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Nuclear Physics B 977 (2022) 115733

NUCLEAR PHYSICS B

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Minimal gauge invariant couplings at order ℓ_p^6 in M-theory

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Received 24 January 2022; received in revised form 6 March 2022; accepted 8 March 2022

Available online 14 March 2022

Editor: Stephan Stieberger

Abstract

Removing the field redefinitions, the Bianchi identities and the total derivative freedoms from the general form of the gauge invariant couplings at order ℓ_p^6 for the bosonic fields of M-theory, we have found that the minimum number of independent couplings in the structures with even number of the three-form, is 1062. We find that there are schemes in which there is no coupling involving R , $R_{\mu\nu}$, $\nabla_\mu F^{\alpha\beta\gamma}$. In these schemes, there are sub-schemes in which, except one coupling which has the second derivative of $F^{(4)}$, the couplings can have no term with more than two derivatives. We find some of the parameters by dimensionally reducing the couplings on a circle and comparing them with the known couplings of the one-loop effective action of type IIA superstring theory. In particular, we find the coupling which has term with more than two derivatives is zero.

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1. Introduction

M-theory is a consistent quantum theory of gravity which includes all types of superstring theories at different limits [1]. In particular, the compactification of M-theory on a circle produces the type IIA superstring theory. A convenient way to study different phenomena in this theory is to use an effective action which is a derivative-expansion of the theory in terms of its massless fields [2,3]. The leading order terms in this expansion is the 11-dimensional supergravity and the next to the leading order terms are at eight-derivative order or ℓ_p^6 -order in which we are interested. There are different techniques to find such couplings [4–14]. One of them is the S-matrix method in which various S-matrix elements are calculated in the M-theory and then they are compared with the corresponding S-matrix elements in the M-theory effective action. Another method is to use the dimensional reduction of the ℓ_p^6 -order couplings on the circle and compare them with the one-loop effective action of the type IIA superstring theory at order α'^3 . To use these methods, one needs to know the independent gauge invariant couplings at order ℓ_p^6 .

To find such independent couplings, one has to impose various Bianchi identities, use field redefinitions freedom [15–17] and remove total derivative terms from the most general gauge invariant couplings. Hence, one should first write all gauge invariant couplings at order ℓ_p^6 and then imposes the above freedoms to reduce them to the minimal couplings. The parameters in the gauge invariant action are either ambiguous or unambiguous depending on whether or not they are changed under these freedoms. Some combinations of the ambiguous parameters, however, remain invariant. This allows one to separate the ambiguous parameters to essential parameters, and some arbitrary parameters. Depending on which set of parameters are choosing as essential parameters and how to choose the arbitrary parameters, one has different schemes. The minimum number of independent couplings are found in the schemes that all the arbitrary parameters are set to zero. This method has been used in [18] to find 60 independent gauge invariant couplings at order α'^2 in the bosonic string theory, and in [19] to find 872 independent NS-NS couplings at order α'^3 in type II superstring theories. These parameters are then fixed in the tree-level effective action by the T-duality [20–22]. We are interested in finding such independent couplings at order ℓ_p^6 in M-theory for bosonic fields. The effective action of M-theory at each order of ℓ_p has two sectors: The Chern-Simons sector which has odd number of the three-form and another sector which has even number of the three-form. Each sector should be invariant under the parity transformation which changes the sign of the three-form [23]. In this paper we are interested in finding the independent couplings in the second sector at order ℓ_p^6 .

The outline of the paper is as follows: In section 2, using the package “xAct” [24], we write the most general gauge invariant couplings involving the 11-dimensional metric $g_{\mu\nu}$ and the three-form $A^{(3)}$ at order ℓ_p^6 . There are 17746 such couplings. Then we add to them the most general total derivative terms and field redefinitions with arbitrary parameters. To impose various Bianchi identities, we rewrite them in the local inertial frame in which the partial derivative of the metric is zero, and rewrite the terms which have derivatives of three-field strength $F^{(4)}$, in terms of potential, *i.e.*, $F^{(4)} = dA^{(3)}$. We then use the arbitrary parameters in the total derivative terms and in the field redefinitions to show that there are only 1062 unambiguous and essential parameters and all other parameters are arbitrary which can be set to zero. We show that there are minimal schemes in which there are 1061 couplings which have no term with more than two

derivatives and no term involving R , $R_{\mu\nu}$, $\nabla_\mu F^{\mu\alpha\beta\gamma}$. There is one essential coupling which has two derivatives on F . We write the explicit form of this coupling, and the couplings with structures $F^6 R$, $F^4 R^2$, $F^2 R^3$, R^4 , $F^2 (\nabla F)^2$, and $(\nabla F)^4$ where R stands for the Riemann curvature, in this section, and the remaining couplings with the structures F^8 , $F^4 (\nabla F)^2$, and $R F^2 (\nabla F)^2$, in Appendix A. In section 3, we briefly discuss the dimensional reduction of the couplings on a circle to find some of the parameters involving four fields by comparing them with the known couplings in the one-loop effective action of type IIA superstring theory. In particular, this comparison dictates that the coupling which has term with more than two derivatives is zero.

2. Minimal couplings at order ℓ_p^6

The bosonic part of the effective action of M-theory has the following derivative-expansion or ℓ_p -expansion where ℓ_p is the 11-dimensional Plank-length:

$$S_{\text{eff}} = \frac{1}{2\kappa_{11}^2} \left[\int d^{11}x \sqrt{-g} (\mathcal{L}_0 + \ell_p^6 \mathcal{L}_6 + \dots) + \int (\mathcal{L}_0^{CS} + \ell_p^6 \mathcal{L}_6^{CS} + \dots) \right] \quad (1)$$

where we have used the fact that the M-theory has no effective action at four and six derivative orders. In fact, as we will argue in the next section, the orders of the derivative terms in the M-theory effective action are at $\ell_p^0, \ell_p^6, \ell_p^{12}, \ell_p^{18}, \ell_p^{24}, \dots$.

The effective action must be invariant under the coordinate transformations and under the A -field gauge transformations. The metric and A -field must appear in the Lagrangian \mathcal{L}_n through their field strengths and their covariant derivatives, *e.g.*, the Lagrangian \mathcal{L} at the leading order of ℓ_p is

$$\mathcal{L}_0 = R - \frac{1}{2 \cdot 4!} F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta} \quad (2)$$

The Chern-Simons form at the leading order is not invariant under the gauge transformation, *i.e.*,

$$\mathcal{L}_0^{CS} = -\frac{1}{6} A^{(3)} \wedge F^{(4)} \wedge F^{(4)} \quad (3)$$

However, its corresponding action is invariant. The 11-dimensional supergravity is invariant under the parity transformation which changes the sign of the A -field [23]. This should be the symmetry of all higher-derivative terms in (1). The parity symmetry then constrains \mathcal{L} to have even number of A -field and \mathcal{L}^{CS} to have odd number of A -field. In this paper, we are interested only on the couplings in \mathcal{L}_6 . A systematic method has been used in [18,19] to find the minimum number of independent couplings at order α'^2 and α'^3 in the effective action of the bosonic string theory. It has been found that there are 60 couplings at order α'^2 and 872 couplings at order α'^3 . In this section, we are going to use this method to find similar couplings in \mathcal{L} at order ℓ_p^6 in the M-theory.

Following [18], one first should write all gauge invariant couplings at eight-derivative order which has even number of the three-form. Using the package “xAct” [24], one finds there are 17746 such couplings in 40 different structures, *i.e.*,

$$L'_6 = m'_1 F_{\alpha\nu}^{\delta\epsilon} F^{\alpha\beta\gamma\nu} F_{\beta\phi}^{\varepsilon\mu} F_{\gamma}^{\zeta\eta\phi} F_{\delta\varepsilon\sigma}^{\theta} F_{\epsilon\zeta}^{\iota\sigma} F_{\mu\iota}^{\kappa\lambda} F_{\eta\theta\kappa\lambda} + \dots \quad (4)$$

where m'_1, \dots, m'_{17746} are some parameters.¹ The above couplings however are not all inde-

¹ Using a computer with 32 GB RAM, the package can generate all couplings excepts the couplings with structure F^8 . The couplings in this structure which include $F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta}$, can be also generated by finding all couplings with structure

pendent. Some of them are related by total derivative terms, some of them are related by field redefinitions, and some others are related by various Bianchi identities.

To remove the total derivative terms from the above couplings, we consider the most general total derivative terms at order ℓ_p^6 which have the following structure:

$$\frac{\ell_p^6}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g} \mathcal{J}_6 = \frac{\ell_p^6}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g} \nabla_\alpha (\mathcal{I}_6^\alpha) \quad (5)$$

where the vector \mathcal{I}_6^α is all possible covariant and gauge invariant terms at seven-derivative level which has even number of the three-form, *i.e.*,

$$\mathcal{I}_6^\alpha = J_1 F^{\gamma\delta\epsilon\mu} R^{\alpha\beta} R_{\beta\epsilon\epsilon\theta} \nabla_\delta F_{\gamma\mu}^{\;\;\;\varepsilon\theta} + \dots \quad (6)$$

where the coefficients J_1, \dots, J_{7760} are 7760 arbitrary parameters. Adding the total derivative terms with arbitrary coefficients to L'_6 , one finds the same Lagrangian but with different parameters m''_1, m''_2, \dots . We call the new Lagrangian L''_6 . Hence

$$\Delta''_6 - \mathcal{J}_6 = 0 \quad (7)$$

where $\Delta''_6 = L''_6 - L'_6$ is the same as L'_6 but with coefficients $\delta m''_1, \delta m''_2, \dots$ where $\delta m''_i = m''_i - m'_i$. Solving the above equation, one finds some linear relations between only $\delta m''_1, \delta m''_2, \dots$ which indicate how the couplings are related among themselves by the total derivative terms. The above equation also gives some relations between the coefficients of the total derivative terms and $\delta m''_1, \delta m''_2, \dots$ in which we are not interested.

The couplings in (7), however, are in a fixed field variables. One is free to change the field variables as

$$\begin{aligned} g_{\mu\nu} &\rightarrow g_{\mu\nu} + \ell_p^6 \delta g_{\mu\nu}^{(6)} \\ A_{\mu\nu\alpha} &\rightarrow A_{\mu\nu\alpha} + \ell_p^6 \delta A_{\mu\nu\alpha}^{(6)} \end{aligned} \quad (8)$$

where the tensors $\delta g_{\mu\nu}^{(6)}$ and $\delta A_{\mu\nu\alpha}^{(6)}$ are all possible covariant and gauge invariant terms at 6-derivative level. The parity symmetry constrains $\delta g_{\mu\nu}^{(6)}$ to have even number of the three-form and $\delta A_{\mu\nu\alpha}^{(6)}$ to have odd number of the three-form, *i.e.*,

$$\begin{aligned} \delta g_{\alpha\beta}^{(6)} &= g_1 F_{\{\alpha}^{\gamma\delta\mu} F_{\beta\}\gamma}^{\;\;\;\epsilon\nu} F_{\delta\mu}^{\;\;\;\varepsilon\theta} F_{\epsilon\epsilon\nu}^{\;\;\;\eta} F_\theta^{\lambda\kappa\sigma} F_{\eta\lambda\kappa\sigma} + \dots \\ \delta A_{\alpha\beta\mu}^{(6)} &= e_1 R^{\gamma\delta} R_{\delta\epsilon\epsilon[\alpha} \nabla_\beta F_{\mu]\gamma}^{\;\;\;\epsilon\epsilon} + \dots \end{aligned} \quad (9)$$

The coefficients g_1, \dots, g_{1987} and e_1, \dots, e_{2679} are arbitrary parameters. When the field variables in \mathcal{L}_6 are changed according to the above field redefinitions, they produce some couplings at orders ℓ_p^8 and higher in which we are not interested in this paper. However, when the field variables in S_0 are changed, up to some total derivative terms, the following couplings at order ℓ_p^6 are produced:

⁶ and then multiplying them with $F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta}$. We found the couplings which have no factor $F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta}$, as follows: We first find all couplings with structure H^8 where H is a three-form. The package can generate such couplings. We then replace each three-form H with the four-form F with one free index. The resulting couplings then each has 8 free indices. We then contract all possible contractions of the free indices, and remove the couplings which have the factor $F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta}$. Note that the number of latter couplings is two less than the number of the couplings constructed by multiplying all contractions of F^6 with $F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta}$.

$$\begin{aligned}
\delta S_0 &= \frac{\delta S_0}{\delta g_{\alpha\beta}} \delta g_{\alpha\beta}^{(3)} + \frac{\delta S_0}{\delta A_{\alpha\beta\mu}} \delta A_{\alpha\beta\mu}^{(3)} \equiv \frac{\ell_p^6}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g} \mathcal{K}_6 \\
&= \frac{\ell_p^6}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g} \left[\left(\frac{1}{6} \nabla_\gamma F^{\gamma\alpha\beta\mu} - \frac{1}{48.144} \epsilon^{\alpha\beta\mu\nu\gamma\lambda\theta\tau\kappa\sigma\zeta} F_{\nu\gamma\lambda\theta} F_{\tau\kappa\sigma\zeta} \right) \delta A_{\alpha\beta\mu}^{(6)} \right. \\
&\quad \left. - (R^{\alpha\beta} - \frac{1}{12} F^{\alpha\gamma\delta\mu} F_{\gamma\delta\mu}^\beta) \delta g_{\alpha\beta}^{(6)} + (\frac{1}{2} R - \frac{1}{48} F_{\alpha\beta\gamma\nu} F^{\alpha\beta\gamma\nu}) \delta g_{\mu\mu}^{(6)} \right]
\end{aligned} \tag{10}$$

Note that if $\delta A_{\mu\nu\alpha}^{(6)}$ included the even number of the A -field, then the couplings in the second line would not be invariant under the parity. The second term in the second line above produces couplings in the Chern-Simons sector in which we are not interested in this paper, hence, we do not consider the effect of field redefinition $\delta A_{\alpha\beta\mu}^{(6)}$ on this term. Adding the total derivative terms and field redefinition terms to L'_6 , one finds the same Lagrangian but with different parameters m_1, m_2, \dots . We call the new Lagrangian \mathcal{L}_6 . Hence

$$\Delta_6 - \mathcal{J}_6 - \mathcal{K}_6 = 0 \tag{11}$$

where $\Delta_6 = \mathcal{L}_6 - L'_6$ is the same as L'_6 but with coefficients $\delta m_1, \delta m_2, \dots$ where $\delta m_i = m_i - m'_i$. Solving the above equation, one finds some linear relations between only $\delta m_1, \delta m_2, \dots$ which indicate how the couplings are related among themselves by the total derivative and field redefinition terms. There are also many relations between $\delta m_1, \delta m_2, \dots$ and the coefficients of total derivative terms and field redefinitions in which we are not interested.

However, to solve the equation (11) one should write it in terms of independent couplings, *i.e.*, one has to impose the following Bianchi identities as well:

$$\begin{aligned}
R_{\alpha[\beta\gamma\delta]} &= 0 \\
\nabla_{[\mu} R_{\alpha\beta]\gamma\delta} &= 0 \\
\nabla_{[\mu} F_{\nu\alpha\beta\gamma]} &= 0 \\
[\nabla, \nabla] \mathcal{O} - R \mathcal{O} &= 0
\end{aligned} \tag{12}$$

To impose the Bianchi identities in non-gauge invariant form, one may rewrite the terms in (11) in the local frame in which the first partial derivative of metric is zero, and rewrite the terms in (11) which have derivatives of F in terms of A -field, *i.e.*, $F = dA$. In this way, the Bianchi identities satisfy automatically [18]. In fact, writing the couplings in terms of potential rather than field strength, there would be no Bianchi identity at all. This way of imposing the Bianchi identities is very easy to perform by the computer.

Using the above steps, one can rewrite the different terms on the left-hand side of (11) in terms of independent but non-gauge invariant couplings. The solution to the equation (11) then has two parts. One part is 1062 relations between only δm_i 's, and the other part is some relations between the coefficients of the total derivative terms, field redefinitions and δm_i 's in which we are not interested. The number of relations in the first part gives the number of independent couplings in \mathcal{L}_6 . In a particular scheme, one may set some of the coefficients in L'_6 to zero, however, after replacing the non-zero terms in (11), the number of relations between only δm_i 's should not be changed, *i.e.*, there must be always 1062 relations. We set the coefficients of the couplings in L'_6 in which each term that has $R, R_{\mu\nu}$ or $\nabla_\mu F^{\mu\nu\alpha\beta}$ to be zero. After setting these coefficients to zero, there are still 1062 relations between δm_i 's. This means we are allowed to remove these terms.

We then try to set zero the couplings in L'_6 which have term with more than two derivatives. Imposing this condition and then solving (11) again, one would find 1061 relations between only δm_i 's. It means that at least one of the independent couplings has terms with more than two derivatives. We have found this independent coupling to be

$$\mathcal{L}_6 \supset m_{315} F_\epsilon^{\mu\nu\sigma} R_{\alpha\gamma}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} \nabla_\sigma \nabla_\delta F_{\beta\epsilon\mu\nu} \quad (13)$$

The way we have found the above coupling is that we divided the couplings involving more than two derivatives to two parts. We then set the coefficients of one part to zero. If the corresponding equations in (11) gives 1062 relations between the remaining δm_i 's then that choice is allowed, otherwise the other part is allowed to be zero. Again we divided the non-zero part to two parts and set half of them to zero. If the corresponding equations in (11) gives 1062 relations between the remaining δm_i 's then that choice is allowed, otherwise the other part is allowed to be zero. Repeating this strategy one finds the above couplings is one of the independent couplings. Apart from the above coupling, all other couplings which have terms with more than two derivatives are allowed to be zero. There are still 3304 couplings which have no term with more than two derivatives and have no terms with structures R , $R_{\mu\nu}$, $\nabla_\mu F^{\mu\nu\alpha\beta}$. Hence, there are still many choices for choosing the non-zero coefficients such that they satisfy the 1062 relations $\delta m_i = 0$. In the particular scheme that we have chosen, there is one coupling appears in (13), and the other 1061 couplings appear in the 9 structures $F^6 R$, $F^4 R^2$, $F^2 R^3$, R^4 , $F^2 (\nabla F)^2$, $(\nabla F)^4$, F^8 , $F^4 (\nabla F)^2$, $R F^2 (\nabla F)^2$.

We have found there are 47 couplings with structure of one Riemann curvature and six F , i.e.,

$$\begin{aligned} \mathcal{L}_6^{F^6 R} = & m_{135} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} F_{\gamma\epsilon}^{\kappa\tau} F_{\delta\nu}^{\omega\varphi} F_{\varepsilon\sigma\kappa\omega} F_{\mu\lambda\tau\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{136} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} F_{\gamma\epsilon\nu}^{\kappa} F_\delta^{\tau\omega\varphi} F_{\varepsilon\sigma\kappa\tau} F_{\mu\lambda\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{144} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon\epsilon}^{\nu} F_\gamma^{\sigma\lambda\kappa} F_\delta^{\tau\omega\varphi} F_{\mu\sigma\lambda\tau} F_{\nu\kappa\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{145} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} F_\varepsilon^{\tau\omega\varphi} F_{\mu\sigma\lambda\tau} F_{\nu\kappa\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{146} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} F_\epsilon^{\sigma\lambda\kappa} F_\varepsilon^{\tau\omega\varphi} F_{\mu\sigma\lambda\tau} F_{\nu\kappa\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{139} F_{\alpha\beta}^{\epsilon\epsilon} F_\gamma^{\mu\nu\sigma} F_\delta^{\lambda\kappa\tau} F_{\epsilon\mu}^{\omega\varphi} F_{\varepsilon\lambda\omega\varphi} F_{\nu\sigma\kappa\tau} R^{\alpha\beta\gamma\delta} + \\ & m_{140} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} F_\delta^{\tau\omega\varphi} F_{\mu\lambda\tau\omega} F_{\nu\sigma\kappa\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{141} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} F_{\delta\lambda}^{\tau\omega\varphi} F_{\mu\tau\omega}^\varphi F_{\nu\sigma\kappa\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{137} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} F_{\gamma\epsilon\epsilon}^{\kappa} F_\delta^{\tau\omega\varphi} F_{\mu\tau\omega\varphi} F_{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} + \\ & m_{138} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} F_\delta^{\tau\omega\varphi} F_{\mu\tau\omega\varphi} F_{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} + \\ & m_{142} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} F_\gamma^{\lambda\kappa\tau} F_{\delta\lambda}^{\omega\varphi} F_{\varepsilon\mu\kappa\tau} F_{\nu\sigma\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{143} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} F_\delta^{\tau\omega\varphi} F_{\mu\lambda\kappa\tau} F_{\nu\sigma\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{153} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon\epsilon}^{\nu} F_\gamma^{\sigma\lambda\kappa} F_{\delta\sigma\lambda}^{\tau} F_{\mu\kappa}^{\omega\varphi} F_{\nu\tau\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{154} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} F_{\varepsilon\sigma\lambda}^{\tau} F_{\mu\kappa}^{\omega\varphi} F_{\nu\tau\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{155} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} F_\epsilon^{\sigma\lambda\kappa} F_{\varepsilon\sigma\lambda}^{\tau} F_{\mu\kappa}^{\omega\varphi} F_{\nu\tau\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{150} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon\epsilon}^{\nu} F_\gamma^{\sigma\lambda\kappa} F_{\delta\sigma}^{\tau\omega} F_{\mu\lambda\kappa}^\varphi F_{\nu\tau\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{151} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} F_{\varepsilon\sigma}^{\tau\omega} F_{\mu\lambda\kappa}^\varphi F_{\nu\tau\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{152} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} F_\epsilon^{\sigma\lambda\kappa} F_{\varepsilon\sigma}^{\tau\omega} F_{\mu\lambda\kappa}^\varphi F_{\nu\tau\omega\varphi} R^{\alpha\beta\gamma\delta} + \\ & m_{147} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon\epsilon}^{\nu} F_\gamma^{\sigma\lambda\kappa} F_\delta^{\tau\omega\varphi} F_{\mu\sigma\lambda\kappa} F_{\nu\tau\omega\varphi} R^{\alpha\beta\gamma\delta} + \end{aligned}$$

$$\begin{aligned}
& m_{148} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} F_{\delta}^{\sigma\lambda\kappa} F_{\varepsilon}^{\tau\omega\varphi} F_{\mu\sigma\lambda\kappa} F_{\nu\tau\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{149} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} F_{\epsilon}^{\sigma\lambda\kappa} F_{\varepsilon}^{\tau\omega\varphi} F_{\mu\sigma\lambda\kappa} F_{\nu\tau\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{177} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma}^{\mu\nu\sigma} F_{\delta}^{\lambda\kappa\tau} F_{\epsilon\mu\nu}^{\omega} F_{\varepsilon\lambda\omega}^{\varphi} F_{\sigma\kappa\tau\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{176} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma}^{\mu\nu\sigma} F_{\delta}^{\lambda\kappa\tau} F_{\epsilon\mu\nu}^{\omega} F_{\varepsilon\lambda\omega}^{\varphi} F_{\sigma\kappa\tau\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{178} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma}^{\mu\nu\sigma} F_{\delta}^{\lambda\kappa\tau} F_{\epsilon\mu\nu}^{\omega\varphi} F_{\mu\nu\lambda\omega} F_{\sigma\kappa\tau\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{179} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma}^{\mu\nu\sigma} F_{\delta\mu}^{\lambda\kappa} F_{\epsilon\nu}^{\tau\omega} F_{\varepsilon\lambda\tau}^{\varphi} F_{\sigma\kappa\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{156} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta}^{\nu\sigma\lambda} F_{\gamma\epsilon}^{\kappa\tau} F_{\delta\nu}^{\omega\varphi} F_{\varepsilon\mu\omega\varphi} F_{\sigma\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} + \\
& m_{157} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma}^{\mu\nu\sigma} F_{\delta}^{\lambda\kappa\tau} F_{\epsilon\mu}^{\omega\varphi} F_{\varepsilon\mu\nu\omega\varphi} F_{\sigma\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} + \\
& m_{158} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma}^{\mu\nu\sigma} F_{\delta\mu}^{\lambda\kappa} F_{\epsilon\nu}^{\tau\omega} F_{\varepsilon\tau\omega}^{\varphi} F_{\sigma\lambda\kappa\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{159} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta}^{\nu\sigma\lambda} F_{\gamma\epsilon\nu}^{\kappa} F_{\delta}^{\tau\omega\varphi} F_{\mu\nu\tau\omega} F_{\sigma\lambda\kappa\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{160} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} F_{\delta}^{\tau\omega\varphi} F_{\mu\nu\tau\omega} F_{\sigma\lambda\kappa\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{161} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} F_{\delta}^{\sigma\lambda\kappa} F_{\varepsilon}^{\tau\omega\varphi} F_{\mu\nu\tau\omega} F_{\sigma\lambda\kappa\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{162} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta}^{\nu\sigma\lambda} F_{\gamma\epsilon\nu}^{\kappa} F_{\delta\epsilon}^{\tau\omega} F_{\mu\tau\omega}^{\varphi} F_{\sigma\lambda\kappa\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{163} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta}^{\nu\sigma\lambda} F_{\gamma\epsilon\nu}^{\kappa} F_{\delta\nu}^{\tau\omega} F_{\mu\tau\omega}^{\varphi} F_{\sigma\lambda\kappa\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{164} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} F_{\delta\nu}^{\tau\omega} F_{\mu\tau\omega}^{\varphi} F_{\sigma\lambda\kappa\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{165} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} F_{\delta}^{\sigma\lambda\kappa} F_{\epsilon\mu}^{\tau\omega} F_{\nu\tau\omega}^{\varphi} F_{\sigma\lambda\kappa\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{166} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta}^{\nu\sigma\lambda} F_{\gamma\epsilon}^{\kappa\tau} F_{\delta\nu}^{\omega\varphi} F_{\varepsilon\mu\kappa\omega} F_{\sigma\lambda\tau\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{169} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma}^{\mu\nu\sigma} F_{\delta\mu\nu}^{\lambda} F_{\epsilon}^{\kappa\tau\omega} F_{\varepsilon\kappa\tau}^{\varphi} F_{\sigma\lambda\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{167} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta}^{\nu\sigma\lambda} F_{\gamma\epsilon}^{\kappa\tau} F_{\delta\nu}^{\omega\varphi} F_{\varepsilon\mu\kappa\tau} F_{\sigma\lambda\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{168} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta}^{\nu\sigma\lambda} F_{\gamma\epsilon\nu}^{\kappa} F_{\delta}^{\tau\omega\varphi} F_{\varepsilon\mu\kappa\tau} F_{\sigma\lambda\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{171} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta}^{\nu\sigma\lambda} F_{\gamma\epsilon\epsilon\nu} F_{\delta}^{\kappa\tau\omega} F_{\mu\kappa\tau}^{\varphi} F_{\sigma\lambda\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{170} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta}^{\nu\sigma\lambda} F_{\gamma\epsilon\nu}^{\kappa} F_{\delta}^{\tau\omega\varphi} F_{\mu\nu\kappa\tau} F_{\sigma\lambda\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{172} F_{\alpha}^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu} F_{\gamma\mu}^{\sigma\lambda} F_{\delta}^{\kappa\tau\omega} F_{\nu\kappa\tau}^{\varphi} F_{\sigma\lambda\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{173} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} F_{\delta\mu}^{\sigma\lambda} F_{\varepsilon}^{\kappa\tau\omega} F_{\nu\kappa\tau}^{\varphi} F_{\sigma\lambda\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{174} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\mu}^{\sigma\lambda} F_{\varepsilon}^{\kappa\tau\omega} F_{\nu\kappa\tau}^{\varphi} F_{\sigma\lambda\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{175} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\epsilon}^{\sigma\lambda} F_{\mu}^{\kappa\tau\omega} F_{\nu\kappa\tau}^{\varphi} F_{\sigma\lambda\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{180} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\epsilon\mu}^{\sigma} F_{\kappa\tau\omega\varphi} F_{\nu}^{\lambda\kappa\tau} F_{\sigma\lambda}^{\omega\varphi} R^{\alpha\beta\gamma\delta} + \\
& m_{181} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\epsilon\mu\nu} F_{\kappa\tau\omega\varphi} F_{\sigma\lambda}^{\omega\varphi} F_{\sigma\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} \tag{14}
\end{aligned}$$

There are 63 couplings with structure of two Riemann curvatures and four F , i.e.,

$$\begin{aligned}
\mathcal{L}_6^{F^4 R^2} = & m_{182} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} F_{\mu\sigma}^{\kappa\tau} F_{\nu\lambda\kappa\tau} R_{\alpha\gamma\beta\delta} R^{\alpha\beta\gamma\delta} + \\
& m_{183} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} F_{\mu\nu}^{\kappa\tau} F_{\sigma\lambda\kappa\tau} R_{\alpha\gamma\beta\delta} R^{\alpha\beta\gamma\delta} + \\
& m_{184} F_{\epsilon\epsilon\mu}^{\sigma} F^{\epsilon\epsilon\mu\nu} F_{\nu}^{\lambda\kappa\tau} F_{\sigma\lambda\kappa\tau} R_{\alpha\gamma\beta\delta} R^{\alpha\beta\gamma\delta} + \\
& m_{188} F_{\beta}^{\mu\nu\sigma} F_{\delta\mu\nu\sigma} F_{\epsilon}^{\lambda\kappa\tau} F_{\varepsilon\lambda\kappa\tau} R_{\alpha}^{\epsilon} \gamma^{\varepsilon} R^{\alpha\beta\gamma\delta} + \\
& m_{187} F_{\beta}^{\mu\nu\sigma} F_{\delta\mu\nu}^{\lambda} F_{\epsilon\sigma}^{\kappa\tau} F_{\varepsilon\lambda\kappa\tau} R_{\alpha}^{\epsilon} \gamma^{\varepsilon} R^{\alpha\beta\gamma\delta} + \\
& m_{186} F_{\beta}^{\mu\nu\sigma} F_{\delta\mu}^{\lambda\kappa} F_{\epsilon\nu\lambda}^{\tau} F_{\varepsilon\sigma\kappa\tau} R_{\alpha}^{\epsilon} \gamma^{\varepsilon} R^{\alpha\beta\gamma\delta} +
\end{aligned}$$

$$\begin{aligned}
& m_{255} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\epsilon\mu}^{\lambda} F_{\delta\sigma}^{\kappa\tau} F_{\nu\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{256} F_{\alpha\beta\epsilon\epsilon} F_{\gamma\mu}^{\sigma\lambda} F_{\delta\sigma}^{\kappa\tau} F_{\nu\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{261} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\delta\mu\sigma} F_{\varepsilon}^{\lambda\kappa\tau} F_{\nu\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{259} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\delta\sigma}^{\lambda} F_{\varepsilon\mu}^{\kappa\tau} F_{\nu\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{258} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\delta}^{\lambda\kappa} F_{\varepsilon\mu\sigma}^{\tau} F_{\nu\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{260} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\delta\mu}^{\lambda} F_{\varepsilon\sigma}^{\kappa\tau} F_{\nu\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{262} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\delta\varepsilon}^{\lambda} F_{\mu\sigma}^{\kappa\tau} F_{\nu\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{263} F_{\alpha\beta\epsilon\epsilon} F_{\gamma\delta}^{\sigma\lambda} F_{\mu\sigma}^{\kappa\tau} F_{\nu\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{249} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\epsilon\mu}^{\lambda} F_{\delta\lambda}^{\kappa\tau} F_{\nu\sigma\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{248} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\varepsilon}^{\lambda\kappa} F_{\delta\mu\lambda}^{\tau} F_{\nu\sigma\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{250} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\delta}^{\lambda\kappa} F_{\varepsilon\mu\lambda}^{\tau} F_{\nu\sigma\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{247} F_{\alpha\beta}^{\sigma\lambda} F_{\gamma\epsilon\mu}^{\kappa} F_{\delta\varepsilon\kappa}^{\tau} F_{\nu\sigma\lambda\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{264} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\epsilon\mu}^{\lambda} F_{\delta\nu}^{\kappa\tau} F_{\sigma\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{265} F_{\alpha\beta\epsilon\epsilon} F_{\gamma\mu}^{\sigma\lambda} F_{\delta\nu}^{\kappa\tau} F_{\sigma\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{266} F_{\alpha\beta\epsilon}^{\sigma} F_{\gamma\delta\varepsilon}^{\lambda} F_{\mu\nu}^{\kappa\tau} F_{\sigma\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{267} F_{\alpha\beta\epsilon\epsilon} F_{\gamma\delta}^{\sigma\lambda} F_{\mu\nu}^{\kappa\tau} F_{\sigma\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{268} F_{\alpha\beta\epsilon\epsilon} F_{\gamma\delta\mu}^{\sigma} F_{\nu}^{\lambda\kappa\tau} F_{\sigma\lambda\kappa\tau} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{269} F_{\alpha\beta\epsilon\epsilon} F_{\gamma\delta\mu\nu} FFR^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + m_{198} F_{\beta\varepsilon}^{\mu\nu} F_{\delta\epsilon\mu\nu} FFR_{\alpha}^{\epsilon\gamma\delta} R^{\alpha\beta\gamma\delta} + \\
& m_{199} F_{\beta\delta}^{\mu\nu} F_{\epsilon\mu\nu} FFR_{\alpha}^{\epsilon\gamma\delta} R^{\alpha\beta\gamma\delta} + m_{209} F_{\beta\gamma\epsilon}^{\nu} F_{\delta\epsilon\mu\nu} FFR_{\alpha}^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} + \\
& m_{185} FFR^2 R_{\alpha\gamma\beta\delta} R^{\alpha\beta\gamma\delta} \tag{15}
\end{aligned}$$

There are 24 couplings with structure of three Riemann curvatures and two F , i.e.,

$$\begin{aligned}
\mathcal{L}_6^{F^2R^3} = & m_{211} F_{\delta\nu}^{\sigma\lambda} F_{\epsilon\mu\sigma\lambda} R_{\alpha}^{\epsilon\gamma\delta} R_{\beta}^{\mu\epsilon\nu} + m_{212} F_{\delta\mu}^{\sigma\lambda} F_{\epsilon\nu\sigma\lambda} R_{\alpha}^{\epsilon\gamma\delta} R_{\beta}^{\mu\epsilon\nu} + \\
& m_{213} F_{\delta\epsilon}^{\sigma\lambda} F_{\mu\nu\sigma\lambda} R_{\alpha}^{\epsilon\gamma\delta} R_{\beta}^{\mu\epsilon\nu} + m_{215} F_{\varepsilon}^{\nu\sigma\lambda} F_{\mu\nu\sigma\lambda} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\gamma\epsilon\delta}^{\mu} + \\
& m_{216} F_{\delta\nu}^{\sigma\lambda} F_{\epsilon\mu\sigma\lambda} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\gamma}^{\mu\epsilon\nu} + m_{217} F_{\delta\mu}^{\sigma\lambda} F_{\epsilon\nu\sigma\lambda} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\gamma}^{\mu\epsilon\nu} + \\
& m_{218} F_{\delta\varepsilon}^{\sigma\lambda} F_{\mu\nu\sigma\lambda} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\gamma}^{\mu\epsilon\nu} + m_{219} F_{\beta\mu\nu}^{\lambda} F_{\delta\epsilon\sigma\lambda} R_{\alpha}^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} R_{\gamma}^{\nu\epsilon\sigma} + \\
& m_{220} F_{\beta\delta\mu}^{\lambda} F_{\epsilon\nu\sigma\lambda} R_{\alpha}^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} R_{\gamma}^{\nu\epsilon\sigma} + m_{221} F_{\beta\delta\epsilon}^{\lambda} F_{\mu\nu\sigma\lambda} R_{\alpha}^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} R_{\gamma}^{\nu\epsilon\sigma} + \\
& m_{222} F_{\beta\epsilon\epsilon\nu} F_{\delta\mu\sigma\lambda} R_{\alpha}^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} R_{\gamma}^{\nu\sigma\lambda} + m_{223} F_{\beta\delta\epsilon\nu} F_{\epsilon\mu\sigma\lambda} R_{\alpha}^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} R_{\gamma}^{\nu\sigma\lambda} + \\
& m_{224} F_{\varepsilon}^{\nu\sigma\lambda} F_{\mu\nu\sigma\lambda} R_{\alpha\gamma\beta}^{\epsilon} R^{\alpha\beta\gamma\delta} R_{\delta}^{\epsilon\mu} + m_{225} F_{\epsilon\epsilon}^{\sigma\lambda} F_{\mu\nu\sigma\lambda} R_{\alpha\gamma\beta}^{\epsilon} R^{\alpha\beta\gamma\delta} R_{\delta}^{\epsilon\mu\nu} + \\
& m_{231} F_{\beta\mu}^{\sigma\lambda} F_{\delta\nu\sigma\lambda} R_{\alpha\gamma}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\epsilon}^{\mu\epsilon\nu} + m_{232} F_{\beta\delta}^{\sigma\lambda} F_{\mu\nu\sigma\lambda} R_{\alpha\gamma}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\epsilon}^{\mu\epsilon\nu} + \\
& m_{233} F_{\beta\delta\mu}^{\lambda} F_{\epsilon\nu\sigma\lambda} R_{\alpha\gamma}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\epsilon}^{\mu\nu\sigma} + m_{270} F_{\epsilon\epsilon}^{\sigma\lambda} F_{\mu\nu\sigma\lambda} R_{\alpha\gamma\beta\delta} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} + \\
& m_{272} F_{\delta\epsilon\mu}^{\lambda} F_{\epsilon\nu\sigma\lambda} R_{\alpha\gamma\beta}^{\epsilon} R^{\alpha\beta\gamma\delta} R^{\epsilon\mu\nu\sigma} + m_{273} F_{\beta\epsilon\mu\nu} F_{\delta\epsilon\sigma\lambda} R_{\alpha}^{\epsilon\gamma\delta} R^{\alpha\beta\gamma\delta} R^{\mu\nu\sigma\lambda} + \\
& m_{274} F_{\beta\epsilon\mu\nu} F_{\delta\epsilon\sigma\lambda} R_{\alpha}^{\epsilon\gamma\delta} R^{\alpha\beta\gamma\delta} R^{\mu\nu\sigma\lambda} + m_{275} F_{\beta\delta\mu\nu} F_{\epsilon\sigma\lambda} R_{\alpha}^{\epsilon\gamma\delta} R^{\alpha\beta\gamma\delta} R^{\mu\nu\sigma\lambda} + \\
& m_{210} FFR_{\alpha}^{\epsilon\gamma\delta} R^{\alpha\beta\gamma\delta} R_{\beta\epsilon\delta\epsilon} + m_{214} FFR_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_{\gamma\epsilon\delta\epsilon} \tag{16}
\end{aligned}$$

There are 7 couplings with structure of four Riemann curvatures, *i.e.*,

$$\begin{aligned} \mathcal{L}_6^{R^4} = & m_{226} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_\gamma^\mu \epsilon^\nu R_{\delta\mu\nu} + m_{227} R_\alpha^\epsilon \gamma^\epsilon R^{\alpha\beta\gamma\delta} R_\beta^\mu \epsilon^\nu R_{\delta\nu\epsilon\mu} + \\ & m_{228} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_\gamma^\mu \epsilon^\nu R_{\delta\nu\epsilon\mu} + m_{229} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_\gamma^\mu \delta^\nu R_{\epsilon\mu\nu} + \\ & m_{230} R_{\alpha\gamma\beta}^\epsilon R^{\alpha\beta\gamma\delta} R_\delta^{\epsilon\mu\nu} R_{\epsilon\mu\nu} + m_{234} R_\alpha^\epsilon \gamma^\epsilon R^{\alpha\beta\gamma\delta} R_\beta^\mu \delta^\nu R_{\epsilon\nu\epsilon\mu} + \\ & m_{271} R_{\alpha\gamma\beta\delta} R^{\alpha\beta\gamma\delta} R_{\epsilon\mu\nu} R^{\epsilon\mu\nu} \end{aligned} \quad (17)$$

There are 24 couplings with structure of two Riemann curvatures and two ∇F , *i.e.*,

$$\begin{aligned} \mathcal{L}_6^{R^2(\partial F)^2} = & m_{282} R_\alpha^\epsilon \gamma^\epsilon R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_\beta^{\mu\nu\sigma} \nabla_\epsilon F_{\delta\mu\nu\sigma} + m_{283} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_\gamma^{\mu\nu\sigma} \nabla_\epsilon F_{\delta\mu\nu\sigma} + \\ & m_{284} R_\alpha^\epsilon \gamma^\epsilon R^{\alpha\beta\gamma\delta} \nabla_\delta F_\beta^{\mu\nu\sigma} \nabla_\epsilon F_{\epsilon\mu\nu\sigma} + m_{285} R_\alpha^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon}^{\nu\sigma} \nabla_\mu F_{\delta\epsilon\nu\sigma} + \\ & m_{286} R_\alpha^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon}^{\nu\sigma} \nabla_\mu F_{\delta\epsilon\nu\sigma} + m_{287} R_\alpha^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\beta\gamma}^{\nu\sigma} \nabla_\mu F_{\delta\epsilon\nu\sigma} + \\ & m_{292} R_\alpha^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\gamma}^{\nu\sigma} \nabla_\mu F_{\epsilon\epsilon\nu\sigma} + m_{294} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} \nabla_\mu F_{\alpha\beta\epsilon}^\sigma \nabla_\nu F_{\gamma\delta\epsilon\sigma} + \\ & m_{295} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} \nabla_\epsilon F_{\alpha\beta\epsilon}^\sigma \nabla_\nu F_{\gamma\delta\mu\sigma} + m_{296} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} \nabla_\gamma F_{\alpha\beta\epsilon}^\sigma \nabla_\nu F_{\delta\epsilon\mu\sigma} + \\ & m_{316} R^{\alpha\beta\gamma\delta} R^{\epsilon\epsilon\mu\nu} \nabla_\sigma F_{\beta\delta\epsilon\nu} \nabla^\sigma F_{\alpha\gamma\epsilon\mu} + m_{317} R_\alpha^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} \nabla_\mu F_{\delta\epsilon\nu\sigma} \nabla^\sigma F_{\beta\gamma\epsilon}^\nu + \\ & m_{318} R_\alpha^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\gamma\delta\nu\sigma} \nabla^\sigma F_{\beta\epsilon\mu}^\nu + m_{319} R_\alpha^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} \nabla_\nu F_{\gamma\delta\epsilon\sigma} \nabla^\sigma F_{\beta\epsilon\mu}^\nu + \\ & m_{320} R_\alpha^{\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} \nabla_\sigma F_{\gamma\delta\epsilon\nu} \nabla^\sigma F_{\beta\epsilon\mu}^\nu + m_{321} R_\alpha^\epsilon \gamma^\epsilon R^{\alpha\beta\gamma\delta} \nabla_\nu F_{\delta\epsilon\mu\sigma} \nabla^\sigma F_{\beta\epsilon}^{\mu\nu} + \\ & m_{322} R_\alpha^\epsilon \gamma^\epsilon R^{\alpha\beta\gamma\delta} \nabla_\sigma F_{\delta\epsilon\mu\nu} \nabla^\sigma F_{\beta\epsilon}^{\mu\nu} + m_{323} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} \nabla_\nu F_{\epsilon\epsilon\mu\sigma} \nabla^\sigma F_{\gamma\delta}^{\mu\nu} + \\ & m_{324} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} \nabla_\sigma F_{\epsilon\epsilon\mu\nu} \nabla^\sigma F_{\gamma\delta}^{\mu\nu} + m_{325} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} \nabla_\nu F_{\delta\epsilon\mu\sigma} \nabla^\sigma F_{\gamma\epsilon}^{\mu\nu} + \\ & m_{326} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} \nabla_\sigma F_{\delta\epsilon\mu\nu} \nabla^\sigma F_{\gamma\epsilon}^{\mu\nu} + m_{327} R_{\alpha\gamma\beta}^\epsilon R^{\alpha\beta\gamma\delta} \nabla_\nu F_{\epsilon\epsilon\mu\sigma} \nabla^\sigma F_{\delta}^{\epsilon\mu\nu} + \\ & m_{328} R_{\alpha\gamma\beta}^\epsilon R^{\alpha\beta\gamma\delta} \nabla_\sigma F_{\epsilon\epsilon\mu\nu} \nabla^\sigma F_{\delta}^{\epsilon\mu\nu} + \\ & m_{329} R_{\alpha\gamma\beta\delta} R^{\alpha\beta\gamma\delta} \nabla_\sigma F_{\epsilon\epsilon\mu\nu} \nabla^\sigma F^{\epsilon\epsilon\mu\nu} \end{aligned} \quad (18)$$

There are 15 couplings with structure of four ∇F , *i.e.*,

$$\begin{aligned} \mathcal{L}_6^{(\partial F)^4} = & m_{375} \nabla_\epsilon F_{\alpha\beta}^{\epsilon\mu} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\mu\nu\sigma} \nabla^\lambda F_{\gamma\delta}^{\nu\sigma} + \\ & m_{378} \nabla_\epsilon F_{\alpha\beta}^{\epsilon\mu} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\delta\mu\nu\sigma} \nabla^\lambda F_{\gamma\epsilon}^{\nu\sigma} + \\ & m_{384} \nabla_\epsilon F_{\alpha\beta\gamma}^{\epsilon\nu} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\mu\nu\sigma} \nabla^\lambda F_{\delta}^{\mu\nu\sigma} + \\ & m_{385} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla^\epsilon F_{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\mu\nu\sigma} \nabla^\lambda F_\epsilon^{\mu\nu\sigma} + \\ & m_{386} \nabla_\delta F_{\mu\nu\sigma\lambda} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla^\epsilon F_{\alpha\beta\gamma\epsilon} \nabla^\lambda F_\epsilon^{\mu\nu\sigma} + \\ & m_{387} \nabla_\epsilon F_{\alpha\beta\gamma\delta} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\mu\nu\sigma} \nabla^\lambda F^\epsilon\mu\nu\sigma + \\ & m_{379} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\mu\nu\sigma} \nabla^\lambda F_\delta^\epsilon \nabla^\sigma \nabla^\mu F_{\alpha\beta\gamma}^\epsilon + \\ & m_{380} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\mu\nu\sigma} \nabla^\lambda F_\delta^\epsilon \nabla^\sigma \nabla^\mu F_{\alpha\beta\gamma}^\epsilon + \\ & m_{381} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\epsilon\nu\sigma\lambda} \nabla^\lambda F_\delta^\mu \nabla^\sigma \nabla^\mu F_{\alpha\beta\gamma}^\epsilon + \\ & m_{383} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\epsilon\nu\sigma} \nabla^\lambda F_\delta^\mu \nabla^\nu \nabla^\mu F_{\alpha\beta\gamma}^\epsilon + \\ & m_{293} \nabla_\epsilon F_\delta^{\nu\sigma\lambda} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla_\mu F_{\epsilon\nu\sigma\lambda} \nabla^\mu F_{\alpha\beta\gamma}^\epsilon + \\ & m_{374} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\epsilon\mu\sigma} \nabla^\lambda F_{\gamma\delta\nu}^\sigma \nabla^\nu F_{\alpha\beta}^{\epsilon\mu} + \\ & m_{377} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla^\lambda F_{\gamma\epsilon\nu}^\sigma \nabla_\mu F_{\delta\epsilon\sigma\lambda} \nabla^\nu F_{\alpha\beta}^{\epsilon\mu} + \\ & m_{376} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla^\lambda F_{\gamma\epsilon\epsilon}^\sigma \nabla^\nu F_{\alpha\beta}^{\epsilon\mu} \nabla_\sigma F_{\delta\mu\nu\lambda} + \\ & m_{382} \nabla^\epsilon F^{\alpha\beta\gamma\delta} \nabla^\lambda F_{\delta\mu}^{\nu\sigma} \nabla^\mu F_{\alpha\beta\gamma}^\epsilon \nabla_\sigma F_{\epsilon\epsilon\nu\lambda} \end{aligned} \quad (19)$$

We have also found there are 134 couplings with structure F^8 , 530 couplings with structure $F^4(\nabla F)^2$ and 217 couplings with structure $RF^2(\nabla F)^2$ that appear in Appendix A.

Even though the total number of minimal gauge invariant couplings at order ℓ_p^6 are fixed, *i.e.*, 1062, the number of couplings in each structure are not fixed. In different schemes, one may find different structures and different number of couplings in each structure. The above structures and the number of terms in each structure are fixed in the scheme that we have chosen. Note, however, that 104 couplings in the structure $\mathcal{L}_6^{F^8}$ are invariant under field redefinition, Bianchi identities and total derivative terms. They are scheme independent. All other couplings dependent on the scheme that one uses for the couplings. The values of the 1062 parameters may be fixed by various techniques in M-theory.

They may be fixed by reducing the couplings on a circle to produce the type IIA couplings at one-loop. Then one may find the 1062 parameters by calculating various S-matrix elements in the resulting type IIA effective field theory and comparing them with the corresponding S-matrix elements in the type IIA superstring theory which has no arbitrary parameters. In this method one has to calculate in the string theory various S-matrix elements which produces 1062 independent contact terms. In the next section we briefly discuss the dimensional reduction of the couplings on a circle to fix some of the parameters.

3. Reduction on a circle

The dimensional reduction of the 11-dimensional couplings on a circle can be done by using the following Kaluza-Klein (KK) reduction of the metric:

$$g_{\mu\nu} = e^{-2\Phi/3} \begin{pmatrix} G_{ab} + e^{2\Phi} C_a C_b & e^{2\Phi} C_a \\ e^{2\Phi} C_b & e^{2\Phi} \end{pmatrix}; g^{\mu\nu} = e^{2\Phi/3} \begin{pmatrix} G^{ab} & -C^a \\ -C^b & e^{-2\Phi} + C_a C^a \end{pmatrix} \quad (20)$$

where G^{ab} is the inverse of the 10-dimensional metric which raises the index of the R-R vector C_a , and the following reductions for the three-form:

$$A_{abc} = C_{abc}; A_{aby} = B_{ab} \quad (21)$$

where $C^{(3)}$ is the R-R three-form and B is the antisymmetric B -field of the type IIA superstring theory. Using these reduction one can calculate the reduction of different 11-dimensional couplings to the 10 dimensions, *e.g.*, the reduction of the overall factor $\sqrt{-g}$ and the scalar curvature in S_0 are

$$\begin{aligned} \sqrt{-g} &= e^{-8\Phi/3} \sqrt{-G} \\ R &= e^{2\Phi/3} \left(R - \frac{16}{3} \nabla_a \Phi \nabla^a \Phi + \frac{14}{3} \nabla_a \nabla^a \Phi - \frac{1}{2.2!} e^{2\Phi} F_{ab} F^{ab} \right) \end{aligned} \quad (22)$$

where F_{ac} if field strength of the R-R one-form. Up to a total derivative term they produce the standard reduction, *i.e.*,

$$\sqrt{-g} R = e^{-2\Phi} \sqrt{-G} \left(R + 4 \nabla_a \Phi \nabla^a \Phi - \frac{1}{2.2!} e^{2\Phi} F_{ab} F^{ab} \right) \quad (23)$$

The reduction of the coupling in the action S_0 involving the field strength of the three-form is

$$-\frac{1}{2.4!} \sqrt{-g} F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta} = e^{-2\Phi} \sqrt{-G} \left(-\frac{1}{2.3!} H_{abc} H^{abc} - \frac{1}{2.4!} e^{2\Phi} \bar{F}_{abcd} \bar{F}^{abcd} \right) \quad (24)$$

where the R-R field strength $\bar{F}^{(4)}$ is $\bar{F}_{abcd} = F_{abcd} + H_{[abc}C_{d]}$. Note that the dilaton factor indicates that the reduction of S_0 correspond to the sphere-level effective action of type IIA. There are stringy corrections to the sphere-level effective action of type IIA which are related to the non-zero modes of the KK mass spectrum [5].

Using the relation between type IIA coupling g_s , the string length ℓ_s and the 11-dimensional Plank length ℓ_p , *i.e.*, $\ell_p = g_s^{1/3} \ell_s$, and the fact that the dilaton factor in the n_h -loop effective action of type IIA superstring theory is given by $e^{-(2-2n_h)\Phi}$, one finds the relation $6n_h = n$ between the derivative couplings in the M-theory at the level ℓ_p^n , and the n_h -loop couplings in the type IIA theory. Then the allowed couplings in the ℓ_p -expansion are at $n = 0, 6, 12, 18, 24, \dots$. They are correspond to the loop-level couplings in type IIA theory with $n_h = 0, 1, 2, 3, 4, \dots$, respectively. In other words, the couplings at each loop-level has no higher-loop corrections. However, there are stringy corrections at each loop-level which are related to the non-zero modes of the KK mass spectrum.

The reduction of the couplings in S_6 which have no three-form is

$$\begin{aligned} & \frac{\ell_p^6}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g} \mathcal{L}_6^{(R^4)} \\ &= \frac{2\pi R_{11}\ell_p^6}{2\kappa_{11}^2} \int d^{10}x \sqrt{-G} \left[m_{226} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_\gamma^{\mu\epsilon\nu} R_{\delta\mu\nu} \right. \\ &+ m_{228} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_\gamma^{\mu\epsilon\nu} R_{\delta\nu\epsilon\mu} + m_{229} R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_\gamma^{\mu\delta\nu} R_{\epsilon\mu\nu} \\ &+ m_{230} R_{\alpha\gamma\beta}^{\epsilon} R^{\alpha\beta\gamma\delta} R_\delta^{\epsilon\mu\nu} R_{\epsilon\mu\nu} + m_{234} R_\alpha^{\epsilon\gamma} R^{\alpha\beta\gamma\delta} R_\beta^{\mu\delta\nu} R_{\epsilon\nu\mu} \\ &+ m_{271} R_{\alpha\gamma\beta\delta} R^{\alpha\beta\gamma\delta} R_{\epsilon\mu\nu} R^{\epsilon\mu\nu} + m_{227} R_\alpha^{\epsilon\gamma} R^{\alpha\beta\gamma\delta} R_\beta^{\mu\epsilon\nu} R_{\delta\nu\mu} + \dots \left. \right] \end{aligned} \quad (25)$$

where $R_{11} = \ell_s g_s$ is the radius of the circle and dots represent the R-R one-form and the dilaton couplings in the effective action of the type IIA theory. Note that as expected there is no overall dilaton factor which indicates that the above couplings correspond to the tours-level effective action of type IIA. On the other hand, the one-loop gravity couplings in type IIA theory are given in a scheme which includes the Ricci curvature and the Ricci scalar, as [25–27]

$$S_3(G) = \frac{\ell_s^6 g_2^2}{2\kappa^2} \frac{a}{3.27} \int d^{10}x \sqrt{-G} (t_8 t_8 - \frac{1}{4} \epsilon_8 \epsilon_8) R^4 \quad (26)$$

where a is a constant number, $\kappa_{11}^2 = 2\pi \ell_s g_s \kappa^2$ and the tensors $\epsilon_8 \epsilon_8$ and t_8 are defined as

$$\begin{aligned} \epsilon_8^{\mu_1 \dots \mu_8} \epsilon_8^{\nu_1 \dots \nu_8} &= \frac{1}{2} \epsilon_{10}^{\mu_1 \dots \mu_8 \alpha\beta} \epsilon_{10}^{\nu_1 \dots \nu_8 \alpha\beta} \\ t_8^{\mu_1 \dots \mu_8} M_{\mu_1 \mu_2}^1 M_{\mu_3 \mu_4}^2 M_{\mu_5 \mu_6}^3 M_{\mu_7 \mu_8}^4 \\ &= 8 \text{Tr}(M^1 M^2 M^3 M^4) + 8 \text{Tr}(M^1 M^3 M^4 M^2) + 8 \text{Tr}(M^1 M^3 M^2 M^4) \\ &- 2 \text{Tr}(M^1 M^2) \text{Tr}(M^3 M^4) - 2 \text{Tr}(M^1 M^3) \text{Tr}(M^2 M^4) - 2 \text{Tr}(M^1 M^4) \text{Tr}(M^2 M^3) \end{aligned} \quad (27)$$

where M^1, \dots, M^4 are four arbitrary antisymmetric matrices. The Ricci curvature and the Ricci scalar in above couplings can be removed by a field redefinition. The Riemann curvature couplings can then be compared with the couplings in (25). One finds the following parameters for the couplings in (17):

$$m_{227} = 0; m_{226} = m_{228} = -m_{229} = \frac{1}{4} m_{230} = -m_{234} = -4m_{271} = -a \quad (28)$$

The gravity couplings are then fixed as

$$\begin{aligned} \mathcal{L}_6^{R^4} = & a \left[-R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_\gamma^{\mu\epsilon\nu} R_{\delta\mu\epsilon\nu} - R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_\gamma^{\mu\epsilon\nu} R_{\delta\nu\epsilon\mu} \right. \\ & + R_{\alpha\beta}^{\epsilon\epsilon} R^{\alpha\beta\gamma\delta} R_\gamma^{\mu\delta\nu} R_{\epsilon\mu\epsilon\nu} - 4R_{\alpha\gamma\beta}^{\epsilon} R^{\alpha\beta\gamma\delta} R_\delta^{\epsilon\mu\nu} R_{\epsilon\mu\epsilon\nu} \\ & \left. + R_\alpha^{\epsilon\gamma} R^{\alpha\beta\gamma\delta} R_\beta^{\mu\delta\nu} R_{\epsilon\nu\epsilon\mu} + \frac{1}{4} R_{\alpha\gamma\beta\delta} R^{\alpha\beta\gamma\delta} R_{\epsilon\mu\epsilon\nu} R^{\epsilon\epsilon\mu\nu} \right] \end{aligned} \quad (29)$$

The complete one-loop effective action of type IIA for other NS-NS or R-R fields are not known. Hence, the other parameters in the M-theory effective action can not be fixed completely in this way. However, the couplings involving four NS-NS fields are known to be given by (26) in which the Riemann curvature is replaced by the following expression [30]:

$$R_{\mu\nu\alpha\beta} = \mathcal{R}_{\mu\nu\alpha\beta} + \frac{1}{2}\partial_\beta H_{\mu\nu\alpha} - \frac{1}{2}\partial_\alpha H_{\mu\nu\beta} \quad (30)$$

where $\mathcal{R}_{\mu\nu\alpha\beta}$ is the linearized Riemann curvature. The last term in (26) has no effect in four-point functions. One can compare the four-point functions resulting from the first term in (13) with the corresponding four-point functions in the dimensional reduction of the couplings in (13), (18) and (19). This S-matrix constraint fixes the parameter in (13) to be zero, i.e.,

$$m_{315} = 0 \quad (31)$$

and fixes the following relations between the couplings in (18):

$$\begin{aligned} m_{283} &= -a/6 + m_{282}/2, m_{284} = a/3 - m_{282}, m_{286} = -2a + 2m_{285}, m_{287} = a - 2m_{285}, \\ m_{292} &= -a + m_{285}, m_{296} = 2m_{295}, m_{317} = 4a - 4m_{285} - 4m_{294} - 4m_{295}, \\ m_{318} &= -a/2, m_{320} = -a/2 - m_{285}/2 + m_{294} - m_{295}, m_{321} = 3m_{282} - 2m_{285} - 2m_{295}, \\ m_{322} &= -a/2 - 3m_{282}/2 + m_{285}, m_{324} = a/8 + 3m_{282}/8 - m_{285}/4, \\ m_{325} &= m_{285} - m_{294} - 2m_{323}, m_{326} = -3m_{282}/4, m_{327} = a - 6m_{282} + m_{285}, \\ m_{328} &= -2a/3 + 2m_{282}, m_{329} = a/48 - m_{282}/16 \end{aligned} \quad (32)$$

and the following relations between the couplings in (19):

$$\begin{aligned} m_{375} &= a/32 - 9m_{293}/4 - m_{374}/2, m_{379} = -a/48 + 2m_{374}/3 - 2m_{378}/3, \\ m_{381} &= a/24 - m_{377}/6 + 4m_{378}/3 + 2m_{380}, \\ m_{382} &= a/12 - 2m_{376}/3 + 8m_{378}/3 + 4m_{380}, \\ m_{383} &= -a/48 + 2m_{374}/3 + 2m_{378}, m_{384} = -a/48 + m_{293} + 2m_{374}/9, \\ m_{385} &= a/288 + m_{374}/18 - m_{376}/36 + 4m_{378}/9 + m_{380}/3, \\ m_{386} &= 5a/144 + m_{293} - 2m_{374}/9 - m_{376}/9 + 10m_{378}/9 + 4m_{380}/3, \\ m_{387} &= a/576 - m_{293}/16 - m_{374}/48 \end{aligned} \quad (33)$$

It is extremely difficult to fix all 1062 parameters by the S-matrix method. One may use symmetries of the effective action to fix them all.

The sphere-level gravity couplings in type II theory at order α'^3 is known to be

$$\int d^{10}x \sqrt{-G} e^{-2\Phi} (t_8 t_8 + \frac{1}{4}\epsilon_8 \epsilon_8) R^4 \quad (34)$$

In this case the reduction of the classical theory on a circle has a $O(1, 1)$ -symmetry [28,29]. This symmetry may be used to find all couplings in type II effective action. In fact the Z_2 -subgroup of this symmetry has been used in [21] to fix all 872 parameters of the NS-NS couplings. There is no such bosonic symmetry in the one-loop effective action. The supersymmetry of the effective actions, however, exists in the classical and loop levels. It has been shown in [4,12] that the R^4 couplings and the Chern-Simons couplings $C \wedge R \wedge R \wedge R$ transform into each other under the supersymmetry transformations. It would be interesting to impose the supersymmetry constraint to fix all 1062 parameters in \mathcal{L}_6 and also the parameters in the Chern-Simons sector \mathcal{L}_6^{CS} .

CRediT authorship contribution statement

Mohammad R. Garousi: Conceptualization, Methodology, Software, Writing, Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

In this appendix we write the independent couplings with the structures F^8 , $F^4(\nabla F)^2$, and $RF^2(\nabla F)^2$. There are 134 couplings with structure F^8 , i.e.,

$$\begin{aligned} \mathcal{L}_6^{F^8} = & m_1 F_\alpha^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_\beta^{\nu\sigma\lambda} F_\gamma^{\kappa\tau\omega} F_\delta^{\varphi\xi\zeta} F_{\epsilon\nu\kappa\varphi} F_{\varepsilon\sigma\tau\xi} F_{\mu\lambda\omega\zeta} + \\ & m_{28} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} F_\varepsilon^{\tau\omega\varphi} F_{\kappa\omega\varphi\zeta} F_{\mu\sigma\tau}^\xi F_{\nu\lambda\xi}^\zeta + \\ & m_{39} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} F_\varepsilon^{\tau\omega\varphi} F_{\kappa\varphi\xi\zeta} F_{\mu\sigma\tau}^\xi F_{\nu\lambda\omega}^\zeta + \\ & m_{40} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_\epsilon^{\sigma\lambda\kappa} F_\varepsilon^{\tau\omega\varphi} F_{\kappa\varphi\xi\zeta} F_{\mu\sigma\tau}^\xi F_{\nu\lambda\omega}^\zeta + \\ & m_{29} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} F_\varepsilon^{\tau\omega\varphi} F_{\kappa\omega\varphi\zeta} F_{\mu\sigma\lambda}^\xi F_{\nu\tau\xi}^\zeta + \\ & m_{31} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} F_{\varepsilon\sigma}^{\tau\omega} F_{\kappa\omega\xi\zeta} F_{\mu\lambda}^\varphi F_{\nu\tau\varphi}^\zeta + \\ & m_{32} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_\epsilon^{\sigma\lambda\kappa} F_{\varepsilon\sigma}^{\tau\omega} F_{\kappa\omega\xi\zeta} F_{\mu\lambda}^\varphi F_{\nu\tau\varphi}^\zeta + \\ & m_{41} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_\epsilon^{\sigma\lambda\kappa} F_\varepsilon^{\tau\omega\varphi} F_{\kappa\varphi\xi\zeta} F_{\mu\sigma\lambda}^\xi F_{\nu\tau\omega}^\zeta + \\ & m_{18} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} F_{\varepsilon\sigma}^{\tau\omega} F_{\kappa\tau\omega\xi} F_{\mu\lambda}^\varphi F_{\nu\varphi\xi}^\zeta + \\ & m_{42} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} F_\varepsilon^{\tau\omega\varphi} F_{\kappa\varphi\xi\zeta} F_{\mu\sigma\lambda\tau} F_{\nu\omega}^\xi F_{\nu\omega}^\zeta + \\ & m_{43} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_\epsilon^{\sigma\lambda\kappa} F_\varepsilon^{\tau\omega\varphi} F_{\kappa\varphi\xi\zeta} F_{\mu\sigma\lambda\tau} F_{\nu\omega}^\xi F_{\nu\omega}^\zeta + \\ & m_{23} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_\epsilon^{\sigma\lambda\kappa} F_{\varepsilon\sigma\lambda}^\tau F_{\kappa\tau\xi\zeta} F_\mu^{\omega\varphi\xi} F_{\nu\omega\varphi}^\zeta + \\ & m_{24} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_{\delta\mu}^{\sigma\lambda} F_{\varepsilon\sigma}^{\kappa\tau} F_{\kappa\tau\xi\zeta} F_{\lambda\omega\varphi}^\zeta F_\nu^{\omega\varphi\xi} + \\ & m_{25} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\mu}^{\sigma\lambda} F_{\varepsilon\sigma}^{\kappa\tau} F_{\kappa\tau\xi\zeta} F_{\lambda\omega\varphi}^\zeta F_\nu^{\omega\varphi\xi} + \\ & m_{26} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_{\delta\varepsilon}^{\sigma\lambda} F_{\kappa\tau\xi\zeta} F_{\lambda\omega\varphi}^\zeta F_{\mu\sigma}^{\kappa\tau} F_\nu^{\omega\varphi\xi} + \\ & m_{27} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\varepsilon}^{\sigma\lambda} F_{\kappa\tau\xi\zeta} F_{\lambda\omega\varphi}^\zeta F_{\mu\sigma}^{\kappa\tau} F_\nu^{\omega\varphi\xi} + \\ & m_5 F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_{\delta\mu}^{\sigma\lambda} F_\varepsilon^{\kappa\tau\omega} F_{\lambda\omega\xi\zeta} F_\nu^{\varphi\xi\zeta} F_{\sigma\kappa\tau\varphi} + \end{aligned}$$

$$\begin{aligned}
& m_{123} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_{\delta\mu}^{\sigma\lambda} F_{\varepsilon\sigma}^{\kappa\tau} F_{\nu\lambda\kappa\tau} FF + \\
& m_{124} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\mu}^{\sigma\lambda} F_{\varepsilon\sigma}^{\kappa\tau} F_{\nu\lambda\kappa\tau} FF + \\
& m_{125} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_{\delta\epsilon}^{\sigma\lambda} F_{\mu\sigma}^{\kappa\tau} F_{\nu\lambda\kappa\tau} FF + \\
& m_{126} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\epsilon}^{\sigma\lambda} F_{\mu\sigma}^{\kappa\tau} F_{\nu\lambda\kappa\tau} FF + \\
& m_{127} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\mu}^{\sigma\lambda} F_{\varepsilon\nu}^{\kappa\tau} F_{\sigma\lambda\kappa\tau} FF + \\
& m_{128} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\epsilon}^{\sigma\lambda} F_{\mu\nu}^{\kappa\tau} F_{\sigma\lambda\kappa\tau} FF + \\
& m_{129} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_{\delta}^{\epsilon\mu\nu} F_{\epsilon\epsilon}^{\sigma\lambda} F_{\mu\nu}^{\kappa\tau} F_{\sigma\lambda\kappa\tau} FF + \\
& m_{133} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta\epsilon}^{\kappa} F_{\varepsilon}^{\lambda\mu\nu} F_{\kappa\lambda}^{\sigma\tau} F_{\mu\nu\sigma\tau} FF + \\
& m_{134} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta\epsilon\epsilon} F_{\kappa\lambda}^{\sigma\tau} F_{\kappa\lambda}^{\mu\nu} F_{\mu\nu\sigma\tau} FF + \\
& m_{130} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_{\delta\epsilon\mu\nu} FF^2 + \\
& m_{131} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\epsilon\mu\nu} FF^2 + m_{132} FF^4
\end{aligned} \tag{35}$$

There are 530 couplings with structure of four F and two ∇F , i.e.,

$$\begin{aligned}
\mathcal{L}_6^{F^4(\partial F)^2} = & m_{433} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma}^{\mu\nu\sigma} F_{\mu\nu}^{\lambda\kappa} \nabla_{\epsilon} F_{\delta\sigma}^{\tau\omega} \nabla_{\kappa} F_{\varepsilon\lambda\tau\omega} + \\
& m_{464} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_{\epsilon\mu}^{\lambda\kappa} F^{\epsilon\mu\nu\sigma} \nabla_{\epsilon} F_{\delta\nu}^{\tau\omega} \nabla_{\kappa} F_{\sigma\lambda\tau\omega} + \\
& m_{465} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma}^{\mu\nu\sigma} F_{\mu\nu}^{\lambda\kappa} \nabla_{\epsilon} F_{\delta\epsilon}^{\tau\omega} \nabla_{\kappa} F_{\sigma\lambda\tau\omega} + \\
& m_{466} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma}^{\mu\nu\sigma} F_{\epsilon\mu}^{\lambda\kappa} \nabla_{\epsilon} F_{\delta\nu}^{\tau\omega} \nabla_{\kappa} F_{\sigma\lambda\tau\omega} + \\
& m_{431} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma}^{\mu\nu\sigma} F_{\mu\nu}^{\lambda\kappa} \nabla_{\kappa} F_{\varepsilon\sigma\tau\omega} \nabla_{\lambda} F_{\delta\epsilon}^{\tau\omega} + \\
& m_{394} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\mu\nu}^{\kappa\tau} F^{\mu\nu\sigma\lambda} \nabla_{\kappa} F_{\gamma\epsilon\sigma}^{\omega} \nabla_{\lambda} F_{\delta\epsilon\tau\omega} + \\
& m_{396} F_{\alpha}^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_{\beta}^{\nu\sigma\lambda} F_{\epsilon}^{\kappa\tau\omega} \nabla_{\kappa} F_{\gamma\epsilon\nu\sigma} \nabla_{\lambda} F_{\delta\mu\tau\omega} + \\
& m_{397} F_{\alpha}^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_{\beta}^{\nu\sigma\lambda} F_{\epsilon\nu}^{\kappa\tau} \nabla_{\kappa} F_{\gamma\epsilon\sigma}^{\omega} \nabla_{\lambda} F_{\delta\mu\tau\omega} + \\
& m_{421} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_{\epsilon\mu}^{\lambda\kappa} F^{\epsilon\mu\nu\sigma} \nabla_{\kappa} F_{\epsilon\sigma\tau\omega} \nabla_{\lambda} F_{\delta\nu}^{\tau\omega} + \\
& m_{432} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma}^{\mu\nu\sigma} F_{\epsilon\mu}^{\lambda\kappa} \nabla_{\kappa} F_{\varepsilon\sigma\tau\omega} \nabla_{\lambda} F_{\delta\nu}^{\tau\omega} + \\
& m_{453} F_{\alpha}^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} \nabla_{\kappa} F_{\mu\sigma\tau\omega} \nabla_{\lambda} F_{\delta\nu}^{\tau\omega} + \\
& m_{450} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_{\delta}^{\epsilon\mu\nu} F_{\varepsilon}^{\sigma\lambda\kappa} \nabla_{\kappa} F_{\mu\nu\tau\omega} \nabla_{\lambda} F_{\epsilon\sigma}^{\tau\omega} + \\
& m_{451} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_{\delta}^{\sigma\lambda\kappa} \nabla_{\kappa} F_{\mu\nu\tau\omega} \nabla_{\lambda} F_{\varepsilon\sigma}^{\tau\omega} + \\
& m_{452} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_{\epsilon}^{\sigma\lambda\kappa} \nabla_{\kappa} F_{\mu\nu\tau\omega} \nabla_{\lambda} F_{\varepsilon\sigma}^{\tau\omega} + \\
& m_{363} F_{\alpha}^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_{\beta}^{\nu\sigma\lambda} F_{\epsilon}^{\kappa\tau\omega} \nabla_{\epsilon} F_{\gamma\delta\nu\sigma} \nabla_{\lambda} F_{\mu\kappa\tau\omega} + \\
& m_{371} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_{\epsilon\mu\nu}^{\lambda} F^{\epsilon\mu\nu\sigma} \nabla_{\epsilon} F_{\delta}^{\kappa\tau\omega} \nabla_{\lambda} F_{\sigma\kappa\tau\omega} + \\
& m_{372} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma}^{\mu\nu\sigma} F_{\epsilon\mu\nu}^{\lambda} \nabla_{\epsilon} F_{\delta}^{\kappa\tau\omega} \nabla_{\lambda} F_{\sigma\kappa\tau\omega} + \\
& m_{373} F_{\alpha}^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} \nabla_{\lambda} F_{\sigma\kappa\tau\omega} \nabla_{\mu} F_{\delta}^{\kappa\tau\omega} + \\
& m_{395} F_{\alpha}^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_{\beta}^{\nu\sigma\lambda} F_{\epsilon}^{\kappa\tau\omega} \nabla_{\kappa} F_{\gamma\epsilon\nu\sigma} \nabla_{\mu} F_{\delta\lambda\tau\omega} + \\
& m_{467} F_{\alpha}^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} \nabla_{\kappa} F_{\sigma\lambda\tau\omega} \nabla_{\mu} F_{\delta\nu}^{\tau\omega} + \\
& m_{456} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_{\delta}^{\epsilon\mu\nu} F_{\varepsilon}^{\sigma\lambda\kappa} \nabla_{\kappa} F_{\nu\lambda\tau\omega} \nabla_{\mu} F_{\epsilon\sigma}^{\tau\omega} + \\
& m_{457} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_{\delta}^{\sigma\lambda\kappa} \nabla_{\kappa} F_{\nu\lambda\tau\omega} \nabla_{\mu} F_{\varepsilon\sigma}^{\tau\omega} + \\
& m_{458} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_{\epsilon}^{\sigma\lambda\kappa} \nabla_{\kappa} F_{\nu\lambda\tau\omega} \nabla_{\mu} F_{\varepsilon\sigma}^{\tau\omega} +
\end{aligned}$$

$$\begin{aligned}
& m_{468} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_{\epsilon\mu}^{\lambda\kappa} \nabla_\kappa F_{\sigma\lambda\tau\omega} \nabla_\nu F_{\delta\epsilon}^{\tau\omega} + \\
& m_{430} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_{\epsilon\mu}^{\lambda\kappa} \nabla_\kappa F_{\varepsilon\sigma\tau\omega} \nabla_\nu F_{\delta\lambda}^{\tau\omega} + \\
& m_{434} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\mu^{\sigma\lambda\kappa} \nabla_\kappa F_{\varepsilon\lambda\tau\omega} \nabla_\nu F_{\delta\sigma}^{\tau\omega} + \\
& m_{469} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_{\delta\mu}^{\lambda\kappa} \nabla_\kappa F_{\sigma\lambda\tau\omega} \nabla_\nu F_{\epsilon\epsilon}^{\tau\omega} + \\
& m_{359} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\mu\nu}^{\kappa\tau} F^{\mu\nu\sigma\lambda} \nabla_\lambda F_{\varepsilon\kappa\tau\omega} \nabla_\sigma F_{\gamma\delta\epsilon}^{\omega} + \\
& m_{364} F_\alpha^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_\beta^{\nu\sigma\lambda} F_\gamma^{\kappa\tau\omega} \nabla_\lambda F_{\mu\kappa\tau\omega} \nabla_\sigma F_{\delta\epsilon\epsilon\nu} + \\
& m_{435} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_{\mu\nu}^{\lambda\kappa} \nabla_\kappa F_{\varepsilon\lambda\tau\omega} \nabla_\sigma F_{\delta\epsilon}^{\tau\omega} + \\
& m_{459} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\mu^{\sigma\lambda\kappa} \nabla_\kappa F_{\nu\lambda\tau\omega} \nabla_\sigma F_{\delta\epsilon}^{\tau\omega} + \\
& m_{354} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_{\varepsilon\mu\nu}^{\lambda} F^{\varepsilon\mu\nu\sigma} \nabla_\lambda F_{\epsilon\kappa\tau\omega} \nabla_\sigma F_{\delta}^{\kappa\tau\omega} + \\
& m_{360} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_{\epsilon\mu\nu}^{\lambda} \nabla_\lambda F_{\varepsilon\kappa\tau\omega} \nabla_\sigma F_{\delta}^{\kappa\tau\omega} + \\
& m_{422} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_{\varepsilon\mu}^{\lambda\kappa} F^{\varepsilon\mu\nu\sigma} \nabla_\kappa F_{\epsilon\lambda\tau\omega} \nabla_\sigma F_{\delta\nu}^{\tau\omega} + \\
& m_{436} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_{\epsilon\mu}^{\lambda\kappa} \nabla_\kappa F_{\varepsilon\lambda\tau\omega} \nabla_\sigma F_{\delta\nu}^{\tau\omega} + \\
& m_{437} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\mu^{\sigma\lambda\kappa} \nabla_\kappa F_{\varepsilon\lambda\tau\omega} \nabla_\sigma F_{\delta\nu}^{\tau\omega} + \\
& m_{454} F_\alpha^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} \nabla_\kappa F_{\mu\lambda\tau\omega} \nabla_\sigma F_{\delta\nu}^{\tau\omega} + \\
& m_{418} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_\mu^{\lambda\kappa\tau} \nabla_\kappa F_{\delta\nu\lambda}^{\omega} \nabla_\sigma F_{\epsilon\epsilon\tau\omega} + \\
& m_{333} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F^{\varepsilon\mu\nu\sigma} F^{\lambda\kappa\tau\omega} \nabla_\lambda F_{\delta\epsilon\mu\nu} \nabla_\sigma F_{\epsilon\kappa\tau\omega} + \\
& m_{365} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_\delta^{\varepsilon\mu\nu} F_{\varepsilon\mu}^{\sigma\lambda} \nabla_\lambda F_{\nu\kappa\tau\omega} \nabla_\sigma F_{\epsilon}^{\kappa\tau\omega} + \\
& m_{460} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_\delta^{\varepsilon\mu\nu} F_\varepsilon^{\sigma\lambda\kappa} \nabla_\kappa F_{\nu\lambda\tau\omega} \nabla_\sigma F_{\epsilon\mu}^{\tau\omega} + \\
& m_{438} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_{\delta\mu}^{\lambda\kappa} \nabla_\kappa F_{\varepsilon\lambda\tau\omega} \nabla_\sigma F_{\epsilon\nu}^{\tau\omega} + \\
& m_{351} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_{\epsilon\mu\nu}^{\lambda} \nabla_\lambda F_\delta^{\kappa\tau\omega} \nabla_\sigma F_{\epsilon\kappa\tau\omega} + \\
& m_{349} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_{\epsilon\mu}^{\lambda\kappa} \nabla_\lambda F_{\delta\nu}^{\tau\omega} \nabla_\sigma F_{\epsilon\kappa\tau\omega} + \\
& m_{353} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_{\delta\mu}^{\lambda\kappa} \nabla_\lambda F_{\epsilon\nu}^{\tau\omega} \nabla_\sigma F_{\epsilon\kappa\tau\omega} + \\
& m_{311} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_\mu^{\lambda\kappa\tau} \nabla_\nu F_{\delta\epsilon\lambda}^{\omega} \nabla_\sigma F_{\epsilon\kappa\tau\omega} + \\
& m_{366} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} \nabla_\lambda F_{\nu\kappa\tau\omega} \nabla_\sigma F_{\epsilon}^{\kappa\tau\omega} + \\
& m_{461} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_\delta^{\sigma\lambda\kappa} \nabla_\kappa F_{\nu\lambda\tau\omega} \nabla_\sigma F_{\epsilon\mu}^{\tau\omega} + \\
& m_{462} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_\delta^{\varepsilon\mu\nu} F_\epsilon^{\sigma\lambda\kappa} \nabla_\kappa F_{\nu\lambda\tau\omega} \nabla_\sigma F_{\epsilon\mu}^{\tau\omega} + \\
& m_{401} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_\mu^{\lambda\kappa\tau} \nabla_\kappa F_{\delta\epsilon\lambda}^{\omega} \nabla_\sigma F_{\epsilon\nu\tau\omega} + \\
& m_{400} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F^{\lambda\kappa\tau\omega} \nabla_\kappa F_{\delta\epsilon\mu\lambda} \nabla_\sigma F_{\epsilon\nu\tau\omega} + \\
& m_{410} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_\gamma^{\mu\nu\sigma} F_\epsilon^{\lambda\kappa\tau} \nabla_\kappa F_{\delta\mu\lambda}^{\omega} \nabla_\sigma F_{\epsilon\nu\tau\omega} + \\
& m_{455} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta\epsilon}^{\mu} F^{\nu\sigma\lambda\kappa} \nabla_\kappa F_{\mu\lambda\tau\omega} \nabla_\sigma F_{\epsilon\nu}^{\tau\omega} + \\
& m_{350} F_\alpha^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_{\beta\epsilon}^{\nu\sigma} F_{\gamma\epsilon}^{\lambda\kappa} \nabla_\lambda F_{\delta\nu}^{\tau\omega} \nabla_\sigma F_{\mu\kappa\tau\omega} + \\
& m_{367} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\epsilon}^{\mu\nu} F_{\delta\epsilon}^{\sigma\lambda} \nabla_\lambda F_{\nu\kappa\tau\omega} \nabla_\sigma F_{\mu}^{\kappa\tau\omega} + \\
& m_{368} F_{\alpha\beta}^{\epsilon\epsilon} F^{\alpha\beta\gamma\delta} F_{\gamma\delta}^{\mu\nu} F_{\epsilon\epsilon}^{\sigma\lambda} \nabla_\lambda F_{\nu\kappa\tau\omega} \nabla_\sigma F_{\mu}^{\kappa\tau\omega} + \\
& m_{369} F_{\alpha\beta\gamma}^{\epsilon} F^{\alpha\beta\gamma\delta} F_\delta^{\varepsilon\mu\nu} F_{\epsilon\epsilon}^{\sigma\lambda} \nabla_\lambda F_{\nu\kappa\tau\omega} \nabla_\sigma F_{\mu}^{\kappa\tau\omega} + \\
& m_{314} F_\alpha^{\epsilon\epsilon\mu} F^{\alpha\beta\gamma\delta} F_{\beta\epsilon}^{\nu\sigma} F^{\lambda\kappa\tau\omega} \nabla_\varepsilon F_{\gamma\delta\lambda\kappa} \nabla_\sigma F_{\mu\nu\tau\omega} +
\end{aligned}$$

$$\begin{aligned}
& m_{658} F^{\epsilon\epsilon\mu\nu} F^{\sigma\lambda\kappa\tau} FF\nabla_\sigma F_{\epsilon\epsilon\mu}^\omega \nabla_\tau F_{\nu\lambda\kappa\omega} + \\
& m_{928} F^{\epsilon\epsilon\mu\nu} F^{\sigma\lambda\kappa\tau} FF\nabla_\omega F_{\sigma\lambda\kappa\tau} \nabla^\omega F_{\epsilon\epsilon\mu\nu} + \\
& m_{931} F^{\epsilon\epsilon\mu\nu} F^{\sigma\lambda\kappa\tau} FF\nabla_\tau F_{\nu\lambda\kappa\omega} \nabla^\omega F_{\epsilon\epsilon\mu\sigma} + \\
& m_{934} F^{\epsilon\epsilon\mu\nu} F^{\sigma\lambda\kappa\tau} FF\nabla_\omega F_{\nu\lambda\kappa\tau} \nabla^\omega F_{\epsilon\epsilon\mu\sigma} + \\
& m_{945} F^{\epsilon\epsilon\mu\nu} F^{\sigma\lambda\kappa\tau} FF\nabla_\omega F_{\mu\nu\kappa\tau} \nabla^\omega F_{\epsilon\epsilon\sigma\lambda} + \\
& m_{982} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} FF\nabla_\tau F_{\sigma\lambda\kappa\omega} \nabla^\omega F_{\epsilon\mu\nu}^\tau + \\
& m_{986} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} FF\nabla_\omega F_{\sigma\lambda\kappa\tau} \nabla^\omega F_{\epsilon\mu\nu}^\tau + \\
& m_{991} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} FF\nabla_\kappa F_{\nu\lambda\tau\omega} \nabla^\omega F_{\epsilon\mu\sigma}^\tau + \\
& m_{995} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} FF\nabla_\tau F_{\nu\lambda\kappa\omega} \nabla^\omega F_{\epsilon\mu\sigma}^\tau + \\
& m_{999} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} FF\nabla_\omega F_{\nu\lambda\kappa\tau} \nabla^\omega F_{\epsilon\mu\sigma}^\tau + \\
& m_{1030} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} FF\nabla_\tau F_{\sigma\lambda\kappa\omega} \nabla^\omega F_{\mu\nu}^{\kappa\tau} + \\
& m_{1036} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} FF\nabla_\omega F_{\sigma\lambda\kappa\tau} \nabla^\omega F_{\mu\nu}^{\kappa\tau} + \\
& m_{1043} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} FF\nabla_\tau F_{\nu\lambda\kappa\omega} \nabla^\omega F_{\mu\sigma}^{\kappa\tau} + \\
& m_{1047} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} FF\nabla_\omega F_{\nu\lambda\kappa\tau} \nabla^\omega F_{\mu\sigma}^{\kappa\tau} + \\
& m_{1054} F_{\epsilon\epsilon\mu}^\sigma F^{\epsilon\epsilon\mu\nu} FF\nabla_\tau F_{\sigma\lambda\kappa\omega} \nabla^\omega F_v^{\lambda\kappa\tau} + \\
& m_{1058} F_{\epsilon\epsilon\mu}^\sigma F^{\epsilon\epsilon\mu\nu} FF\nabla_\omega F_{\sigma\lambda\kappa\tau} \nabla^\omega F_v^{\lambda\kappa\tau} + \\
& m_{1062} FF^2 \nabla_\omega F_{\sigma\lambda\kappa\tau} \nabla^\omega F^{\sigma\lambda\kappa\tau}
\end{aligned} \tag{36}$$

Finally, there are 217 couplings with structure of one Riemann curvature, two F and two ∇F , i.e.,

$$\begin{aligned}
\mathcal{L}_6^{RF^2(\partial F)^2} = & m_{276} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\epsilon\mu\sigma} \nabla_\delta F_{\beta\nu\lambda\kappa} + \\
& m_{277} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\mu\sigma}^\kappa \nabla_\delta F_{\beta\nu\lambda\kappa} + \\
& m_{278} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\epsilon\mu\nu} \nabla_\delta F_{\beta\sigma\lambda\kappa} + \\
& m_{279} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\mu\nu}^\kappa \nabla_\delta F_{\beta\sigma\lambda\kappa} + \\
& m_{280} F_{\epsilon\epsilon\mu}^\sigma F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\nu}^{\lambda\kappa} \nabla_\delta F_{\beta\sigma\lambda\kappa} + \\
& m_{388} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\beta\nu\sigma} \nabla_\kappa F_{\gamma\delta\epsilon\mu} + \\
& m_{389} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\beta\epsilon\nu\sigma} \nabla_\kappa F_{\gamma\delta\mu\lambda} + \\
& m_{398} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\nu\sigma\lambda} \nabla_\kappa F_{\delta\epsilon\mu} + \\
& m_{402} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon\nu\sigma} \nabla_\kappa F_{\delta\epsilon\mu\lambda} + \\
& m_{403} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\beta\gamma\nu\sigma} \nabla_\kappa F_{\delta\epsilon\mu\lambda} + \\
& m_{406} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\beta\epsilon\sigma} \nabla_\kappa F_{\delta\mu\nu\lambda} + \\
& m_{407} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon\epsilon\nu} \nabla_\kappa F_{\delta\mu\sigma\lambda} + \\
& m_{408} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\beta\gamma\epsilon\nu} \nabla_\kappa F_{\delta\mu\sigma\lambda} + \\
& m_{411} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\beta\epsilon\mu} \nabla_\kappa F_{\delta\nu\sigma\lambda} + \\
& m_{412} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\beta\mu}^\kappa \nabla_\kappa F_{\delta\nu\sigma\lambda} + \\
& m_{413} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon\epsilon\mu} \nabla_\kappa F_{\delta\nu\sigma\lambda} +
\end{aligned}$$

$$\begin{aligned}
& m_{414} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon\mu}^\kappa \nabla_\kappa F_{\delta\nu\sigma\lambda} + \\
& m_{415} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\mu}^{\lambda\kappa} \nabla_\kappa F_{\delta\nu\sigma\lambda} + \\
& m_{419} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\gamma\nu\sigma} \nabla_\kappa F_{\epsilon\epsilon\mu\lambda} + \\
& m_{423} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\gamma\epsilon\nu} \nabla_\kappa F_{\epsilon\mu\sigma\lambda} + \\
& m_{424} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\gamma\nu}^\kappa \nabla_\kappa F_{\epsilon\mu\sigma\lambda} + \\
& m_{426} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\gamma\delta\mu}^\kappa \nabla_\kappa F_{\epsilon\nu\sigma\lambda} + \\
& m_{439} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\gamma\epsilon\epsilon} \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{440} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\gamma\epsilon}^\kappa \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{441} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\gamma}^{\lambda\kappa} \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{442} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\epsilon\epsilon}^\kappa \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{443} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\epsilon}^{\lambda\kappa} \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{444} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\gamma\delta\epsilon}^\kappa \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{445} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\gamma\delta\epsilon}^\kappa \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{446} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\gamma\delta}^{\lambda\kappa} \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{447} F_{\alpha\beta}^{\epsilon\epsilon} F_\epsilon^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\gamma\delta}^{\lambda\kappa} \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{448} F_{\alpha\beta}^{\epsilon\epsilon} F_\gamma^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\delta\epsilon}^{\lambda\kappa} \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{449} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_\delta^{\sigma\lambda\kappa} \nabla_\kappa F_{\mu\nu\sigma\lambda} + \\
& m_{471} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\beta\delta\sigma\lambda} \nabla^\kappa F_{\alpha\gamma\mu\nu} + \\
& m_{473} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\beta\delta\nu\lambda} \nabla^\kappa F_{\alpha\gamma\mu\sigma} + \\
& m_{476} F_{\epsilon\epsilon\mu}^\sigma F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\beta\delta\sigma\lambda} \nabla^\kappa F_{\alpha\gamma\nu}^\lambda + \\
& m_{484} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\epsilon\mu\lambda} \nabla^\kappa F_{\beta\epsilon\nu\sigma} + \\
& m_{486} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\gamma\delta\sigma\lambda} \nabla^\kappa F_{\beta\epsilon\mu\nu} + \\
& m_{492} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\mu\sigma\lambda} \nabla^\kappa F_{\beta\epsilon\nu}^\lambda + \\
& m_{489} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\gamma\delta\mu\lambda} \nabla^\kappa F_{\beta\epsilon\nu\sigma} + \\
& m_{495} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\gamma\delta\sigma\lambda} \nabla^\kappa F_{\beta\mu\nu}^\lambda + \\
& m_{498} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\gamma\delta\epsilon\mu} \nabla^\kappa F_{\beta\nu\sigma\lambda} + \\
& m_{499} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\epsilon\epsilon\mu} \nabla^\kappa F_{\beta\nu\sigma\lambda} + \\
& m_{504} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\gamma\delta\mu\lambda} \nabla^\kappa F_{\beta\nu\sigma}^\lambda + \\
& m_{505} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\epsilon\mu\lambda} \nabla^\kappa F_{\beta\nu\sigma}^\lambda + \\
& m_{509} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon\mu}^\nu R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\gamma\delta\sigma\lambda} \nabla^\kappa F_{\beta\nu}^{\sigma\lambda} + \\
& m_{510} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon\epsilon}^\nu R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\mu\sigma\lambda} \nabla^\kappa F_{\beta\nu}^{\sigma\lambda} + \\
& m_{512} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon\epsilon\mu} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\nu\sigma\lambda} \nabla^\kappa F_\beta^{\nu\sigma\lambda} + \\
& m_{514} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\mu\nu\sigma\lambda} \nabla^\kappa F_{\gamma\delta\epsilon\epsilon} + \\
& m_{515} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\mu\nu\sigma\lambda} \nabla^\kappa F_{\gamma\delta\epsilon\epsilon} + \\
& m_{517} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\epsilon\nu\sigma\lambda} \nabla^\kappa F_{\gamma\delta\epsilon\mu} +
\end{aligned}$$

$$\begin{aligned}
& m_{583} F_{\alpha\beta}^{\epsilon\epsilon} F_\gamma^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\nu\sigma\lambda\kappa} \nabla^\kappa F_{\epsilon\epsilon\mu}^\lambda + \\
& m_{585} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\mu\nu\sigma\lambda} \nabla^\kappa F_{\epsilon\epsilon}^\sigma{}^\lambda + \\
& m_{586} F_{\alpha\beta}^{\epsilon\epsilon} F_\gamma^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\varepsilon\sigma\lambda\kappa} \nabla^\kappa F_{\epsilon\mu\nu}^\lambda + \\
& m_{588} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\varepsilon\nu\sigma\lambda} \nabla^\kappa F_{\epsilon\mu}^\sigma{}^\lambda + \\
& m_{589} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\nu\sigma\lambda\kappa} \nabla^\kappa F_{\epsilon\mu}^\sigma{}^\lambda + \\
& m_{591} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta\epsilon}^\mu R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\mu\nu\sigma\lambda} \nabla^\kappa F_\varepsilon^\nu{}^\sigma{}^\lambda + \\
& m_{592} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon\epsilon}^\mu R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\nu\sigma\lambda\kappa} \nabla^\kappa F_\mu^\nu{}^\sigma{}^\lambda + \\
& m_{593} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta\epsilon\epsilon} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\mu\nu\sigma\lambda} \nabla^\kappa F^{\mu\nu\sigma\lambda} + \\
& m_{399} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\epsilon\mu} \nabla_\lambda F_{\beta\gamma\nu\sigma} + \\
& m_{472} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\alpha\gamma\mu\sigma} \nabla_\lambda F_{\beta\delta\nu\kappa} + \\
& m_{470} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\alpha\gamma\mu\nu} \nabla_\lambda F_{\beta\delta\sigma\kappa} + \\
& m_{475} F_{\epsilon\epsilon\mu}^\sigma F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\alpha\gamma\nu}^\lambda \nabla_\lambda F_{\beta\delta\sigma\kappa} + \\
& m_{330} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\varepsilon F_{\beta\nu\sigma}^\kappa \nabla_\lambda F_{\gamma\delta\mu\kappa} + \\
& m_{488} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu\sigma} \nabla_\lambda F_{\gamma\delta\mu\kappa} + \\
& m_{502} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu\sigma}^\lambda \nabla_\lambda F_{\gamma\delta\mu\kappa} + \\
& m_{485} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\epsilon\mu\nu} \nabla_\lambda F_{\gamma\delta\sigma\kappa} + \\
& m_{494} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\mu\nu}^\lambda \nabla_\lambda F_{\gamma\delta\sigma\kappa} + \\
& m_{507} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon\mu}^{\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu}^{\sigma\lambda} \nabla_\lambda F_{\gamma\delta\sigma\kappa} + \\
& m_{334} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\nu\sigma}^\kappa \nabla_\lambda F_{\delta\epsilon\mu\kappa} + \\
& m_{335} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\epsilon F_{\beta\nu\sigma}^\kappa \nabla_\lambda F_{\delta\epsilon\mu\kappa} + \\
& m_{480} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\gamma\nu\sigma} \nabla_\lambda F_{\delta\epsilon\mu\kappa} + \\
& m_{483} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\epsilon\nu\sigma} \nabla_\lambda F_{\delta\epsilon\mu\kappa} + \\
& m_{503} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu\sigma}^\lambda \nabla_\lambda F_{\delta\epsilon\mu\kappa} + \\
& m_{547} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\epsilon\mu\nu} \nabla_\lambda F_{\delta\epsilon\sigma\kappa} + \\
& m_{337} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon\nu}^\kappa \nabla_\lambda F_{\delta\mu\sigma\kappa} + \\
& m_{338} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\varepsilon F_{\beta\gamma\nu}^\kappa \nabla_\lambda F_{\delta\mu\sigma\kappa} + \\
& m_{339} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\varepsilon F_{\beta\epsilon\nu}^\kappa \nabla_\lambda F_{\delta\mu\sigma\kappa} + \\
& m_{340} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\varepsilon F_{\gamma\epsilon\nu}^\kappa \nabla_\lambda F_{\delta\mu\sigma\kappa} + \\
& m_{479} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\gamma\nu} \nabla_\lambda F_{\delta\mu\sigma\kappa} + \\
& m_{491} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\epsilon\nu}^\lambda \nabla_\lambda F_{\delta\mu\sigma\kappa} + \\
& m_{508} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon\epsilon}^{\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu}^{\sigma\lambda} \nabla_\lambda F_{\delta\mu\sigma\kappa} + \\
& m_{544} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\epsilon\nu} \nabla_\lambda F_{\delta\mu\sigma\kappa} + \\
& m_{343} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\beta\mu}^\kappa \nabla_\lambda F_{\delta\nu\sigma\kappa} + \\
& m_{344} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon\mu}^\kappa \nabla_\lambda F_{\delta\nu\sigma\kappa} + \\
& m_{511} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon\mu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_\beta^{\nu\sigma\lambda} \nabla_\lambda F_{\delta\nu\sigma\kappa} +
\end{aligned}$$

$$\begin{aligned}
& m_{539} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\epsilon\epsilon\mu} \nabla_\lambda F_{\delta\nu\sigma\kappa} + \\
& m_{540} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\epsilon\epsilon\mu} \nabla_\lambda F_{\delta\nu\sigma\kappa} + \\
& m_{552} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\epsilon\mu}{}^\lambda \nabla_\lambda F_{\delta\nu\sigma\kappa} + \\
& m_{553} F_{\alpha\beta}^{\epsilon\epsilon} F_\epsilon^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\epsilon\mu}{}^\lambda \nabla_\lambda F_{\delta\nu\sigma\kappa} + \\
& m_{564} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon\epsilon}^{\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\mu}{}^{\sigma\lambda} \nabla_\lambda F_{\delta\nu\sigma\kappa} + \\
& m_{565} F_{\alpha\beta}^{\epsilon\epsilon} F_{\epsilon\epsilon}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\mu}{}^{\sigma\lambda} \nabla_\lambda F_{\delta\nu\sigma\kappa} + \\
& m_{526} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\delta\mu\nu} \nabla_\lambda F_{\epsilon\epsilon\sigma\kappa} + \\
& m_{577} F_{\alpha\beta}^{\epsilon\epsilon} F_\gamma^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\delta\mu\nu}{}^\lambda \nabla_\lambda F_{\epsilon\epsilon\sigma\kappa} + \\
& m_{355} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\gamma\nu}{}^\kappa \nabla_\lambda F_{\epsilon\mu\sigma\kappa} + \\
& m_{356} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\epsilon\nu}{}^\kappa \nabla_\lambda F_{\epsilon\mu\sigma\kappa} + \\
& m_{516} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\delta\epsilon\mu} \nabla_\lambda F_{\epsilon\nu\sigma\kappa} + \\
& m_{530} F_{\alpha\beta}^{\epsilon\epsilon} F_\epsilon^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\delta\mu}{}^\lambda \nabla_\lambda F_{\epsilon\nu\sigma\kappa} + \\
& m_{572} F_{\alpha\beta}^{\epsilon\epsilon} F_\gamma^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\delta\epsilon\mu}{}^\lambda \nabla_\lambda F_{\epsilon\nu\sigma\kappa} + \\
& m_{580} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\delta\mu}{}^{\sigma\lambda} \nabla_\lambda F_{\epsilon\nu\sigma\kappa} + \\
& m_{587} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\epsilon\mu}{}^{\sigma\lambda} \nabla_\lambda F_{\epsilon\nu\sigma\kappa} + \\
& m_{361} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\gamma\varepsilon}{}^\kappa \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{362} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\epsilon\varepsilon}{}^\kappa \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{513} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\delta\epsilon\varepsilon} \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{522} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\delta\varepsilon}{}^\lambda \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{523} F_{\alpha\beta}^{\epsilon\epsilon} F_\epsilon^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\delta\varepsilon}{}^\lambda \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{535} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon\varepsilon}^{\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\delta}{}^{\sigma\lambda} \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{536} F_{\alpha\beta}^{\epsilon\epsilon} F_{\epsilon\epsilon}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\delta}{}^{\sigma\lambda} \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{569} F_{\alpha\beta}^{\epsilon\epsilon} F_\gamma^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\delta\epsilon\varepsilon}{}^\lambda \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{574} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\delta\varepsilon}{}^{\sigma\lambda} \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{584} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\epsilon\epsilon}{}^{\sigma\lambda} \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{590} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\delta\varepsilon}^{\mu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_\epsilon{}^{\nu\sigma\lambda} \nabla_\lambda F_{\mu\nu\sigma\kappa} + \\
& m_{390} F_\epsilon{}^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\gamma\delta\nu\lambda} \nabla_\mu F_{\alpha\beta\varepsilon\sigma} + \\
& m_{416} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\nu\sigma\lambda} \nabla_\mu F_{\beta\gamma\varepsilon\epsilon} + \\
& m_{345} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\delta\nu\sigma\kappa} \nabla_\mu F_{\beta\gamma\varepsilon}{}^\kappa + \\
& m_{346} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\delta\nu\sigma\kappa} \nabla_\mu F_{\beta\epsilon\varepsilon}{}^\kappa + \\
& m_{392} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\gamma\delta\sigma\lambda} \nabla_\mu F_{\beta\epsilon\varepsilon\nu} + \\
& m_{331} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\gamma\delta\sigma\kappa} \nabla_\mu F_{\beta\epsilon\nu}{}^\kappa + \\
& m_{427} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\epsilon\nu\sigma\lambda} \nabla_\mu F_{\gamma\delta\varepsilon}{}^\kappa + \\
& m_{358} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\nu\sigma\kappa} \nabla_\mu F_{\gamma\delta\varepsilon}{}^\kappa + \\
& m_{496} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu\sigma\lambda} \nabla_\mu F_{\gamma\delta\varepsilon\kappa} +
\end{aligned}$$

$$\begin{aligned}
& m_{487} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu\sigma} \nabla_\mu F_{\gamma\delta\lambda\kappa} + \\
& m_{500} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu\sigma}{}^\lambda \nabla_\mu F_{\gamma\delta\lambda\kappa} + \\
& m_{428} F_{\alpha\beta}^{\epsilon\epsilon} F_\epsilon^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\nu\sigma\lambda} \nabla_\mu F_{\gamma\delta}{}^{\lambda\kappa} + \\
& m_{417} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\nu\sigma\lambda} \nabla_\mu F_{\gamma\delta}{}^\kappa + \\
& m_{347} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\delta\nu\sigma\kappa} \nabla_\mu F_{\gamma\epsilon\epsilon}{}^\kappa + \\
& m_{348} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\delta\nu\sigma\kappa} \nabla_\mu F_{\gamma\epsilon\epsilon}{}^\kappa + \\
& m_{497} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu\sigma\lambda} \nabla_\mu F_{\delta\epsilon\epsilon\kappa} + \\
& m_{429} F_{\alpha\beta}^{\epsilon\epsilon} F_\gamma^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\nu\sigma\lambda} \nabla_\mu F_{\delta\epsilon}{}^{\lambda\kappa} + \\
& m_{288} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon\nu\sigma} \nabla_\mu F_{\delta\epsilon\lambda\kappa} + \\
& m_{482} F_\alpha^{\epsilon\epsilon\mu} F_\gamma^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\epsilon\nu\sigma} \nabla_\mu F_{\delta\epsilon\lambda\kappa} + \\
& m_{501} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu\sigma}{}^\lambda \nabla_\mu F_{\delta\epsilon\lambda\kappa} + \\
& m_{289} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon\nu\sigma} \nabla_\mu F_{\delta\sigma\lambda\kappa} + \\
& m_{290} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\epsilon\nu}{}^\kappa \nabla_\mu F_{\delta\sigma\lambda\kappa} + \\
& m_{291} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\nu}{}^{\lambda\kappa} \nabla_\mu F_{\delta\sigma\lambda\kappa} + \\
& m_{506} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon\epsilon}^{\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\nu}{}^{\sigma\lambda} \nabla_\mu F_{\delta\sigma\lambda\kappa} + \\
& m_{543} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\epsilon\epsilon\nu} \nabla_\mu F_{\delta\sigma\lambda\kappa} + \\
& m_{393} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\gamma\delta\sigma\lambda} \nabla_\nu F_{\alpha\beta\epsilon\mu} + \\
& m_{332} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\gamma\delta\sigma\kappa} \nabla_\nu F_{\alpha\beta\mu}{}^\kappa + \\
& m_{409} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\mu\sigma\lambda} \nabla_\nu F_{\beta\gamma\epsilon\epsilon} + \\
& m_{341} F_\alpha^{\epsilon\epsilon\mu} F_\epsilon^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\delta\mu\sigma\kappa} \nabla_\nu F_{\beta\gamma\epsilon}{}^\kappa + \\
& m_{425} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\epsilon\mu\sigma\lambda} \nabla_\nu F_{\gamma\delta\epsilon}{}^\kappa + \\
& m_{357} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\mu\sigma\kappa} \nabla_\nu F_{\gamma\delta\epsilon}{}^\kappa + \\
& m_{420} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\epsilon\epsilon\sigma\lambda} \nabla_\nu F_{\gamma\delta\mu}{}^\kappa + \\
& m_{352} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\epsilon\epsilon\sigma\kappa} \nabla_\nu F_{\gamma\delta\mu}{}^\kappa + \\
& m_{342} F_\alpha^{\epsilon\epsilon\mu} F_\beta^{\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\delta\mu\sigma\kappa} \nabla_\nu F_{\gamma\epsilon\epsilon}{}^\kappa + \\
& m_{336} F_{\alpha\beta}^{\epsilon\epsilon} F^{\mu\nu\sigma\lambda} R^{\alpha\beta\gamma\delta} \nabla_\lambda F_{\delta\epsilon\sigma\kappa} \nabla_\nu F_{\gamma\epsilon\mu}{}^\kappa + \\
& m_{297} F_{\epsilon\epsilon}^{\sigma\lambda} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\beta\mu}{}^\kappa \nabla_\nu F_{\delta\sigma\lambda\kappa} + \\
& m_{298} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon\mu}^{\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_\beta{}^{\sigma\lambda\kappa} \nabla_\nu F_{\delta\sigma\lambda\kappa} + \\
& m_{562} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon\epsilon}^{\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\mu}{}^{\sigma\lambda} \nabla_\nu F_{\delta\sigma\lambda\kappa} + \\
& m_{563} F_{\alpha\beta}^{\epsilon\epsilon} F_{\epsilon\epsilon}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\mu}{}^{\sigma\lambda} \nabla_\nu F_{\delta\sigma\lambda\kappa} + \\
& m_{299} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon\epsilon}^{\nu} R^{\alpha\beta\gamma\delta} \nabla_\mu F_\gamma{}^{\sigma\lambda\kappa} \nabla_\nu F_{\delta\sigma\lambda\kappa} + \\
& m_{300} F_{\alpha\beta}^{\epsilon\epsilon} F_{\epsilon\epsilon}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\mu F_\gamma{}^{\sigma\lambda\kappa} \nabla_\nu F_{\delta\sigma\lambda\kappa} + \\
& m_{579} F_{\alpha\beta}^{\epsilon\epsilon} F_{\gamma\epsilon}^{\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\delta\mu}{}^{\sigma\lambda} \nabla_\nu F_{\epsilon\sigma\lambda\kappa} + \\
& m_{301} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon\epsilon}^{\nu} R^{\alpha\beta\gamma\delta} \nabla_\delta F_\beta{}^{\sigma\lambda\kappa} \nabla_\nu F_{\mu\sigma\lambda\kappa} + \\
& m_{391} F_\epsilon^{\sigma\lambda\kappa} F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\gamma\delta\nu\lambda} \nabla_\sigma F_{\alpha\beta\epsilon\mu} +
\end{aligned}$$

$$\begin{aligned}
& m_{404} F_\alpha^{\epsilon\epsilon\mu} F^{\nu\sigma\lambda\kappa} R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\delta\epsilon\mu\lambda} \nabla_\sigma F_{\beta\gamma\epsilon\nu} + \\
& m_{474} F_{\epsilon\epsilon\mu}^\sigma F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\alpha\gamma\nu}^\lambda \nabla_\sigma F_{\beta\delta\lambda\kappa} + \\
& m_{493} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\mu\nu}^\lambda \nabla_\sigma F_{\gamma\delta\lambda\kappa} + \\
& m_{302} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\mu F_{\beta\nu}^{\lambda\kappa} \nabla_\sigma F_{\gamma\delta\lambda\kappa} + \\
& m_{303} F_{\epsilon\epsilon\mu}^\sigma F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\nu F_{\alpha\beta}^{\lambda\kappa} \nabla_\sigma F_{\gamma\delta\lambda\kappa} + \\
& m_{304} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\nu}^{\lambda\kappa} \nabla_\sigma F_{\delta\mu\lambda\kappa} + \\
& m_{305} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\varepsilon F_{\beta\nu}^{\lambda\kappa} \nabla_\sigma F_{\delta\mu\lambda\kappa} + \\
& m_{481} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\gamma\nu}^\lambda \nabla_\sigma F_{\delta\mu\lambda\kappa} + \\
& m_{490} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\beta\epsilon\nu}^\lambda \nabla_\sigma F_{\delta\mu\lambda\kappa} + \\
& m_{557} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\epsilon\nu}^\lambda \nabla_\sigma F_{\delta\mu\lambda\kappa} + \\
& m_{306} F_{\epsilon\epsilon\mu}^\sigma F^{\epsilon\epsilon\mu\nu} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\alpha\beta}^{\lambda\kappa} \nabla_\sigma F_{\delta\nu\lambda\kappa} + \\
& m_{307} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\gamma F_{\beta\mu}^{\lambda\kappa} \nabla_\sigma F_{\delta\nu\lambda\kappa} + \\
& m_{550} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\epsilon\mu}^\lambda \nabla_\sigma F_{\delta\nu\lambda\kappa} + \\
& m_{551} F_{\alpha\beta}^{\epsilon\epsilon} F_\epsilon^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\epsilon\mu}^\lambda \nabla_\sigma F_{\delta\nu\lambda\kappa} + \\
& m_{308} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\mu F_{\beta\nu}^{\lambda\kappa} \nabla_\sigma F_{\delta\nu\lambda\kappa} + \\
& m_{309} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\mu F_{\gamma\nu}^{\lambda\kappa} \nabla_\sigma F_{\delta\nu\lambda\kappa} + \\
& m_{310} F_{\alpha\beta}^{\epsilon\epsilon} F_\epsilon^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\mu F_{\gamma\nu}^{\lambda\kappa} \nabla_\sigma F_{\delta\nu\lambda\kappa} + \\
& m_{576} F_{\alpha\beta}^{\epsilon\epsilon} F_\gamma^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\delta\mu\nu}^\lambda \nabla_\sigma F_{\epsilon\epsilon\lambda\kappa} + \\
& m_{529} F_{\alpha\beta}^{\epsilon\epsilon} F_\epsilon^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\delta\mu}^\lambda \nabla_\sigma F_{\epsilon\nu\lambda\kappa} + \\
& m_{571} F_{\alpha\beta}^{\epsilon\epsilon} F_\gamma^{\mu\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\delta\epsilon\mu}^\lambda \nabla_\sigma F_{\epsilon\nu\lambda\kappa} + \\
& m_{312} F_\alpha^{\epsilon\epsilon\mu} F_{\epsilon\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\nu}^{\lambda\kappa} \nabla_\sigma F_{\mu\nu\lambda\kappa} + \\
& m_{313} F_\alpha^{\epsilon\epsilon\mu} F_{\gamma\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla_\delta F_{\beta\epsilon}^{\lambda\kappa} \nabla_\sigma F_{\mu\nu\lambda\kappa} + \\
& m_{521} F_\alpha^{\epsilon\epsilon\mu} F_{\beta\epsilon}^{\nu\sigma} R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\gamma\delta\epsilon}^\lambda \nabla_\sigma F_{\mu\nu\lambda\kappa} + \\
& m_{281} FF R^{\alpha\beta\gamma\delta} \nabla_\gamma F_\alpha^{\sigma\lambda\kappa} \nabla_\delta F_{\beta\sigma\lambda\kappa} + m_{478} FF R^{\alpha\beta\gamma\delta} \nabla_\kappa F_{\beta\delta\sigma\lambda} \nabla^\kappa F_{\alpha\gamma}^{\sigma\lambda} + \\
& m_{477} FF R^{\alpha\beta\gamma\delta} \nabla^\kappa F_{\alpha\gamma}^{\sigma\lambda} \nabla_\lambda F_{\beta\delta\sigma\kappa} \tag{37}
\end{aligned}$$

where $FF = F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta}$. Note that the number of all contractions of F^8 without imposing the field redefinition is 176. There are 104 couplings in (35) that their coefficients are unambiguous. The coefficients of the couplings in (35) which have FF or $F_{\mu\alpha\beta\gamma} F_\nu^{\alpha\beta\gamma}$ are essential parameters.

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