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Testing capability index C_{pk} with its application in automobile engine manufacturing industry

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ABSTRACT
Hypotheses testing is an effective technique for making a decision about the process capability. The adversity to test C_{pk} based on determining the distribution of its natural estimator is very complex, even under the Normal distribution. Thus, the Monte Carlo simulation approach is an executable procedure for the hypotheses testing of C_{pk} based on the natural estimator. The proposed approach is a testing technique to evaluate whether products quality meets the preset specification limits. This procedure implies a Monte Carlo statistical test, which is applicable to Normal processes. A case study in an automobile factory is presented to check out the products, and numerical computations are presented to show the effect of the Monte Carlo critical values for making a reliable decision on the hypotheses testing of C_{pk}.

KEYWORDS
capability analysis; critical value; hypotheses testing; Monte Carlo simulation

Introduction

Process capability indices (PCIs) have been extensively applied in the industrial processes to evaluate whether products quality meets the preset specification limits (SLs). The two most popular indices C_p and C_{pk} are introduced as

$$C_p = \frac{USL - LSL}{6\sigma}$$

and

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, and σ is the process standard deviation.

Fundamentally, μ and σ are unknown parameters, and so we need a random sample to estimate the unknown parameters. Therefore, μ and σ can be estimated by $\overline{X} = \sum_{i=1}^{n} X_i/n$ and $S_{n-1} = \sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2/n-1}$, which are the mean and standard deviation of the sample, respectively. In what follows, by substituting these estimators, one can obtain the natural estimator $\widehat{C}_{pk}$. Using the reality $\min\{a,b\} = (a+b)/2 - |a-b|/2$, C_{pk} can be rewritten mathematically as

$$C_{pk} = \frac{d - |\mu - M|}{3\sigma}, \tag{1}$$

where $M = \frac{LSL+USL}{2}$ is the midpoint of the engineering tolerance and $d = \frac{USL-LSL}{2}$ is the half length of the tolerance range. Therefore, the natural estimator is given by

$$\widehat{C}_{pk} = \frac{d - |\overline{X} - M|}{3S_{n-1}}. \tag{2}$$

Three estimators have been suggested to estimate the C_{pk} index, namely (*i*) Bissell's estimator $\widehat{C'}_{pk}$ (Bissell 1990), (*ii*) the natural estimator $\widehat{C}_{pk}$ (Kotz, Pearn, and Johnson 1993), and (*iii*) the Bayesian-like estimator $\widehat{C''}_{pk}$ (Pearn and Chen 1996). Pearn and Chen (1997, 1999) proposed a procedure for making a decision based on an unbiased estimator of the Bayesian-like estimator $\widehat{C''}_{pk}$, and also they determined the critical value of the quality test. A new hypotheses testing approach for C_{pk} on the basis of the generalized p-value was presented in (Meng, Yang, and Huang 2021). Pearn and Lin (2004) did the

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statistical hypotheses testing for analyzing the index C_{pk} and determined the critical values with the presence of process parameter C_p. Also, the second author, Parchami, Ivani, and Mashinchi (2011) applied a fuzzy testing based on the fuzzy p-value method. The fuzzy qualities and reviewing some fuzzy process capability indices were discussed in (Parchami and Mashinchi 2012). Kargar, Mashinchi, and the second author (Kargar, Mashinchi, and Parchami 2014) used the Bayesian method to test the C_{pk} index. Arefi (2018) utilized a method to test the hypotheses for a mean of the Normal distribution with known/unknown variance based on fuzzy data.

A more powerful test is presented in this study based on the Monte Carlo (MC) simulation approach to specifying the components of the decision-making in testing quality using the PCI C_{pk}. Most importantly, the critical value of Pearn and Lin's test (Pearn and Lin 2004) is dependent on the parameter C_p. The contribution of the present paper is to provide an approach using an MC simulation method to determine the critical value for testing the most popular capability index C_{pk} without the presence of the parameter C_p. This paper is organized as follows. In Section 2, definitions of hypotheses testing for analyzing the PCI C_{pk} are given as a statistical method. The MC approach is provided in Section 3 to make inference about the natural estimator $\widehat{C}_{pk}$. Numerical results and concluding remarks are discussed in Section 4, and Section 5 presents conclusions.

Statistical hypotheses testing

The main goal in testing quality is to determine a suitable critical value for testing hypotheses on the PCI C_{pk} at the given significance level. Therefore, the aim of this section is to offer an approach to make a reliable decision based on the natural estimator $\widehat{C}_{pk}$. To test whether the producing process is capable of producing items within the preset SLs, the following hypotheses test is considered:

$$\begin{cases} H_0 : C_{pk} \le c_0 & (\textit{process is not capable}), \\ H_1 : C_{pk} > c_0 & (\textit{process is capable}), \end{cases} \tag{3}$$

in which c_0 is the standard minimal criterion for the PCI C_{pk}.

Herein, we intend to consider the following test function:

$$\phi(x_1, x_2, \ldots, x_n) = \begin{cases} 1, & \widehat{c}_{pk} > c, \\ 0 & \textit{otherwise}, \end{cases} \tag{4}$$

where c is the critical value of the proposed quality test. The decision rule is presented as follows: $\phi(x_1, x_2, \ldots, x_n)$ rejects the null hypothesis H_0 $(C_{pk} \le c_0)$ if $\widehat{c}_{pk} > c$ (i.e., the process is capable). Also,

$$\alpha = Pr\left(\widehat{C}_{pk} > c | C_{pk} = c_0\right), \tag{5}$$

is the probability of type I error, and we have

$$1 - \alpha = Pr\left(\widehat{C}_{pk} \le c | C_{pk} = c_0\right). \tag{6}$$

Therefore, the critical value of this proposed quality test is the $(1-\alpha)$th quantile of $\widehat{C}_{pk}$ distribution under the condition $C_{pk} = c_0$.

On the other hand, the p-value of this proposed quality test is equal to

$$\begin{aligned} p - value &= Pr(\widehat{C}_{pk} > \widehat{c}_{pk} | C_{pk} = c_0) \\ &= E[I(\widehat{C}_{pk} > \widehat{c}_{pk} | C_{pk} = c_0)], \end{aligned} \tag{7}$$

where $\widehat{c}_{pk}$, computed by Eq. (2), using observations $x_1, \ldots, x_n$, is the observed capability index C_{pk} while the indicator function of an event A is denoted by $I(A)$. Also, the probability of type II error can be calculated as

$$\begin{aligned} \beta(c^*_{pk}) &= Pr\left(\widehat{C}_{pk} \le c | C_{pk} = c^*_{pk}\right) \\ &= E\left[I\left(\widehat{C}_{pk} \le c \mid C_{pk} = c^*_{pk}\right)\right], \end{aligned} \tag{8}$$

for any arbitrary point $c^*_{pk} > c_0$. Hence,

$$\begin{aligned} \Pi(C_{pk}) = 1 - \beta(C_{pk}) &= Pr\left(\widehat{C}_{pk} > c | C_{pk}\right) \\ &= E\left[I\left(\widehat{C}_{pk} > c \mid C_{pk}\right)\right]. \end{aligned} \tag{9}$$

is the power function of the quality test based on the index C_{pk}.

Monte Carlo simulation approach

In this section, we intend to propose the MC simulation approach for any Normal process. To simulate the components of the decision-making—such as the probability of type II error, the p-value, and the critical value—for the hypotheses testing of C_{pk} at the considered significance level, α, we proposed the following procedure:

Step 1: By Eq. (2), calculate the observed index $\widehat{c}_{pk}$ using the observations $x_1, \ldots, x_n$.

Step 2: Compute the sequence $\{\mu_1, \mu_2, \ldots, \mu_s\}$ to cover the distance $[x_{(1)}, x_{(n)}]$ by the means of the formula

$$\mu_j = x_{(1)} + \frac{j-1}{s-1}(x_{(n)} - x_{(1)}), \ j = 1, 2, \ldots, s, \tag{10}$$

where $x_{(1)}$ and $x_{(n)}$ are the first and the last ordered observations $x_1, x_2, ..., x_n$, respectively. The positive natural number s specifies the number of the μ_js that can be selected according to the applicant sensitivity.

Step 3: Follow the below (I)–(VI) parts for all $\mu_j \in \{\mu_1, \mu_2, ..., \mu_s\}$:

I. calculate the unknown root σ_j on the basis of μ_j, by the following equation:

$$\frac{d - |\mu_j - M|}{3\sigma_j} = c_0, \tag{11}$$

II. considering $m = 10^4$, simulate 10000 random samples with size n from $N(\mu_j, \sigma_j^2)$,

III. estimate $\widehat{c}_{pk,j}^{[1]}, \widehat{c}_{pk,j}^{[2]}, ..., \widehat{c}_{pk,j}^{[m]}$, by Eq. (2) for every simulated sample in Part (II),

IV. by means of Eq. (6), the critical value based on 10000 simulated samples is equivalent to the $(1-\alpha)$th quantile of $\widehat{C}_{pk}$ distribution, that is,

$$c_j = \widehat{c}_{pk,j}^{(m(1-\alpha))}, \quad j = 1, ..., s, \tag{12}$$

where $\widehat{c}_{pk,j}^{(1)}, \widehat{c}_{pk,j}^{(2)}, ..., \widehat{c}_{pk,j}^{(m)}$ are the ordered indices from Part (III),

V. the simulated p-value is equal to

$$p - value_j = \frac{1}{m}\sum_{r=1}^{m} I\left(\widehat{c}_{pk,j}^{[r]} > \widehat{c}_{pk} \mid \mu = \mu_j, \sigma = \sigma_j\right), \quad j = 1, ..., s, \tag{13}$$

where the estimated indices are indicated by $\widehat{c}_{pk,j}^{[1]}, ..., \widehat{c}_{pk,j}^{[m]}$ and σ_j is the obtainable root of the equation $C_{pk} = c_0$ from Part (I).

VI. the simulated β for every point $c_{pk}^* > c_0$, is

$$\beta(c_{pk}^*)_j = \frac{1}{m}\sum_{r=1}^{m} I\left(\widehat{c}_{pk,j}^{[r]} \le c_j \mid \mu = \mu_j, \sigma = \sigma_j^*\right), \; j = 1, ..., s, \tag{14}$$

where σ_j^* is the obtainable root of the equation $C_{pk} = c_{pk}^*$. Likewise, the estimated indices based on μ_j and σ_j^* are indicated by $\widehat{c}_{pk,j}^{[1]}, ..., \widehat{c}_{pk,j}^{[m]}$.

Step 4: The MC critical value of this proposed quality test is equivalent to

$$c_{MC} = \frac{1}{s}\sum_{j=1}^{s} c_j. \tag{15}$$

Step 5: The null hypothesis in Eq. (3) is rejected at the significance level α, if $\widehat{c}_{pk} > c_{MC}$ (i.e., the process is capable); otherwise the process is incapable.

Step 6: The MC p-value in the proposed quality test is equivalent to

$$p{-}value_{MC} = \frac{1}{s}\sum_{j=1}^{s} p - value_j. \tag{16}$$

Step 7: The MC probability of type II error for any point $c_{pk}^* > c_0$ can be simulated in the proposed quality test by the total average of s calculated probability of type II errors at the point c_{pk}^* in repetitions of Part (VI), that is,

$$\beta_{MC}\left(c_{pk}^*\right) = \frac{1}{s}\sum_{j=1}^{s} \beta\left(c_{pk}^*\right)_j. \tag{17}$$

Remark 1 Note that the MC power function of the quality test for every C_{pk} is equal to

$$\Pi_{MC}(C_{pk}) = 1 - \beta_{MC}(C_{pk}). \tag{18}$$

Remark 2 Note that theoretical Pearn and Lin's method on the statistical hypotheses testing attempts to find the critical value by writing the equation $\alpha = P\left(\widehat{C}_{pk} > c | C_{pk} = c_0\right)$ on the basis of the PCI C_p. Under the assumption of Normality, they obtained the critical value c by means of solving the equation

$$\alpha = \int_0^{3C_p\sqrt{n}} G\left(\frac{(n-1)(3C_p\sqrt{n} - y)^2}{9nc^2}\right) \times \left(\phi^*\left[y + 3(C_p - c_0)\sqrt{n}\right] + \phi^*\left[y - 3(C_p - c_0)\sqrt{n}\right]\right)dy \tag{19}$$

where c_0 is as expressed in Eq. (3), $\phi^*(\cdot)$ is the probability density function of the standard Normal distribution $N(0, 1)$, and $G(\cdot)$ is the cumulative distribution function of the chi-square distribution χ_{n-1}^2.

A case study for the quality test is investigated in the next section in detail to illustrate the proposed approach.

A case study in automobile engine manufacturing industry

In order to become a successful engine producer in the automobile engine manufacturing process, it is necessary to check all their clearances and the custom settings in the manufacturing line. Essentially, the crankshaft is the spine of the interior combustion engine. Converting a linear movement to a rotational movement is the task of the crankshaft for the suitable operation of the engine.

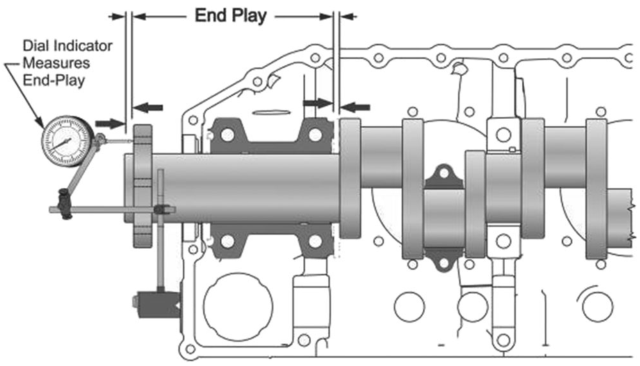

Figure 1. Measuring crankshaft end play.

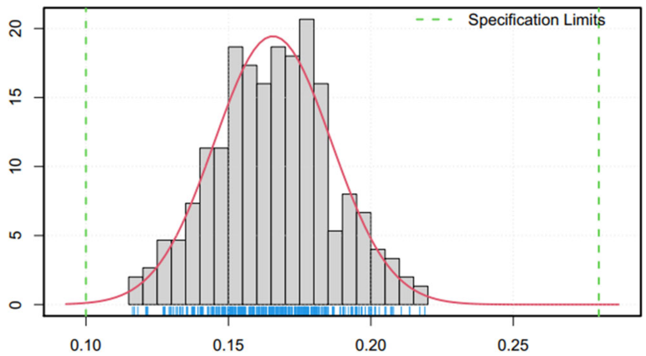

Figure 2. Histogram of 300 observed crankshaft end play measurements.

Table 1. Simulated components of the decision-making for all repetitions of the proposed procedure, in the case study.

j	μ_j	σ_j	c_j	p-value$_j$	$\beta(c^*pk)_j$
1	0.116	0.005	1.115	0.082	0.032
2	0.126	0.009	1.115	0.086	0.030
3	0.135	0.012	1.112	0.084	0.023
4	0.144	0.015	1.117	0.086	0.032
5	0.154	0.018	1.115	0.087	0.027
6	0.163	0.021	1.115	0.090	0.024
7	0.172	0.024	1.117	0.082	0.030
8	0.182	0.027	1.113	0.089	0.023
9	0.191	0.030	1.098	0.049	0.010
10	0.200	0.027	1.114	0.081	0.027
11	0.210	0.023	1.113	0.083	0.021
12	0.219	0.020	1.117	0.089	0.026
Total average			c_{MC} = 1.114	p-value$_{MC}$ = 0.082	β_{MC} (1.21) = 0.025

Most importantly, crankshaft "end play" is important because it restricts the fore and aft motion of the crankshaft in the block. If the automobile engine is assembled with too large end play in the crank, the forward motion of the crankshaft in the block can chew up the main bearing caps and the block. Also, if the crankshaft end play is too small, then it can be caused insufficient lubrication and thermal expansion of parts. In this work, the crankshaft end play is measured as the clearance between the crankcase thrust washer and the axial mating surface of the crankshaft. Figure 1 indicates the details of a crankshaft in the engine.

In the quality analysis for one of the engine mechanical systems in the automobile factory, the LSL and USL of crankshaft end play should be 0.10 and 0.28 millimeters, respectively. A random sample of the end play measurements $(n = 300)$ is collected from the crankshafts of the interior combustion engines; see Figure 2. Also, the blue signs in this figure show observed end play measurements of 300 crankshafts.

Now, we are going to investigate the quality of the crankshaft end play based on C_{pk}. The MC simulation method is considered for testing $H_0 : C_{pk} \leq 1$, against $H_1 : C_{pk} > 1$, using observations $x_1, x_2, \ldots, x_{300}$. Lilliefors (Kolmogorov–Smirnov) Normality test with $D = 0.0336$ and p-value $= 0.5626$, confirms that data follow the Normal distribution.

Meanwhile, the estimator of unknown parameters μ and σ is estimated by $\hat{\mu} = \overline{x} = 0.1656$ and $\hat{\sigma} = s_{n-1} = 0.0205$, respectively. Hence, the estimated value of the PCI C_{pk} is

$$\widehat{c}_{pk} = \frac{d - |\overline{x} - M|}{3s_{n-1}} = \frac{0.09 - |0.1656 - 0.19|}{3 \times 0.0123} = 1.066.$$

Therefore, $1.066 < c_{MC}$ is the MC critical value of testing quality based on the PCI C_{pk} at the significance level 0.010, where the critical value should be simulated using the MC method. We perform the proposed procedure, which is presented in Section 3, to simulate the MC critical value in the quality test using the natural estimator $\widehat{C}_{pk}$, where the mean is changed over the following sequence 0.116, 0.126, 0.135, 0.144, 0.154, 0.163, 0.172, 0.182, 0.191, 0.200, 0.210, 0.219.

The unknown root σ_j, from the equation $\sigma_j = \frac{d-|\mu_j - M|}{3c_0}$, is computed in Table 1 for all $s = 12$ considered cases by Eq. (11). Therefore, for each case, 10000 independent random samples are simulated with size 300 from $N\left(\mu_j, \sigma_j^2\right)$. In every simulated sample, we estimate the index C_{pk} based on observations $x_1, x_2, \ldots, x_{300}$ using Eq. (2). Histograms of simulated indices under H_0 and H_1 hypotheses are shown in Figure 3, for the first six repetitions of the proposed procedure in the case study.

Next, we order 10000 estimated indices; the 9900th index is selected as the critical value; and the results are recorded in Table 1 for all twelve considered cases. These simulated critical values are indicated by red dashed lines in Figure 3. Also, Table 1 contains the MC simulated results for twelve considered cases. For

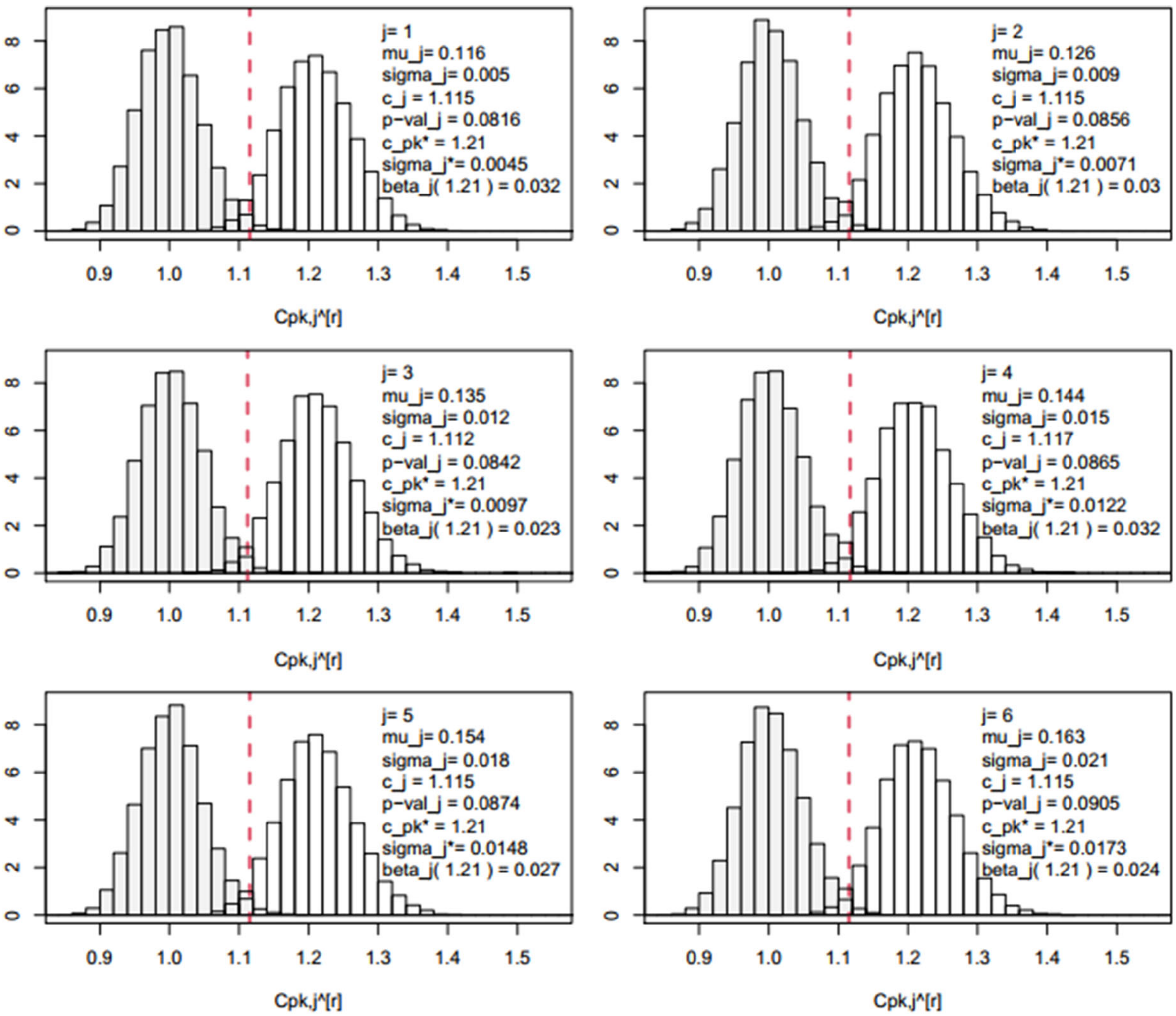

Figure 3. Histograms of the simulated capability indices ($\hat{c}^{[r]}_{pk,j}$, for $r = 1, \ldots, 10000$) by Normal data under H_0 and H_1 hypotheses, for the first six repetitions of the proposed procedure in the case study.

instance, in the third row of Table 1, the following results are obtained:

- $\mu_3 = 0.135$ is the third considered value for μ in simulation using Step 2 from the proposed procedure,
- $\sigma_3 = 0.012$ is the computed value of σ for $j = 3$ using Part (I),
- $c_3 = 1.112$ is the simulated critical value using Part (IV),
- $p - \text{value}_3 = 0.084$ is the simulated p-value using Part (V), and
- $\beta(1.21)_3 = 0.023$ is the simulated β at point 1.21 using Part (VI) of the proposed procedure.

The average of twelve obtained critical values is equivalent to $c_{MC} = 1.114$, which is determined as the MC critical value for making a decision on the quality test in this study. The last row of Table 1 contains the MC critical value, the MC p-value, and the MC β at point 1.21, respectively, which are the average of twelve simulated values during the proposed procedure. Additionally, $c = 1.115$ is the critical value of Pearn and Lin's test for $c_0 = 1.00$ with $n = 300$ at significance level $= 0.010$.

Regarding the case study, the observed index $\hat{c}_{pk} = 1.066$ is less than the critical value of both tests and the process is incapable at the considered significance level. Hence, with regard the preset SLs, one can conclude that this process cannot meet the capability requirement at the significance level 0.010.

Also, the power function of the quality test for the PCI C_{pk} based on Pearn and Lin's method, which is denoted by $\Pi_{PL}(C_{pk})$, can be computed as

$$\Pi_{PL}(C_{pk}) = \int_0^{3C_p\sqrt{n}} G\left(\frac{(n-1)\left(3C_p\sqrt{n}-y\right)^2}{9nc^2}\right) \times \left(\phi^*\left[y+3(C_p-C_{pk})\sqrt{n}\right] + \phi^*\left[y-3(C_p-C_{pk})\sqrt{n}\right]\right)dy \qquad (19)$$

For the desired quality condition with $c_0 = 1.00$, $\alpha = 0.010$, and sample size $n = 300$, the power function curves of both tests are drawn in Figure 4. Using Pearn and Lin's method for the samples with size $n \geq 100$, we can consider $C_p = c_0 + 0.12$ for solving Eq. (19) to obtain the critical value $c = 1.115$. It must be noted that the power function of Pearn and Lin's test cannot plot completely in Figure 4 considering $C_p \geq C_{pk}$, in which $C_{pk} = 1.12$ is shown in Figure 4 by the green vertical line. Furthermore, the critical value using our proposed approach can be computed as $c_{MC} = 1.114$ for $c_0 = 1.00$ with sample size 300 at the significance level 0.010. Also, we can plot the MC power function of the quality test $\Pi_{MC}(C_{pk})$ completely (see Figure 4). We observe from Figure 4 that the power function of the quality test by means of the proposed procedure is larger than the power function of the quality test based on Pearn and Lin's method.

Table 2 contains all components of the decision-making for testing quality using both approaches (i.e., Pearn and Lin's method and our proposed procedure) with the standard minimal criterion $c_0 = 1.00$, under various significance levels. Also, for various contractual values of c_0 and $\alpha = 0.010$, the counter curve of C_{pk} is plotted in Figure 5. Moreover, the coordinates of the considered points (μ_j, σ_j) in Part (I) of the proposed procedure are shown over each contour for $j = 1, \ldots, 12$.

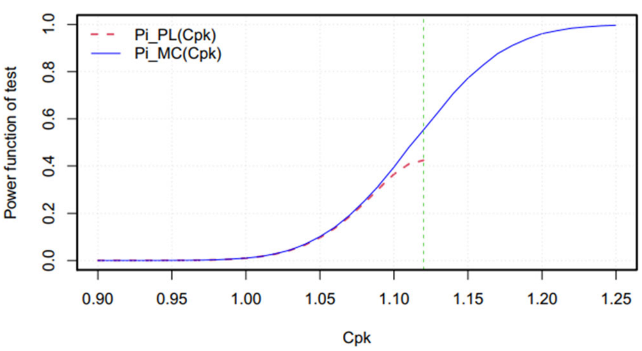

Figure 4. Plots of the MC power function of the quality test $\Pi_{MC}(C_{pk})$ and Pearn and Lin's power function of the quality test $\Pi_{PL}(C_{pk})$ for $c_0 = 1.00$, $C_p = 1.12$ with $n = 300$ and $\alpha = 0.010$.

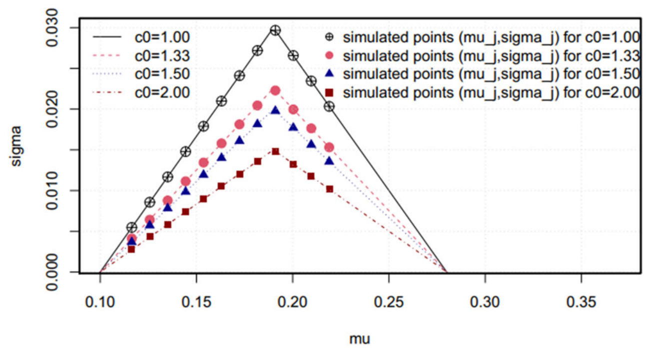

Figure 5. Counters of C_{pk} for various values of standard minimal criterion c_0 and $\alpha = 0.010$ with all coordinates (μ_j, σ_j) in simulation.

Table 3. Comparison of running time between two methods on the quality test with sample size $n = 300$ for the standard minimal criterion $c_0 = 1.00$ and $\alpha = 0.01$ in the case study.

Methods	Critical value of test	Running time (second)
Pearn and lin's method	1.115	0.085
Our proposed method	1.114	3.900

The principal characteristics of our personal computer (PC): Processor (Intel Core(TM) i7-10700 CPU @ 2.90 GHz 2.90 GHz), Installed RAM (16 GB).

Table 2. Results of comparison between Pearn and Lin's method (Pearn's method) and the proposed procedure (our procedure) on the quality test with sample size $n = 300$ for the standard minimal criterion $c_0 = 1.00$ and different significance levels in the case study.

		$\alpha = 0.010$	$\alpha = 0.025$	$\alpha = 0.050$	$\alpha = 0.100$
		Quality test with $c0 = 1.00$ and $Cp = 1.12$ for the preset SLs $LSL = 0.10$ and $USL = 0.28$			
Pearn's method	c	1.115	1.096	1.080	1.062
	Decision by $\hat{c}pk = 1.066$	Not reject $H0$	Not reject $H0$	Not reject $H0$	Reject $H0$
	Process	Incapable	Incapable	Incapable	Capable
	p-value	0.085	0.085	0.085	0.085
	$\beta(c^*pk)$	$\beta(1.140) = 0.635$	$\beta(1.100) = 0.480$	$\beta(1.070) = 0.573$	$\beta(1.040) = 0.670$
		Quality test with $c0 = 1.00$ for the preset SLs $LSL = 0.10$ and $USL = 0.28$ without the presence of Cp			
Our procedure	cMC	1.114	1.095	1.079	1.061
	Decision by $\hat{c}pk = 1.066$	Not reject $H0$	Not reject $H0$	Not reject $H0$	Reject $H0$
	Process	Incapable	Incapable	Incapable	Capable
	p-valueMC	0.082	0.082	0.082	0.082
	$\beta MC\ (c^*pk)$	$\beta MC\ (1.210) = 0.025$	$\beta MC\ (1.170) = 0.061$	$\beta MC\ (1.140) = 0.099$	$\beta MC\ (1.110) = 0.143$

Table 4. MC critical values obtained by the proposed approach for $c_0 = 1.00$, 1.33, 1.50, and various sample sizes n at different significance levels $\alpha = 0.010$, 0.025, 0.050, 0.100.

	$c0 = 1.00$				$c0 = 1.33$				$c0 = 1.50$			
n	0.010	0.025	0.050	0.100	0.010	0.025	0.050	0.100	0.010	0.025	0.050	0.100
16	1.729	1.574	1.455	1.337	1.776	1.617	1.494	1.374	1.800	1.639	1.514	1.392
20	1.613	1.486	1.387	1.288	1.656	1.526	1.425	1.323	1.678	1.546	1.445	1.341
24	1.528	1.424	1.340	1.255	1.569	1.463	1.377	1.289	1.590	1.482	1.395	1.307
30	1.450	1.363	1.295	1.223	1.489	1.399	1.329	1.256	1.508	1.417	1.347	1.273
50	1.324	1.262	1.214	1.163	1.751	1.668	1.608	1.541	1.970	1.877	1.811	1.735
105	1.206	1.169	1.140	1.108	1.595	1.548	1.510	1.468	1.795	1.743	1.701	1.654
160	1.159	1.132	1.110	1.084	1.534	1.500	1.470	1.437	1.728	1.690	1.657	1.620
215	1.137	1.113	1.093	1.072	1.505	1.474	1.449	1.421	1.696	1.661	1.633	1.603

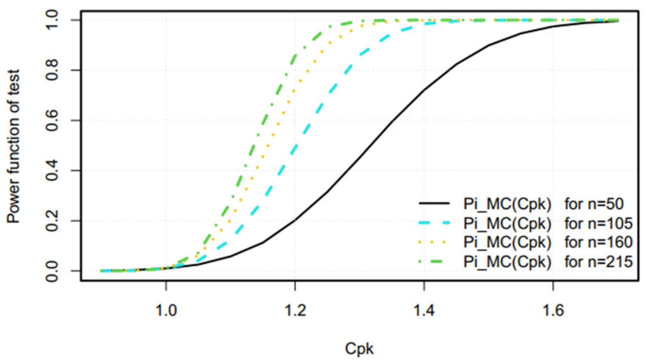

Figure 6. Plots of the MC power function of quality test $\Pi_{MC}(C_{pk})$ for the computed critical values based on the given values of $\alpha = 0.010$, $c_0 = 1.00$, and various sample sizes $n = 50(55)215$.

To perform the comparison between the proposed method and the Pearn and Lin's method, we compute the running time for calculating the critical values by R software. In this case, to compute the critical values, the running times (in terms of the second) are shown in Table 3. This Table shows that the performance of the Pearn and Lin's method is slightly faster than our proposed method.

The calculations of this study can be collected for different sample sizes at different significance levels, as shown in Table 4. Also, for the desired quality condition with $c_0 = 1.00$, $\alpha = 0.010$, and various sample sizes $n = 50(55)215$, one can compute the appropriate critical value c_{MC} from Eq. (15), and the MC power function of the quality test can be drawn in Figure 6, which shows that the larger sample size can increase the power of the quality test.

Conclusions

The capability index C_{pk} was utilized in industrial processes to assess the performance of a process. This study attempted to propose an MC simulation method for the hypotheses testing of C_{pk}, based on the natural estimator $\widehat{C}_{pk}$. Of particular significance is the MC simulation approach for solving the equation $\alpha = Pr\left(\widehat{C}_{pk} > c | C_{pk} = c_0\right)$ without the presence of the process parameter C_p for any Normal process. This proposed approach implies a more powerful test, which is a useful method for specifying whether the process meets the simulated capability requirement on the basis of the preset SLs. An application was investigated to show the performance of the proposed procedure in the automobile engine manufacturing industry.

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