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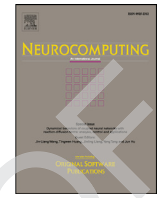

Bipartite consensus control for fractional-order nonlinear multi-agent systems: An output constraint approach

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ABSTRACT

Bipartite consensus of multiple fractional-order nonlinear systems with output constraints is assessed under signed graph. The agents' model is completely unknown with high-order heterogeneous strict-feedback dynamics and external disturbances, which cover single- and double-integrator integer-order systems as special forms. To ensure the bipartite consensus task, a novel fully distributed controller is developed based on backstepping technique and neuro-adaptive update mechanism. A barrier Lyapunov function is introduced to limit the followers' outputs within the preset bounds. Algebraic graph theory and Lyapunov fractional-order stability theorem are employed to deal with the analysis difficulties caused by the network of fractional-order dynamics. Sufficient conditions on bipartite consensus is established, and it is also shown that all the closed-loop error signals are uniformly ultimately bounded. The simulation results are carried out to demonstrate the effectiveness of the proposed approach.

1. Introduction

In the past decade, different perspectives of design and analysis on collective behavior of multi-agent systems (MASs) have been considered, such as consensus (or synchronization) of multiple manipulators [26], formation control of robot teams [30], connectivity preserving for a group of Euler–Lagrange systems [48], rendezvous of multiple spacecrafts [8], etc. Among them, consensus of MASs aims at designing distributed control laws to make follower agents reach an agreement on some desired value or trajectory, see, for instance [6,31–33,53], and the references therein). However, the aforementioned control protocols are only applicable on the collaborative networks, where interactions between all the agents are represented via nonnegative classical graphs.

In numerous realistic systems, such as social networks and multi-robotic systems, the collective behavior of multiple agents is modeled over networks with cooperative and competitive communications. In this case, a signed bipartite graph is introduced to represent the communication among agents. In a signed bipartite graph, the adjacency matrixes entries are capable of being both positive and negative. Cooperative and competitive relationship between the agents are associated with the positive and negative weights, respectively. The bipartite consensus deals with extending distributed control protocols for agents such that the outputs/states converge to a common value asymptotically in modulus, but different in sign for antagonistic agents. The bipartite consensus control design for first-order MASs was first introduced in [1]. Subsequently, some effective bipartite consensus approaches have been established for different classes of linear MASs under signed graph, [13,24,45,52]. The bipartite consensus problem of MASs with nonlinear dynamics has recently received significant attention due to practical demands. In [14,43], the uniform ultimate bound stability method was utilized for bipartite consensus control problem of multiple nonlinear systems with external disturbances, where the unknown nonlinearities in the followers' dynamics were handled with the adaptive compensator technique and robust control mechanism. In [46], an adaptive control approach was studied for strict-feedback MASs to achieve bipartite consensus in a fixed time. The output-feedback bipartite consensus control problem was investigated in [50] for strict-feedback nonlinear systems.

Although the distributed control design for consensus of MASs have extensively been considered, the followers' outputs constraint is rarely studied. This problem is still a technical challenge to be solved due to the existence of many physical constraints in real-world systems. It is well known that using the barrier Lyapunov function (BLF) is an effective strategy to cope with output constraint issue. In contrast to the conventional quadratic Lyapunov

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Q2
[1] This paper attempts to design adaptive output constraint controllers for a class of uncertain MASs described by fractional-order uncertain strict-feedback dynamics.

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functions [13,14,24,43,45,46,52], a BLF is not radially unbounded, however it grows to infinity whenever its arguments tend to some finite limits. According to this property and based on the Lyapunov method, it is guaranteed that the system's output/states are limited to predefined bounds. Moreover, compared to the conventional quadratic Lyapunov function based designs, it is shown in [24] that the BLF based control design approaches require less restrictive initial conditions on closed-loop state variables. Recently, some adaptive control schemes using the BLF design approach have been developed in [11,12,21,22,38,49] for nonlinear uncertain systems with output or states constraints. Nevertheless, these control schemes do not consider the network communication for uncertain nonlinear systems. In [7], a distributed output constraint control method was presented to realize the consensus performance in a finite time for multiple uncertain Euler-Lagrange systems. However, this result is limited to the agents with nonlinear dynamics in Brunovsky structure. In [40], a distributed adaptive control structure was studied based on the BLFs to cope with the constrains on states for strict-feedback MASs. An event-triggered consensus scheme with state constraints was studied for strict-feedback MASs by utilizing the BLF in [54]. However, the aforementioned synchronization methods are limited to integer-order MASs.

Many real-world coordination behaviors such as robot formation and multi-vehicle systems in complex environments are modeled by a network of fractional-order dynamics. This includes autonomous underwater vehicles moving on the top of macromolecule fluids, high-speed aircraft traveling in dust storm, rain, or snow and ground vehicles moving on top of sand, grass, or muddy road. The design of distributed control protocols and their consensus stability analysis are more challenging for the fractional-order systems, compared to the integer-order ones. This is because some well-known mathematical tools, such as Leibniz rule, are not well established for the fractional-order derivatives. As a result, it is not straightforward to adopt the classical stability analysis procedures from the integer-order systems for the fractional-order dynamics. To solve this problem, some interesting distributed control approaches have been presented for the fractional-order MASs. The distributed consensus control for a group of linear fractional-order systems with first-order dynamics is studied in [5]. The finite-time consensus for linear fractional-order MASs was also investigated in [17].

The aforementioned control approaches are appropriate for the linear fractional-order MASs, however almost all real systems are intrinsically nonlinear. Therefore, the study of nonlinear fractional-order MASs is of much importance. In the preceding years, the researchers have placed more emphasis on designing distributed controllers for the nonlinear fractional-order MASs, however there are still few reported results [3,4,9,10,23,39,51]. The earliest study in this field was investigated in [51], where a linear leader–follower distributed protocol was studied for nonlinear fractional-order MASs. Continuous and discontinuous distributed leader-following control structures were also considered in [10] for nonlinear fraction-order MASs. A discontinuous distributed leader–follower control structure was established for nonlinear fractional-order MASs based on sliding mode design approach in [4]. Other different control designs such as impulsive method [23], and event-based control structure [39] are also considered for the nonlinear fractional-order MASs. In [4,10,23,39,51], the dynamics of agents have a simple single integrator structure. Besides, the design of distributed architectures require some global information of the Laplacian matrix associated with the graph topology. Recently, in order to eliminate these shortcomings, the distributed consensus of nonlinear double integrator fractional-order MASs using an adaptive method was addressed in [9]. However, in [4,9,10,23,39,51], not only the considered follower agents require to satisfy matching conditions, but they also should communicate cooperatively. Moreover, we have to note that in [4,9,10,23,39,51], the representing system's functions for follower agent dynamics are supposed to satisfy the Lipschitz condition. Moreover, up to now, a consensus control method has not been developed for fractional-order MASs with output constraints. Hence, despite of the existing results, the following problems are till now remained

1. How can we design a fully distributed control structure for a network of fractional-order systems under both cooperative-competitive interactions?
2. How can we design a fully distributed control architecture for fractional-order MASs with high-order nonlinear strict-feedback structure with completely unknown dynamics?
3. How to limit the followers' outputs to preset bounds and meet the physical constraints in realistic systems?

These three questions motivate us to develop the theoretical results. Hence, this paper attempts to design the adaptive output constraint controller for a class of uncertain MASs described by fractional-order uncertain strict-feedback dynamics. Another motivation for considering this kind of MASs lies on the industrial situations. For instance, for synchronization of power systems with multi-machine power systems and different synchronous generators, a network of doubly-fed induction generators or synchronization of multiple flexible robot manipulators, fractional-order uncertain strict-feedback dynamics is required. To the best of our knowledge, no existing work has considered the consensus problem of networked fractional-order uncertain strict-feedback agents with followers' output constraints. The main contributions of this paper are three-fold

1. This is the first time that an adaptive bipartite consensus control in a fully distributed manner independent of any global information of the Laplacian matrix for a class of fractional-order nonlinear MASs under both cooperative and competitive interactions is introduced.
2. In contrast to all previous works, this paper not only investigates the high-order strict-feedback fractional-order MASs, but also, it considers a distributed adaptive neural control protocol to approximate the unknown nonlinearities by employing the minimal learning parameter (MLP) approach.
3. Compared to the existing results, a distributed adaptive neural bipartite consensus control scheme is proposed for nonlinear fractional-order MASs with outputs' constraints. By employing BLFs in the distributed controller design, the follower agents' outputs constraints are well satisfied within the limits. It is notable that the consensus control methods for the fractional-order MASs presented in references such as [4,9,10,23,39,51] are restricted to the MASs without outputs constraints.

Notations: In this paper, $\Re$ ($\Re^+$) represents the real number set (positive real number set), $\Re^N$ denotes the real N-vectors set, and $\Re^{N\times N}$ indicates the real $N\times N$ matrices set. For a scalar value of x, $|x|$ indicates the absolute value and for a vector $\mathbf{x}$, $||\mathbf{x}||$ denotes the 2-norm. The operator diag$(\cdot)$ is considered to show a diagonal matrix of the arguments and symbol arg$(\cdot)$ is used to represent the argument of complex number. $Ln(\cdot)$ stands for the natural logarithm of its argument and the superscript "T" denotes transposition of matrix or vector. The sign$(\cdot)$ is defined as the standard signum function.

The rest of this paper is organized as follows. The technical background is presented in Section 2. In Section 3, the problem formulation is given. A distributed adaptive bipartite consensus tracking control approach with followers' output constraints is proposed in Section 4. The simulation results are carried out in Section 5 to show the effectiveness of the main results. Finally, the conclusion is described in Section 6.

2. Technical background

2.1. Fractional calculus

In this subsection, we provide some necessary Definitions, Properties and Lemmas related to the fractional calculus, including the Caputo fractional derivative and the Mittag-Leffler function (M-LF), alongside with a Lyapunov-based stability criterion for fractional-order systems.

Two operators are mainly associated with fractional calculus, namely the Riemann–Liouville and the Caputo. The most important reason for popularity of the Caputo's fractional derivative is that its Laplace transform only requires the integer-order derivatives of the initial conditions. Therefore, the Caputo fractional derivative is exploited to model the dynamics of the fractional-order agents in this paper.

For any real number $q \in (0\,1)$, the q-order Caputo fractional derivative of $f(t)$ is defined as [27]

$$ {}_0^{\mathcal{C}}D_t^q f(t) = \frac{1}{\Gamma(1-q)} \int_0^t \frac{f'(\tau)}{(t-\tau)^q} d\tau, \tag{1} $$

where $f'(t)$ is the first integer-order derivative of $f(t)$ and $\Gamma(1-q) = \int_0^\infty \tau^{-q}\exp(-\tau)d\tau$.

The Laplace transform of the q-order Caputo fractional derivative is represented as [27]

$$ \mathcal{L}\{{}^{\mathcal{C}}{}_0D_t^q f(t)\} = s^q F(s) - s^{q-1} f(0), \tag{2} $$

where $q \in (0\,1)$ and $F(s)$ is the Laplace transform of $f(t)$.

Property 1 [27]**.** For any $q \in (0\,1)$, the following hold

1. ${}_0^{\mathcal{C}}D_t^q(c_1 f(t) \pm c_2 g(t)) = c_1 {}_0^{\mathcal{C}}D_t^q f(t) \pm c_2 {}_0^{\mathcal{C}}D_t^q g(t)$,
2. ${}_0^{\mathcal{C}}D_t^q\left(\frac{f(t)}{g(t)}\right) \le \frac{{}_0^{\mathcal{C}}D_t^q f(t) g(t) - f(t) {}_0^{\mathcal{C}}D_t^q g(t)}{g^2(t)}$,

where c_1 and c_2 are constants.

In the following, we define the M-LF, which is used in Section 4 to analyze the stability of the closed-loop system.

Definition 1 [27]**.** The M-LF is expressed as

$$ E_{(q,\gamma)}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(qk+\gamma)}, \tag{3} $$

where $q \in (0\,1)$, $\gamma \in \Re^+$ and z is a complex number. The Laplace transform of M-LF is given by Podlubny [27]

$$ \mathcal{L}\left\{t^{\gamma-1} E_{(q,\gamma)}(-\zeta t^q)\right\} = \frac{s^{q-\gamma}}{s^q + \zeta}, \qquad \Re e(s) > |\zeta|^{\frac{1}{q}}, \tag{4} $$

where $\Re e(s)$ is the real part of s and $\zeta \in \Re$.

The following Lemma gives an upper bound on the M-LF which is used to obtain the ultimate bounds for Lyapunov variables.

Lemma 1 [27]**.** *If $\gamma \in \Re$, $q \in \Re^+$ and $\phi \in \Re^+$ satisfying $q \in (0\,1)$, and $\frac{\pi q}{2} < \phi < \pi q$, then there exists $\Upsilon \in \Re^+$, such that the M-LF is bounded by*

$$ |E_{(q,\gamma)}(z)| \le \frac{\Upsilon}{1+|z|}, \quad \gamma \le |\arg(z)| \le \pi, \quad |z| \ge 0. \tag{5} $$

The following Lemmas are used in Section 4 to analyze the stability of the closed-loop network of fractional-order MASs with model uncertainties and unknown external disturbances.

Lemma 2 [27]**.** *If $\mathbf{x}(t) = [x_1(t), \ldots, x_n(t)]^{\mathrm{T}} \in \Re^n$ is a smooth vector function, $q \in (0,\,1)$, and $t \ge 0$, then, there exists a positive definite matrix $P \in \Re^{n\times n}$ such that*

$$ {}_0^{\mathcal{C}}D_t^q\left(\mathbf{x}^{\mathrm{T}}(t) P \mathbf{x}(t)\right) \le 2\mathbf{x}^{\mathrm{T}}(t)\ P\ {}_0^{\mathcal{C}}D_t^q \mathbf{x}(t). \tag{6} $$

Lemma 3 [10]**.** *Let the q-order derivative of a smooth function $V(t) : \Re^+ \to \Re$ satisfy*

$$ {}_0^{\mathcal{C}}D_t^q V(t) + \eta V(t) \le \varrho, \tag{7} $$

where $q \in (0\,1)$, $\eta > 0$, and $\varrho \ge 0$. Then, the following holds

$$ V(t) \le V(0) E_{(q,1)}(-\eta t^q) + \frac{\varrho \varpi}{\eta}, \qquad t \ge 0, \tag{8} $$

where ϖ is the max $= \{1, \Upsilon\}$ and Υ is defined in Lemma 1.

2.2. Graph theory

Commonly, an algebraic graph theory as a mathematical approach is employed to illustrate the communication network of a MAS. A signed bipartite graph is considered in order to show the relationship between different agents. Consider $\mathcal{G} \triangleq \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ as a signed bipartite directed graph (diagraph), and $\mathcal{V} = \{\nu_i : i = 1, \ldots, N\}$ as the set of followers. $\mathcal{E} \subseteq \{e_{ij} : i = 1, \ldots, N, j = 1, \ldots, N, i \ne j\}$ is a set of edges in which $e_{ij} = (\nu_i, \nu_j) \in \mathcal{E}$ if and only if there exists an information exchange from i^{th} follower to j^{th} follower, and adjacency matrix is described interactions of followers in signed bipartite diagraph as $\mathcal{A} = [a_{ij}] \in \Re^{N\times N}$, where $a_{ij} \ne 0$ if $e_{ji} \in \mathcal{E}$. The sign a_{ij} represents the collective behavior type, i.e., for a competitive relationship between the ith and the jth follower, a negative value is reported for a_{ij}; and for cooperative behaviors this sign is the positive. Moreover, in situations that no directed paths from the follower jth to the follower ith is designed, a_{ij} is set to zero. For the ith follower, the set of neighbors is denoted by $N_i = \{j | a_{ij} \ne 0\}$. $\mathcal{L} = \mathcal{C} - \mathcal{A}$, is defined as Laplacian matrix and $\mathcal{L} \in \Re^{N\times N}$, where the weighted degree matrix of ith follower is denoted by $\mathcal{C} = \mathrm{diag}(c_i)$ and $c_i = \sum_{j\in N_i} |a_{ij}|$.

We now define another graph $\bar{\mathcal{G}}$ to associate a network of N followers with a leader. The adjacency matrix for the leader is defined $\mathcal{B} = \mathrm{diag}(b_i) \in \mathbb{R}^{N\times N}$, with $b_i > 0$ (or $b_i < 0$) if only if ith follower directly receives cooperative (or competitive) information from the leader, otherwise $b_i = 0$. A digraph is said to have a spanning tree if there exists at least one agent (called root note) that has a direct path to every other agent.

Definition 2 [43]**.** Structurally balanced property is defined for a signed bipartite diagraph $\mathcal{G}$ if it includes a bipartition of the sets of followers $\mathcal{V}_1$ and $\mathcal{V}_2$, where $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \varnothing$ such that $a_{ij} > 0, \forall i, j \in \mathcal{V}_m (m = 1, 2)$; $a_{ij} < 0, \forall i \in \mathcal{V}_m, \forall j \in \mathcal{V}_n, m \ne n\ (m, n = 1, 2)$. Otherwise, $\mathcal{G}$ is called unstructurally balanced.

Assumption 1. The diagraph $\bar{\mathcal{G}}$ has a directed spanning tree with the leader as the root node and $\mathcal{G}$ is structurally balanced.

2.3. Uniform ultimate bounded consensus

The uniform ultimate bound consensus in bipartite manner is established for a leader–follower case if there exists a compact set $\Omega \subset \Re^n$, so that $\forall \mathbf{x}_{i,1}(t_0) \in \Omega$, there exists $T \ge 0$ and $\epsilon > 0$ such that [43]

$$ ||\mathbf{x}_{i,1}(t) - \mu_i \mathbf{x}_{0,1}(t)|| \le \epsilon, \qquad \text{for} \qquad \forall t \ge t_0 + T, \tag{9} $$

where $\mathbf{x}_{i,1}(t)$ is the output of i^{th} follower, $\mathbf{x}_{0,1}(t)$ is the leader's output, $\mu_i = 1$ if $i \in \mathcal{V}_1$ and $\mu_i = -1$ if $i \in \mathcal{V}_2$. Moreover, if $\epsilon = 0$, for $t \to \infty$, it is said that asymptotical bipartite consensus is achieved.

2.4. Neural networks

Among many different applications such as approximating unknown functions, neural networks are considered as a powerful tool in control system design. The detailed study of NNs can be

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found in [16]. Based on the universal approximation property of NNs, $f(\mathbf{Q})$ is expressed as

$$f(\mathbf{Q}) = \mathbf{\Pi}^* \boldsymbol{\phi}(\mathbf{Q}) + \varepsilon(\mathbf{Q}), \qquad \forall \mathbf{Q} \in \Omega_{\mathbf{Q}} \subset \Re^q, \tag{10}$$

where $\mathbf{\Pi}^* = [w_1^*, \dots, w_\varsigma^*]$ is the ideal constant weight vector, $\boldsymbol{\phi}(\mathbf{Q}) = [\boldsymbol{\phi}_1(\mathbf{Q}_1) \dots, \boldsymbol{\phi}_\varsigma(\mathbf{Q}_\varsigma)]^{\mathrm{T}}$ is the basis function vector, $\varsigma > 1$ denotes the number of neurons, and $\varepsilon(\mathbf{Q})$ is the minimum approximation error. Generally, the basis function $\boldsymbol{\phi}_i(\mathbf{Q}_i)$ for $i = 1, 2, \dots, \varsigma$, can be selected as Gaussian, hyperbolic tangent or sigmoid, etc. In this paper, due to applying radial basis function neural networks (RBF NNs), Gaussian basis functions are used.

Assumption 2. The minimum approximation error of NNs and the ideal constant weight vector over the compact set $\Omega_{\mathbf{Q}}$ are respectively bounded by unknown positive constants ε^* and w_m as

$$|\varepsilon(\mathbf{Q})| \le \varepsilon^*, \quad ||\mathbf{\Pi}^*|| \le w_m, \quad \mathbf{Q} \in \Omega_{\mathbf{Q}}. \tag{11}$$

Lemma 4 [15]**.** *For any vector ($\boldsymbol{k}$, $\mathbf{m}$)$\in \Re^{\mathrm{n}}$, the following inequality holds*

$$\mathbf{k}^{\mathrm{T}}\mathbf{m} \le \frac{\kappa^p}{p}||\mathbf{k}||^p + \frac{1}{q\kappa^q}||\mathbf{m}||^q, \tag{12}$$

where $\kappa > 0$, $p > 1$, $q > 1$, and $(p-1)(q-1) = 1$.

Definition 3 [38]**.** A barrier Lyapunov function (BLF) (i.e., $V(x,\ t)$) is a continues, scalar, positive definite and $C^{1,1}$ function defined for the dynamical systems $\dot{\chi} = F(\chi)$, on an open region W including the origin, which for $k_b > 0$ as a boundary of region W, has the following property

$$V(x,t) \to \infty \quad as \quad x \to \pm k_b, \tag{13}$$

and $\forall t \ge t_0$ ensures that $V(x,\ t) \in L_\infty$ according to $\dot{\chi} = F(\chi)$ for $\chi(t_0) \in W$.

Similar to [38], in this paper, a BLF is utilized as

$$Ln\frac{k_b^2}{k_b^2 - \beta^2(t)}, \tag{14}$$

where $\beta(t)$ is bounded by k_b.

In order to resolve the control problem of followers' output constraints, the following Lemma is employed.

Lemma 5 [38]**.** *For existing the arbitrary positive constant k_b,the following inequality holds*

$$Ln\frac{k_b^2}{k_b^2 - \beta^2(t)} \le \frac{\beta^2(t)}{k_b^2 - \beta^2(t)}, \tag{15}$$

if $|\beta(t)| \le k_b$ is satisfied for all times.

3. Problem formulation

The dynamics of ith follower labeled from 1 to N is described by the following fractional-order nonlinear uncertain systems

$$\begin{cases} {}_0^C D_t^q x_{i,k} = x_{i,k+1} + f_{i,k}(\mathbf{x}_{i,k}) + \rho_{i,k}(t), \qquad k = 1, 2, \dots, n_i - 1, \\ {}_0^C D_t^q x_{i,n_i} = u_i + f_{i,n_i}(\mathbf{x}_{i,n_i}) + \rho_{i,n_i}(t), y_i = x_{i,1}, \end{cases} \tag{16}$$

where ${}_0^C D_t^q x_{i,k}$ for $k = 1, \dots, n_i$ denotes the q-order Caputo fractional derivative of follower's state, and $0 < q < 1$. $\mathbf{x}_{i,n_i} = [x_{i,1}, x_{i,2}, \dots, x_{i,n_i}]^{\mathrm{T}} \in \Re^{n_i}$ is the state vector of i^{th} follower, $u_i \in \Re$ is the control input and $y_i \in \Re$ is the output of ith follower agent. $\mathbf{x}_{i,k} = [x_{i,1}, x_{i,2}, \dots, x_{i,k}]^{\mathrm{T}} \in \Re^k$. $f_{i,k}(\mathbf{x}_{i,k})$: $\Re^k \to \Re$ for $k = 1, \dots, n_i$ is unknown smooth function and $\rho_{i,k}(t) : \Re^+ \to \Re$ for $k = 1, \dots, n_i$ is an unknown bounded external disturbance, i.e., $|\rho_{i,k}(t)| \le \rho_{i,k}^*$, where $\rho_{i,k}^*$ is an unknown constant.

The leader dynamics labeled as 0 is described by

$$\begin{cases} {}_0^C D_t^q x_{0,k} = x_{0,k+1} + f_{0,k}(\mathbf{x}_{0,k}, t), \quad k = 1, 2, \dots, n_i - 1, \\ {}_0^C D_t^q x_{0,n_i} = f_{0,n_i}(\mathbf{x}_{0,n_i}, t), \\ y_0 = x_{0,1}, \end{cases} \tag{17}$$

where ${}_0^C D_t^q x_{0,k}$ for $k = 1, \dots, n_0$ denotes the q-order Caputo fractional derivative of the leader's state and $0 < q < 1$. $\mathbf{x}_{0,n_i} = [x_{0,1}, x_{0,2}, \dots, x_{0,n_0}]^{\mathrm{T}} \in \Re^{n_0}$ is the state vector of the leader, $\mathbf{x}_{0,k} = [x_{0,1}, x_{0,2}, \dots, x_{0,k}]^{\mathrm{T}} \in \Re^k$ and $f_{0,k}(\mathbf{x}_0, t) : \Re^k \times \Re^+ \to \Re$ for $k = 1, \dots, n_0$ is locally Lipschitz in $\mathbf{x}_{0,k}$ and piecewise continues in t, and it is also a bounded function.

Remark 1. If the q-order Caputo fractional derivative in the dynamics (16) and (17) is replaced by the conventional integer-order derivative and the interactions are also only cooperative, extensive results have been studied, see for example [47]. However, the control design for nonlinear fractional-order MASs, especially in strict-feedback dynamical form, till now is an open problem, which is one of our motivations in preparing this paper.

The *Control objective* is declared in this paper as constructing a fully distributed adaptive neural control architecture for a network of uncertain nonlinear follower agents (16) considering bounded dynamic leader (17) such that

1. All the closed-loop network signals are uniform ultimate bounded.
2. All the local bipartite tracking errors

$$\lambda_i = x_{i,1} - \mu_i x_{0,1}, \quad \mu_i \in \{1, -1\}, \quad \text{for} \quad i = 1, \dots, N, \tag{18}$$

are confined to preset bounds.

Assumption 3. There exist positive constants k_c, A_1, and A_2 such that the leader output and its q-order fractional derivative is continuous and bounded, such that, $|x_{0,1}| \le A_1 \le k_c$ and $|{}_0^C D_t^q x_{0,1}| \le A_2$.

Remark 2. The following statements are scrutinized.

- Assumption 1 is a necessary condition to obtain the leader-following bipartite consensus problem. Assumption 2 is a statement about the boundedness of ideal weight vector and approximation error for RBF neural network, see for example, [16]. In Assumption 3, the boundedness of leader output and its q-order fractional derivative is emphasized. This Assumption is less restrictive than that considered in [7,40,54], where the boundedness of n integer-order derivative of desired signal are required for employing the backstepping design.
- The fractional-order system (16) can be utilized to represent a large class of nonlinear dynamical systems such as
  1. Robotic systems: a network of two-DOF robotic manipulators, a group of single-link flexible-joint robots, and a network of robots with two revolute joints in the vertical plane, [55].
  2. Power systems: a multi-machine-infinite bus power system [25], a network of doubly-fed induction generators [2], and multiple hydro-turbine governing systems, [44].
  3. Mechanical systems: multiple two-inverted pendulums connected by an unknown device, [55].
  4. Chaotic systems: Chuaâs circuit, Gyroscope systems, Duffing and Holmes systems, [27].

Remark 3. The proposed output constraint control strategy is general enough to cover both cooperative and bipartite consensus of networked nonlinear fractional-order (or integer-order) systems. The first motivation for studying the distributed bipartite output constraint controller is derived from industrial applications such as bipartite consensus in formation and flocking of multiple mechanical systems or polarization in opinion dynamics. One example is the bipartite consensus problem for a network of fractional-order robots which collect information from both teammates and antagonists to achieve agreement with the own team. Another typical example is the fractional-order model of social networks in which a pair of agents can be friends or rivals depending on their relationship such as trust/distrust, like/dislike, etc. Moreover, bipartite consensus in a network of fractional-order chaotic systems can be considered as one common example for the proposed control approach.

4. Main results

4.1. Control structure

In this subsection, a fully distributed adaptive control structure using the backstepping methodology and BLF scheme is proposed for the network of fractional-order systems (16) and (17) under signed bipartite diagraph.

Hereafter, symbol t in all equations is omitted for simplicity. To construct the proposed controller, the following coordination transformations are adopted

$$\beta_{i,1} = \sum_{j\in N_i} |a_{i,j}|(x_{i,1} - \mathrm{sign}(a_{i,j})x_{j,1}) + b_i(x_{i,1} - \mu_i x_{0,1}), \tag{19}$$

$$\beta_{i,k} = x_{i,k} - \tau_{i,k-1}, \qquad k = 2, \ldots, n_i, \tag{20}$$

where $\beta_{i,1}$ is the distributed bipartite tracking error, $\beta_{i,k}$ is the error surface, and $\tau_{i,k-1}$ is the virtual control law.

Remark 4. Distributed bipartite graph-based error surface (19) is used for the backstepping bipartite consensus tracking control design such that the followers' outputs ($x_{i,1}$) ultimately synchronize to the leader output ($x_{0,1}$) in modulus but different in sign for antagonistic agents. Moreover, the effects of graph signals for ith follower agent (i.e., $a_{i,j}\mathrm{sign}(a_{i,j}){}_0^C D_t^q x_{j,1}$ and $|b_i|{}_0^C D_t^q x_{0,1}$) are compensated in first virtual control law.

Step one: From (16), (19) and (20), one follows that

$${}_0^C D_t^q \beta_{i,1} = (c_i + b_i)\Big(\beta_{i,2} + \tau_{i,1} + F_{i,1}(\mathbf{Q}_{i,1}) + \bar{\rho}_{i,1}(t)\Big), \tag{21}$$

where $F_{i,1}(\mathbf{Q}_{i,1}) = f_{i,1}(x_{i,1}) - \frac{\sum_{j\in N_i}|a_{i,j}|\mathrm{sign}(a_{i,j})}{c_i+b_i}(x_{j,2} + f_{j,1}(x_{j,1})) - \frac{b_i\mu_i}{c_i+b_i}{}_0^C D_t^q x_{0,1}\big)$, $\bar{\rho}_{i,1} = \rho_{i,1} - \big(\frac{\sum_{j\in N_i}|a_{i,j}|\mathrm{sign}(a_{i,j})}{c_i+b_i}\big)\rho_{j,1}$ and $\mathbf{Q}_{i,1} = [x_{0,1}, x_{i,1}, x_{j,1}, x_{j,2}]^{\mathrm{T}}$, $j \in N_i$.

According to the universal approximation property of RBF NNs, (21) is rewritten as

$${}_0^C D_t^q \beta_{i,1} = (c_i + b_i)\Big(\beta_{i,2} + \tau_{i,1} + \mathbf{\Pi}_{i,1}^{*\mathrm{T}}\boldsymbol{\phi}_{i,1}(\mathbf{Q}_{i,1}) + \varepsilon_{i,1} + \bar{\rho}_{i,1}\Big). \tag{22}$$

Now we design the virtual controllers and the adaptive laws as

$$\tau_{i,1} = \frac{1}{c_i + b_i}\Big(-\alpha_{i,1}\beta_{i,1} - \frac{\chi_i\beta_{i,1}}{k_b^2 - \beta_{i,1}^2}\Big) - \frac{1}{2\omega_{i,1}}\frac{\beta_{i,1}}{k_b^2 - \beta_{i,1}^2}\hat{\pi}_{i,1}, \tag{23}$$

$${}_0^C D_t^q \hat{\pi}_{i,1} = \frac{\gamma_{i,1}}{2\omega_{i,1}}(b_i + c_i)\frac{\beta_{i,1}^2}{(k_{i,b}^2 - \beta_{i,1}^2)^2} - \gamma_{i,1}\sigma_{i,1}\hat{\pi}_{i,1}, \tag{24}$$

where $\hat{\pi}_{i,1}$ is the estimate of $\pi_{i,1}^* = \varsigma_{i,1}||\mathbf{\Pi}_{i,1}^*||^2$, $\varsigma_{i,1} \geq \boldsymbol{\phi}_{i,1}^{\mathrm{T}}(\mathbf{Q}_{i,1})\boldsymbol{\phi}_{i,1}(\mathbf{Q}_{i,1})$, $\omega_{i,1}$ is a positive constant, $\alpha_{i,1}$ and χ_i are positive control gains, and $\gamma_{i,1}$ and $\sigma_{i,1}$ are positive adaption gain and sigma modification factor, respectively.

Remark 5. In comparison to [6,31,34,42,43,47,53], in order to eliminate the over-parameterization drawback in conventional neural approximators, by employing the MLP approach in proposed method, the scalar fractional-order adaptive laws for each follower are only updated, and there is no need to determine the centers of receptive fields and widths of basis functions. Besides, in contrast with [3,4,9,10,23,39,51], due to using the MLP scheme, in (23) and (24) only relative output information is required to design the distributed control protocol. Therefore, communications between neighborhoods are significantly reduced, and the extended bipartite consensus approach can be easily implemented for a network with large number of followers.

Choose the BLF candidate as

$$V_{i,1} = \frac{1}{2}Ln\frac{k_b^2}{k_b^2 - \beta_{i,1}^2} + \frac{1}{2\gamma_{i,1}}\tilde{\pi}_{i,1}^2, \tag{25}$$

where $\tilde{\pi}_{i,1} = \pi_{i,1}^* - \hat{\pi}_{i,1}$ and k_b is the preset bound of $|\beta_{i,1}|$ for $\forall t \geq 0$.

Remark 6. The Lyapunov method is a basic approach for investigating the stability of closed-loop nonlinear MASs. For the analysis of distributed adaptive fuzzy or neural control designs of integer-order MASs, the Lyapunov quadratic functions have been frequently utilized. According to Zouari et al. [55,56], using this conventional class of Lyapunov functions to design controllers and analyze the stability of fractional-order nonlinear MASs is very complicated because of the unlimited series are produced by Lyapunov quadratic functions with fractional-order derivative. Recently, to deal with this problem, the Lyapunov fractional-order stability has been developed based on the fact that in stable fractional-order systems, the generalized energy does not decrease exponentially [56]. In this paper, the bipartite consensus stability analysis for nonlinear fractional-order MASs will be resolved by applying the Lyapunov fractional-order stability theorem and related lemmas.

Using Lemma 2 and Property 1, the following is obtained

$${}_0^C D_t^q V_{i,1} \leq \frac{\beta_{i,1}}{k_b^2 - \beta_{i,1}^2}{}_0^C D_t^q \beta_{i,1} - \frac{1}{\gamma_{i,1}}\tilde{\pi}_{i,1}\,{}_0^C D_t^q \hat{\pi}_{i,1}, \tag{26}$$

then along with (22), one has

$$\begin{aligned}{}_0^C D_t^q V_{i,1} \leq{}& \frac{\beta_{i,1}}{k_b^2 - \beta_{i,1}^2}\Big((c_i + b_i)\big(\beta_{i,2} + \tau_{i,1} + \mathbf{\Pi}_{i,1}^{*\mathrm{T}}\boldsymbol{\phi}_{i,1}(\mathbf{Q}_{i,1}) \\ &+ \varepsilon_{i,1} + \bar{\rho}_{i,1}\big)\Big) - \frac{1}{\gamma_{i,1}}\tilde{\pi}_{i,1}\,{}_0^C D_t^q \hat{\pi}_{i,1}.\end{aligned} \tag{27}$$

Via Lemma 4, one obtains

$$\frac{\beta_{i,1}}{k_b^2 - \beta_{i,1}^2}\mathbf{\Pi}_{i,1}^{*\mathrm{T}}\boldsymbol{\phi}_{i,1}(\mathbf{Q}_{i,1}) \leq \frac{1}{2\omega_{i,1}}\frac{\beta_{i,1}^2}{(k_b^2 - \beta_{i,1}^2)^2}\pi_{i,1}^* + \frac{1}{2}\omega_{i,1}, \tag{28}$$

where $\omega_{i,1}$ is defined under (24).

Substituting (23), (24), and (28) into (27), results in

$$\begin{aligned}{}_0^C D_t^q V_{i,1} \leq{}& \frac{\beta_{i,1}}{k_b^2 - \beta_{i,1}^2}\Big(-\alpha_{i,1}\beta_{i,1} - \frac{\chi_i\beta_{i,1}}{k_b^2 - \beta_{i,1}^2} + (c_i + b_i) \\ &\Big(\beta_{i,2} + \varepsilon_{i,1} + \bar{\rho}_{i,1} + \frac{1}{2}\omega_{i,1}\Big)\Big) + \sigma_{i,1}\tilde{\pi}_{i,1}\hat{\pi}_{i,1}.\end{aligned} \tag{29}$$

Via Lemma 4, one can obtain

$$\frac{\beta_{i,1}}{k_b^2 - \beta_{i,1}^2}\Big(\beta_{i,2} + \varepsilon_{i,1} + \bar{\rho}_{i,1}\Big) \leq \frac{3}{2}\frac{\beta_{i,1}^2}{(k_b^2 - \beta_{i,1}^2)^2} + \frac{1}{2}\beta_{i,2}^2 + \frac{1}{2}\varepsilon_{i,1}^{*2} + \frac{1}{2}\bar{\rho}_{i,1}^{*2}, \tag{30}$$

where $|\bar{\rho}_{i,1}| \leq \bar{\rho}_{i,1}^*$ and $|\varepsilon_{i,1}| \leq \varepsilon_{i,1}^*$.

Combining (30) with (29) gives

$$\begin{aligned}{}_0^C D_t^q V_{i,1} \leq{}& -\alpha_{i,1}\frac{\beta_{i,1}^2}{k_b^2 - \beta_{i,1}^2} - \Big(\chi_i - \frac{3}{2}\Big)\frac{\beta_{i,1}^2}{(k_b^2 - \beta_{i,1}^2)^2} \\ &+ \frac{1}{2}(c_i + b_i)\beta_{i,2}^2 + \frac{1}{2}(c_i + b_i)\varepsilon_{i,1}^{*2} + \frac{1}{2}(c_i + b_i)\bar{\rho}_{i,1}^{*2} \\ &+ \frac{1}{2}(c_i + b_i)\omega_{i,1} + \sigma_{i,1}\tilde{\pi}_{i,1}\hat{\pi}_{i,1}.\end{aligned} \tag{31}$$

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Inductive Step: From (16) and (20), we have

$$ {}_0^C D_t^q \beta_{i,k} = \beta_{i,k+1} + \tau_{i,k} + F_{i,k}(\mathbf{Q}_{i,k}) + \rho_{i,k}, \tag{32}$$

where $F_{i,k}(\mathbf{Q}_{i,k}) = f_{i,k}(\mathbf{x}_{i,k}) - {}_0^C D_t^q \tau_{i,k-1}$.

According to the universal approximation capability of RBF NNs, (32) becomes

$$ {}_0^C D_t^q \beta_{i,k} = \beta_{i,k+1} + \tau_{i,k} + \mathbf{\Pi}_{i,k}^{*\mathrm{T}} \boldsymbol{\phi}_{i,k}(\mathbf{Q}_{i,k}) + \varepsilon_{i,k} + \rho_{i,k}. \tag{33}$$

Now the virtual control and the adaptive law are designed as

$$ \tau_{i,k} = -\alpha_{i,k}\beta_{i,k} - \frac{1}{2\omega_{i,k}}\beta_{i,k}\hat{\pi}_{i,k}, \tag{34}$$

$$ {}_0^C D_t^q \hat{\pi}_{i,k} = \frac{\gamma_{i,k}}{2\omega_{i,k}}\beta_{i,k}^2 - \gamma_{i,k}\sigma_{i,k}\hat{\pi}_{i,k}, \tag{35}$$

where $\hat{\pi}_{i,k}$ is the estimate of $\pi_{i,k}^* = \varsigma_{i,k}||\mathbf{\Pi}_{i,k}^*||^2$, $\varsigma_{i,k} \geq \boldsymbol{\phi}_{i,1}^{\mathrm{T}}(\mathbf{Q}_{i,k})\boldsymbol{\phi}_{i,k}(\mathbf{Q}_{i,k})$, $\omega_{i,k}$ is a positive constant, $\alpha_{i,k}$ is positive control gain, and $\gamma_{i,k}$ and $\sigma_{i,k}$ are positive adaption gain and sigma modification factor, respectively.

Define a Lyapunov function candidate as

$$ V_{i,k} = \frac{1}{2}\beta_{i,k}^2 + \frac{1}{2\gamma_{i,k}}\tilde{\pi}_{i,k}^2, \tag{36}$$

where $\tilde{\pi}_{i,k} = \pi_{i,k}^* - \hat{\pi}_{i,k}$.

Considering (33) and (36), it follows that

$$ {}_0^C D_t^q V_{i,k} \leq \beta_{i,k}\Big(\beta_{i,k+1} + \tau_{i,k} + \mathbf{\Pi}_{i,k}^{*\mathrm{T}} \boldsymbol{\phi}_{i,k}(\mathbf{Q}_{i,k}) + \varepsilon_{i,k} + \rho_{i,k}\Big) - \frac{1}{\gamma_{i,k}}\tilde{\pi}_{i,k}\, {}_0^C D_t^q \hat{\pi}_{i,k}. \tag{37}$$

Similar to first step, one has

$$ \beta_{i,k}\mathbf{\Pi}_{i,k}^{*\mathrm{T}} \boldsymbol{\phi}_{i,k}(\mathbf{Q}_{i,k}) \leq \frac{1}{2\omega_{i,k}}\beta_{i,k}^2\pi_{i,k}^* + \frac{1}{2}\omega_{i,k}, \tag{38}$$

where $\omega_{i,k}$ is defined under (35).

Substituting (34), (35), and (38) into (37) yields

$$ {}_0^C D_t^q V_{i,k} \leq \beta_{i,k}\Big(- \alpha_{i,k}\beta_{i,k} + \beta_{i,k+1} + \varepsilon_{i,k} + \rho_{i,k}\Big) + \frac{1}{2\omega_{i,k}} + \sigma_{i,k}\tilde{\pi}_{i,k}\hat{\pi}_{i,k}. \tag{39}$$

Moreover, the following inequality holds

$$ \beta_{i,k}\Big(\beta_{i,k+1} + \varepsilon_{i,k} + \rho_{i,k}\Big) \leq \frac{3}{2}\beta_{i,k}^2 + \frac{1}{2}\beta_{i,k+1}^2 + \frac{1}{2}\varepsilon_{i,k}^{*2} + \frac{1}{2}\rho_{i,k}^{*2}, \tag{40}$$

where $|\rho_{i,k}| \leq \rho_{i,k}^*$ and $|\varepsilon_{i,k}| \leq \varepsilon_{i,k}^*$.

Substituting (40) in (39), we have

$$ {}_0^C D_t^q V_{i,k} \leq -\Big(\alpha_{i,k} - \frac{3}{2}\Big)\beta_{i,k}^2 + \frac{1}{2}\beta_{i,k+1}^2 + \frac{1}{2\omega_{i,k}} + \frac{1}{2}\varepsilon_{i,k}^{*2} + \frac{1}{2}\rho_{i,k}^{*2} + \sigma_{i,k}\tilde{\pi}_{i,k}\hat{\pi}_{i,k}. \tag{41}$$

Final step: According to (20), one has

$$ {}_0^C D_t^q \beta_{i,n_i} = u_i + F_{i,n_i}(\mathbf{Q}_{i,n_i}) + \rho_{i,n_i}, \tag{42}$$

where $F_{i,n_i}(\mathbf{Q}_{i,n_i}) = f_{i,n_i}(\mathbf{x}_{i,n_i}) - {}_0^C D_t^q \tau_{i,n_i-1}$. Using RBF NNs to approximate unknown nonlinearities, (42) we have

$$ {}_0^C D_t^q \beta_{i,n_i} = u_i + \mathbf{\Pi}_{i,n_i}^{*\mathrm{T}} \boldsymbol{\phi}_{i,n_i}(\mathbf{Q}_{i,n_i}) + \varepsilon_{i,n_i} + \rho_{i,n_i}. \tag{43}$$

Now the actual control and adaptive laws are designed as follows

$$ u_i = -\alpha_{i,n_i}\beta_{i,n_i} - \frac{1}{2\omega_{i,n_i}}\beta_{i,n_i}\hat{\pi}_{i,n_i}, \tag{44}$$

$$ {}_0^C D_t^q \hat{\pi}_{i,n_i} = \frac{\gamma_{i,n_i}}{2\omega_{i,n_i}}\beta_{i,n_i}^2 - \gamma_{i,n_i}\sigma_{i,n_i}\hat{\pi}_{i,n_i}, \tag{45}$$

where $\hat{\pi}_{i,n_i}$ is the estimate of $\pi_{i,n_i}^* = \varsigma_{i,n_i}||\mathbf{\Pi}_{i,n_i}^*||^2$, $\varsigma_{i,n_i} \geq \boldsymbol{\phi}_{i,n_i}^{\mathrm{T}}(\mathbf{Q}_{i,n_i})\boldsymbol{\phi}_{i,n_i}(\mathbf{Q}_{i,n_i})$, ω_{i,n_i} is a positive constant, α_{i,n_i} is a positive control gain, and γ_{i,n_i} and σ_{i,n_i} are positive adaption gain and sigma modification factor, respectively.

The Lyapunov function candidate is considered as

$$ V_{i,n_i} = \frac{1}{2}\beta_{i,n_i}^2 + \frac{1}{2\gamma_{i,n_i}}\tilde{\pi}_{i,n_i}^2, \tag{46}$$

where $\tilde{\pi}_{i,n_i} = \pi_{i,n_i}^* - \hat{\pi}_{i,n_i}$.

Via Lemma 2 and (43), one can obtain

$$ {}_0^C D_t^q V_{i,n_i} \leq \beta_{i,n_i}\Big(u_i + \mathbf{\Pi}_{i,n_i}^{*\mathrm{T}} \boldsymbol{\phi}_{i,n_i}(\mathbf{Q}_{i,n_i}) + \varepsilon_{i,n_i} + \rho_{i,n_i}\Big) - \frac{1}{\gamma_{i,n_i}}\tilde{\pi}_{i,n_i}\, {}_0^C D_t^q \hat{\pi}_{i,n_i}. \tag{47}$$

Similar to previous steps, one has

$$ \beta_{i,n_i}\mathbf{\Pi}_{i,n_i}^{*\mathrm{T}} \boldsymbol{\phi}_{i,n_i}(\mathbf{Q}_{i,n_i}) \leq \frac{1}{2\omega_{i,n_i}}\beta_{i,n_i}^2\pi_{i,n_i}^* + \frac{1}{2}\omega_{i,n_i}, \tag{48}$$

where ω_{i,n_i} is defined under (45).

Considering (44), (45), (48), and (47), we have

$$ {}_0^C D_t^q V_{i,n_i} \leq \beta_{i,n_i}\Big(- \alpha_{i,n_i}\beta_{i,n_i} + \varepsilon_{i,n_i} + \rho_{i,n_i}\Big) + \frac{1}{2}\omega_{i,n_i} + \sigma_{i,n_i}\tilde{\pi}_{i,n_i}\hat{\pi}_{i,n_i}. \tag{49}$$

Via Lemma 4, one has

$$ \beta_{i,n_i}\Big(\varepsilon_{i,n_i} + \rho_{i,n_i}\Big) \leq \beta_{i,n_i}^2 + \frac{1}{2}\varepsilon_{i,n_i}^{*2} + \frac{1}{2}\rho_{i,n_i}^{*2}, \tag{50}$$

where $|\rho_{i,n_i}| \leq \rho_{i,n_i}^*$ and $|\varepsilon_{i,n_i}| \leq \varepsilon_{i,n_i}^*$.

Note to (50), (49) can be rewritten as

$$ {}_0^C D_t^q V_{i,n_i} \leq -\Big(\alpha_{i,n_i} - 1\Big)\beta_{i,n_i}^2 + \frac{1}{2}\varepsilon_{i,n_i}^{*2} + \frac{1}{2}\rho_{i,n_i}^{*2} + \frac{1}{2\omega_{i,n_i}} + \sigma_{i,n_i}\tilde{\pi}_{i,n_i}\hat{\pi}_{i,n_i}. \tag{51}$$

4.2. Bipartite consensus analysis

To consider the bipartite consensus tracking analysis of the overall closed-loop network system, select the following Lyapunov candidate function

$$ V = \sum_{i=1}^{N}\sum_{k=1}^{n_i} V_{i,k}. \tag{52}$$

Theorem 1. *Consider the closed-loop network system including the fractional-order agents (16), (17) and the fully distributed adaptive neural bipartite consensus control laws (23), (34), (44) with the neural laws (24), (35), (45) under Assumptions 1–3. If* $\alpha_{i,1} > 0, \alpha_{i,2} > \frac{3}{2} - \frac{1}{2}(c_i + b_i), \alpha_{i,3} > 2, \ldots, \alpha_{i,n_i-1} > 2, \alpha_{i,n_i} > \frac{3}{2}$, *and* $\sigma_{i,1}\gamma_{i,1} > 0, \ldots, \sigma_{i,n_i}\gamma_{i,n_i} > 0$, *then all signals of the closed-loop network systems are uniformly ultimate bounded, while the followers' outputs constraints* $|y_i| \leq k_c$, $\forall t \geq 0$ *are not violated.*

Proof. Considering (31), (41), and (51), one can obtain

$$ \begin{aligned} {}_0^C D_t^q V \leq \sum_{i=1}^{N}\Bigg\{ & - \alpha_{i,1}\frac{\beta_{i,1}^2}{k_b^2 - \beta_{i,1}^2} - \Big(\alpha_{i,2} - \frac{3}{2} - \frac{1}{2}(c_i + b_i)\Big)\beta_{i,2}^2 \\ & - \sum_{m=3}^{n_i-1}\Big(\alpha_{i,m} - 2\Big)\beta_{i,m}^2 - \Big(\alpha_{i,n_i} - \frac{3}{2}\Big)\beta_{i,n_i}^2 \\ & - \sum_{m=1}^{n_i}\sigma_{i,m}\tilde{\pi}_{i,m}^2 + \frac{1}{2}(c_i + b_i)\varepsilon_{i,1}^{*2} \\ & + \frac{1}{2}(c_i + b_i)\bar{\rho}_{i,1}^{*2} + \frac{1}{2}(c_i + b_i)\omega_{i,1} + \frac{1}{2}\sum_{m=2}^{n_i}\varepsilon_{i,m}^{*2} \\ & + \sum_{m=2}^{n_i}\frac{1}{2}\omega_{i,m} + \frac{1}{2}\sum_{m=2}^{n_i}\rho_{i,m}^{*2} + \sum_{m=1}^{n_i}\sigma_{i,m}\pi_{i,m}^{*2}\Bigg\}. \end{aligned} \tag{53}$$

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Then according to Lemma 5, (53) can be rewritten as follows

$$
\begin{aligned}
{}_0^CD_t^qV \le \sum_{i=1}^{N}\Bigg\{ & -\alpha_{i,1}Ln\frac{k_b^2}{k_{i,b}^2-\beta_{i,1}^2}-\left(\alpha_{i,2}-\frac{3}{2}-\frac{1}{2}(c_i+b_i)\right)\beta_{i,2}^2\\
& -\sum_{m=3}^{n_i-1}\left(\alpha_{i,m}-2\right)\beta_{i,m}^2-\left(\alpha_{i,n_i}-\frac{3}{2}\right)\beta_{i,n_i}^2\\
& -\sum_{m=1}^{n_i}\sigma_{i,m}\tilde{\pi}_{i,m}^2+\frac{1}{2}(c_i+b_i)\varepsilon_{i,1}^{*2}\\
& +\frac{1}{2}(c_i+b_i)\bar{\rho}_{i,1}^{*2}+\frac{1}{2}(c_i+b_i)\omega_{i,1}+\frac{1}{2}\sum_{m=2}^{n_i}\varepsilon_{i,m}^{*2}\\
& +\sum_{m=2}^{n_i}\frac{1}{2}\omega_{i,m}+\frac{1}{2}\sum_{m=2}^{n_i}\rho_{i,m}^{*2}+\sum_{m=1}^{n_i}\sigma_{i,m}\pi_{i,m}^{*2}\Bigg\}.
\end{aligned} \tag{54}
$$

Now by defining

$$
\begin{aligned}
\eta = \min_{i=1,\ldots,N} \Bigg\{ & 2\alpha_{i,1}, 2\left(\alpha_{i,2}-\frac{3}{2}-\frac{1}{2}(c_i+b_i)\right), 2\left(\alpha_{i,3}-2\right),\ldots,\\
& 2\left(\alpha_{i,n_i-1}-2\right), 2\left(\alpha_{i,n_i}-\frac{3}{2}\right), \sigma_{i,1}\gamma_{i,1}, \sigma_{i,2}\gamma_{i,2},\ldots,\\
& \sigma_{i,n_i-1}\gamma_{i,n_i-1}, \sigma_{i,n_i}\gamma_{i,n_i}\Bigg\},
\end{aligned} \tag{55}
$$

$$
\begin{aligned}
\varrho = \sum_{i=1}^{N}\Bigg\{ & \frac{1}{2}(c_i+b_i)\varepsilon_{i,1}^{*2}+\frac{1}{2}(c_i+b_i)\bar{\rho}_{i,1}^{*2}+\frac{1}{2}(c_i+b_i)\omega_{i,1}\\
& +\frac{1}{2}\sum_{m=2}^{n_i}\varepsilon_{i,m}^{*2}+\sum_{m=2}^{n_i}\frac{1}{2}\omega_{i,m}+\frac{1}{2}\sum_{m=2}^{n_i}\rho_{i,m}^{*2}+\sum_{m=1}^{n_i}\sigma_{i,m}\pi_{i,m}^{*2}\Bigg\},
\end{aligned} \tag{56}
$$

(54) becomes

$$
{}_0^CD_t^qV \le -\eta V+\varrho. \tag{57}
$$

From Lemma 3, we have

$$
V \le V(0)E_{(q,1)}(-\eta t^q)+\frac{\varrho\varpi}{\eta}. \tag{58}
$$

Based on Lemma 1, it is easy to obtain that $V(t)\le\frac{\varrho\varpi}{\eta}$, for $t\to\infty$. So, $V(t)$ is bounded. Therefore, it is ensured that $\beta_{i,1},\ldots,\beta_{i,n_i}$ and $\tilde{\pi}_{i,1},\ldots,\tilde{\pi}_{i,n_i}$ are bounded. According to Assumption 2 and the definition $\pi_{i,k}^*=\varsigma_{i,k}||\Pi_{i,k}^*||^2$ for $k=1,\ldots,n_i$, the boundedness of $\pi_{i,1}^*,\ldots,\pi_{i,n_i}^*$ is obvious. Due to $\hat{\pi}_{i,k}=\pi_{i,k}^*+\tilde{\pi}_{i,k}$, it implies that $\hat{\pi}_{i,1},\ldots,\hat{\pi}_{i,n_i}$ are also bounded. Therefore, the q-order fractional derivative of adaptive laws ${}_0^CD_t^q\hat{\pi}_{i,1},\ldots,{}_0^CD_t^q\hat{\pi}_{i,n_i}$ are bounded according to (24), (35) and (45). On the other hand, the following inequalities hold

$$
\begin{aligned}
\sum_{i=1}^{N}\frac{k_b^2}{k_b^2-\beta_{i,1}^2} &\le \exp\left(2V(0)E_{(q,1)}(-\eta t^q)+2\frac{\varrho\varpi}{\eta}\right),\\
|\beta_{i,1}| &\le k_b\sqrt{1-\exp\left(-2V(0)E_{(q,1)}(-\eta t^q)-2\frac{\varrho\varpi}{\eta}\right)},
\end{aligned} \tag{59}
$$

for $i=1,2,.,N$. Then, from (59) and based on Lemma 1, one has

$$
|\beta_{i,1}| \le k_b\sqrt{1-\exp\left(-2\frac{\varrho\varpi}{\eta}\right)}, \tag{60}
$$

as $t\to\infty$, hence, distributed bipartite tracking errors are restricted to preset bounds for all times, i.e., $|\beta_{i,1}|\le k_b$, $\forall t\ge 0$. It is clear that $||\beta_1||\le\sqrt{N}k_b$, where $\beta_1=[\beta_{1,1},\ldots,\beta_{N,1}]^{\mathrm{T}}$. Then, from (19), it is obtained that $|\lambda_i|\le||\lambda||\le\frac{\sqrt{N}k_b}{\bar{\sigma}(L+B)}$, where $\lambda=[\lambda_1,\ldots,\lambda_N]^{\mathrm{T}}$. From $\lambda_i=x_{i,1}-\mu_i x_{0,1}$ and $|x_{0,1}|\le A_1$, it shown that $|x_{i,1}|\le|\lambda_i|+|x_{0,1}|\le\frac{\sqrt{N}k_b}{\bar{\sigma}(L+B)}+A_1$. Define $k_c=\frac{\sqrt{N}k_b}{\bar{\sigma}(L+B)}+A_1$, and then $|x_{i,1}|\le k_c$. Therefore, the followers' outputs are not violated. From (58), it is verified that $\beta_{i,k}$ is bounded for $i=1,.,N,k=2,.,n_i$ as follows

$$
|\beta_{i,k}| \le \sqrt{2V(0)E_{(q,1)}(-\eta t^q)+2\frac{\varrho\varpi}{\eta}}. \tag{61}
$$

According to the virtual and actual control laws in (23), (34), and (44), one infers that $\tau_{i,1},\ldots,\tau_{i,n_i}$ and u_i are all bounded. Then, based on (20), $x_{i,k}$ for $i=1,.,N,k=2,.,n_i$ is also bounded. On the other hand, because $f_{i,k}(\mathbf{x}_{i,k})$ is a real smooth function and $\rho_{i,k}$ is a real bounded external disturbance, from (16), it is clear that ${}_0^CD_t^{\alpha_l}x_l$ is bounded. In summary, all signals in the closed-loop network system is bounded. This completes the proof. □

Remark 7. The control approaches in [18–20,37,41,55,56] are only valuable for nonlinear fractional-order systems without communication graph. However, in this work we consider the problem of bipartite consensus tracking for multiple nonlinear fractional-order systems with followers' output constraints. Hence, compared to these mentioned studies, in the proposed control approach, it is necessary to consider communication between the agents, coupling dynamics from neighborhoods and so forth. Moreover, in some existing fractional-order results [36], unknown fractional derivatives of virtual controls are appeared in control laws, due to fact that the Leibniz rule is not satisfied for the fractional derivatives. Hence, in this work by defining composite uncertainties (i.e., $F_{i,k}(\mathbf{x}_{i,k})$) and employing MLP approach in RBF NNs, this problem is effectively resolved.

Remark 8. In MASs, to reduce the undesirable effects of external disturbances and neural approximation errors on the consensus stability of closed-loop networked system and furthermore, in order to improve the consensus control performance, the tuning of design parameters should be appropriately done. An effective selection of the design parameters is only a sufficient condition to guarantee the consensus stability of the networked system. From (58), we see that large values of $\alpha_{i,k}$,$\gamma_{i,k}$ and small value of $\sigma_{i,k}$ provide faster convergence and also smaller ultimate bounds. However, in this situation, the control cost becomes too large and the transient state behavior may be oscillating.

For the bipartite consensus problem of multiple uncertain fractional-order systems without any output constraints, the following Corollary is derived by eliminating the BLF from Theorem 1.

Corollary 1. *Consider the networked fractional-order systems (16) and (17) controlled by*

$$
\tau_{i,k}=-\alpha_{i,k}\beta_{i,k}-\frac{1}{2\pi_{i,k}}\beta_{i,k}\hat{\pi}_{i,k}, \tag{62}
$$

$$
{}_0^CD_t^q\hat{\pi}_{i,k}=\frac{\gamma_{i,k}}{2\omega_{i,k}}\beta_{i,k}^2-\gamma_{i,k}\sigma_{i,k}\hat{\pi}_{i,k}, \tag{63}
$$

for $i=1,\ldots,N$ and $k=1,.,n_i$, where $\tau_{i,n_i}=u_i$. Under Assumptions 1–3, it can be proved that the distributed bipartite tracking errors are converged to an adjustable neighborhood of the origin.

Proof. To prove the boundedness of closed-loop signals, one follows the proof procedure of Theorem 1 considering the Lyapunov candidate function as $V=\sum_{i=1}^{N}\sum_{k=1}^{n_i}\left(\frac{1}{2}\beta_{i,k}^2+\frac{1}{2\gamma_{i,k}}\tilde{\pi}_{i,k}^2\right)$. □

5. Simulation results

In this section, to demonstrate the applicability and effectiveness of the introduced control approach, the simulation results for three examples are derived. Based on [36], for numerical solution of nonlinear fractional-order differential equations, Grunwald

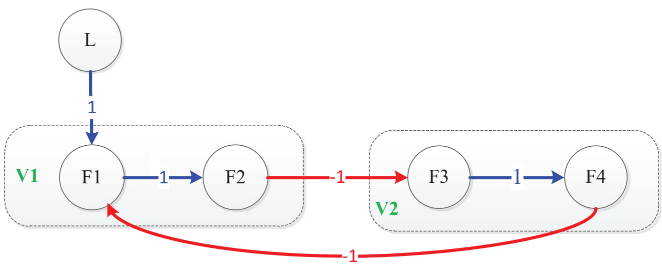


Fig. 1. Communication graph of first example.

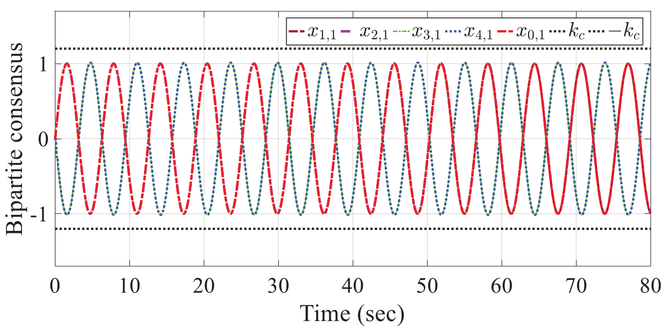


Fig. 2. Bipartite consensus performance, example one.

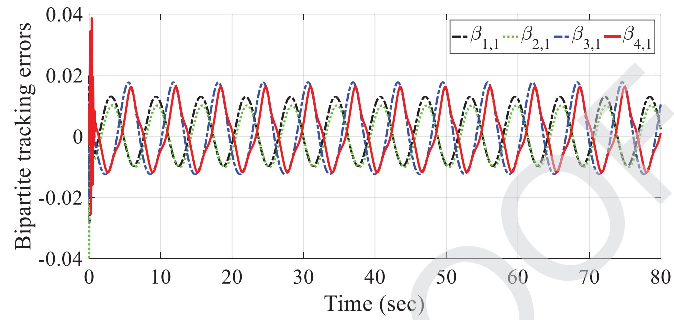


Fig. 3. Distributed bipartite tracking errors, example one.

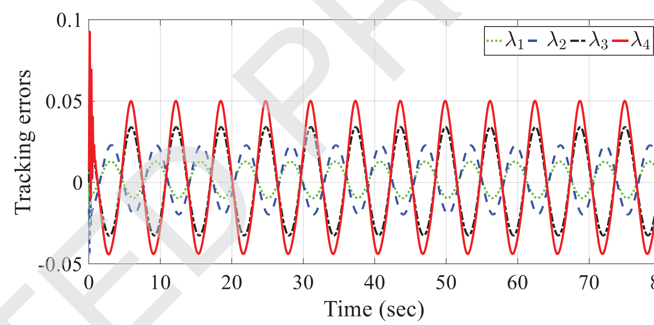


Fig. 4. Local bipartite tracking errors, example one.

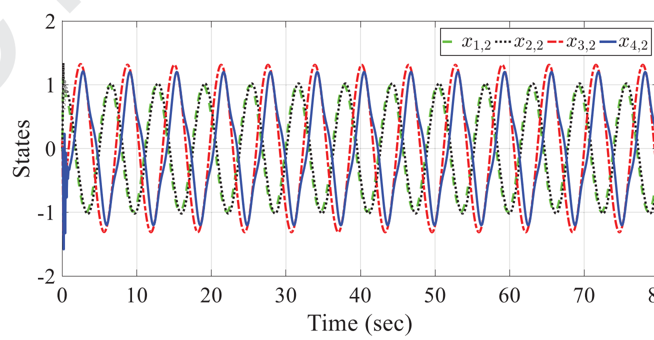


Fig. 5. States, example one.

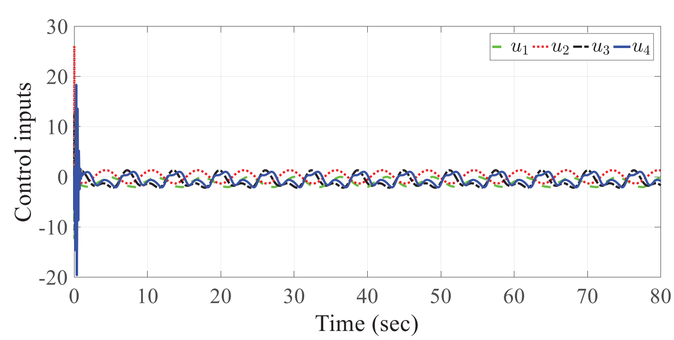


Fig. 6. Control inputs, example one.

Letnikov definition is utilized with sample time 1 (ms) in MATLAB© software.

Example one (Numerical simulation). The nonlinear dynamics of a fractional-order follower are described as

$$\begin{cases} {}_0^C D_t^{0.9} x_{i,1} = x_{i,2} + f_{i,1}(\mathbf{x}_{i,1}) + \rho_{i,1}, \\ {}_0^C D_t^{0.9} x_{i,2} = u_i + f_{i,2}(\mathbf{x}_{i,2}) + \rho_{i,2}, \\ y_i = x_{i,1}, \end{cases} \tag{64}$$

where the followers' outputs are restricted to $|x_{i,1}| \le k_c = 1.2, \forall t \ge 0$.

The nonlinear functions in (64) are selected as $f_{1,1}(\mathbf{x}_{1,1}) = 0.1x_{1,1}, f_{1,2}(\mathbf{x}_{1,2}) = x_{1,1}x_{1,2} + \exp(-x_{1,1}x_{1,2}), f_{2,1}(\mathbf{x}_{2,1}) = -0.1x_{2,1}, f_{2,2}(\mathbf{x}_{2,2}) = x_{2,2}, f_{3,1}(\mathbf{x}_{3,1}) = x_{3,1}, f_{3,2}(\mathbf{x}_{3,2}) = x_{3,2}^2, f_{4,1}(\mathbf{x}_{4,1}) = x_{4,1} \cos^2(x_{4,1}), f_{4,2}(\mathbf{x}_{4,2}) = x_{4,2}, \rho_{1,1}(t) = \ldots = \rho_{4,2}(t) = 0.1\sin(t)$.

The initial states of the agents and controller are $\mathbf{x}_1(0) = [0.02, 0.03]^{\mathrm{T}}, \mathbf{x}_2(0) = [-0.02, 0]^{\mathrm{T}}, \mathbf{x}_3(0) = [0, 0.05]^{\mathrm{T}}, \mathbf{x}_4(0) = [0.01, 0]^{\mathrm{T}}$ and $\hat{\pi}_{i,k}(0) = 0$ for all the follower agents. Control parameters are also selected as $\alpha_{i,1} = 15, \alpha_{i,2} = 7, \gamma_{i,k} = 1, \sigma_{i,k} = 0.1$ and $\omega_{i,k} = 0.1$ for $i = 1, \ldots, 4$ and $k = 1, 2$.

For the simulation study, the connected signed bipartite digraph consisting of four follower agents and one leader as depicted in Fig. 1 is proposed. The graph is structurally balanced, containing a directed spanning tree with leader as a root and only the first follower has directly access to the information of leader. In Fig. 1, the blue and red edges display the cooperative and competitive interactions among the followers, respectively. Figs. 2–7 are depicted to show the followers' output constraint performance for the bipartite consensus of a networked fractional-order nonlinear systems. From Fig. 2, it is obvious that the bipartite consensus tracking performance of distributed control protocol between the followers and leader is well obtained within two sub-networks $\mathcal{V}_1 = \{1, 2\}$ and $\mathcal{V}_2 = \{3, 4\}$, where $x_{i,1} \to x_{0,1}, \forall i \in \mathcal{V}_1$ and $x_{i,1} \to -x_{0,1}, \forall i \in \mathcal{V}_2$. Moreover, it can be seen that the followers' outputs stays within the preset bounds when the proposed control protocol is applied. In Fig. 3, the distributed bipartite tracking errors are shown, from which, it can be obtained that $\beta_{i,1} < k_b$ and $\beta_{i,1} > -k_b, \forall t \ge 0$. Similar to Fig. 3, Fig. 4 is depicted to demonstrate that the local bipartite tracking errors are limited to a fairly small neighborhood of the origin. From Figs. 2–4, it can be deduced that the states constraints are not overstepped. The trajectories of states $x_{i,2}$ for $i = 1, ., N$ are described in Fig. 5. From Fig. 6, it is clear that the control inputs are also bounded. The trajectories of adaptive parameters are depicted in Fig. 7, and it is obvious that the approximated parameters are all bounded. According to the simulation results, it is confirmed that the bipartite consensus tracking performance is achieved, the followers' outputs constraints are not violated, and all the closed-loop network signals are also bounded.

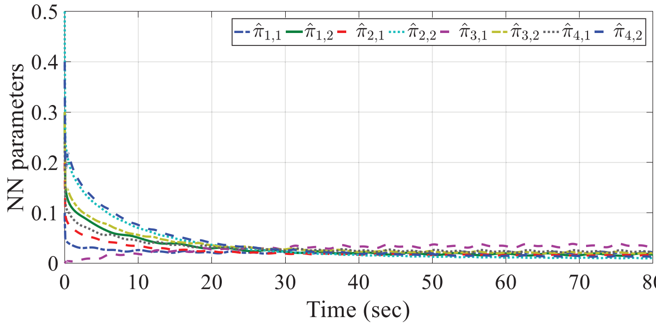


Fig. 7. Nueral network parameters, example one.

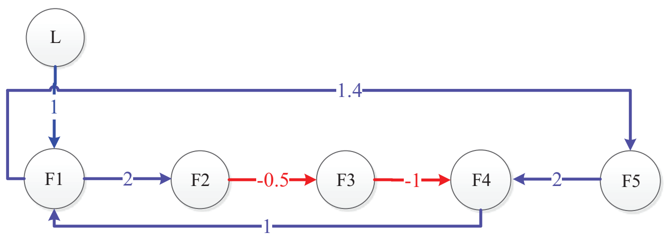


Fig. 8. Communication graph of second example.

Example two (Performance comparison). In order to illustrate the applicability and advantages of the proposed control strategy in Theorem 1, the performance of investigated control architecture in Theorem 1 is compared to the algorithm in Corollary 1 for multiple fractional-order systems described by (65) under the graph topology displayed in Fig. 8.

All follower agents are described by a fractional-order differential equations in strict-feedback form, and their dynamics for $i = 1, \ldots, 4$ are written as

$$\begin{cases} {}_0^C D_t^{0.75} x_{i,1} = x_{i,2} + f_{i,1}(\mathbf{x}_{i,1}) + \rho_{i,1}, \\ {}_0^C D_t^{0.75} x_{i,2} = u_i + f_{i,2}(\mathbf{x}_{i,2}) + \rho_{i,2}, \\ y_i = x_{i,1}, \end{cases} \tag{65}$$

in which the nonlinear functions are selected as $f_{1,1}(\mathbf{x}_{1,1}) = 0.1x_{1,1}\cos^2(x_{1,1})$, $f_{1,2}(\mathbf{x}_{1,2}) = \sin(x_{1,1})x_{1,2}$, $f_{2,1}(\mathbf{x}_{2,1}) = -0.1x_{2,1}^2$, $f_{2,2}(\mathbf{x}_{2,2}) = \exp(x_{2,2})$, $f_{3,1}(\mathbf{x}_{3,1}) = \frac{1}{x_{3,1}^2+1}$, $f_{3,2}(\mathbf{x}_{3,2}) = x_{3,1}x_{3,2}$, $f_{4,1}(\mathbf{x}_{4,1}) = x_{4,1}\tanh^2(x_{4,1})$, $f_{4,2}(\mathbf{x}_{4,2}) = 0.1x_{4,1}$, and $\rho_{1,1} = \ldots = \rho_{4,2} = 0$.

The initial states of the agents and controller are chosen as before. The design parameters are selected as $\alpha_{i,1} = 15, \alpha_{i,2} = 7$ for $i = 1, 2, 3$, $\alpha_{4,1} = 20, \alpha_{4,2} = 20$ and $\gamma_{i,k} = 1, \sigma_{i,k} = 0.7, \omega_{i,k} = 1$ for $i = 1, \ldots, 4$ and $k = 1, 2$. Two cases are of interest

Case 1. The proposed control method in Theorem 1 is used for the multiple fractional-order systems described in (65), where the followers' outputs are restricted to $|x_{i,1}| \leq k_c = 1.1, \forall t \geq 0$.

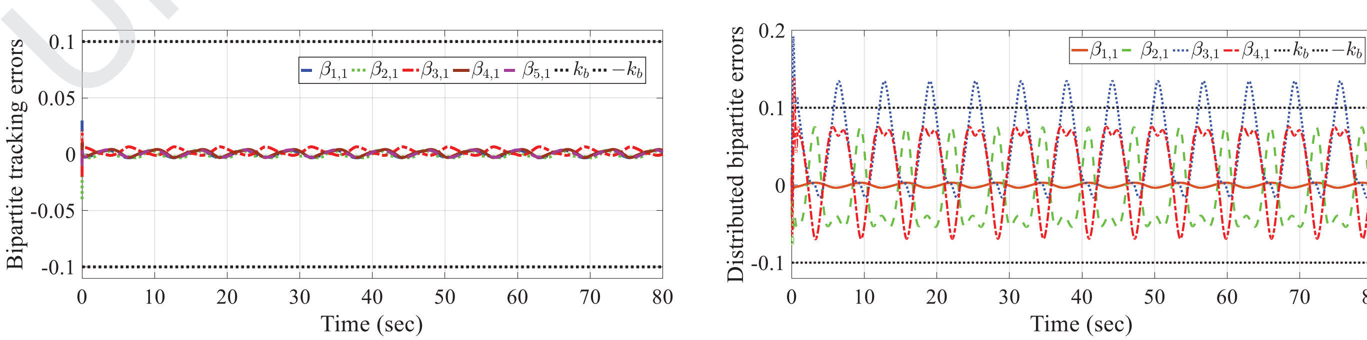
Case 2. The proposed control method in Corollary 1, which is the general distributed backstepping design without employing the barrier Lyapunov function scheme is used for the multiple fractional-order systems described in (65).

Figs. 9 and 10 display the corresponding curves of the distributed and local bipartite consensus errors, respectively. According to Figs. 9 and 10, it is deduced that a better bipartite consensus tracking performance is obtained for case 1 due to employing BLF approach. From Figs. 9a and 10 a, it is clear that the distributed bipartite consensus errors satisfy the constraint $|\beta_{i,1}| < 0.1$ for $i = 1, ., 4$ and the advantage of utilizing the BLF strategy is well perceived. However, in case 2, the distributed consensus bipartite errors for third and fourth agents violate the preset error constraints. In Figs. 10a and b, the local bipartite tracking errors are given under case 1 and 2, respectively. It can be deduced that in both cases, the bipartite consensus tracking objective is obtained, and the local bipartite tracking errors stay strictly within the different constrained bounds. However, in case 1, more accuracy for bipartite consensus performance is achieved because of employing BLF strategy.

Example three (Practical example). To show the effectiveness of the proposed method in practical situations, bipartite consensus problem for a network of homogeneous chaotic Chua–Hartleys systems is considered. The fractional-order dynamics of the Chua–Hartleys systems for $i = 1, \ldots, 4$ are described as [27]

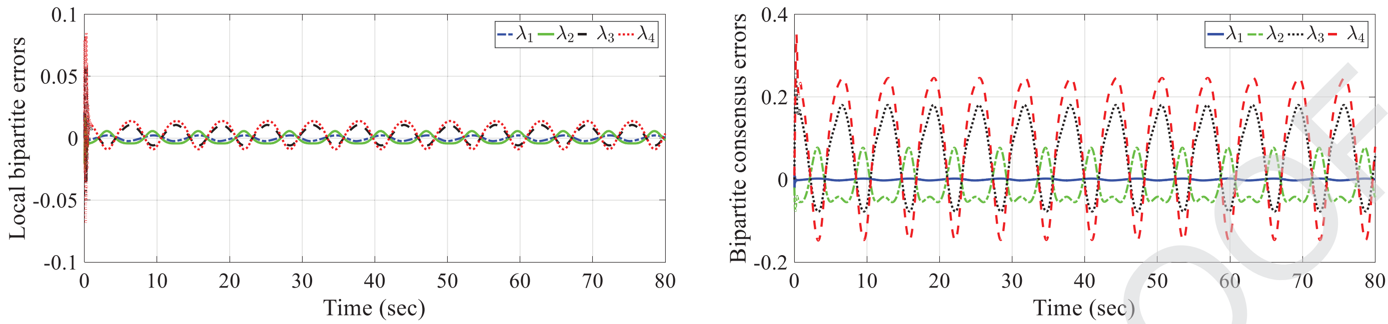
$$\begin{cases} {}_0^C D_t^{0.98} x_{i,1} = x_{i,2} + f_{i,1}(\mathbf{x}_{i,1}) + \rho_{i,1}, \\ {}_0^C D_t^{0.98} x_{i,1} = x_{i,3} + f_{i,2}(\mathbf{x}_{i,2}) + \rho_{i,2}, \\ {}_0^C D_t^{0.98} x_{i,2} = u_i + f_{i,3}(\mathbf{x}_{i,3}) + \rho_{i,3}, \\ y_i = x_{i,1}, \end{cases} \tag{66}$$

where $f_{i,1}(\mathbf{x}_{i,1}) = \frac{10}{7}(x_{i,1} - x_{i,1}^3)$, $f_{i,2}(\mathbf{x}_{i,2}) = 10x_{i,1} - x_{i,2}$, $f_{i,3}(\mathbf{x}_{i,3}) = -\frac{100}{7}x_{i,2}$, $\rho_{i,1} = 0$, $\rho_{i,2} = 0$ and $\rho_{i,3} = \sin(0.5t)$. In this example, the communication graph is considered the same as in example one, i.e., Fig. 1. The design parameters are chosen as $\alpha_{i,1} = 15, \alpha_{i,2} = 7, \gamma_{i,k} = 1, \sigma_{i,k} = 0.1$ and $\omega_{i,k} = 0.1$ for $i = 1, \ldots, 4$ and $k = 1, 2, 3$. The simulation is derived and the results are shown in Figs. 11–14. Fig. 11, Fig. 12 and Fig. 13 show the state variables $x_{i,1}$, $x_{i,2}$ and $x_{i,3}$ for $k = 1, \ldots 4$, respectively. Fig. 14 shows the distributed bipartite tracking errors. From Figs. 11–14 it is obvious that the signals are all bounded.



(a) Case 1. (b) Case 2.

Fig. 9. Distributed bipartite tracking errors, example two.



(a) Case 1. (b) Case 2.

Fig. 10. Local bipartite tracking errors, example two.

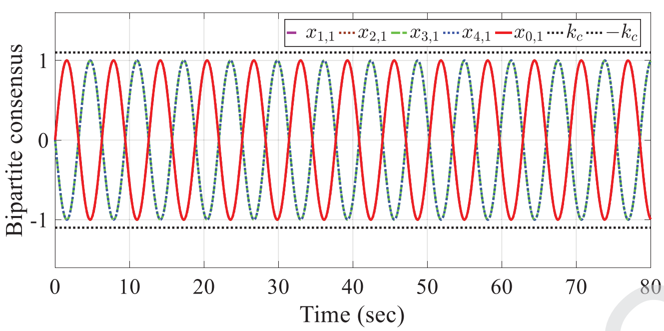


Fig. 11. Bipartite consensus performance, example three.

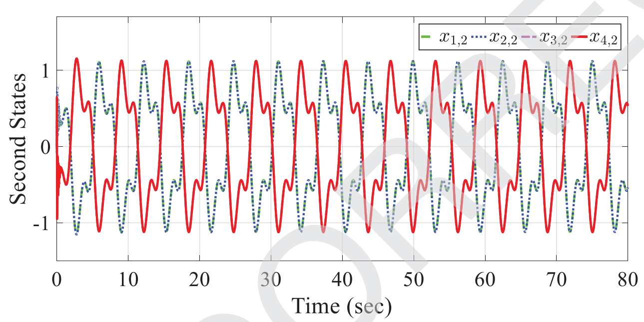


Fig. 12. Second states, example three.

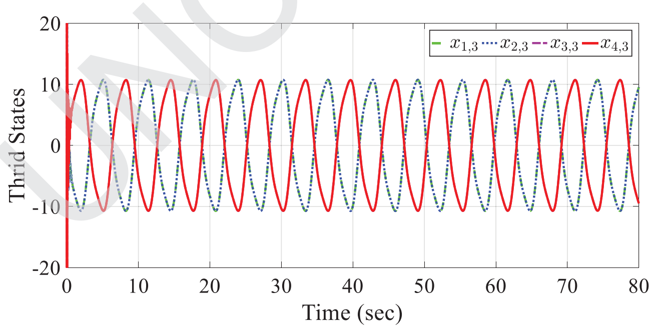


Fig. 13. Third states, example three.

Remark 9. It would be more practical to consider unknown control direction problem [28,29,35] for a network of fractional-order systems. Such a design is important and challenging because the Nussbaum function design for multiple fractional-order systems, requires designing infinite dimensional Nussbaum functions. It will be considered as one of our future works.

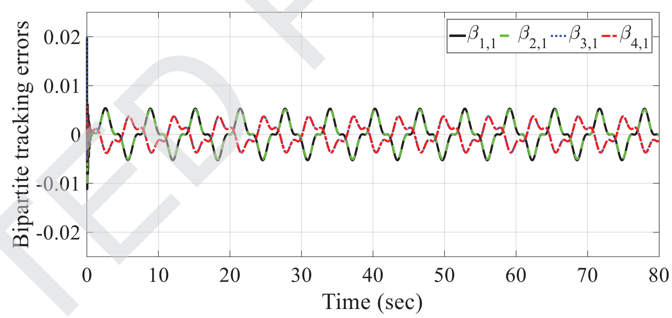


Fig. 14. Distributed bipartite tracking errors, example three.

6. Conclusion

This work has considered the distributed adaptive bipartite consensus problem for multiple uncertain nonlinear fractional-order systems in strict-feedback form with both unknown function and the output constraints. The problem of followers output constraint is resolved using the barrier Lyapunov function method. Multiple fractional-order strict-feedback dynamics have been studied by employing backstepping approach and neural networks where less learning parameters have been adjusted online in controller design. In this paper, the problem of less learning parameters for multiple fractional-order systems with both nonlinear uncertainties and output/state constraints have been investigated for the first time. By employing barrier Lyapunov function scheme and some appropriate Lemmas, proof of the proposed control strategy is derived such that the followers' outputs constraints have been ensured. Moreover, all the closed-loop network signals are SGUUB. Additionally, the distributed (or local) bipartite tracking errors are converged to a small neighborhood of zero. Finally, three simulation examples are given to verify the effectiveness of proposed method in theory and practice.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Milad Shahvali: Conceptualization, Methodology, Software, Validation, Formal analysis, Writing - original draft, Writing - review & editing. **Ali Azarbahram:** Software, Methodology, Writing - original draft, Writing - review & editing. **Mohammad-Bagher Naghibi-Sistani:** Supervision, Writing - original draft, Writing - review &

editing. **Javad Askari:** Writing - original draft, Writing - review & editing.

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