# A fuzzy stochastic immediate assembly supply chain with uncertain demand

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## Abstract
This study aimed at examining a three-echelon supply chain including manufacturers, warehouses and customers with uncertain demand. Following the proposed models, the goal was to minimize all supply chain costs and shortages. Stochastic fuzzy scenario-based programming in the form of chance constrained was utilized to control and deal with the assumed uncertainty which was considered to manifest itself through customer demand. To achieve more flexibility in the supply chain network and due to uncertain demand, immediate assembly in warehouses was applied. Furthermore, important supply chain decisions such as warehouse use, production quantities and distribution were examined. Subsequently and through solving a numerical example including one manufacturer, three warehouses and seven customers, the performance of the uncertain model was investigated.

**Keywords:** Stochastic programming, Assembly in warehouse, Supply chain under uncertainty, Fuzzy chance constrained programming.

## 1. Introduction

The increasing complexity of supply chain networks, shorter product life cycle, and unstable situation faced the supply chain with uncertainty. These are currently the main obstacles in achieving time delivery, increasing customer satisfaction, improving efficiency and reducing costs [1]. Coyle et al. acknowledge that the past decade has been a period of rapid change for organizations, especially businesses, forcing them to engage in exciting interactions to be much more innovative, flexible, and responsive [2]. From the point of view of Simchi-Levi et al., supply chain management, is a set of approaches to effectively integrate suppliers, manufacturers, warehouses and shops, so that commodity are produced and transferred in the right quantity and in the right places at the right time to reduce system costs, which bring the required level of the service satisfaction [3]. Supply chain management is consist of managing and coordinating the various flows within it. This fact highlights the tremendous importance of supply chain management emphasis on increasing flexibility, the ability to respond quickly and effectively to market changes [4]. In order to apply supply chain management, it is necessary to develop the internal cognition aspects of the common measures performance to build acceptance of broad participation aspects, and to avoid a unilateral and centralized approach [5].

The supply chain studied in this paper includes a single product and multi-echelon supply chain that is considered and analyzed over several periods. The three echelon supply chain network is considered including the manufacturer and elected distribution centers and the customer area, where distribution centers also act as warehouses and customer demand is uncertain. In addition to the final product (ready for delivery), the manufacturer is able to send semi-finished products to distribution centers. Customer demand is forecasted according to their demand from previous periods. To transport the product from the manufacturer to the warehouses, shipment by the container has been used for cost efficiency. Immediate assembly in distribution centers is used to flex the supply chain. In this way, warehousing centers can deliver the product to the customer by assembling different parts, but the cost of products assembled in warehouses is usually more than the cost of producing the final product by the factory. The shortage of customer demand is included in the problem. The aim of this study is to model the supply chain in order to create the least cost and shortage due to the existing uncertainty.

## 2. Literature Review

The practical and theoretical fascinations of the supply chain issue have created great interest among researchers in this field, and there is a very rich literature in this field. Gholamian et al. presented a multi-level and multi-product fuzzy planning model for the closed-loop supply chain under demand

and cost uncertainties. In the objective function of the model, they are evaluated the level of customer satisfaction and minimized the waiting time, greenhouse gases and carbon dioxide emitted from the vehicles. Interactive fuzzy programming was used to solve the model [6]. Nayeri et al. designed a closed-loop supply chain network under uncertainty that simultaneously made both operational and strategic decisions. The aims of the model included financial, environmental and social impact optimization. Fuzzy robust optimization was used to deal with uncertainty [7]. Cao et al. examined a multi-period distribution network with fuzzy supply. A hybrid method, using the primary-secondary algorithm, the expected value and the branch and boundary approach was used to solve the model [8]. Tsao et al. designed a power distribution network with uncertain parameters. A robust multi-objective stochastic programming method was presented. The goal was to minimize economic, environmental, and social costs. The problem was formulated as a fuzzy three-objective optimization model [9]. Chen et al. examined the smart supply chain management, which consisted of two parts: determining the weight of benchmarks and ranking of suppliers. Their study included the study of the simultaneous use of internal uncertainty and external uncertainty that are involved in the supplier selection process. They proposed a fuzzy-TOPSIS combined method for selecting a stable supplier for a smart supply chain [10]. Wang et al. provided a framework for designing large complex product supply chains using fuzzy performance development and the gray decision approach. Gray weighted decision making approach helped decision makers to identify the optimal quality procedure with uncertain information [11]. Tayyab and Sarkar proposed an advanced integrated multi-dimensional evaluation approach to support the textile industry in effective supplier selection and quantity allocation. To achieve supplier evaluation and order allocation at the same time, a multi-objective textile supply chain management model was developed. The interactive fuzzy method was used to control the uncertainty [12].

## 3. Stochastic fuzzy problem

The nomenclature of sets, parameters and variables are as follows:

Indices

| | |
|---|---|
| $m, b \in \{1,2,\dots,M'\}$ | Potential warehouses that can be used ($M'$number of potential warehouses for use) |
| $l \in \{1,2,\dots,L\}$ | Customer Area ($L$ Number of Customers) |
| $t \in \{1,2,\dots,T\}$ | Time Period ($T$ Number of Time Periods) |
| $r, s \in \{1,2,\dots,ns\}$ | Scenarios ($ns$ Number of Available Scenarios) |

Parameters

| | |
|---|---|
| $O$ | The rate occupancy area of the final product |
| $G$ | The rate occupancy area of semi-finished parts (under assembly) |
| $E$ | The total area of each container |
| $R$ | Cost of shortage per unit of the product (currency) |
| $U_m$ | Container shipping cost from manufacturer to warehouse *m* (currency) |
| $H$ | Cost of immediate product assembly in the warehouse of prefabricated parts (currency) |
| $J$ | Cost of maintaining prefabricated parts in the warehouse (currency) |
| $\Psi_s$ | The probability of scenario *s* |
| $C_m$ | Fixed cost of using the warehouse at point *m* (currency) |
| $C_{ml}^{TR}$ | Cost of shipping each unit of product from the warehouse *m* to the customer area *l* |
| $C_{mt}^{I}$ | Cost per unit of product inventory maintenance in warehouse *m* during period t (currency) |
| $S_m$ | Maximum area of warehouse *m* |
| $D_{lts}$ | Product demand in customer region *l* in time period *t* under scenario *s* (goods unit) |
| $P_t$ | Cost of production each unit of final product in the factory in period *t* |
| $P'_t$ | Cost of production each unit of product sub-assembly in the factory in period *t* |
| $\pi$ | Minimum quantity of product required to establish the transportation flow |
| $M$ | A large positive number |
| $Hw$ | Number of commodities that can be stacked in each containers |
| $Hm$ | Number of commodities that can be stacked in each warehouses |
| $S_j$ | A set of scenarios in category *j* in which the scenarios have the same amount of demand |

$\alpha$ Degree of credibility

Binary variables

$Y_m$ If warehouse *m* is used, equal to 1, otherwise get zero value

$X_{ml}$ If the warehouse *m* is allocated to the customer *l* is equal to 1, otherwise get zero value

$\lambda_b$ If *b* warehouses used, is equal to 1, otherwise get zero value

Continuous variables

$V_{lts}$ Product shortage in period *t* of customer *l* under scenario *s*

$Qsub'_{mt}$ Prefabricate quantity of product transferred from the factory to the warehouse *m* in period *t*

$Tsub_{mts}$ Number of products assembled in warehouse *m* in period *t* under scenario *s*

$W_{mt}$ Number of containers sent to warehouse *m* in period *t*

$W$ Total number of shipping containers

$I_{mt}^{min}$ Minimum inventory of product stored in warehouse *m* during period *t* (safety stock)

$Q_{mlts}$ The amount of product transferred from the warehouse *m* to the customer *l* in period *t* under scenario *s* (including the complete product sent from the manufacturer to the warehouse and the product assembled in the warehouse)

$Q_{mt}$ Quantity of complete product shipped from factory to warehouse *m* in period *t* (includes only complete product made in the manufacturer)

$I_{mts}$ Inventory level of the final product in warehouse *m* in period *t* under scenario *s*

$Iq\mathrm{sub}_{\mathrm{mts}}$ Inventory level of sub-assembled parts of the product in warehouse m in period *t* under scenario *s*

In the proposed model, the total costs of the supply chain network are minimized according to the uncertain environment by achieving an ideal solution. The proposed model, in order to reduce costs, determine the true quantity of product transferred to each warehouses in each time periods. Also, the model determines the quantity of the semi-finished product that should be sent directly to the warehouses in each period or remain in the warehouse. In addition, specifies which products and quantity of products transferred to the customer in each period. In hypothetical supply chain planning, production and shipping quantities, including final and prefabricated products for the manufacturer, are independent of the scenario and can only be planned and changed over different time periods.

The mathematical model of the problem is as follows:

$$Min\ Z = z_1 + \sum_t (z_2 + z_3) + \sum_{s=1}^{NS} \Psi_s \left( \sum_t (z_4 + z_5 + z_6 + z_7 + z_8) \right) \quad (1)$$

$$z_1 = \sum_m C_m * Y_m \quad (2) \qquad z_2 = \sum_m U_m * W_{mt} \quad (3)$$

$$z_3 = \left(\sum_m P_t * Q_{mt}\right) + \left(\sum_m P'_t * Qsub'_{mt}\right) \quad (4)$$

$$z_4 = \sum_m \sum_l C_{ml}^{TR} * Q_{mlts} \quad (5) \qquad z_5 = \sum_m C_{mt}^{I} * I_{mts} \quad (6)$$

$$z_6 = \sum_l V_{lts} * R \quad (7) \qquad z_7 = \sum_m Tsub_{mts} * H \quad (8)$$

$$z_8 = \sum_m Iqsub_{mts} * J \quad (9)$$

Equation (1) is the objective function, the first term investigated using candidate warehouses, which are independent from the scenarios and time periods, because it is a strategic decision and take into account in the beginning of the planning horizon. In the second term, the operation is related to the level of the producer to the warehouse, which is planned during the horizon of the time cannot be changed in each scenario due to the costs and production plans, and is independent from the scenarios. In the third term, the warehouse level to the customers is considered, which are planned according to the scenarios and the time period. Equation (2) indicates that if a warehouse is used, a fixed cost is considered and it does not charge any cost to the model if a warehouse is not used. Equation (3) calculates the shipping cost for each number of containers shipped. Equation (4) indicates the cost of production per unit of complete product and the cost of production per unit of prefabricated product in the manufacturer. Equation (5) shows the cost of transferring each unit of product (including the final product sent from the manufacturer to the warehouses and the product assembled in the warehouses) from the warehouses to the customers. Equation (6) shows the cost of each unit of product storage (the final product produced by the manufacturer that is sent to the warehouses) in the warehouse, in different scenarios warehouses can have different levels of inventory. Equation (7) also considered customer shortages, and this relationship shows the cost per unit shortage. Equation (8) shows the cost per unit of product assembled in each warehouses. Equation (9) shows that all prefabricated products shipped from the manufacturer are not assembled, and may be used later. This expression shows the prefabricated product maintenance cost in warehouses. The assembled products are assembled and immediately transferred according to the excess demand, so the maintenance cost is not considered for these products.

$$V_{lts} \geq D_{lts} - \sum_m Q_{mlts} \qquad \forall\, ,t,l,s \qquad (10)$$

$$Cr(V_{lts} + \sum_m Q_{mlts} \geq D_{lts}) \geq \alpha \qquad \forall\, ,t,l,s \qquad (11)$$

Constraints (10) indicates the shortage of the model. To account for fuzzy demand uncertainty, constraints (10) with the credibility measure is written as constraints (11). Constraints (11) will be set to alpha degree. The trapezoidal fuzzy number of the demand parameter is defined as $\widetilde{D_{lts}} = (D^1_{lts}, D^2_{lts}, D^3_{lts}, D^4_{lts})$ and constraints (11) can be written as constraints (12) according to the [13].

$$V_{lts} + \sum_m Q_{mlts} \geq (2 - 2 * \alpha) * D^3_{lts} + (2 * \alpha - 1) * D^4_{lts} \qquad \forall\, ,t,l,s \qquad (12)$$

$$I^{min}_{mt} \geq \frac{\sum_s \sum_l \Psi_s * V_{tls}}{\sum_m Y_m} \qquad \forall m,t \qquad (13)$$

Safety stock is considered as a nonlinear relation (13). These constraints, which represent the amount of safety stock in the problem, is removed from the modeling and replaced by constraints (14) -(16) so that the model can be expressed linearly.

$$\sum_m Y_m = \sum_{b=1}^{M'} b * \lambda_b \qquad (14)$$

First, the relation (14) is added to the constraints of the problem so that if a warehouse is used, the usage variable number $b$ of the warehouse $\lambda_b$ become 1, because the value of this variable needs to be determined to be used in constraints (13). For example, if three warehouses are used, in the left of the relation, $Y_1$, $Y_2$ and $Y_3$ which are the binary variables corresponding to warehouses 1, 2 and 3, respectively, become 1, and their sum value become 3. Then in right side of the relation, the usage variables b of the warehouse take the value of one. Two modes can be occurred. In the first case, only the use variable of three warehouses becomes one, and in the second case, the use variable of two warehouses and the use variable of one warehouse simultaneous become one. In the correct way, the first case should occur and to prevent the second case, relation (15) is presented.

$$\sum_{b=1}^{M'} \lambda_b = 1 \qquad (15)$$

Candidate warehouses, for example, either three or four warehouses are definitely used, so only one of $\lambda_b$ variables should be 1 and the other $\lambda_b$ variables should be 0. For this purpose, Equation (15) is introduced. According to the correct operation of the variable $\lambda_b$, constraints (13) changed as constraints (16). Constraints (16) has a linear form because the denominator is integer and turns the problem into linear modeling. As a result, constrains (13) is removed from the model and constraints (14), (15) and (16) are added to the model.

$$I^{min}_{mt} \geq \frac{\sum_s \sum_l \Psi_s * V_{lts}}{b} - S_m * (1 - \lambda_b) \qquad \forall m,t,b \qquad (16)$$

$$\sum_m \sum_t W_{mt} = W \qquad (17)$$

$$O * Q_{mt} + G * Qsub'_{mt} \leq E * Hw * W_{mt} \qquad \forall m,t \qquad (18)$$

$$X_{ml} \leq Y_m \qquad \forall\, m,l \qquad (19)$$

Equation (16) determines the safety stock of the problem. Equation (17) calculates the total number of containers which are used. Constraints (18) shows that a coefficient of product states is considered for transportation that the number of containers become integer. Constraints (19) indicates that in order to assign a warehouse to the customer, first, the warehouse must be used.

$$\pi * X_{ml} \leq Q_{mlts} \leq M * X_{ml} \qquad \forall\, m, l, t \qquad (20)$$
$$Q_{mt} \leq M * Y_m \qquad \forall\, m, t \qquad (21)$$
$$Qsub'_{mt} \leq M * Y_m \qquad \forall m, t \qquad (22)$$
$$I_{mts} = Q_{mt} + Tsub_{mts} - \sum_l Q_{mlts} \qquad \forall m, s, t = 1 \qquad (23)$$
$$I_{mts} = I_{m.t-1.s} + (Q_{mt} + Tsub_{mts} - \sum_l Q_{mlts}) \qquad \forall m, s, t \geq 2 \qquad (24)$$
$$Iqsub_{mts} = Qsub'_{mt} - 3 * Tsub_{mts} \qquad \forall m, s, t = 1 \qquad (25)$$
$$Iqsub_{mts} = Iqsub_{m.t-1.s} + (Qsub'_{mt} - 3 * Tsub_{mts}) \qquad \forall m, s, t \geq 2 \qquad (26)$$
$$O * I_{mts} + G * Iqsub_{mts} \leq S_m * Hm \qquad \forall m, t, s \qquad (27)$$
$$I_{mts} \geq I_{mt}^{min} \qquad \forall\, m, t, s \qquad (28)$$

Constraints (20) shows that the quantity of commodities can be predefined to a maximum and minimum values of $M$ and $\pi_i$, respectively, which are sent from the warehouses to the customers (the number of commodities to be sent must be more than a specified quantity in order to be economically viable), The greatest demand is considered as $M$. Constraints (21) shows that the quantity of goods sent from the factory to the warehouses is limited to a maximum of $M$, because warehouses have a certain capacity. Constraints (22) determines that the maximum amount of sub-assembled commodities sent from the manufacturer to the warehouses is limited to the value of $M$. Constraints (23) represent the equilibrium equation of the inventory level of final products in the first period for warehouses. Constraints (24) represents the equilibrium equation of the inventory level of final products for subsequent periods in warehouses. Equations (25) is the equilibrium constraints of the inventory level of the sub-assembled parts in the first period. A complete product is assembled, from each of the three sub-assembled parts. Constraints (26) represents the equilibrium equation of the inventory level of sub-assembled parts in warehouse for subsequent periods. Constraints (27) indicates warehouse capacities. Constraints (28) indicates that the inventory level is at least equal to the safety stock.

$$Q_{imlts} = Q_{imltr} \qquad \forall i, m, l \ \ and\ r, s \in S_j, t = 1 \qquad (29)$$
$$Tsub_{imts} = Tsub_{imtr} \qquad \forall i, m, l \ \ and\ r, s \in S_j, t = 1 \qquad (30)$$

Equations (29) shows that the amount of products sent from the warehouses to the customers is the same in the category of specific scenarios for the first time period. Since in this category of scenarios, there are consecutive scenarios that have the same amount of demand, therefore the amounts which are sent are considered same. Equations (30) is for the amount of products assembled in the warehouses, which is similar to Equations (29).

$$Q_{imlts} = Q_{imltr} \qquad \forall i, m, l \ \ and\ r, s \in S_j, t = 2 \qquad (31)$$
$$Tsub_{imts} = Tsub_{imtr} \qquad \forall i, m, l \ \ and\ r, s \in S_j, t = 2 \qquad (32)$$

Equations (31) and (32) are similar to Equations (29) and (30), respectively, for the second time period.

$$V_{lts},\ Qsub'_{mt},\ Tsub_{mts},\ W_{mt},\ I_{mt}^{min},\ Q_{mlts},\ Q_{mt},\ I_{mts},\ Iq\mathrm{sub}_{\mathrm{mts}} \geq 0, int \qquad (33)$$
$$Y_m, X_{ml}, \lambda_b \epsilon (0,1) \qquad (34)$$

## 4. Numerical Example

In this section, a numerical example with one manufacturer, three warehouses and seven customers with the data shown in the Tables 1-5 is considered.

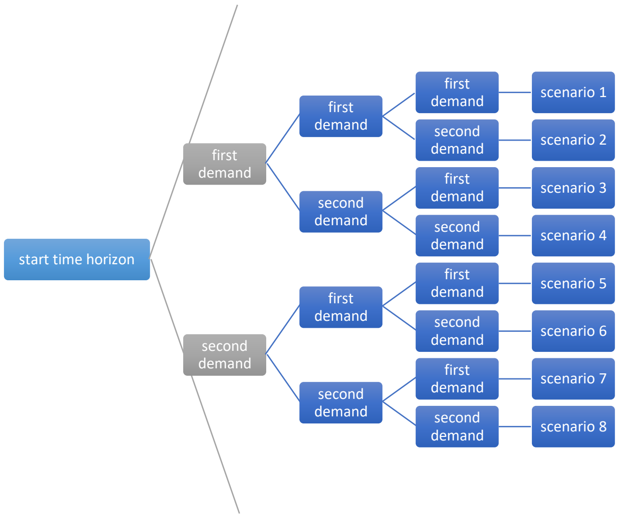


**Figure 1- Scenario tree**

The Equations (31) and (32) have been used to obtain the $D^3_{lts}$ and $D^4_{lts}$ parameters, respectively.

$$D^3_{lts} = D^1_{lts} + 0.2 * D^1_{lts} \qquad \forall l,t,s \tag{31}$$

$$D^4_{lts} = D^1_{lts} + 0.5 * D^1_{lts} \qquad \forall l,t,s \tag{32}$$

**Table 1 – $D^1_{lts}$**

| Customer | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Scenario (first period) | 1-2-3-4 | 246 | 250 | 246 | 286 | 281 | 283 | 290 |
| | 5-6-7-8 | 220 | 210 | 212 | 215 | 230 | 240 | 250 |
| Scenario (second period) | 1-2-5-6 | 246 | 275 | 250 | 286 | 281 | 283 | 286 |
| | 3-4-7-8 | 210 | 220 | 200 | 210 | 220 | 210 | 200 |
| Scenario (third period) | 1-3-5-7 | 446 | 384 | 446 | 686 | 481 | 583 | 573 |
| | 2-4-6-8 | 257 | 293 | 228 | 437 | 229 | 218 | 184 |

**Table 2 - Cost of each sub-assembly production**

| period | first | second | third |
|---|---|---|---|
| product | 17 | 16 | 17 |

**Table 4 - Cost of each complete product production**

| period | first | second | third |
|---|---|---|---|
| product | 61 | 60 | 60 |

**Table 3- Cost of product storage in warehouse**

| Warehouse | | 1 | 2 | 3 |
|---|---|---|---|---|
| product | First period | 10 | 10 | 10 |
| | Second period | 10 | 10 | 10 |
| | Third period | 12 | 12 | 12 |

**Table 5 – Ship ping costs from warehouses to customers**

| customer | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Warehouse 1 | 5 | 4 | 6 | 7 | 6 | 5 | 6 |
| Warehouse 2 | 6 | 5 | 7 | 8 | 7 | 4 | 5 |
| Warehouse 3 | 4 | 3 | 5 | 6 | 5 | 7 | 8 |

In order to establish the transportation flow from warehouses to customers, the minimum amount of products for economies of scale is assumed be equal to 10. The total space of each container is 30 $m^2$. The costs of transferring the containers to the warehouses are equal to 100, 110 and 120 currencies, respectively. The area in containers and warehouses are such that products can be stacked on top of each other. The number of products that can be stacked in containers is equal to 5. Occupancy rate of final products is 2 $m^2$. The occupancy rate of semi-manufactured products for each product is 0.5 $m^2$.

The cost of each shortage unit for both products is 50 monetary units. Three candidate warehouses are considered in the supply chain network, and the fixed cost of each warehouses are equal to 15,500, 13,000 and 11,000 monetary units, respectively. Each warehouses have an area for inventory capacity, which are equal to 5000, 4000 and 3000 $m^2$ for each warehouses, respectively. The cost of assembling the product in the warehouses of prefabricated parts is 18 monetary units. The cost of maintaining prefabricated parts in the warehouse is 3 monetary units. The number of products that can be stacked in warehouses is equal to 5. The probability of each scenarios is equal to 0.125 and the degree of credibility was considered 0.5.

## 5. Results

Proposed model coded in CPLEX software and run in an Intel Core i7 with 2.3 GHz CPU and 6 GB of RAM.

Due to the volume of data and to summarize the answers, the expected value of the scenarios was used in the tables 6,9,10,11,13 where the answer depended on the scenarios. The value of the objective function obtained from solving the numerical example is 563712.5. Table 6 shows the number of products assembled in each period and in each warehouses. Table 7 shows the safety stock required to be stored in each warehouse for each period. The number of final products transferred from the manufacturer to the warehouses is shown in Table 8. The amount of expected value of inventory held in each warehouses for each period is shown in Table 9. In Table 10 the expected value of the number of products transferred from different warehouses to different customers in each period is presented. Table 11 shows the expected values of the product stock level under the warehouse assembly. Table 12 shows the model solution, the number of prefabricated products transferred from the manufacturer to each warehouses in each period. Table 13 shows the solution obtained from solving the model and the number of products shortage. Table 14 shows the number of containers that transferred to each warehouse in each period.

**Table 6- expected values of assembly in warehouses**

| Warehouse | 1 | 2 | 3 |
|---|---|---|---|
| First period | 0 | 0 | 6 |
| Second period | 0 | 0 | 115 |
| Third period | 40 | 0 | 110 |

**Table 8- Number of complete products sent from the factory to the warehouses**

| factory \ warehouse | 1 | 2 | 3 |
|---|---|---|---|
| First period | 325 | 325 | 1973 |
| Second period | 0 | 0 | 587 |
| Third period | 0 | 240 | 500 |

**Table 7- safety stoke values**

| warehouse | 1 | 2 | 3 |
|---|---|---|---|
| First period | 291 | 291 | 291 |
| Second period | 0 | 0 | 0 |
| Third period | 0 | 180 | 0 |

**Table 9 - expected Values Inventory Level of complete Product in Warehouses**

| warehouse | 1 | 2 | 3 |
|---|---|---|---|
| First period | 291 | 291 | 291 |
| Second period | 0 | 0 | 0 |
| Third period | 50 | 0 | 0 |

**Table 11- expected Values of Sub-Assembly Inventory Level in Warehouse**

| warehouse | **1** | **2** | **3** |
|---|---|---|---|
| First period | 0 | 0 | 2 |
| Second period | 0 | 0 | 346 |
| Third period | 0 | 0 | 10 |

**Table 12- Sub-Assembly quantities sent from the manufacturer to the warehouse**

| warehouse | 1 | 2 | 3 |
|---|---|---|---|
| First period | 0 | 0 | 18 |
| Second period | 0 | 0 | 690 |
| Third period | 0 | 0 | 50 |

**Table 10 – expected value of products transferred from the warehouse to the customer**

| Warehouse | Customer / period | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 1 | first | 24 | 0 | 0 | 0 | 10 | 0 | 0 |
| | second | 160 | 0 | 0 | 0 | 130 | 0 | 0 |
| | third | 260 | 0 | 0 | 0 | 150 | 0 | 0 |
| 2 | first | 0 | 0 | 0 | 0 | 0 | 17 | 17 |
| | second | 0 | 0 | 0 | 0 | 0 | 198 | 93 |
| | third | 0 | 0 | 0 | 0 | 0 | 140 | 50 |
| 3 | first | 328 | 338 | 387 | 148 | 189 | 125 | 172 |
| | second | 65 | 240 | 215 | 225 | 94 | 27 | 127 |
| | third | 40 | 180 | 290 | 320 | 180 | 120 | 160 |

**Table 13- expected value of shortage**

| customer | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| First period | 0 | 0 | 0 | 313 | 156 | 227 | 176 |
| Second period | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Third period | 0 | 0 | 0 | 0 | 50 | 0 | 40 |

**Table 14 - Number of containers sent from the manufacturer to the warehouses**

| warehouse / period | First | Second | Third |
|---|---|---|---|
| first | 13 | 0 | 10 |
| second | 13 | 0 | 18 |
| Third | 79 | 32 | 0 |

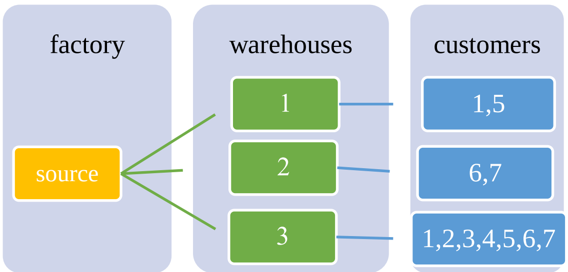


**Figure 2. Relation of flows**

To show the structure of the supply chain network, the number of warehouses used among the candidate warehouses, the set of retailers, as well as how to allocate the manufacturer to each warehouse and the allocation of each warehouses to each retailers are shown in Figure 2.

## 6. Conclusion

This study examined a three-echelon supply chain including a producer, warehouses, and customers under demand uncertainty. The adopted Supply Chain Network in was multi-level, multi-period and single-product. Considering the supply chain network under uncertainty, stochastic programming based on a combination of scenario and fuzzy numbers was utilized. Delivery of products from the producer level to the warehouse level was used by the container to have an economical transportation system. The main assumption was that the manufacturer is able to produce products in both final (fully produced) and sub-assembly states. Sub-assembly products refer to the ones that are produced in separate parts and ready to be assembled and create a complete product at the next level of the supply chain. Sub-assembly parts are important due to the uncertainty of demand so that if demand surpassed the available inventory, the supply chain network, to a large extent, is capable of supplying the product. A number of warehouses were candidates for the supply chain some of which were located regarding the solution of the model. These warehouses were deemed centralized which were responsible for assembling and distributing products besides warehousing products. To evaluate the efficiency and effectiveness of the modeling, a numerical example with hypothetical data was solved by CPLEX software version 12.6. The results showed that the planning and assembly mode in the warehouse has greatly contributed to the shortage of customer demand and has led to a flexible supply chain.

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