G}_i{}^k\dot{B}_{kj}\dot{B}_l{}^m\dot{B}_m{}^n\dot{B}_n{}^p\dot{B}_{pl} + \cdots. \end{aligned} \qquad (39)$$

The coefficients $n_1, \ldots, n_{748}$, $e_1, \ldots, e_{748}$, $d_1, \ldots, d_{1105}$, and $f_1, \ldots, f_{665}$ are arbitrary parameters. When the field variables in $\mathbf{S}_3^c$ are changed according to the above field redefinitions, they produce some couplings at orders α'^6 and higher in which we are not interested in this paper. However, when the NS-NS field variables in $\mathbf{S}_0^c$ are changed, the following couplings at order α'^3 are produced:

$$\begin{aligned} \delta\mathbf{S}_0^c = &-\frac{2\alpha'^3}{\kappa^2}\int dte^{-\Phi}\bigg[\delta n^{(3)}\left(-\frac{1}{4}\dot{B}_{ij}\dot{B}^{ij} - \frac{1}{4}\dot{G}_{ij}\dot{G}^{ij} + \dot{\Phi}^2\right) + \delta\Phi^{(3)}\left(-\frac{1}{4}\dot{B}_{ij}\dot{B}^{ij} - \frac{1}{4}\dot{G}_{ij}\dot{G}^{ij} + \dot{\Phi}^2\right) - 2\dot{\Phi}\frac{d}{dt}\delta\Phi^{(3)} \\ &+ \delta G_{ij}^{(3)}\left(-\frac{1}{2}\dot{B}_k{}^j\dot{B}^{ki} - \frac{1}{2}\dot{G}_k{}^j\dot{G}^{ki}\right) + \frac{1}{2}\dot{G}^{ij}\frac{d}{dt}\delta G_{ij}^{(3)} + \frac{1}{2}\dot{B}^{ij}\frac{d}{dt}\delta B_{ij}^{(3)}\bigg], \\ \equiv &-\frac{2\alpha'^3 c}{\kappa^2}\int dte^{-\Phi}\mathcal{K}_3, \end{aligned} \qquad (40)$$

where we have used the fact that the lapse function appears in the action (15) by replacing $dt \to dt/n$ [22].

Adding the total derivative terms and the field redefinition terms to the action (35), one finds new action S_3^c, i.e.,

$$S_3^c = -\frac{2c}{\kappa^2}\int dt[e^{-\Phi}\mathcal{L}_3(G, B, \Phi) + \cdots], \qquad (41)$$

where the Lagrangian $\mathcal{L}_3(G, B, \Phi)$ is related to the Lagrangian $\mathcal{L}_3(G, B)$ as

$$\mathcal{L}_3 = L_3 + \mathcal{J}_3 + \mathcal{K}_3. \qquad (42)$$

The action $\mathbf{S}_3^c$ and S_3^c are physically equivalent. They appear in different schemes. Choosing different values for the arbitrary parameters in $\mathcal{J}_3, \mathcal{K}_3$, one would find different forms of couplings for the Lagrangian $\mathcal{L}_3$. We choose these parameters such that all terms that have any derivative of Φ, second and higher derivatives of G, B, and all terms that have $\mathrm{Tr}(L^2)$, $\mathrm{Tr}(M^2)$, $\mathrm{Tr}(L)$ or $\mathrm{Tr}(L^3)$ to be zero. Inserting the resulting relations between the parameters into (42), we find the following scheme for the Lagrangian $\mathcal{L}_3$:

$$\begin{aligned}\mathcal{L}_3 = &(\mathrm{Tr}(L^2M^2))^2/4 - 3\mathrm{Tr}(L^2M^2L^2M^2)/32 + 3\mathrm{Tr}(L^2M^2LMLM)/8 - 3\mathrm{Tr}(L^2M^3L^2M)/16\\
&+ 3\mathrm{Tr}(L^2M^6)/16 - 3\mathrm{Tr}(L^2MLM^2LM)/16 - 3\mathrm{Tr}(L^3M^2LM^2)/16 + 3\mathrm{Tr}(L^3M^3LM)/8\\
&- 3\mathrm{Tr}(L^3ML^3M)/32 - \mathrm{Tr}(L^2M^2)\mathrm{Tr}(L^4)/8 + (\mathrm{Tr}(L^4))^2/64 - 3\mathrm{Tr}(L^4M^4)/16\\
&+ 3\mathrm{Tr}(L^4ML^2M)/16 - 3\mathrm{Tr}(L^5MLM)/16 + 3\mathrm{Tr}(L^6M^2)/16 - 3\mathrm{Tr}(L^8)/128\\
&+ 3\mathrm{Tr}(LM^2LM^4)/16 - 3\mathrm{Tr}(LM^3LM^3)/32 - 3\mathrm{Tr}(LM^5LM)/16\\
&- \mathrm{Tr}(L^2M^2)\mathrm{Tr}(LMLM)/4 + \mathrm{Tr}(L^4)\mathrm{Tr}(LMLM)/16 + (\mathrm{Tr}(LMLM))^2/16\\
&- 3\mathrm{Tr}(LMLMLMLM)/64 - \mathrm{Tr}(L^2M^2)\mathrm{Tr}(M^4)/8 + \mathrm{Tr}(L^4)\mathrm{Tr}(M^4)/32\\
&+ \mathrm{Tr}(LMLM)\mathrm{Tr}(M^4)/16 + (\mathrm{Tr}(M^4))^2/64 - 3\mathrm{Tr}(M^8)/128,\end{aligned} \tag{43}$$

which has the same 28 structures as in (34) but with different coefficients. Note that, in this scheme, there is no term in $\mathcal{L}_3$ other than the above 28 couplings.

Now using the definition of the generalized metric in (16), one finds

$$\begin{aligned}\mathrm{tr}(\dot{\mathcal{S}}^4) = &\,2\mathrm{Tr}(L^4) + 2\mathrm{Tr}(M^4) - 8\mathrm{Tr}(L^2M^2) + 4\mathrm{Tr}(LMLM),\\
\mathrm{tr}(\dot{\mathcal{S}}^8) = &\,8\mathrm{Tr}(L^2M^2L^2M^2) - 32\mathrm{Tr}(L^2M^2LMLM) + 16\mathrm{Tr}(L^2M^3L^2M) - 16\mathrm{Tr}(L^2M^6)\\
&+ 16\mathrm{Tr}(L^2MLM^2LM) + 16\mathrm{Tr}(L^3M^2LM^2) - 32\mathrm{Tr}(L^3M^3LM) + 8\mathrm{Tr}(L^3ML^3M)\\
&+ 16\mathrm{Tr}(L^4M^4) - 16\mathrm{Tr}(L^4ML^2M) + 16\mathrm{Tr}(L^5MLM) - 16\mathrm{Tr}(L^6M^2) + 2\mathrm{Tr}(L^8)\\
&- 16\mathrm{Tr}(LM^2LM^4) + 8\mathrm{Tr}(LM^3LM^3) + 16\mathrm{Tr}(LM^5LM)\\
&+ 4\mathrm{Tr}(LMLMLMLM) + 2\mathrm{Tr}(M^8).\end{aligned} \tag{44}$$

Using the above $O(9,9)$-invariant expressions, one can write (43) as

$$\mathcal{L}_3 = \frac{1}{256}(\mathrm{tr}(\dot{\mathcal{S}}^4))^2 - \frac{3}{256}\mathrm{tr}(\dot{\mathcal{S}}^8), \tag{45}$$

which is consistent with the cosmological action (6). We have done the same calculations with couplings in [14], and found exactly the same result. The form of the one-dimensional field redefinitions and total derivative terms, however, are different in the two cases. The above calculations confirm the NS-NS couplings at order α'^3, which has been found in [14,15].

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