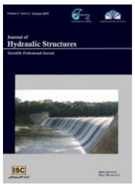
Journal of Hydraulic Structures
J. Hydraul. Struct., 2021; 7(3):33-41
DOI: 10.22055/JHS.2021.38330.1188



Introduce an approach to computing mean velocity and discharge using entropy velocity concept and a data-driven technique and only one single measured value of mean velocity

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Abstract

Discharges in hydrometric stations are estimated by converting the stage values to the discharge using a stage-discharge relationship or by multiplying mean velocity with flow cross-sectional area. Estimation of mean velocity in hydrometric stations, especially during flood events, is not easily possible. Therefore, the method of estimating mean velocity by converting the maximum velocity to mean velocity using a conversion factor is a desirable method. The velocity convert factor estimation in stations without enough valuable measured discharge data is a challenging issue. Present study develops a method for determining the conversion factor by combining the entropy velocity profile and a data-driven technique (genetic programming) by knowing only one mean velocity value, and thus develops a method to determine discharge at the weak gauging sites. The advantage of the method introduced in this study is the simplicity of application and the use of parameters that can be easily measured to estimate the mean velocity values. The performance of the method was evaluated by comparing the computed and the observed mean velocity values and the Root Mean Square Error and the Mean Absolute Error were found to be 0.05 m s-1 and 0.04 m s-1, respectively. The results showed that the introduced method estimates a suitable conversion factor compared to similar methods and is applicable for stations without measurement.

Keywords: Water Width to Water Depth Ratio, Data-Driven Technique, Entropy Velocity Profile, Stage-Discharge, Mean Velocity.

Received: 23 August 2021; Accepted: 17 October 2021

1. Introduction

Evaluation of water discharge in open channels is required in irrigation, river monitoring and control, water resources management, water balance assessment at the basin scale, design of hydraulic structures, and calibration and validation of runoff and flood routing models [1].

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Measurement of discharge at gaging stations requires information about the mean velocity in a number of subsections across the channel and their cross-section geometry. In wide channels flow velocity increases monotonically from 0 at the channel bed to the maximum value near or below the water surface, and the velocity distribution may be considered one-dimensional (1-D) [2]. In narrow channels, the velocity is influenced by the boundaries and varies even along the transverse direction and the velocity distribution may become predominantly two-dimensional. The maximum velocity may occur at or below the water surface, inducing dip-phenomenon and the position of maximum velocity is influenced by the aspect ratio (*B/D*) where *B* is the channel width and *D* is the water depth [3-5]. Maghrebi [6] proposed a method to estimate the mean velocity using the single point velocity measurement. Kavousizadeh and Ahmadi [7] proposed a novel approach to improve the model of Ahmadi et al. [8] (in which the rating curve was identified by utilizing the information of discharge at a referenced water level) by modifying its basic assumptions and using multi-objective optimization. Recently, several studies have estimated mean velocity by applying entropy. Using Shannon entropy, Chiu [9] introduced a relation between the ratio of mean to maximum velocity and entropy parameter *M* as:

$$\phi_M = \frac{u_m}{u_{max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} \tag{1}$$

Where M is the dimensionless entropy parameter; u_m is the mean velocity and u_{max} is the maximum velocity in a channel cross section. Several authors have investigated the accuracy of this relationship using field data [10-14]. Choo et al. [15] proposed a theoretical formula for estimating maximum velocity using the entropy concept. Moramarco and Singh [16] investigated the dependence of *M* on hydraulic and geometric characteristics at a river site and showed that *M* was not dependent on the dynamics of flood, because it did not depend on the energy or water surface slope, S_f. Hence, the formula expressing *M* as a function of hydraulic radius, Manning’s roughness coefficient, and elevation, y_0, where the horizontal velocity is hypothetically equal to zero was identified. During high floods, the measurement of velocity is possible only in the upper portion of the flow area where the maximum velocity occurs. In this way, the knowledge of *M*, and hence of the entropic relationship, Eq. (1), can be useful for computing mean velocity. It is noted that the estimation of *M* is easy if there is a robust velocity dataset containing the pairs (u_m, u_{max}). The dependence of *M* on the water depth and roughness height is more evidenced in shallow water conditions where *M* can be influenced by roughness height, but in high flow conditions it is found constant [16-18]. Based on the entropy-based velocity profile law and classical relationships for uniform flow and friction factor, Greco [2] proposed a general logarithmic relationship between parameter Φ_M, Eq. (1), and the water depth to bed roughness ratio (*D/d*). Greco and Moramarco [19] introduced a similar relationship between Φ_M, and the ratio between water width, *B*, and flow depth, *D*:

$$\Phi_M = A_B ln\frac{B}{D} + C_B \tag{2}$$

where A_B and C_B are the appropriate coefficients. Using Eq. (2), one can easily estimate Φ_M by measuring only channel width and flow depth, and hence, through Eq. (1) the mean velocity and discharge can be inferred. Therefore, it is of considerable interest to estimate parameters A_B and C_B at river sites where velocity data are not available. Abdolvandi et al. [20] proposed a simple method for estimating parameters in Eq. (2) at ungauged river sites. The method is also extended to the case of Φ_M constant as expressed by Eq. (1) and results on mean velocity assessment are compared with those of Eq. (2). The present study proposes a simple novel method for determining the velocity conversion factor at ungauged river sites, knowing only one mean velocity value. This method is practical and requires little measurement compared to conventional methods, and uses easily measurable parameters.

2. Methods

2.1. Brief Theoretical background on M and entropy velocity profile

For a gauged site where a dataset of pairs (u_m, u_{max}) is available, *M* can be easily estimated. For ungauged sites, Greco [2] introduced an equation between velocity ratio and water depth to roughness height ratio:

$$\Phi_M = \frac{U_m}{U_{max_\Phi}\, ln\frac{D}{d_\Phi}} \tag{3}$$

where A_Φ and B_Φ are the numerical coefficients, *D* is the maximum water depth, and *d* is the characteristic dimension of roughness elements. Equation (3) expresses the potential effect of bed roughness on the entropy-based velocity distribution in open channel flow, depending on the scale of large, medium, and small roughness elements. Studies have shown that it is possible to express average flow width, *B*, and depth, as well as the ratio *D/d*, as functions of water discharge, *Q* [21-23]:

$$\begin{cases} B = \alpha Q^a \\ D = \beta Q^b \\ \frac{D}{d} = \frac{\gamma}{i\%^j} Q^c \end{cases} \tag{4}$$

in which *i%* represents the local bed slope; and *a, b, c, α, β, γ, j* are the numerical coefficients. After a little algebra, the relative submergence can be reported in terms of the aspect ratio, *B/D*, as follows [19]:

$$\frac{D}{d} = \frac{\gamma}{i\%^j}\left[\frac{\beta}{\alpha}, \frac{B}{D}\right]^{\frac{c}{\alpha-b}} = K^*\left(\frac{B}{D}\right)^{\alpha^*} \tag{5}$$

where *k** and *a** are the coefficients. Finally, Eq. (3) can be reformulated taking into accountEq. (5), and the velocity ratio Φ_M can be derived as a function of *B/D* as expressed by Eq. (2).

Eq. (2) can be useful in the mean velocity and discharge estimation, especially in high flow conditions, because by knowing the channel section, water depth and water width and using proper coefficients (A_B, $C_{B,}$), Φ_M can be estimated by (2) and then the mean velocity if u_{max} is measured.

2.2. The proposed method for estimating the coefficients of Equation 2

This study proposes a simple but effective method for the estimation of mean velocity and discharge at ungauged sites by knowing only one mean velocity value. Measurement of hydraulic parameters, such as water surface width, water depth, and cross-sectional flow area in cross-sections, is easy, but the estimation of discharge needs mean velocity, u_m. The knowledge of *M* along with the measurement of u_{max} makes it possible to apply Eq.1 so that u_m can be estimated. In this case, an accurate estimate of *M* is necessary. For a gauged river site, *M* can be inferred if a robust dataset of pairs (u_{m}, u_{max}) is available. However, for ungauged sites, *M* should be assessed in a different way. In order to determine the Φ_M value, this study proposes a method based on the coupling of Eq.2 and stage-discharge relation in an iterative loop to obtain Φ_M using only a single point of mean velocity measurement. Hence, Eq. (2) can be rewritten as:

$$\phi_{Mi} = A_B ln\frac{B_i}{D_i} + C_B \tag{6}$$

where B_i is the channel width corresponding to the different flow depths, D_i. The method consists of the following steps:

a) Let's assume that at an ungauged site only u_{maxi} is measured at different water depths, D_i and width, B_i; in addition, the mean velocity and discharge have been measured only at one depth. For the convenience of this measurement, it can be done at the lowest depth (D_1).

b) A set of equations for Φ_{M1} and Φ_{avg} is created as follows:

The average Φ is equal to:

$$\phi_{avg} = \frac{A_B \sum_{i=1}^{n} \ln\left({}^{B_i}/_{D_i}\right) + nC_B}{n} = A_B \frac{\sum_{i=1}^{n} \ln\left({}^{B_i}/_{D_i}\right)}{n} + C_B = A_B(ln\, {}^{B}/_{D})_{avg} + C_B \tag{7}$$

As a result:

$$\begin{cases} \phi_{M1} = A_B ln\frac{B_1}{D_1} + C_B \\ \phi_{avg} = A_B(ln\, {}^{B}/_{D})_{avg} + C_B \end{cases}$$

By showing the natural logarithm of the aspect ratio ($\ln B_i/D_i$) as AR_{lni}, after mathematical simplifications, the following relations for calculating the coefficients A_B and C_B in terms of Φ_M and the ratio of width to depth are obtained:

$$A_B = \frac{\phi_{M1} - \phi_{Mavg}}{AR_{ln_1} - AR_{ln_{avg}}} \tag{8}$$

$$C_B = \frac{\phi_{Mavg} AR_{ln_1} - \phi_{M1} AR_{ln_{avg}}}{AR_{ln_1} - AR_{ln_{avg}}} \tag{9}$$

Since the measured values of water surface width and water depth are available, the AR_{ln1} and AR_{lnavg} values are calculated, and also, since the mean velocity values are measured at a reference point (here at point i=1), the value Φ_{M1} is available.

c) By assuming a value for Φ_{avg}, the coefficients A_B and C_B are estimated.

d) The observed dataset of (u_{maxi}, D_i, B_i, A_i) at the site is split in two parts: Set 1 and Set 2;

e) B_i, D_i, A_i and u_{maxi} are considered from set1. Φ_{Mi} is estimated by applying the values of A_B, C_B computed in the previous step in Eq.2. Therefore, the mean velocity, u_{mi} and discharge Q_i for each of the stage values can be assessed as:

$$B_i D_i \rightarrow \phi_{Mi} = A_B ln\frac{B_i}{D_i} + C_B \rightarrow u_{mi} = u_{maxi}\phi_{Mi} \rightarrow Q_i = u_{m_i} A_i$$

f) Based on Q_i and D_i, the relationship Q=f(D) is identified by using Genetic Programing;

g) B_j, D_j and A_j are considered from set 2. Discharge according to the relationship obtained in step f) is assessed and Φ_{Mj} is estimated as well. Therefore, the maximum velocity, u'_{maxj} for each of the stage values can be assessed as:

$$\begin{cases} \phi_{Mj} = A_B ln \dfrac{B_j}{D_j} + C_B \\ u_{mj} = \dfrac{Q_j}{A_j} \end{cases} \rightarrow u'_{maxj} = {}^{u_{mj}}/_{\phi_{Mi}}$$

h) New computed maximum velocity values estimated in g) , u'_{maxj} at the river site are compared with the observed ones, u_{maxj}. Performance measures are assessed in terms of Root Mean Square Error (E_{RMS}) and Mean Absolute Error (E_{MA});

i) If optimal performance is not achieved, return to step c), modify Φ_{Mavg} ($\Phi_{avg\ new}$= Φ_{avg}+0.01) and repeat steps d to h. (Φ_M is a number between 0 and 1 and is calculated up to two decimal places, so new Φ_{Mavg}= previous Φ_{Mavg} + 0.01 is reasonable)

The above steps involved in the procedure are depicted in Figure 1. In stations where there are no mean velocity observational values, common methods of estimating Φ, such as regression between mean velocity data and maximum velocity, cannot be used. The proposed method creates a mechanism that estimates the Φ_M values without the need for a large number of measured mean velocity data and only by measuring at one point. To evaluate the performance of the proposed method, the Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) were used.

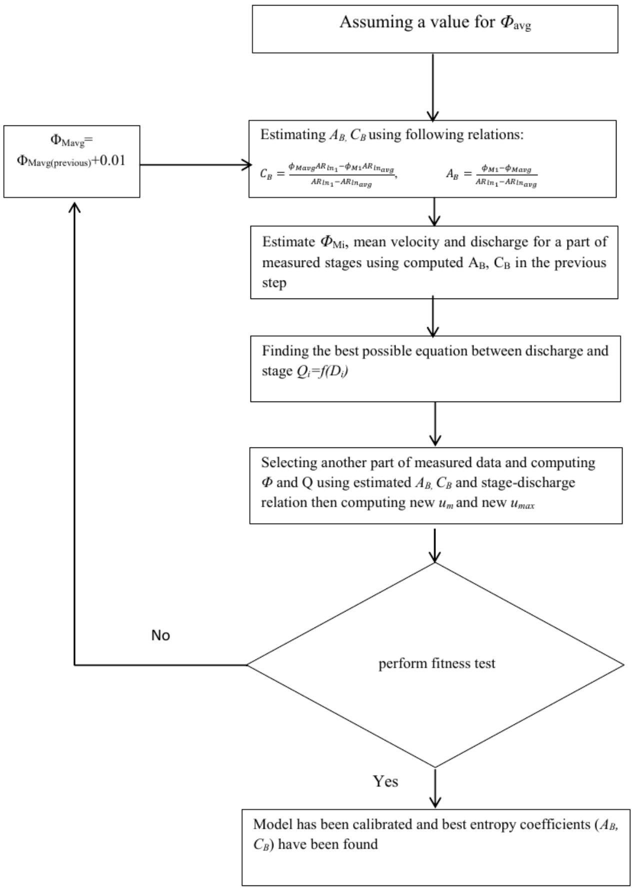

Figure 1. Flowchart of the method

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}((X_e)_i - (X_o)_i)^2} \tag{10}$$

$$MAE = \frac{1}{n}\sum_{i=0}^{n}|X_{o_i} - X_{e_i}| \tag{11}$$

where X_0 is the observed value, and X_e is the value estimated by the proposed method.

3. Results and discussion

3.1. Data

The proposed method was tested using data from the Ponte Nuovo station on the Tiber River, Italy. The velocity dataset consists of velocity points data whose number along each sampled vertical is sufficient (more than eight for some verticals) to reconstruct accurately the vertical velocity profile. For each measurement, u_m and u_{max} are available along with the flow depth, D [20]. In this study, thirty measurements were used and divided into two datasets each of which consisted of fifteen measurements. Table 1 shows the main hydraulic characteristics of the two datasets. The selection of two sets was done in a random way.

Table 1. Range of Variation of the Main Hydraulic Characteristics of Two Datasets

	Q **(m^3 s^{-1})**	***D*** **(m)**	***B*** **(m)**	***U*****MAX (m s^{-1})**	***U*****M (m s^{-1})**
Set 1	9-475	1.3-5.4	41-56	0.31-2.77	0.19-1.84

3.2. Evaluation of the performance of the method

Using the observed maximum velocity as an indicator of performance, Φ_{Mavg} equal to 0.65 produced the best results, and the coefficients A_B and C_B were estimated to be -0.0613 and 0.8193, respectively. Therefore, the final relationship for *Φ* estimated as a function of the ratio between water width and water depth is:

$$\phi_M = 0{,}819 - 0{,}061 \ln {}^{B}\!/_{D} \tag{12}$$

Since it was assumed that there was only one measured value of the mean velocity, in order to estimate Equation (12), the measured mean velocity value (u_{m1}) and the observed measured values of B, D, and u_{max} were used. The performance of the method was evaluated by comparing the computed and the observed mean velocity values and RMSE and MAE were found to be 0.05 m s-1 and 0.04 m s-1, respectively (Table 2).

Table 2. Evaluation of the performance of the method by the evaluation criteria

R**^{2}**	***MAE*** **(m s^{-1})**	***RMSE*** **(m s^{-1})**
0.98	0.04	0.05

In figure 2, the values of computational mean velocity are plotted against the observed mean velocity values, line 1:1 is plotted and the correlation coefficient is calculated. It is observed that there is a high correlation between the computed and measured values. Furthermore, as shown in Figure 3, the values of the computed mean velocity and measured at different depths are compared.

As can be seen, the measured and computed values of mean velocity from shallow to high depths are very close and show the good performance of the proposed method at different depths.

Therefore, by having the values of water depth and channel width at the corresponding depth, it is possible to compute the values of Φ for any station using an equation (such as equation (12)) and calculate the mean velocity and discharge using Φ values. This study has some limitations, to test the proposed method, we need measured data of mean velocity, flow discharge, cross-section geometry, maximum velocity, and depth of flow. collecting measured data accurately, especially mean and maximum velocities at different depths (shallow to high), is not easy. In this study, the data of one station were used for validation, but for future studies, it is suggested to evaluate the introduced method with other sections in different rivers.

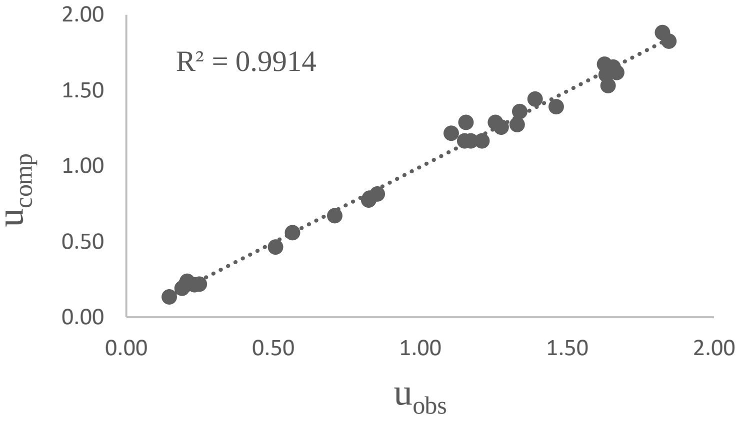

Figure 2. Comparison between computed and observed mean velocity; the line 1:1 is also shown

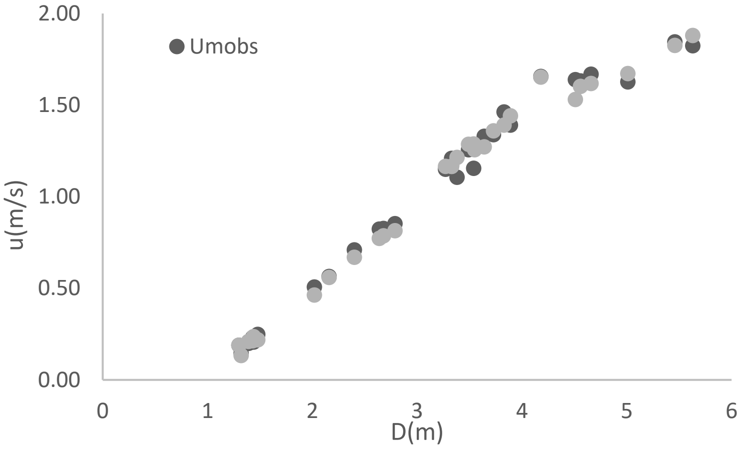

Figure 3. Comparison between computed and observed mean velocity in different depths

4. Conclusions

In this study, a simple method is proposed based on a combination of entropy-based velocity distribution and one data-driven technique to convert maximum velocity to mean velocity at non-measured stations (only one measured mean velocity value for one depth is required). The most important advantage of the introduced method compared to other common methods is that the values of mean velocity and discharge can be estimated only by using geometric and measurable parameters. In fact, this method is practical and requires little measurement compared to conventional methods, and uses parameters which are easily measurable. The mean absolute error and root mean square error for the computed mean velocity values by the proposed method were equal to 0.04 ms^{-1} and 0.05ms^{-1}, respectively. The results show that the performance of the method has been appropriate and acceptable. Therefore, the method can be conveniently adapted to estimate the entropy quantity Φ_M starting from easy measurements of hydraulic variables (water depth, channel width, and maximum velocity) and one mean velocity value, also during high floods.

5. Acknowledgments

The authors thank Professor Tommaso Moramarco, Director of Research, Research Institute for Geo-Hydrological Protection, National Research Council, Perugia, Italy for providing the data

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