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A robust fuzzy optimization approach for reverse logistics network design with buyback offers

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Abstract

Purpose – In this paper, the authors aim to investigate the relationship between buyback policy and the potential number of used products that could be collected by developing a robust fuzzy reverse logistics network.

Design/methodology/approach – In this approach, the authors seek to determine the amount of buyback based on the condition of used products at the time of return. In this process, the authors also take into account that apart from the condition of used products, other factors exist that the actual return rate could be dependent on them. This matter propelled us to make a novel distinction between the probability of return estimated from appropriate buybacks offered to consumers, and the actual return rate of used products using fuzzy mathematical methods. Besides that, a compatible robust fuzzy optimization method has been implemented on the model to deal with uncertain properties of it and simultaneously fortifying its responses against any possible effect of return rate fluctuation.

Findings – To analyze and evaluate the model performance, the authors decided to apply a series of exhaustive randomly generated experiments onto it. Also, the authors introduced a Lagrangian relaxation solution methodology to facilitate and improve the solving process of the model. Then, the evaluation of the results enabled us to demonstrate the model validity, and underscore its utility to deal with problems with more sophisticated used product collection process that practitioners tend to encounter in the real-world circumstances.

Originality/value – This study suggests a novel way to design the return rate of used products in a reverse logistics network with buyback offers through a complete set of factors affecting it. Furthermore, the procedure of developing the model encompasses several important aspects that significantly decrease its complexity and improve its applicability.

Keywords Reverse logistics, Buyback offer, Robust optimization, Flexible constraint programming, Lagrangian relaxation, Logistics, Optimization, Allocation, Mathematical programming, Fuzzy

Paper type Research paper

1. Introduction

Although, for many years, products have been discarded after reaching their end of cycles, the idea of reverse logistics (RL) started to draw attention just from the beginning of the 1990s. In recent years, as the result of globalization and industrialization, worldwide production capacity has increased enormously, leading to a surge in the potential number of used products (Asees Awan and Ali, 2019). In this regard, a survey conducted by a collaboration between the International Solid Waste Association (ISWA), United Nation University (UNU) and International Telecommunication Union (ITU) showed that, in 2016, across the world, a massive amount of 44.7 million metric tons of electronic waste were generated (Baldé *et al.*, 2017). This number is equivalent to 6.1 kg per inhabitant. Moreover, from 2001 up to 2014, in the USA alone, roughly an average of 12.8 million automobiles

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annually had been scrapped, which almost equals 13 million tons of steel (National Transportation Statistics, 2018). Therefore, product recovery has become extremely difficult to ignore in the process of production. According to the Council of Logistics Management (CLM), RL can be defined as follows:

> The process of planning, implementing, and controlling the efficient, cost-effective flow of raw materials, in-process inventory, finished goods, and related information from the point of consumption to the point of origin to recapture value or proper disposal (Rogers and Tibben-Lembke, 1998).

Under this description, every RL network encompasses several important stages. Fleischmann *et al.* (2001) defined those as the collection of used products, sorting and organizing them and processing them based on their conditions. These processes specifically could be recycling, repairing, remanufacturing, dissembling of used products, green strategies for disposing of them and, finally, their redistribution.

Furthermore, the standout purposes behind endeavors in the RL field could be identified as the following three phenomena. The first one could be associated with environmental turbulence and the high rate of waste generated as the result of mass production. This problem has become so acute that in many countries, governments enacted laws to ensure manufacturers and producers significantly reduce their products' harmful impact on the environment. A most renowned example of such legislation is the Waste Electrical and Electronic Equipment (WEEE) set by European Nations in 2003, which enforces all electronic manufactures that have a market in those countries to burden the responsibility of recovery of their used products (Dowlatshahi, 2010). Second, it has been shown that an efficient RL network could provide an opening for a cost-saving effort in many industries. Based on numerous studies by scholars, it was discovered that an RL network implementation could be an opportunity rather than an obligation for producers to decrease their products' cost through recovery and recycling of used products (Ravi Shankar, 2017). Similarly, among some industries, the economic advantages of an RL network proved to be so significant that they voluntarily have decided to integrate their direct and reverse networks, which in turn leads to a closed-loop supply chain (CLSC) network (Östlin *et al.*, 2009; Farrokhi-Asl *et al.*, 2019). This coordination of reverse and direct logistics network yields to overall higher production efficiency. Finally, in recent years, there has been a growing level of consumer awareness and public consciousness regarding the environment (Tibben-Lembke and Rogers, 2002). This matter created an inclination from consumers toward eco-friendly commodities, which encouraged companies to produce a type of product accommodated to fulfill that particular demand (Chen *et al.*, 2018).

Although the concept of RL network implementation is hugely advantageous, it also inherently contains several challenging issues (Peidro *et al.*, 2009). In this regard, an important issue is how to deal with uncertainty in the process of collecting used products (i.e. return flow). Uncertainty exists in both types of direct logistics and RL, but it could be more critical in RL because of the unpredictable nature of return flow, the higher number of external elements affecting it and the unavailability of solid databases in comparison to direct flow (Guide *et al.*, 2003). Besides that, in recent years, the share of valuable and durable products (e.g. electronic devices) in the market has significantly increased, which has spurred to an expectation from product holders to receive some type of compensation to give up their ownerships (i.e. their reservation); together, they compounded the volatility and difficulty of collection process (John *et al.*, 2018). The importance of this issue becomes more evident when we realize that the reliability of responses acquired from any RL mathematical model highly hinges on the precision from which the return rate would be estimated (Wojanowski *et al.*, 2007). Consequently, previous studies proposed various methodologies such as deposit-refund, trade-in rebate, product exchange and buyback offer polices to deal with this problem. They mainly concentrated on developing models that enable the company to control the collection rate at an appropriate limit while optimizing its expenses. This matter derives from the fact that whether the government legislation or economic opportunities or even both would lead to a product recovery effort, the system inevitably requires to collect a certain portion of total used products among consumers.

Without loss of generality, they suggested the process of collection of used products; companies strive to reduce their cost whereas the product holders seek more appealing incentives. This correlation has made the buyback offer policy the most prevalent and effective strategy in the related papers. Consequently, researchers have considered this correlation as the module to estimate the probability of return of used products (Guide *et al.*, 2003). Now, an important question may arise that "why not allocate a fixed offer to all of the used products?" However, in this way, the lowest quality level of used products would be collected more than other levels of quality, which creates unnecessary expenses for a company (Aras *et al.*, 2008). Thus, we understand that to develop a functioning RL network, it is necessary to use a proper procedure to allocate buyback offers to consumers. Thus far, the majority of related researches suggested that factors such as quality of used products or distance of consumers from the closest collection center should be considered to estimate the return rate (Aras and Aksen, 2008; Dutta *et al.*, 2016). Although we acknowledge that for the collection of used products, their condition at the time of return plays the highest meaningful role in determining appropriate buyback offers. However, we argue that, in reality, there are other factors, such as seasonality of the product or easiness of product transportation or warranty services, whose impact on the product holder's willingness to return cannot be ignored. This is a critical shortfall among the relevant literature. Hence, our main motivation in this paper is to address this issue. Here, we seek to develop an RL model in which the return rate would be approximated based on a more comprehensive set of factors. Hence, we decided to divide the factors into two groups on product-based ones (i.e. quality and age) and externally originated ones (e.g. warranty period). Then, we use a combination of a fuzzy flexible constraint method and a specific linearization procedure to implement this configuration onto our proposed RL model. Afterward, we opted to use a specific robust optimization in the model development process to simultaneously fortify its responses against the return rate fluctuation, and calculate the confidence levels of fuzzy constraints more effectively. Finally, a Lagrangian relaxation method has been introduced to help us solving larger scale problems for the model.

In the next section, we conduct a review of the literature concerning product recovery with emphasis on RL networks with buyback offer. In Section 3, we present the problem description. In Section 4, we demonstrate the mechanism for allocating buyback offer to product holders. Section 5 details the schema of the proposed RL model. In Section 6, we explain the process of incorporating the effect of externally originated factors into the model, using the fuzzy flexible constraint technique. In Section 7, we discuss the robust optimization role in model development and its procedural implementation. Section 8 encompasses the Lagrangian relaxation solution methodology. In Section 9, we evaluate the model validity and effectiveness by running several exhaustive experiments. Finally, Section 10 includes our conclusion about the proposed model, and our recommendation regarding some possible directions for future researches.

2. Literature review

To date, the mainstream of endeavors in the RL field mostly concentrated on determining the location of collection centers, their optimal numbers and their capacity, allocating flow between specialized facilities (e.g. recycling centers, disposal centers, etc.) using mixed-integer/continuous linear/non-linear mathematical modeling. The range of these efforts started from simple locating collection centers with unlimited capacity and continues to more advanced stochastic multi-stage models with limited capacity or multi-commodity ones. Furthermore, the prevalent methods to solve *NP-hard* and non-linear models were heuristic algorithms, metaheuristic algorithms and strong combinational optimization (Pishvaee and Torabi, 2010). Some exhaustive RL review papers are presented in Govindan and Soleimani (2017) and Prajapati *et al.* (2018).

The idea of buyback offer inclusion in an RL network was introduced for the first time by Klausner and Hendrickson (2000). They developed a framework to obtain the optimal amount of buyback and each unit cost of the RL process. This model only focused on the cost of the RL network and did not consider the product holder's willingness to return. Guide and Van Wassenhove (2001) were the first researchers who emphasized the importance of an incentive role in recovering used products. Guide *et al.* (2003) developed a model to obtain the optimal amount of buyback and the selling price of the recovered components for a phone company. Mukhopadhyay and Setoputro (2004) studied an e-business market, where customers, because of the condition of sale, might be alert toward the manufacturer return policy. They developed a model to determine optimal prices and the return policy with setting parameters heeding to the market details. Ray *et al.* (2005) decided to find an optimal price for durable and remanufacture-able products. They suggested that a trade-in rebate policy implemented by the company could persuade customers to replace their old products with new ones. They developed three scenarios in which incentive offers could be dependent on the age of products or independent of them or entirely considered as an uniform offer for all of the customers. Aras and Aksen (2008) proposed non-linear mixed-integer programming with a drop-off policy to locate collection centers, simultaneously determining the optimal amount of buyback offer. They used a uniform probability function to approximate the rate of return.

Aras *et al.* (2008) continued their exploration in this area. They proposed a continuous right triangular probability function to approximate the attitude of product holders toward incentive offers. They designed a mathematical model for locating collection centers and collecting used products using a pick-up policy with capacitated vehicles. Vadde *et al.* (2010) proposed a method to calculate the optimal price for reusable and recoverable parts of used products. They also addressed the uncertainty nature of the product recovery process by considering several scenarios in which product recovery facilities passively accept return products or passively collect them. In 2016, Dutta *et al.* studied a CLSC with three-way recovery methods. The goal of this model was to find optimal manufacturing, remanufacturing and recycling quantities. The most notable innovation of this paper was a piece-wise concave function, which was used to approximate the probability of return products based on their quality and age. Fattahi and Govindan (2017) developed an integrated direct/RL network with a buyback policy over a planning period. They considered demand in both reverse and forward flow to be stochastic.

However, a 2014 paper by Tekin Temur *et al.* highlighted an important fact that in the collection process of used products rate of return not only depends on traditionally more investigated factors (e.g. quality, age, and distance), but also a host of other less examined ones. A complete list of these factors is shown in Table 1. As we can see, these factors are diverse and have different sources. Several are associated with the promotion policies of companies themselves, some are contributed to government legislations and others are related to used products condition and features. Here, we argue that previous studies in RL networks with buyback policy are not consistent enough. Thus far, researchers either solely investigated a buyback offer mechanism without incorporating it with an RL network, or they explored this integration while considering only a limited number of factors in their

Micro factors (firm based)		Firm strategy (Mukhopadhyay and Setoputro, 2004; Klausner and Hendrickson, 2000) Advertisements (Klausner and Hendrickson, 2000) Giving information to customers (Hess and Mayhew, 1997) Ability of the company to repair products (Thierry *et al.*, 1995) Warranty period (Thierry *et al.*, 1995)
Product-based	Condition (i.e. quality and age)	Life cycle point of product (Tibben-Lembke, 2002) Economic life of the product (Östlin *et al.*, 2009) Rate of defects (Gomez *et al.*, 2002) e-Waste quantity (Managers)
	Features	Complexity of product modularity (Östlin *et al.*, 2009) Seasonality of product (Tibben-Lembke, 2002) Product initial price (Hess and Mayhew, 1997) Sales amount (Gomez *et al.*, 2002; Tibben-Lembke, 2002) Easiness of product returns (Klausner and Hendrickson, 2000)
Macro factors (government and socioeconomic)		Legal enforcement (Koster *et al.*, 2002) Investment on the environment (Managers) Customer segment (De Brito, 2004) Education status (WEEE, 2003) Population and population density (WEEE, 2003; Hanafi *et al.*, 2007) Income (WEEE, 2003)

Table 1. Factors affecting the return rate of used products

approach for estimating the return rate. Notably, they were mostly concerned with the convenience and net value of companies rather than a more realistic description of product holders' willingness to return. Therefore, we admit that the inclusion of buyback offer concept in RL networks has improved the resultant models compared to those without it, but a lack of broader perspective still restricted their efforts.

Here, we once again stress the importance of addressing uncertainty in an RL mathematical model design. As Luhandjula (2006) pointed out, a deterministic approach in the process of RL mathematical modeling could lead to an ill-defined and incorrect description of the real-world event. Now, the reader should bear in mind that the aforementioned problem contains a high number of factors with different degrees of effect on the product holders' willingness to return or not. Therefore, it is very difficult to exactly define and predict such vague and ambiguous behavior. In such circumstances where a specific parameter behavior exhibits a completely unpredictable trend, the fuzzy mathematical programming method has often proved to be an effective resolution (Mula *et al.*, 2006; Simangunsong *et al.*, 2012; Sadjadi *et al.*, 2019).

Hence, in this paper, we decided to address the intricacies of considering various factors affecting the return rate in an RL model by implementing a fuzzy flexible constraint programming method. This approach enables us to control the collection process by allocating buyback offers dependent on the condition of used products at the time of return while considering the effect of other externally originated factors. To do so, we define the correlation between the approximate amount of return and the actual rate of return with a certain degree of leeway. Additionally, we attempt to determine the extent of externally originated factors' effect onto the model using a specific robust optimization methodology. In the next section, the details of this process are discussed.

3. Problem description

In this paper, we follow two main objectives to develop our model. First, we maintain the solid assumption of approximating the probability of return using buyback offers dependent on the condition of used products at the time of return; however, unlike other researches, we do not assume the resultant estimation as the actual return rate in the model. Here, we decide to distinguish between the chance of the return for buyback offers and the actual return rate of used products. With the help of this distinction, we attempt to address the effect of other factors onto the collection process.

Second, in the procedure of developing the model, we propose a series of particular techniques such as modification of the return probability function and implementing a robust fuzzy optimization that not only provides us an accurate description of the real-world problem circumstances, but also would be highly compatible with each other to significantly reduce the complexity of the outcome model. Here, we highlight our methodology and each step attribution to develop an RL network with buyback offers accommodated to resolve the aforementioned problem:

- Initially, we introduce a piece-wise probability function that provides us the platform to determine the return rate for each buyback offer based on two factors of quality and age of used products. Then, we modify it to its corresponding discrete function. This adjustment makes it more compatible with the following integration with the RL network.
- In the next step, we present a fuzzy mixed-integer programming model for an RL network design with three stages and a drop-off policy. The model's overall goals are to find the location and number of collection centers, flow between specialized facilities, the optimal amount of buyback offers and the number of used products that should be collected.
- In the next step, the fuzzy relation between the actual return rate and the approximated chance of return has been discussed. Using the fuzzy flexible programming method, we managed to define two possible conditions of shortage and surplus, and convert the model into its crisp counterpart.
- Then, we use a compatible robust fuzzy optimization approach for enhancing the performance and solving capability of the model. This procedure enables us to find the optimal amount of membership function of fuzzy parameters of the model by adding two penalties of risk and deficiency to the objective function.
- Afterward, a Lagrangian relaxation method has been implemented on the model to deal with solving the difficulty of large-scale problems that a commercial solver might not be able to handle.
- Finally, we applied the model onto several extensive numerical experiments. The evaluation and analysis of the results proved the model validity and effectiveness.

4. Rate of return in the presence of buyback offers

In our approach, we decided to separate the effect of influential factors on the return rate into two categories of product-based and externally originated ones. The first category factors encompass quality and age levels (i.e. product-based factors) of used products, and have been approximated by a piece-wise concave probability function (Dutta *et al.*, 2016). Based on related papers, we point out the quality and age of used products as the two factors with the highest degree of importance and practicability. Additionally, as we mentioned, RL networks often collect used products and sort them based on their condition. Thus, the assumption of classifying the collected used products into different predefined levels of quality and age is highly justifiable.

Moreover, if we assume that a collecting company uses a buyback policy to prevent unnecessary expenses, it requires to offer consumers a reasonable price. With the help of this relationship between the amount of buyback offer and the condition of the used product at the time of return, we would be able to somewhat anticipate consumers' behavior and make a preliminary estimation of the probability of return. However, the actual amount of returned used products could vary from that initial estimation because of the effect of the second category factors (i.e. externally originated factors). That is why, we initially approximate the probability of return for each used product with certain quality and age, and then incorporate the effect of second category factors onto the model using the flexible constraint method. Here, we consider the effect of second category factors as an uncertain value, which would be determined by the opinion of experts causing two possible states of shortage or surplus in the model.

4.1 Piece-wise probability return function

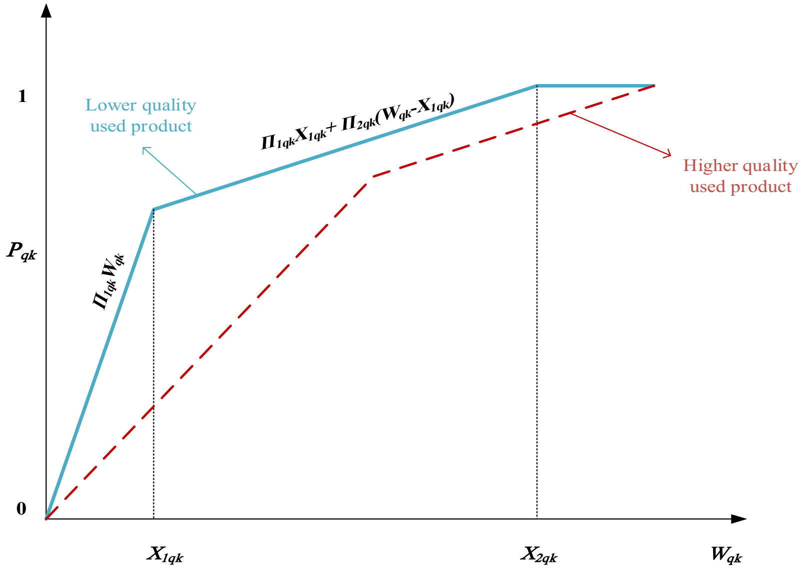
We stated that the product holder's decision regarding the returning of a used product depends on the amount of buyback they would be offered. Then, if we assume W_{qk} is the amount of buyback, which is going to be offered to a used product with the quality of q and age of k at the time of return, Dutta *et al.* (2016) suggested that a consumer decision pattern regarding his/her chance of accepting that offer could be approximated by the following piece-wise probability function:

$$P_{qk} = \begin{cases} \dfrac{\alpha_1 W_{qk}}{\alpha_1 x_{1qk} + \alpha_2 (x_{2qk} - x_{1qk})} & \textit{if} \quad 0 \leq W_{qk} < x_{1qk} \\ \dfrac{\alpha_1 x_{1qk} + \alpha_2 \left(W_{qk} - x_{1qk}\right)}{\alpha_1 x_{1qk} + \alpha_2 (x_{2qk} - x_{1qk})} & \textit{if} \quad x_{1qk} \leq W_{qk} < x_{2qk} \\ 1 & \textit{if} \quad W_{qk} \geq x_{2qk} \end{cases} \tag{1}$$

P_{qk} is the probability of return when a buyback price W_{qk} offered to a used product with q quality and k age. x_{1qk} and x_{2qk} are two breakpoints demonstrating product holders' change of attitude toward buyback prices (see Figure 1). This function can also receive more than two breakpoints as long as the slope of it is decreasing. α_1 and α_2 are two fixed coefficients, and ($\alpha_1 \leq \alpha_2$). If we assume:

$$\begin{cases} \dfrac{\alpha_1}{\alpha_1 x_{1qk} + \alpha_2 (x_{2qk} - x_{1qk})} = \pi_{1qk} \\ \dfrac{\alpha_2}{\alpha_1 x_{1qk} + \alpha_2 (x_{2qk} - x_{1qk})} = \pi_{2qk} \end{cases} \tag{2}$$

where π_{1qk} and π_{2qk} denote the marginal probabilities of acceptance. This means for the first interval (0, π_{1qk}), marginal probability of return is π_{1qk}, and for the second interval (x_{1qk}, x_{2qk}), this measure is equal to π_{2qk}. The quantity of π_{1qk}, π_{2qk} depends on the quality and age level of used products. It should be noted that the amount of α_1, α_2 is associated with the type of used products. In Figure 1, the two graphs of the probability function are associated with two different quality levels of a used product. The parameter x_{2qk} demonstrates the minimum price that the chance of return is about 100%. Therefore, we should not offer buyback prices higher than x_{2qk}, because it would not improve the probability of return, and only increases expenses of the collection.



Source: Dutta *et al.* (2016)

Figure 1.
Piece-wise concave probability function

It also can be seen that for a fixed buyback offer, the used product with higher quality would have a lower probability of return in comparison to a lower quality product, which is completely aligned with real-world circumstances.

Concerning related papers, the uniform probability function had been the most popular function to approximate the return rate of used products. However, the aforementioned piece-wise concave probability function has some significant strength compared to it. The capacity to add more breakpoints to the probability function makes it far stronger to anticipate product holders' heterogeneous behavior. Additionally, for companies that collect more than one type of product, especially third-party RL ones, setting proper values to (α_1, α_2) corresponding to each type of used product could be very advantageous. Therefore, we suggest the piece-wise probability function performance dominates the uniform probability function without necessarily increasing the intricacies of the resultant model.

4.2 Corresponding discrete probability function

A major challenge in the process of integrating an RL network with a buyback offer mechanism has been the complexity of the outcome model.

Even though we indicated that considering buyback offers would improve the product recovery process, we have to remember that the whole idea of buyback offer implementation is a tactical decision-making process that would be incorporated with higher level strategic decisions of an RL planning. Thus, we argue if this procedure would restrict the model applicability, the entire endeavor could be discouraging for practitioners to use it.

Here, we seek to deal with this issue by modifying the piece-wise concave probability function into a discrete function. This approach, for the first time, was introduced by Keyvanshokooh *et al.* (2013). They suggested several disjointed levels of buyback for a uniform probability function to approximate the percentage of returned used products. Here, to modify the piece-wise probability function, we need to implement the following steps. Initially, we suggest that for any used product, the possible buyback offers should be divided onto $O+E$ separated levels (see Figure 2). It means that for each used product with a particular quality and age, only a finite number of buyback offers can be allocated. To do so, we divide the interval of (0, x_{2qk}) or the denominator of the probability function $\alpha_1 x_{1qk} + \alpha_2 (x_{2qk} - x_{1qk})$, which is a fixed number for each used product with a certain quality and age onto $O+E$ disjointed levels. Then, we assume that O number of disjointed levels are allocated to the first part of the probability function, and E number of levels are assigned to the second part of it.

Now, for each used product with specific quality and age, just one buyback offer should be allowed to ensure about this matter the following constraint is required:

$$\sum_{o} \mu_{oqk} = 1 \quad \forall q, k \tag{3}$$

Also, under the piece-wise probability function configuration, the second interval only activates when the optimal amount buyback offer surpasses the quantity of x_{1qk} to ascertain about it, and hence we add the following constraint:

$$\sum_{e} \varphi_{eqk} \leq \mu_{Oqk} \quad \forall q, k \tag{4}$$

where μ_{oqk} and φ_{eqk} are two auxiliary binary variables associated with first and second intervals of the piece-wise probability function, respectively. Afterward, we can use the summation of buyback offer levels to calculate the probability of return for each used product, and write it as follows:

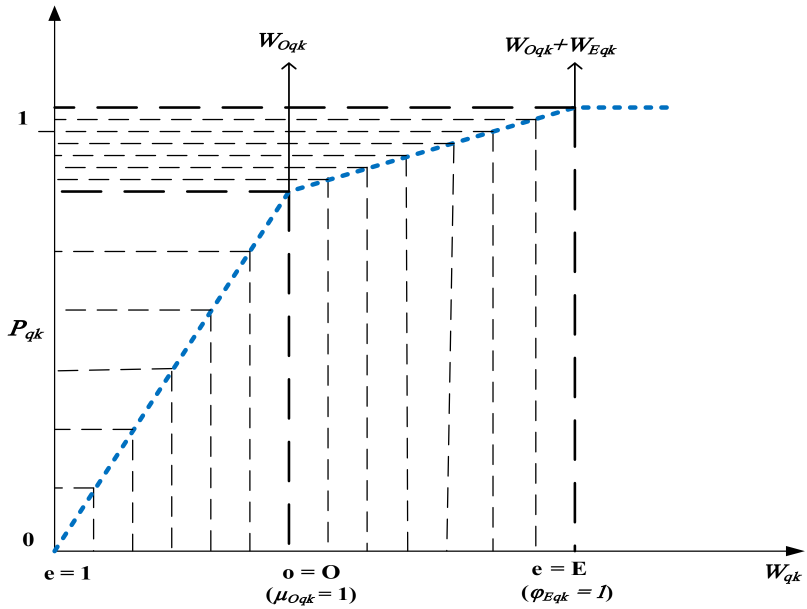


Figure 2.
Modified discrete probability function

$$P_{qk} = \alpha_1 \sum_{o=1}^{O} \mu_{oqk} \left(\frac{o-1}{O-1} \right) + \alpha_2 \sum_{e=1}^{E} \varphi_{eqk} \left(\frac{e}{E} \right) \tag{5}$$

As the result of Constraints (3) and (4), only one of the buyback offer levels could receive a value other than zero, which would indicate the optimal offer. With a simple transformation, we can also calculate the optimal amount of buyback offers as follows:

$$W_{qk} = \frac{\alpha_1}{\pi_{1qk}} \sum_{o=1}^{O} \mu_{oqk} \left(\frac{o-1}{O-1} \right) + \frac{\alpha_2}{\pi_{2qk}} \sum_{e=1}^{E} \varphi_{eqk} \left(\frac{e}{E} \right) \tag{6}$$

It also should be mentioned that by increasing the number of $O + E$, the discrete probability function could provide a closer estimation of its corresponding continuous function.

5. Model formulation of fuzzy reverse logistics network with buyback offer

Here, we formulate a fuzzy mixed-integer non-linear programming model for an RL network with buyback offers. The network structure consists of three stages: customer zones, collection centers and specialized facilities with a drop-off policy (see Figure 3). The used products are picked up by the company from customer zones, and delivered to collection centers where operations of inspection and sorting are going to be implemented on them. Then, based on the adequate process selected in the collection centers, used products are shipped to specialized facilities for operations of remanufacturing or recycling or scrapping.

Similar to related papers, we assumed that the government had set a mandatory minimum threshold for the company to achieve. In return, it would compensate the company by paying a predetermined subsidy for each collected used product (Dutta *et al.*, 2016). Each used product with a certain quality and age level has a specific processing cost and a potential value besides its buyback price. In the model, we decided to write these two measures as the overall value of each used product, which can be calculated by reducing its potential value from its processing cost.

Furthermore, to calculate the actual amount of used products return in the presence of the second category factors influence, we define the relation between the estimated numbers of collected used products and the actual return rate as a flexible constraint. This matter

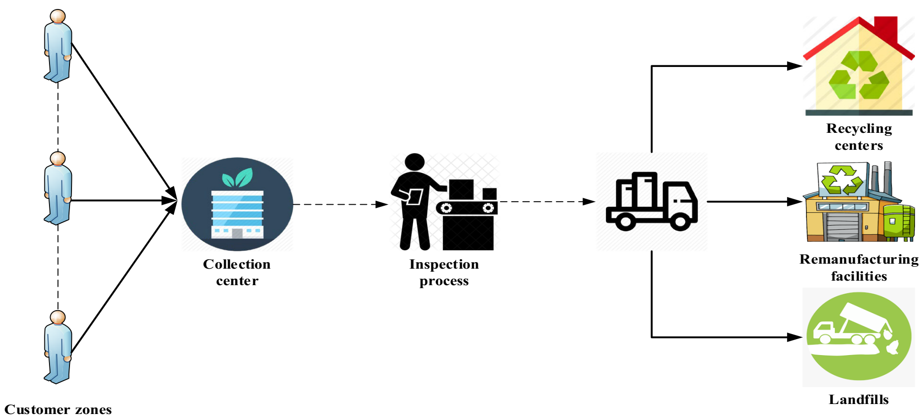


Figure 3. Framework of the reverse logistics network

adds a certain degree of leeway to the model. This capability of the model to achieve a different rate of return (i.e. higher or lower) than the approximated one indicates the effect of second category factors. As we assumed, the model allocates an appropriate buyback to each used product based on its quality and age. Thus, the sole scenario that the return rate would surpass the approximated rate is when consumers would freely return and not receive any compensation. On the other hand, when the return rate falls short of the approximated rate, we associate it with the negative effect of second category factors. This translates to higher reservation levels among product holders. Finally, the model goals are locating collection centers; determining the optimal number of them; and allocating flow between collection centers and specialized facilities, while finding optimal buyback prices for used products and the number of used products that should be gathered.

Furthermore, there are several other essential assumptions in the model that are presented in the below paragraph:

- Potential locations of collection centers are determined.
- The location of remanufacturing, recycling facilities and landfills are determined.
- For each type of used product, several quality and age levels are defined.
- Portions of used products that could be remanufactured or recycled or should be scrapped based on their quality are known.
- The potential number of used products in each customer zone is known and dividable between collection centers.
- Each collection center has a predetermined capacity for used products based on their quality and age at the time of return.

5.1 Indices

J = Set of customer zones;
I = Set of potential collection centers;
M = Set of remanufacturing centers;
R = Set of recycling centers;
L = Set of landfills;
Q = Set of the quality levels of used products; and
K = Set of age levels of used products.

5.2 Parameters description

F_i = Opening and operation cost of each collection center;
C = Unit transportation cost for each unit of used product between customer zones and collection centers;
Cm = Unit transportation cost for each unit of used product between collection centers and remanufacture centers;
Cr = Unit transportation cost for each unit of used product between collection centers and recycling centers;
Cl = Unit transportation cost for each unit of used product between collection centers and landfills;
d_{ij} = Distance between customer zone i and collection center j;
d_{im} = Distance between collection center i and remanufacture facility m;
d_{ir} = Distance between collection center i and recycling facility r;
d_{il} = Distance between collection center i and landfill l;

S_{jqk} = Potential number of product holders in customer zone j with quality q and age k;
H_{iqk} = The capacity of collection center i for used products with quality q and age k;
Θ_q = The portion of used products with quality q which could be remanufactured;
δ_q = The portion of used products with quality q which could be recycled;
ε_q = The portion of used products with quality q which should be scrapped;
v_{qk} = The overall value of a remanufactured, used product with quality q and age k;
re_{qk} = The overall value of a recycled used product with quality q and age k;
t = Government subsidy for each collected, used product;
λ = The minimum rate for collecting used products set by the government; and
M = A large number.

5.3 Decision variables

Y_i = 1, if collection center i set to be established and 0, otherwise;
Z_{ijqk} = Quantity of used products in collection center i with quality q and age k collected from customer zone j;
Um_{im} = Quantity of used products transported from collection center i to remanufacture center m;
Ur_{ir} = Quantity of used products transported from collection center i to recycling center r;
Ul_{il} = Quantity of used products transported from collection center i to landfill l;
W_{qk} Amount of buyback price for a used product with quality q and age k; and
P_{qk} = Percentage of used products with quality q and age k which are collected.

5.4 Objective function

$$Min \sum_i \sum_j \sum_q \sum_k Z_{ijqk} W_{qk} + \sum_i F_i Y_i + \sum_j \sum_i \sum_q \sum_k Cd_{ij} Z_{ijqk} + \sum_i \sum_r Cnd_{ir} Ur_{ir} + \sum_i \sum_m Cmd_{im} Um_{im} - \sum_i \sum_l Cld_{il} Ul_{il} - \sum_i \sum_j \sum_q \sum_k Z_{ijqk} (\theta_q v_{qk} + \delta_q re_{qk} + t) \quad (7)$$

Subject to:

$$\sum_i Z_{ijqk} \cong P_{qk} S_{jqk} \ \forall j, q, k \quad (8)$$

$$\sum_j \sum_q \sum_k Z_{ijqk} \leq MY_i \quad \forall i \quad (9)$$

$$\sum_i \sum_m Um_{im} = \sum_i \sum_j \sum_q \sum_k \theta_q R_{ijqk} \quad (10)$$

$$\sum_i \sum_r Ur_{ir} = \sum_i \sum_j \sum_q \sum_k \delta_q R_{ijqk} \quad (11)$$

$$\sum_i \sum_l Ul_{il} = \sum_i \sum_j \sum_q \sum_k \varepsilon_q R_{ijqk} \quad (12)$$

$$\sum_j Z_{ijqk} \leq Y_i H_{iqk} \quad \forall i, q, k \tag{13}$$

$$\sum_i \sum_j \sum_q \sum_k Z_{ijqk} \geq \lambda \sum_j \sum_q \sum_k S_{jqk} \tag{14}$$

$$P_{qk} \leq 1 \quad \forall q, k \tag{15}$$

$$Y_i, \mu_{oqk}, \varphi_{eqk} = \{1, 0\} \quad \forall i, q, k, o, e \tag{16}$$

$$Z_{ijqk}, Ur_{ir}, Um_{im}, Ul_{il}, P_{qk} \geq 0 \quad \forall i, j, q, k, r, m, l \tag{17}$$

And Constraints (3)–(6).

The objective function (7) is to minimize the total cost of the system. It consists of the cost of buyback offers, cost of transportations (from customer zones to collection centers, and then to specialized facilities), setup and operation cost of collection centers. The revenues come from government subsidies for collected used products, and their recycling or remanufacturing salvage values. Constraint (8) demonstrates the fuzzy relationship between the estimated number of collected used products and the actual number of returns. Constraint (9) ensures that only when a collection center is established, it could be assigned. Constraints (10)–(12) control the balance of flow between customer zones, collection centers and specialized facilities. Constraint (13) warrants that each collection center must not collect more than its capacity. Constraint (14) shows the minimum government rate of return, which has to be met by the company. Constraint (15) guarantees that the chance of return does not surpass one, because it would be meaningless if it happens. Constraints (16) and (17) denote the variable types and their non-negativity. We also note that Constraints (3)–(6) were related to the optimal amount of buyback offers and the probability of return. Finally, we can calculate the total cost collected used products from buyback offers in the objective function as follows:

$$\sum_i \sum_j \sum_q \sum_k S_{jqk} P_{qk} W_{qk} = \sum_j \sum_q \sum_k S_{jqk} \Big(\tfrac{(\alpha_1)^2}{\pi_{1qk}} \sum_{o=1}^{O} \mu_{oqk} \big(\tfrac{o-1}{O-1}\big)^2 + \tfrac{(\alpha_2)^2}{\pi_{2qk}} \sum_{e=1}^{E} \varphi_{eqk} \left(\frac{e}{E}\right) 2 \\ + \left(\frac{\alpha_1 \alpha_2}{\pi_{1qk}} + \frac{\alpha_1 \alpha_2}{\pi_{2qk}}\right) \left(\sum_o^O \sum_e^E \left(\frac{o-1}{O-1}\right) \left(\frac{e}{E}\right) \mu_{oqk} \varphi_{eqk} \right) \tag{18}$$

$$\left(\frac{\alpha_1 \alpha_2}{\pi_{1qk}} + \frac{\alpha_1 \alpha_2}{\pi_{2qk}}\right) \left(\sum_o^O \sum_e^E \left(\frac{o-1}{O-1}\right) \left(\frac{e}{E}\right) \mu_{oqk} \varphi_{eqk} \right) = 0 \quad \textit{if } (o < O) \textit{ then } \sum_e \varphi_{eqk} \\ = 0 \ \forall q, k \tag{19}$$

$$\alpha_1 \alpha_2 \left(\frac{1}{\pi_{1qk}} + \frac{1}{\pi_{2qk}}\right) \left(\sum_o^O \sum_e^E \left(\frac{o-1}{O-1}\right) \left(\frac{e}{E}\right) \mu_{oqk} \varphi_{eqk} \right) \quad \textit{if } (o = O) \textit{ then } \sum_e \varphi_{eqk} = 1 \ \forall q, k \\ = \alpha_1 \alpha_2 \left(\frac{1}{\pi_{1qk}} + \frac{1}{\pi_{2qk}}\right) \left(\sum_e^E \left(\frac{O-1}{O-1}\right) \left(\frac{e}{E}\right) \varphi_{eqk} \right) \tag{20}$$

$$\sum_i\sum_j\sum_q\sum_k R_{ijqk}W_{qk} = \sum_j\sum_q\sum_k S_{jqk}\left(\frac{(\alpha_1)^2}{\pi_{1qk}}\sum_{o=1}^{O}\mu_{oqk}\left(\frac{o-1}{O-1}\right)^2 + \frac{(\alpha_2)^2}{\pi_{2qk}}\sum_{e=1}^{E}\varphi_{eqk}\left(\frac{e}{E}\right)^2\right.$$
$$\left. + \alpha_1\alpha_2\left(\frac{1}{\pi_{1qk}}+\frac{1}{\pi_{2qk}}\right)\left(\sum_e^E\left(\frac{e}{E}\right)\varphi_{eqk}\right)\right) \tag{21}$$

Under Constraint (4), when the optimal buyback offer is lower than the amount of x_{1qk} (i.e. first breakpoint), then φ_{eqk} is nullified. Equation (18) could gain this condition, therefore the only time that its value would be anything than zero is when $\mu_{Oqk} = 1\ \forall q, k$ (i.e. when the chance of return surpasses the maximum amount of the first interval of the piece-wise probability function). The simplified traceable equivalent of equation (18) is written in equation (21). Finally, equation (21) consists of multiply two binary and two bounded integer variables. This issue also can be linearized with the aforementioned technique by Keyvanshokooh *et al.* (2013).

6. Fuzzy flexible constraints programming

In the model development process, we assumed that the company allocates buyback offers based on the quality and age of used products. Then, we attempted to address the effect of second category factors by defining the relation between the actual and approximated return rates as a flexible constraint. Now, we point out that the return rate could exceed the approximated chance of return only when some product holders would return without receiving any buyback offer. This means that consumers are less sensitive toward buyback offers, and the used product is less valuable. On the other hand, in circumstances that consumers are attuned toward buyback offers, the actual return rate could be less than the approximated chance of return.

Consequently, the actual return rate could fluctuate between two possible upper and lower limits. As we mentioned, because of multiple factors influencing product holder expectations toward a buyback offer and their different extent, it would be very difficult to determine their exact measures. However, the specific attitude among product holders' returning pattern could be a solid insight for the experts to determine the values of those two thresholds. Also, we expect the experts to express their opinions linguistically and imprecisely, so we assume the shortage and surplus measures as two fuzzy parameters. In the below section, we briefly introduce the fuzzy flexible constraint method, and then we used it to address the uncertain properties of the model.

Based on Cadenas and Verdegay (1997), we assume that we have a linear programming model as follows:

$$\begin{aligned}&min\ (x)\\&subject\ to\\&g_i(x) \le b_i,\ i = 1,\ldots,m\\&x \in X = \{x \in \Re^n,\ x \ge 0\}\end{aligned} \tag{22}$$

We can modify its constraints into flexible ones by implementing specific a tolerance allowance for them. Therefore, we use fuzzy sets of membership functions as follows: $v_i \ge 0\ \forall i = 1, 2, \ldots, m$

$$v_i(x) = \begin{cases} 1 & g_i(x) \le b_i \\ 1 - \dfrac{g_i(x) - b_i}{\sigma_i} & b_i < g_i(x) \le b_i + \sigma_i \\ 0 & g_i(x) > b_i + \sigma_i \end{cases} \tag{23}$$

Additionally, if we assume $\tilde{\sigma}$ as a fuzzy measure with a triangular distribution function. Then, the triangular prominence points of it can be written as $\tilde{\sigma} = (\sigma^p.\ \ \sigma^m.\ \ \sigma^o)$.

Ultimately, with the help of Yager (1981) method of classification, we can convert this fuzzy number into a crisp equivalent as:

$$\left(\sigma^m + \frac{\sigma^p - 2\sigma^m + \sigma^o}{3}\right) \tag{24}$$

Now, using the abovementioned relations, the crisp counterpart of fuzzy Constraint (8) could be written as:

$$R_{ijqk} \leq S_{jqk}P_{qk} + (1-\beta_1)\tilde{\sigma}_1 - (1-\beta_2)\tilde{\sigma}_2 \tag{25}$$

$$R_{ijqk} \leq S_{jqk}P_{qk} + (1-\beta_1)\left(\sigma_1^m + \frac{\sigma_1^p - 2\sigma_1^m + \sigma_1^o}{3}\right) - (1-\beta_2)\left(\sigma_2^m + \frac{\sigma_2^p - 2\sigma_2^m + \sigma_2^o}{3}\right) \tag{26}$$

$$\beta_1,\ \beta_2 \in [0,1] \tag{27}$$

where $\tilde{\sigma}_1$, $\breve{\sigma}_2$ denote the tolerances of the return rate in two optimistic and realistic scenarios, respectively. Also, β_1, β_2 are indicating the minimum satisfaction levels of fuzzy constraints. It can be seen that because of the effect of second category factors, the actual amount of returned used products could exceed or fall short of the estimated amount.

7. Robust optimization

We pointed out that values of $\tilde{\sigma}_1$, $\tilde{\sigma}_2$ are determined by the opinion of experts, and they account for the effect of second category factors on the return rate of used products. Therefore, their values are signaling their importance on the product holder return decision. Thus, when a product holder's willingness to return is at his/her most favorable case, the highest rate of return could occur, and the membership function of $\tilde{\sigma}_1$ would be zero (i.e. $\beta_1 = 0$). In contrast, in the worst-case scenario, for a product holder to receive a buyback offer, the membership function $\tilde{\sigma}_2$ would be zero (i.e. $\beta_2 = 0$), and the lowest probability of return would happen.

Now, there is a question of "how can we calculate the optimal amounts of fuzzy minimum satisfaction level (i.e. β_1, β_2)?" Regarding this matter, one way could be setting those values subjectively through decision-maker judgment. However, this route has a couple of big disadvantages. First, regardless of how many scenarios that we would consider, the responses could not be verified as the optimal result of the model. Furthermore, because we are forced to test multiple values for the minimum satisfaction levels, an excessive number of experiments would be required to achieve a reliable conclusion. This ultimately succumbs to more time-demanding solution process and analysis (Pishvaee and Fazli Khalaf, 2016).

Consequently, in this paper, we have decided to find a way to calculate minimum satisfaction levels simultaneously with the process of solving the model, and not setting their values subjectively. For this purpose, we took advantage of a robust fuzzy optimization method proposed by Pishvaee and Fazli Khalaf (2016). According to this approach, we need to define two new parameters as γ_1 and γ_2, and then two new penalties should be added to the objective function as follows:

$$\gamma_1\left((1-\beta_1)\left(\sigma_1^m + \frac{\sigma_1^p - 2\sigma_1^m + \sigma_1^o}{3}\right)\right) \tag{28}$$

$$\gamma_2\left(\beta_2\left(\sigma_2^m + \frac{\sigma_2^p - 2\sigma_2^m + \sigma_2^o}{3}\right)\right) \tag{29}$$

Expressions (28) and (29) denote the total penalty of violation of flexible constraint into higher and lower quantities, respectively. They called this method, robust controlling of the fuzzy constraint feasibility. Consequently, we can apply this methodology onto the model, and transform it into a robust linear integer programming problem. This strategy enables us to find the optimal values of β_1, β_2 in the solving process of the model itself. This process also strengthens the model response attitude toward possible fluctuation of the return rate. Notably, we point out that this method would not create two meaningless parameters of γ_1, γ_2, which were added to help us with the solving process of the model. Conversely, the decision-maker can determine the values of γ_1, γ_2 to indicate two penalties of risk and deficiency, respectively. This way, if we assume the return rate optimistically, γ_1 adds a penalty to the cost of the system. This penalty is the risk of expecting willingness among product holders to return optimistically, and the effect of second category factors in our favor. On the other hand, if we set the return rate more realistically, γ_2 would increase the objective function total cost, which could be counted as the penalty of deficiency in the collection process. This shortage is the difference between the actual number of used products collected and the approximated number of them.

8. Solution methodology

In the process of integrating the buyback offer mechanism with the RL network, a critical point is the complexity of the outcome model. In many studies, the researchers had to either suffice to a limited analysis of their model or develop a heuristic algorithm to solve it. For instance, Fattahi and Govindan (2017) proposed a simulated annealing algorithm to solve larger size problems. That is why we decided to implement a Lagrangian relaxation technique onto the model to improve its capability to cope with larger scale problems. Lagrangian relaxation has proven to be an effective way to address *NP-Hard* network problems. Fisher (2004) has conducted a complete survey on this method, and explained the computational aspect of it. Scholars such as Fahimnia *et al.* (2017) and Diabat *et al.* (2018) have used this method for the solving process of mixed-integer programming models with a great success.

Similarly, the proposed model in this paper is a mixed-integer programming problem that the difficulty of solving it derives from circumstances where increasing the scale of model parameters leads to an exponential increase in the processing time of it by a commercial solver. Moreover, we also have to take into account the extra binary variables added to the model for modification of buyback offers, which could compound the solving difficulty of it. This matter prompted us to use the Lagrangian relaxation solution methodology. The specific Lagrangian relaxation adopted in the model is based on Fisher's (2004) research, and contains three main steps. Initially, we need to calculate the optimal amount of lower bound, then with the help of it, the optimal amount of upper bound could be obtained. Next, we examine these two measures differences, and if it would be close enough, the solution methodology is completed. Otherwise, we need to repeat this process by updating the values of lower and upper bounds until reaching a sufficiently close result.

8.1 Optimal amount of lower and upper bounds

The lower bound could be found through relaxing one or several constraints of the model. It might cause the model to be infeasible, but still significantly reduces the complexity of solving it. Here, after several examinations, we have decided to relax Constraint (9), which specifically deals with the allocation of consumer zones to collection centers (Diabat *et al.*, 2018). After relaxing it, we can formulate the Lagrangian dual problem as follows:

$$Min\,L(g_{ijqk}) = OF + \gamma_1\left((1-\beta_1)\left(\sigma_1^m + \frac{\sigma_1^p - 2\sigma_1^m + \sigma_1^o}{3}\right)\right)$$
$$+\gamma_2\left(\beta_2\left(\sigma_2^m + \frac{\sigma_2^p - 2\sigma_2^m + \sigma_2^o}{3}\right)\right) + \sum_i\sum_j\sum_q\sum_k g_{ijqk}\left(\sum_j\sum_q\sum_k Z_{ijqk} - MY_i\right) \tag{30}$$

Subject to: Constraints (3)–(6), (10)–(17), (26) and (27).

g_{ijqk} indicates the non-negative Lagrange coefficient. The initial value of it is determined by the decision-maker. To obtain the amount of lower bound, we need to optimize the Lagrangian dual problem subjected to Constraints (3)–(6), (10)–(17), (26) and (27), and the rest of the necessary constraints related to the linearization process. However, often the relaxation procedure causes the lower bound to be infeasible. To deal with this problem, we could use the value of variables computed from the Lagrangian dual problem to solve the main objective function. Now, we can obtain a feasible solution that demonstrates the possible amount of upper bound.

8.2 Updating the upper and lower bound values

If the upper and lower bound values are close enough to each other, we can consider it as a desirable result. However, similar to other hybrid solution methodologies, to achieve a more accurate solution, repeating the aforementioned process is necessary. Thus, the Lagrangian coefficient should be updated to achieve new values for upper and lower bounds. According to Fisher (2004), for determining the measure of Lagrangian coefficient at *n + 1* iteration, we could use the sub-gradient optimization method as follows:

$$g_{ijqk}^{n+1} = \max\left\{0, g_{ijqk}^{n+1} - change^n\left(\sum_j\sum_q\sum_k Z_{ijqk} - MY_i\right)\right\} \tag{31}$$

In equation (31), n denotes the number of iteration, and $change^n$ is defined as:

$$change^n = \frac{\psi^n(Upper\ bound - Lower\ bound^n)}{\left(\sum_j\sum_q\sum_k Z_{ijqk} - MY_i\right)^2} \tag{32}$$

where the value of the upper bound is the best-achieved result of equation (32), and the lower bound is the value of its number n iteration. Besides, the value of parameter ψ^n is specified by the decision-maker concerning to each particular problem. Notably, after conducting several iterations, if no improvements were achieved, then the value of ψ^n parameter should be altered. In this paper, at the start, we set $\psi^n = 2$, and after four successive runs if required, reduce it to half. This process needs to be continued in sequence until the satisfactory results are obtained.

9. Computational results

In this section, we apply the proposed model onto several exhaustive numerical experiments. In doing so, we aim to examine two important aspects of it. First, we want to ensure the model validity and response timing, which, if acceptable, would underscore the effectiveness of linearization and Lagrangian relaxation procedures. Second, by running several specific experiments encompassing different types of used product, we attempt to analyze the model behavior, and demonstrate its utility and capability to address the effect of externally originated factors in an RL network. Together, if we achieve a desirable evaluation, we could argue the proposed model empowers practitioners to deal with real-world problems more effectively.

Here, we generated the value of parameters randomly using a uniform distribution, and their scales according to recent related studies (Dutta *et al.*, 2016; Sadjadi *et al.*, 2019). Notably, in this study, for computational purposes, we used commercial *GAMS 24.0.1* software and its *CPLEX* solver application. Also, all tests were carried on a computer with a configuration of *Core i7 CPU* processor, *8 GB RAM* and *Windows 10* operator.

9.1 Result of numerical experiment with different scales

Now, we seek to evaluate the model practicability concerning the scale of problems. Thus, we have randomly generated a numerical experiment with six different sets of scales (see Table 2). We attempted to design the scales based on similar researches (Pishvaee *et al.*, 2011; Keyvanshokooh *et al.*, 2013; Fattahi and Govindan, 2017). Without loss of generality, we assume used products that belong to the quality level with the higher label are in better condition than the lower label ones. The same assumption for categorizing used products with different age levels has been considered.

The result of six tests is demonstrated in Table 3. The column "Gap" shows the positive percentage difference between optimal responses from *CPLEX* and the Lagrangian relaxation method. As can be seen in Table 3, for both *CPLEX* and the Lagrangian relaxation method, there is a direct connection between the size of a test and its run-time. However, after the experiment with a scale of 4, the time required by the CPLEX solver to produce a result becomes troublesome.

For example, for the experiment with a scale of 5, the solver is unable to give any response even after 10 h of running. On the other hand, the Lagrangian relaxation method has managed to produce a result in a more acceptable time-frame. This matter, in conjunction with the negligible gap between CPLEX and Lagrangian relaxation results, demonstrates the superiority of the latter method in the process of dealing with the intricacies of larger scale problems.

Table 2.
Scale of parameters for six experiments

Data set no.	Collection centers $\lvert I \rvert$	Customer zones $\lvert J \rvert$	Predefined levels of quality $\lvert Q \rvert$	Predefined levels of age $\lvert K \rvert$	Remanufacturing facilities $\lvert M \rvert$	Recycling centers $\lvert R \rvert$	Landfills $\lvert L \rvert$	Alternative levels of the buyback offer $\lvert O \rvert$	$\lvert E \rvert$
1	3	5	3	3	2	2	2	10	8
2	6	10	3	3	3	3	3	10	8
3	10	15	4	4	4	4	4	16	12
4	15	20	5	5	6	6	6	16	12
5	18	25	6	6	6	6	6	20	16
6	22	30	7	7	8	8	8	20	16

9.2 Buyback offer method analysis

Here, we aim to evaluate the impact of the buyback policy mechanism. Therefore, we decided to assess the model using an experiment for three distinct types of used products. To do so, we assume all the parameters are identical except three parameters of the marginal probability of acceptance, the potential values of recycling and remanufacturing. The scale of parameters is based on Data set 2. The results are depicted in Figure 4. The average buyback for each scenario is illustrated as $\left(\frac{W_{11}+W_{12}+W_{13}}{3}\right)$. If we consider the used product on the right-hand side of the graph as Type 1 and the following two as Types 2 and 3, respectively, we can point out that Type 1 used products have a lower salvage revenue (recycling plus remanufacturing value) in comparison to the other ones. Consequently, Type 1 used products have the highest total cost between themselves. Furthermore, the model responses for Type 1 used products indicate that the system is more inclined to collect inferior quality level used products because it is virtually allocating the same amount of buyback offer to different levels of quality. Noteworthy, this matter is related to Constraint (14) in the model, which enforces the system to collect a minimum threshold assigned by the government.

In this experiment, we considered it to be 50% of the total number of used products. Therefore, the model attempts to ensure meeting this regulated limit at the lowest possible cost. In contrast, Type 3 used product has a higher potential salvage value, which caused the

Table 3. Results of six experiments with different data set

Data set no.	Lagrangian relaxation method results (in \$)	CPLEX solver results (in \$)	Gap	Lagrangian relaxation method delivery time (s)	CPLEX solver delivery time (s)
1	1,902.51	1,854.95	%2.5	162	151
2	3,066.73	2,940.10	%4.1	986	903
3	4,852.19	4,696.92	%3.2	2,072	4,406
4	7,743.06	7,448.82	%3.8	2,833	15,194
5	12,385.52	N/A	–	3,705	>36,000
6	17,433.68	N/A	–	5,268	>36,000

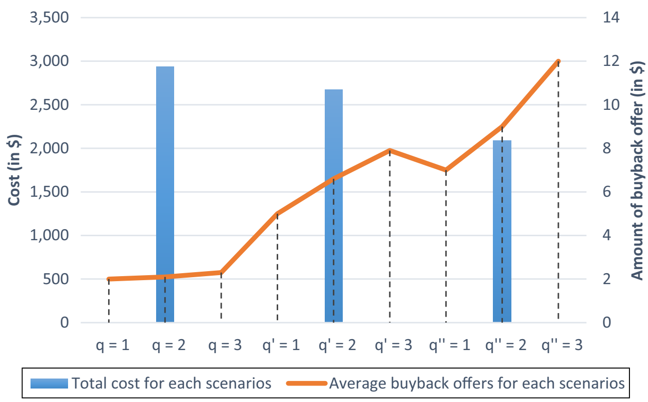


Figure 4. Performance of the model for different types of used products

lowest total cost compared to the other two types of used product. On top of that, the graph trend shows superior quality levels of the used products have received higher buyback offers than lower quality ones. This attitude suggests that unlike Type 1 used products, the system seeks to facilitate the return of better condition Type 3 used products. Finally, Type 2 used products behavior can be described as somewhat between the other two. All in all, this analogy enabled us to highlight the proposed model capability to deal with different types of used product collection processes.

9.3 Sensitivity analysis

In this section, we carried out a sensitivity analysis of several important parameters of the model. By analyzing their behavior, we attempt to draw some important insights from crucial parameters of it. The rest of the analysis is based on an experiment with the configuration of Type 1 used product from the previous section with the Data set 2 scale. Also, for convenience in presentation, we used the average amount of buyback offers, which is $\frac{\sum_q \sum_k W_{qk}}{Q+K}$.

9.3.1 Effect of the marginal probability of acceptance.

In the model, we used a discrete probability function to approximate the probability of return.

Thus, we expect changing the amount of marginal probability of return would alter the amount of optimal buyback offers. For this purpose, we assume that the first interval marginal probability of acceptance $\pi 1_{qk}$ is fixed, and alter the second one π_{2qk} (see Figure 5). We can deduce from the graph that by increasing the marginal probability of acceptance, the system requires lower buyback offers to convince consumers to return. This matter eventually decreases the total cost of system.

9.3.2 Effect of the penalty of deficiency.

In the process of developing a robust objective function, we defined two penalties of risk and deficiency. Here, we attempt to analyze the model attitude toward a change in the amount of penalty of deficiency. Therefore, if we increase this penalty, we would expect the return rate to become more restricted, succumbing to additional inclination in the model to accept shortage. In Figure 6, the result achieved from this procedure is demonstrated. From the

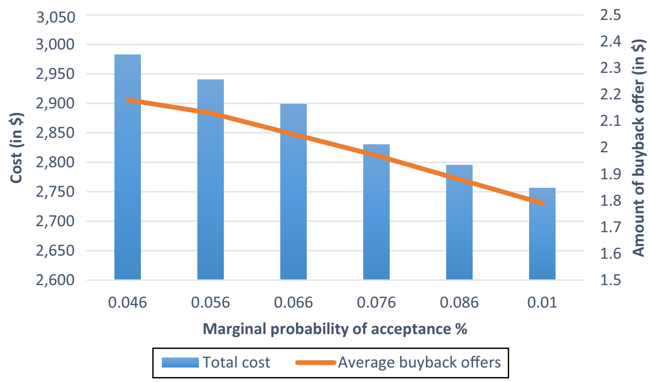


Figure 5.
Effect of the marginal probability of acceptance on the total cost and buyback offers

graph, we can discover that the total cost of the network has gone up, and the minimum satisfaction level has been reduced. Thus, the overall amount of shortage of return has increased.

9.3.3 Analysis of the robust fuzzy optimization strategy.
In our approach, to find the optimal membership function of the fuzzy parameters of the model, we sought to take advantage of a robust fuzzy optimization method. Consequently, we defined two additional parameters of risk and deficiency in the model. Here, we investigate the performance of this methodology. To do so, we randomly generate six different amounts of penalties for the model.

Then, we attempt to solve the fuzzy model with these penalties while using predetermined values for the membership function of fuzzy constraints at the levels of {0.5, 0.7 and 0.9}. Finally, for the sake of creating a comparative analysis, we use the measure of variables computed from the fuzzy method to calculate the total cost of the original robust fuzzy model without the predetermined fuzzy constraints levels (Pishvaee and Fazli Khalaf, 2016). The results of this process are presented in Figure 7, and as we can see, the robust optimization method responses dominate the scenario-based approach solutions, proving its superiority in the process.

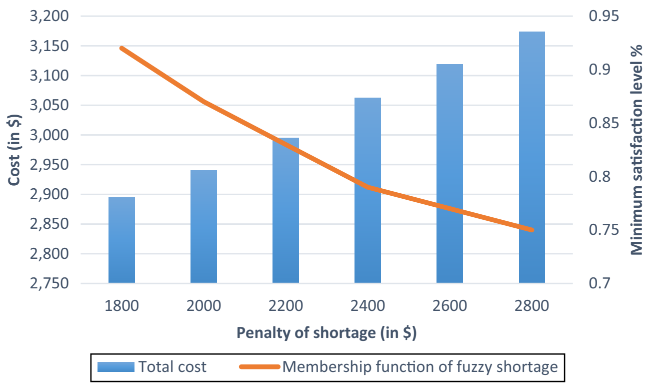


Figure 6.
Effect of the penalty of shortage on the total cost and minimum satisfaction level of constraints

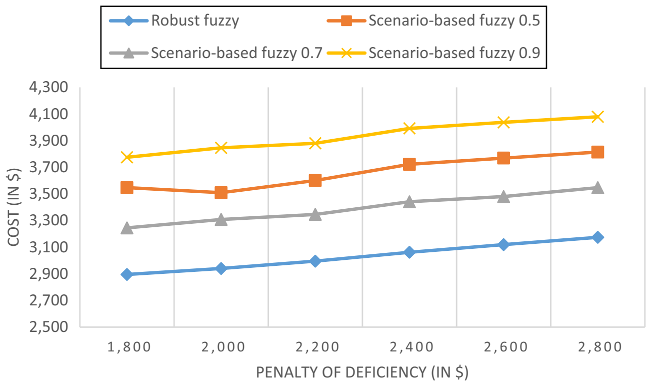


Figure 7.
Comparative analysis of the robust fuzzy and scenario-based optimization methods

10. Conclusion and remarks

Nowadays, product recovery is an inseparable part of the production. This matter arises from three major reasons: environmental concerns, economic opportunities of product recovery concept itself and consumers' consciousness. Additionally, because of the inherent uncertainty of any RL network, and growing level of reservation among product holders to receive an incentive offer to return their used products, the inclusion of a buyback offer has become an efficient way to control and estimate the return rate. In this paper, we set out to address this problem by developing a robust fuzzy RL network with buyback offers. We initially made a preliminary approximation of return rate using buyback offers dependent on the quality and age of used products at the time of return. Then, we extend our approach by incorporating the effect of other factors on the return rate of used products in the model using a fuzzy mathematical method, and, in particular, flexible constraint programming. Additionally, a robust fuzzy optimization method has been implemented on the model to define two scenarios of risk and deficiency. These two scenarios demonstrate the positive and negative impact of externally originated factors on the actual return rate. The effect of these factors has been considered a fuzzy parameter that could be determined by the opinion of experts. During the aforementioned efforts, we also modify the piece-wise probability function that was used to approximate the chance of return into its corresponding discrete function. Then, using the robust fuzzy optimization method, we managed to linearize the model completely. Moreover, the Lagrangian relaxation method has been introduced to improve the model capability to solve larger scale problems. Finally, we ran several extensive numerical experiments on the model. Results have shown the model validity and effectiveness for different types of used products as well as relatively large-scale problems.

Even though, in recent years, the concept of RL with incentive offer policy has drawn more attention to itself, we argue that given the broad and complex nature of this topic, there are still many aspects of it that should be addressed. This matter creates a requirement to add more dimensions to an incentivized product recovery system. For future studies, we could suggest a dynamic approach, in which a collection company might be able to alter its buyback setting levels according to each particular month or season. This could prove to be quite advantageous because consumers do not have a steady pattern of spending, and usually tend to increase their expenditure around holidays. Therefore, even a less appealing offer in that specific time period is likely to satisfy their reservation.

We also suggest that a CLSC network with buyback offers could be a promising step forward because, today, in many industries, product recovery is the responsibility of the producers and manufacturers themselves. Such a model could provide an opportunity to determine buyback offer considering the important factor of the initial product sale price, which, to our knowledge, is yet to be adequately realized.

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