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A Fault-Tolerant Sensor Reconciliation Scheme based on LPV Unknown Input Observers

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Abstract— **This paper presents a fault-tolerant sensor reconciliation scheme for systems equipped with a redundant number of possibly faulty "physical" sensors. The reconciliator is in charge to discover on-line, at each time instant, the possibly faulty physical sensors and exclude their measures from the generation of the "virtual" sensors, which, on the contrary, are supposed to be always healthy and suitably usable for control purposes without requiring the reconfiguration of the nominal control law. Amongst many, the solution proposed here is based on the use of a Linear Parameter Varying Unknown Input Observers (LPV-UIO) coupled with an "ad-hoc" parameter estimator used to identify on-line the current sensor reconciliation matrix. The latter is therefore used to hide the faulty measures from the pool of physical outputs in the generation of the virtual outputs. For simplicity, the sensor faults here considered are limited to variation of sensors' gain and offset values. The scheme is fully described and all of its properties investigated and proved. Finally, a simulation example is reported in details to show the effectiveness of the scheme.**

I. Introduction

Modern technological systems rely on sophisticated components designed to meet performance and safety requirements. However, increased complexity, usage of unmanned vehicles and systems provided by autonomous decision intelligence and a number of additional smart components make these systems more vulnerable to failures that can lead to loss of performance and sometimes may dangerously affect both system and human operator safety.

For this reason, the ability to detect faulty sensors and recover uncorrupted data has gained importance in many different applications. Specifically, in traditional control systems, faulty sensors give wrong information about the system status, which could cause instability. Even if stability is kept, inaccurate sensor values may introduce poor regulation or tracking performance, which may be highly undesirable for many high precision control applications ([1], [2]).

A systematic way to address these issues is to exploit Sensor Reconciliation (SR) schemes ([3]) which may be implemented to recover useful data from the pool of redundant sensors whenever unpredicatable fault events may eventually occur. The SR unit usually behaves as a *virtual sensor* ([4]) translating measurements from the possibly faulty sensors into the reliable signals that the controller can handle confidently.

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In this way, if effective, the need of using complex control reconfiguration strategies to accommodate sensor faults would be avoided and traditional simple controllers could be considered for control purposes. For this reason, many SR approaches proposed in the literature appear as part of Fault-Tolerant Control (FTC) schemes. Among many, it is worth mentioning [5], [6], where the sensor information are fused in a decentralized way by local estimators. Another class of SR FTC-based strategies considered in ([2], [7]) relies on a switching mechanism involving sensors and related observers to implicitly detect the healthy components of the system. The estimates provided by the observers are compared at each sampling time by a switching logic that allows one to select the sensors-observer pair with the smallest estimation error.

All the above mentioned approaches present a common denominator consisting in the accomplishment of two main tasks: (i) identification of faults in the sensors, (ii) rectification of sensor measurements. In this respect many effective methods have been developed for the estimation of either actuator or sensor faults [8], [9], while see [10], [11] for relevant works in sensor rectification.

The aim of this paper is to propose and discuss a general SR method for linear discrete-time systems with redundant physical sensors possibly subject to loss of effectiveness (gain) and offset (bias) faults. To this end, the proposed scheme consists of three interconnected modules: (i) a Parameter Estimator unit implemented via a constrained weighted least-squares batch method used within a windowing data processing approach to estimate the current gain sensor faults, (ii) a polytopic Unknown Input Observer (UIO) ([12]) in charge of combining the corrupted information gathered by multiple sensors to reconstruct, on the basis of the output of the Parameter Estimator, the state of the system and estimating possible bias fault occurrences; (iii) a sensor reconciliation unit used to reconcile sensor measures. The key idea used in the proposed scheme is to consider estimates of current sensor' gains as structural uncertainty in the the plant and consequently to design a polytopic LPV-UIO observer capable to deal with such an uncertainty via a specific Linear Matrix Inequality (LMI) procedure. Recent contributions to polytopic LPV-UIO design methodologies can be found e.g. in [13], [14]. In particular, here we extend the ideas presented in [14] to the design of continuous-time LPV-UIO scheme to the discrete-time case and to the more complex scenario here considered, where the time-varying parameters are not perfectly known. Properties of the presented UIO are formally proved and discussed. A final numerical example is reported to show the effectiveness of the proposed strategy.

Notation

Let $\mathbb{R}$ denote the set of real numbers and $\mathbb{N}$ those of natural numbers. Let $\|\cdot\|_2$ denote the weighted 2-norm of a vector (i.e. $|x|_2 = \sqrt{x^T x}$). Given a matrix $M \in \mathbb{R}^{n\times m}$, the i-th row of M is denoted as $M^{(i)}$. For a matrix $M \in \mathbb{R}^{n\times m}$, the *Moore-Penrose Pseudoinverse* is defined as $M^\dagger \in \mathbb{R}^{n\times m}$ and is computed as $M^\dagger := (A^T A)^{-1} A^T$. For $\mathcal{P} \in \mathbb{R}^p$ and $\mathcal{Q} \in \mathbb{R}^q$ being two polytopes of dimension p and q respectively, their *Cartesian Product* is defined as

$$\mathcal{P} \times \mathcal{Q} = \{(x, y) : x \in \mathcal{P}, y \in \mathcal{Q}\}$$

The Polytope $\mathcal{S}_k := \{\xi \in \mathbb{R}^l | \xi_i \geq 0,\ i = 1, ..., l, \sum_{i=1}^{l} \xi_i = 1\}$ is a k-dimensional *Unit Simplex*. For l matrices $M_i \in \mathbb{R}^{n\times m}$, $i = 1, ..., l$, their *Convex Hull*, denoted by $\text{Co}\{M_i\}, i = 1, ..., l$, is the polytope arising by all convex combinations of matrices M_i i.e $\{\sum_{i=1}^{l} \rho_i M_i, [\rho_1, ..., \rho_l]^T \in \mathcal{S}_l\}$ with $\mathcal{S}_l$ being a l-dimensional unit simplex.

II. Problem Formulation

Let us consider a plant whose dynamics is described by the following discrete-time state-space representation

$$x_p(t + 1) = Ax_p(t) + Bu(t) + Ev(t) \quad (1)$$

$$y(t) = \Delta\big(\gamma(t)\big) C_y x_p(t) + F b(t) \quad (2)$$

$$z(t) = H_z C_y x_p(t) \quad (3)$$

where $x_p(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is a known input while $v(t) \in \mathbb{R}^{n_v}$ is an unknown input. Moreover $y(t) \in \mathbb{R}^m$ represents the *plant output* provided by the physical redundant sensors possibly effected by both bias $b(t) \in \mathbb{R}^q$ and loss of effectiveness faults, the latter modeled by the gain matrix $\Delta(\gamma) \in \mathbb{R}^{m\times m}$ that, for simplicity, hereafter we assume to have the following elementary structure:

$$\Delta(\gamma) = \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \gamma_m \end{bmatrix} \quad (4)$$

Finally, $z(t) \in \mathbb{R}^r$, with $r \leq m$, is defined as the *virtual output* of the system and represents the healthy information we need to get from the plant for control purposes regardless of any fault possibly occurring on the physical sensors.

It is clear that in the absence of faults one would have $\Delta(\gamma) = I_m$ and $b(t) = 0$. However, in the more general case $b(t) \neq 0$ and $\Delta(\gamma)$, changes accounting for all possible occurring values of γ are confined in the generic polytope

$$\Gamma \subseteq \{\gamma : 0_m \leq \gamma \leq 1_m\} \quad (5)$$

For this reason, it is not convenient to evaluate the signal $z(t)$ as $z(t) = H_z y(t)$ because it would be affected by possibly corrupted information brought by $y(t)$. However, because the state $x_p(t)$ is assumed not directly measurable, $z(t)$ cannot be evaluated as simply as in (3), but a more sophisticated machinery is required. This aspect motivates the design of the *Sensor Reconciliator* (virtual sensor) unit that is basically aimed at addressing the following problem:

Sensor Reconciliaton Design Problem (SRDP-Problem) : *Given the system* (1)-(3)*, on the basis of the real output* $y(t)$*, compute at each time* $t \geq 0$ *a virtual output* $z(t)$ *whose value is as close as possible to* $H_z C_y x_p(t)$ *despite both faults occurrences corrupting signal* $y(t)$ *and presence of disturbance* $v(t)$.

III. Virtual Sensor Architecture

The basic approach here considered for solving the **SRDP-Problem** is to compute an estimate $\hat{x}_p(t)$ of the state $x_p(t)$ and then to evaluate the corresponding approximation $\hat{z}(t)$ of $z(t)$ by exploiting the following equation

$$\hat{z}(t) = H_z C_y \hat{x}_p(t) \quad (6)$$

Such an approach is not trivial because it involves two

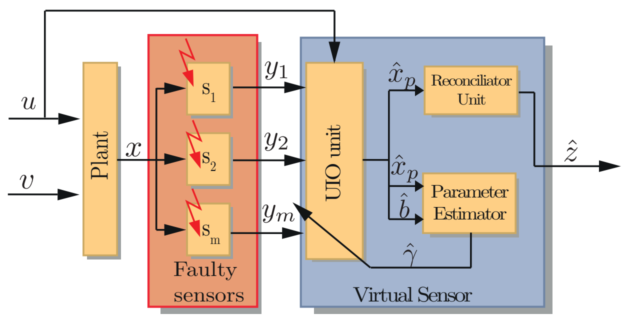

Fig. 1. Virtual Sensor Architecture

critical points: 1) How to estimate the fault occurrences corrupting $y(t)$? 2) How to get a good estimation $\hat{x}_p(t)$ in presence of an unknown input $v(t)$ and time-varying sensor gains and bias?

To deal with these questions, we propose the *virtual sensor* architecture depicted in Fig. 1 consisting of three modules: an *Unknown Input Observer* (UIO) unit which is the core of this scheme and is designed not only to give an estimation of $x_p(t)$ but also to evaluate an approximation to the bias fault $b(t)$; a *Parameter Estimator* whose output is an estimate of effectiveness matrix (4) and a *Reconciliator Unit* that simply performs the computation indicated in (6).

A. Sensor Fault Augmented Model

In order to design the UIO, the following augmented state is considered including the bias fault $b(t)$ among its components

$$x(t) = \begin{bmatrix} x_p(t) \\ b(t) \end{bmatrix} \quad (7)$$

In this way, the related augmented model can be described as

$$\begin{aligned} x(t + 1) &= \bar{A}x(t) + \bar{B}u(t) + \bar{E}v(t) + \bar{F}\Delta b(t) \\ y(t) &= \bar{C}_\gamma x(t) \end{aligned} \quad (8)$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix}, \ \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \ \bar{E} = \begin{bmatrix} E \\ 0 \end{bmatrix}, \ \bar{F} = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$\bar{C}_\gamma = \begin{bmatrix} \triangle(\gamma) C_y & F \end{bmatrix}, \ \Delta b(t) = b(t+1) - b(t) \qquad (9)$$

Moreover, the following technical assumption is required

Assumption *1:*

$$\text{rank}\{\bar{C}_\gamma \bar{E}\} = \text{rank}(\bar{E}), \forall \gamma \in \Gamma \qquad (10)$$

B. Unknown Input Observer

In this section we describe the basic ingredients of the proposed UIO. Let us assume to be provided with an estimation $\hat{\gamma}(t)$ of $\gamma(t)$ at each time t. Then, a possible structure for an unknown input observer for the model (8) is given by

$$\hat{x}(t+1) = T_{\hat{\gamma}(t)} \bar{A} \hat{x}(t) + T_{\hat{\gamma}(t)} \bar{B} u(t) + L_{\hat{\gamma}(t)} \Big(y(t) - \hat{y}(t) \Big) + Q_{\hat{\gamma}(t)} y(t+1) \qquad (11)$$

where $T_\gamma \in \mathbb{R}^{(n+q)\times(n+q)}$, $L_\gamma \in \mathbb{R}^{(n+q)\times m}$ and $Q_\gamma \in \mathbb{R}^{(n+q)\times m}$ represent design parameters all depending on the effectiveness matrix (4). In particular, if T_γ were chosen to satisfy

$$T_\gamma + Q_\gamma \bar{C}_\gamma = I_{n+q} \qquad (12)$$

the system (8) could be represented by

$$x(t+1) = T_{\hat{\gamma}(t)} \bar{A} x(t) + T_{\hat{\gamma}(t)} \bar{B} u(t) + T_{\gamma(t)} \bar{E} v(t) + T_{\hat{\gamma}(t)} \bar{F} \Delta b(t) + Q_{\hat{\gamma}(t)} \tilde{y}(t+1) \qquad (13)$$

where $\tilde{y}(t) := \bar{C}_{\hat{\gamma}(t)} x(t)$. As a consequence, the one-step ahead evolution of the state estimation error

$$e(t) := x(t) - \hat{x}(t) \qquad (14)$$

would take the following form

$$e(t+1) = \left(T_{\hat{\gamma}(t)} \bar{A} - L_{\hat{\gamma}(t)} \bar{C}_{\hat{\gamma}(t)} \right) e(t) + T_{\hat{\gamma}(t)} \bar{E} v(t) + T_{\hat{\gamma}(t)} \bar{F} \Delta b(t) + d(t) \qquad (15)$$

with

$$d(t) := Q_{\hat{\gamma}(t)} (\tilde{y}(t+1) - y(t+1)) + L_{\hat{\gamma}(t)} \left(C_{\gamma(t)} - C_{\hat{\gamma}(t)} \right) x(t)$$

Under the condition

$$T_\gamma \bar{E} = 0, \forall \gamma \in \Gamma \qquad (16)$$

equation (15) becomes

$$e(t+1) = \left(T_{\hat{\gamma}(t)} \bar{A} - L_{\hat{\gamma}(t)} \bar{C}_{\hat{\gamma}(t)} \right) e(t) + T_{\hat{\gamma}(t)} \bar{F} \Delta b(t) + d(t) \qquad (17)$$

and, if $\left(T_{\gamma(t)} \bar{A} - L_{\gamma(t)} \bar{C}_{\gamma(t)} \right)$ were chosen as a stable matrix $\forall \gamma \in \Gamma$, the state estimation error would go to zero when $t \to \infty$, $\Delta b(t) \to 0$ and $\hat{\gamma}(t) \to \gamma(t)$. Hence, the state of the system would be estimated asymptotically and the unknown input would be completely decoupled. It is worth commenting that condition (16), which can be recast in the form $\bar{E} = Q_\gamma \bar{C}_\gamma \bar{E}$, is satisfied if Q_γ is chosen as

$$Q_\gamma := \bar{E} (\bar{C}_\gamma \bar{E})^\dagger, \forall \gamma \in \Gamma \qquad (18)$$

where the existence of the matrix $(\bar{C}_\gamma \bar{E})^\dagger$ is guaranteed $\forall \gamma \in \Gamma$ by **Assumption 1**.

Moving from these considerations, in order to design the UIO (11), it is is sufficient to determine a parameter varying gain $L_{\gamma(t)}$ that robustly stabilizes the system (17) against all possible occurrences of $\Delta b(t)$ and $d(t)$. Such a problem has been addressed in a significant amount of works for different contexts by exploiting well-known results on robust control theory and LMI formalism. In particular, the approach considers systems (17) characterized by a structured uncertainty related to γ and attempts to determine a LPV gain that can be tuned on-line by exploiting an estimate $\hat{\gamma}(t)$ of the true $\gamma(t)$. In this respect, it is worth pointing out that unfortunately the matrix T_γ does not depend linearly on the parameter γ. As a consequence, the related uncertainty representation results non-convex and the system (17) cannot be considered a polytopic LPV form. For this reason, in order to take advantages of existing LMI based design techniques, we assume hereafter to be provided by a polytopic embedding approximation for matrices T_γ and $\bar{C}_\gamma$ given by

$$\begin{cases} \bar{C}_\rho = \sum_{i=1}^{l} \rho_i(\gamma) \tilde{C}_i, \\ T_\rho = \sum_{i=1}^{l} \rho_i(\gamma) \tilde{T}_i \end{cases} \qquad (19)$$

for a certain continuous functions $\rho_i : \Gamma \to \mathbb{R}$ of γ and pair of matrices $(\tilde{T}_i, \tilde{C}_i)$, $i = 1, ..., l$. In addition, we assume that the map $\rho : \Gamma \to \mathcal{R}^l$ given by $\rho := (\rho_1, ..., \rho_l)^T$ always returns values into the unit simplex $\mathcal{S}_l$. Hence, for each $\gamma \in \Gamma$, the pair $(T_\rho, \tilde{C}_\rho)$ lies in the convex hull $\text{Co}\{(\tilde{T}_i, \tilde{C}_i)\}, i = 1, ..., l$.

Moreover, the above representations have to guarantee that the following assumptions hold true:

A1 $\Big(T_\rho \bar{A}, \bar{C}_\rho \Big)$ is detectable $\forall \rho \in \mathcal{S}_l$

A2 $T_\rho \bar{E} = 0$, $\forall \rho \in \mathcal{S}_l$

Now, we have all the ingredients to design a LPV gain $L_{\hat{\rho}}$ defined as follow

$$L_{\hat{\rho}} = \sum_{i=1}^{l} \hat{\rho}_i(\gamma) L_i \qquad (20)$$

where the gains L_i, $i = 1, ..., l$ are properly chosen to stabilize the observer, provided that an estimation $\hat{\rho}(t)$ is available, with the estimation error subject to

$$e(t+1) = N_{\hat{\rho}(t)} e(t) + T_{\hat{\rho}(t)} \bar{E} v(t) + F_{\hat{\rho}(t)} w(t) \qquad (21)$$

with

$$N_\rho := \left(T_\rho \bar{A} - L_\rho \bar{C}_\rho \right), F_\rho := \begin{bmatrix} T_\rho \bar{F} & I \end{bmatrix},$$

$$w(t) := \begin{bmatrix} \Delta b^T(t) d^T(t) \end{bmatrix}^T$$

More formally we are interested to find a parameter-dependent gain $L_{\hat{\rho}(t)}$ such that difference equation (21) is stable for any arbitrary time variation of the parameters

$\hat{\rho}(t) \in \mathcal{S}_l$ and such that, for any input $w(t) \in \ell_2$, the error $e(t)$ is bounded

$$||e(\cdot)||_2 < \sigma ||w(\cdot)||_2 \tag{22}$$

A convex optimization methodology to solve the above stated design problem is provided in the next Theorem 1.

Theorem 1: Assume a symmetric positive definite matrices P_i and matrices G_i and Y_i, $i = 1, ..., l$ exist such that the optimization problem

$$\min_{P_i, G_i, Y_i, \mu} \mu$$

subject to

$$\begin{aligned} \Xi_{ij} &:= \begin{bmatrix} G_i + G_i^T - P_j & Q_{12} & G_i F_i \\ \star & P_i - I & 0 \\ \star & \star & \mu I \end{bmatrix} > 0, \\ Q_{12} &:= G_i \tilde{T}_i \tilde{A} - Y_i \tilde{C}_i, \; i = 1, ..., l, \; j = 1, ..., l \end{aligned} \tag{23}$$

$$\Xi_{ijk} := \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ * & P_i + P_k - I & 0 \\ * & * & \mu I \end{bmatrix} > 0$$

$$i = 1, ..., l-1, j = 1, ..., l, k = i+1, ..., l$$

$$R_{11} := G_i + G_i^T + G_k + G_k^T + P_j$$

$$R_{12} := G_i \tilde{T}_k \bar{A} + G_k \tilde{T}_i \bar{A} - Y_i \tilde{C}_k - Y_k \tilde{C}_i, R_{13} := G_i F_k + G_k F_i \tag{24}$$

has a solution. Then, the convergence of the observer estimation error dynamically characterized by equation (21) is ensured and a guaranteed $\mathcal{H}_\infty$ performance gain (22) given by

$$\sigma \;=\; \sqrt{\mu^\star}, \; \mu^\star \;=\; \min \mu \tag{25}$$

it achieved. Moreover, the observer gain vertices defined in (20) are given by

$$L_i = G_i^{-1} Y_i \tag{26}$$

and stabilize the observer for any arbitrary time variation of the parameter $\hat{\rho}(t)$ in the polytope $\mathcal{S}_l$.

Proof: Omitted for space reasons.

Remark 1: It is worth remarking that it is not hard to get polytopic representation in the form (19) and a number of procedures exist in literature dealing with the above task (see for instance [15], [16])

C. Parameter Estimator

Here we describe the Parameter Estimator unit of Figure 1. It is based on an algorithm that is able to detect constant or slowly-varying gain faults in systems (2). Among many methods that can be used to solve this problem, we consider in this work constrained batch least-mean-squares approach ([17]) used within a windowing data processing strategy. The underlying idea is to estimate a matrix $\Delta(\hat{\gamma}(t))$ that matches as much as possible the plant measured signals and the estimated state in the last N time instants, with N arbitrarily chosen. To this end, we assume to be provided with the last N samples of both the *physical outputs* $y(t)$ and state estimation $\hat{x}(t)$ of the augmented system (8). In this way, by assuming $\hat{x}(t) = x(t)$ (*certainty equivalence hypothesis*), the following consistency equation can be imposed to the matrix $\Delta(\hat{\gamma}(t))$

$$y(t-i) = \Delta(\hat{\gamma}(t)) C_y \hat{x}_p(t-i) + F\hat{b}(t-i), \; i = 0, ..., N-1 \tag{27}$$

that are equivalent to

$$y(t-i) - F\hat{b}(t-i) = X(t-i)\hat{\gamma}(t), \; i = 0, ..., N-1 \tag{28}$$

where

$$X(t-i) := \text{diag}\left(C_y^{(1)} \hat{x}_p(t-i), ..., C_y^{(m)} \hat{x}_p(t-i)\right)$$

This allows one to recast the problem in a classical regressor form:

$$Y(t) = \varphi(t)\gamma(t) \tag{29}$$

where

$$Y(t) := \begin{bmatrix} y(t) - F\hat{b}(t) \\ \vdots \\ y(t-N+1) - F\hat{b}(t-N+1) \end{bmatrix}$$

are the measures and

$$\varphi(t) := \begin{bmatrix} X(t), \ldots, X(t-N+1) \end{bmatrix}^T$$

collects the linear regressors. Then, the variable $\hat{\gamma}(t)$ can be estimated through the resolution of the following quadratic program with linear constraints

$$\begin{aligned} \hat{\gamma}(t) := \arg\min_{\gamma} \quad & \tfrac{1}{2} \|(Y(t) - \varphi(t)\gamma)\|_2^2 \\ \text{subject to} \quad & \gamma \in \Gamma \end{aligned} \tag{30}$$

In [18] it has been proved that, under a constant $\gamma(t) = \gamma^\star$, a sufficient condition to guarantee convergence of $\hat{\gamma}(t)$ to $\gamma^\star$ for some $t^\star \gg N$ is that

$$\text{rank}\{\varphi(t^\star)\} = n \tag{31}$$

In particular, if $\bar{C}_y$ has not zero columns, a sufficient condition to ensure (31) is

$$\text{rank}\{\hat{X}_p(t)\} = n \tag{32}$$

where matrix $\hat{X}_p(t)$ is defined as

$$\hat{X}_p(t) := \left[\hat{x}_p(t), \ldots, \hat{x}_p(t-N)\right]^T \tag{33}$$

Such a property can be guaranteed if the state estimation $\hat{x}_p(t)$ problem is solved under a persistent excitation condition on the measures provided by the physical sensors or by a suitable artificial dither injected in the state estimation $\hat{x}_p(t)$ sent to the Parameter Estimator so as to force that signal to be persistently exciting so as to make (32) to hold true.

D. Reconciliation Algorithm

Finally, the proposed sensor reconciliation method can be summarized in the following algorithm

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UIO based Sensor Reconciliation Algorithm (UIO-SR)

INITIALIZATION:

1: **compute** L_i, $i = 1, ..., l$ according to Theorem 1
2: **chose** horizon N for the Parameter Estimator;
3: **set** $\Delta(\hat{\gamma}(t)) = I_m$ and $\hat{b}(t) = 0$ for $t = 0, ..., N-1$;
4: **store** L_i, $i = 1, ..., l$, N, $\Delta(\hat{\gamma}(t))$ and $\hat{b}(t)$, $t = 0, ..., N-1$.

ON-LINE PHASE (generic time $t \geq N$):

1: **receive** $y(t)$ from the sensors;
2: **compute** $\hat{\rho}(\hat{\gamma}(t-1))$ on the basis of the polytopic representation (19) ▷ This instruction can be skipped in the case of constant gain L to be used;
3: **compute** $Q_{\hat{\gamma}(t-1)}$ as in (18);
4: **set** $T_{\hat{\gamma}(t-1)} := I_{n+q} - Q_{\hat{\gamma}(t-1)}\bar{C}_{\hat{\gamma}(t-1)}$;
5: **estimate** plant state and bias by evaluating

$$\hat{x}(t) = T_{\hat{\gamma}(t-1)}\bar{A}\hat{x}(t-1) + T_{\hat{\gamma}(t-1)}\bar{B}u(t-1) + L_{\hat{\rho}(\hat{\gamma}(t-1))}\Big(y(t-1) - \hat{y}(t-1)\Big) + Q_{\hat{\gamma}(t)}y(t)$$

▷ $L_{\hat{\rho}(\hat{\gamma}(t-1))} = L$ in the case of constant gain to be used
6: **estimate** $\hat{\gamma}(t)$ by solving (30)
7: **compute** the estimated *real output* as $\hat{y}(t) = \bar{C}_{\hat{\gamma}(t)}\hat{x}(t)$
8: **return** the *virtual output* $\hat{z}(t) = H_z\hat{y}(t)$
9: **set** $t := t + 1$
10: **go to** step 1

IV. ILLUSTRATIVE EXAMPLE

In this section, the effectiveness of the proposed **UIO-SR** scheme is investigated by considering a linear stable model in form of (1)-(2) characterized by the following matrices

$$A = \begin{bmatrix} 0.98806 & 0.0096049 \\ -0.32754 & 0.93033 \end{bmatrix}, B = \begin{bmatrix} -0.0001 \\ -0.0921 \end{bmatrix},$$

$$C_y = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}, E = 0.01 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}, F = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

and γ is supposed to be confined within the polytope $\Gamma := \left\{\gamma : [\underline{\gamma}_1, \underline{\gamma}_2, \underline{\gamma}_3]^T \leq \gamma \leq [\overline{\gamma}_1, \overline{\gamma}_2, \overline{\gamma}_3]^T\right\}, \underline{\gamma}_1 = \underline{\gamma}_2 = 0, \underline{\gamma}_3 = 0.1, \overline{\gamma}_i = 1, i = 1, 2, 3.$

The goal of this simulation is to verify the capability of the proposed method in extracting the first component of th state $x_p(t)$ into the *virtual output* $z(t) = H_z C_y x_p(t)$ with sensor reconciliation matrix given by $H_z = \begin{bmatrix} 0.5 & 0.5 & 0 \end{bmatrix}$. Along the simulation the known input $u(t)$ and the unknown input $v(t)$ are supposed to be a square wave with period 625 and amplitude equal to 15 and a white noise with standard deviations equal to 10 respectively. Moreover, the bias profile on the three available physical sensors changes along the simulations according to the profile depicted in Figure 3 (solid black line) and faults on the matrix effectiveness gain will affect the first two sensors as depicted in Figure 2. In this scenario, without any sensor reconciliator block the *virtual output* would result falsified, as depicted in Figure 4 (blue dashed line), because of faults occurrences on the physical sensors. In this scenario, without any sensor reconciliator block the *virtual output* would result falsified, as depicted in Figure 4 (blue dashed line), because of faults occurrences on the physical sensors. In order to exploit the **UIO-SR** with the LPV unknown input observer described in Section IV.B, the above presented plant has to be recast in the augmented form (8) with matrix $\bar{C}_\gamma$ given by

$$\bar{C}_\gamma := \begin{bmatrix} \gamma_1 & 0 & 1 \\ \gamma_2 & 0 & 1 \\ \gamma_3 & \gamma_3 & 1 \end{bmatrix} \quad (34)$$

while the LPV UI observer has been designed as in (11) with

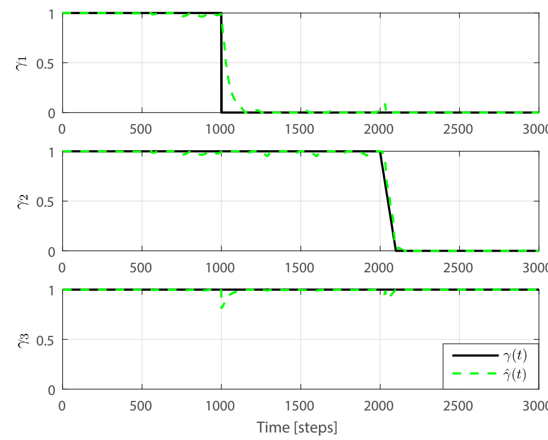

Fig. 2. Effectiveness Matrix Estimation

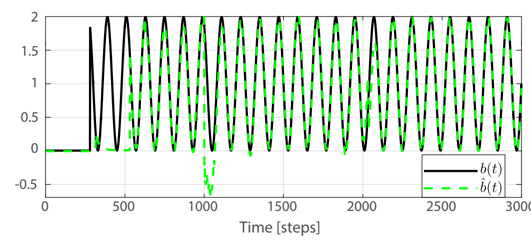

Fig. 3. Bias Estimation

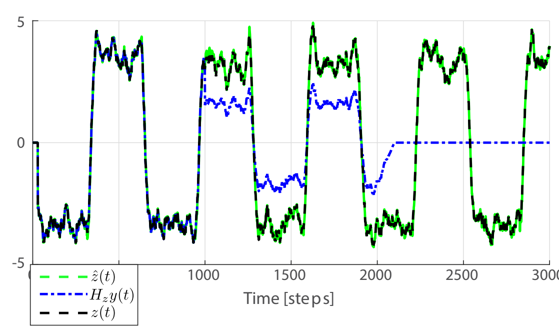

Fig. 4. Virtual Output

Q_γ selected as in (18). The resulting solution is given by

$$T_\gamma := \begin{bmatrix} 1+\beta_1(\gamma) & \beta_2(\gamma) & \beta_3(\gamma) \\ \beta_1(\gamma) & 1+\beta_2(\gamma)) & \beta_3(\gamma) \\ 0 & 0 & 1 \end{bmatrix} \tag{35}$$

with

$$\begin{aligned}
\beta_1(\gamma) &:= -\frac{\gamma_1(t)^2}{100Q(\gamma)} - \frac{3\gamma_3(t)^2}{125Q(\gamma)} \\
\beta_2(\gamma) &:= -\frac{\gamma_2(t)^2}{1000Q(\gamma)} - \frac{\gamma_3(t)^2}{625Q(\gamma)} \\
\beta_3(\gamma) &:= -\frac{\gamma_1(t)}{1000Q(\gamma)} - \frac{\gamma_2(t)}{10000Q(\gamma)} - \frac{\gamma_3(t)}{625Q(\gamma)} \\
Q(\gamma) &:= \frac{{\gamma_1}^2}{100} + \frac{{\gamma_2}^2}{10000} + \frac{16{\gamma_3}^2}{625}
\end{aligned} \tag{36}$$

In order to exploit the LPV observer, an embedding polytopic representation of the form of (19) has been derived. To this end, it is worth observing that matrices T_γ and $\bar{C}_\gamma$ can be embedded in two different polytopes, i.e. $\mathcal{P}_T := Co\{T_1, ..., T_8\}$ and $\mathcal{P}_C := Co\{C_1, ..., C_8\}$ respectively, with related vertices computed by evaluating matrices T_γ and $\bar{C}_\gamma$ on the extremum points of Γ. Then, a suitable polytopic representation (19) can be achieved by first deriving the LPV scheduling parameter $\rho(\gamma)$, that in this example is composed by 64 components, each one having the following structure

$$\rho_{(i-1)8+j}(\gamma) := \rho_i^1(\gamma)\rho_j^2(\gamma)$$

where both ρ^1 and ρ^2 are vectors each consisting of the following 8 elements

$$\begin{aligned}
&\rho_1^{(\cdot)} := \alpha_1^{(\cdot)}\alpha_2^{(\cdot)}\alpha_3^{(\cdot)},\ \rho_2^{(\cdot)} := \alpha_1^{(\cdot)}\alpha_2^{(\cdot)}\left(1-\alpha_3^{(\cdot)}\right), \\
&\rho_3^{(\cdot)} := \alpha_1^{(\cdot)}\left(1-\alpha_2^{(\cdot)}\right)\alpha_3^{(\cdot)},\ \rho_4^{(\cdot)} := \alpha_1^{(\cdot)}\left(1-\alpha_2^{(\cdot)}\right)\left(1-\alpha_3^{(\cdot)}\right) \\
&\rho_5^{(\cdot)} := \left(1-\alpha_1^{(\cdot)}\right)\alpha_2^{(\cdot)}\alpha_3^{(\cdot)},\ \rho_6^{(\cdot)} := \left(1-\alpha_1^{(\cdot)}\right)\alpha_2^{(\cdot)}\left(1-\alpha_3^{(\cdot)}\right), \\
&\rho_7^{(\cdot)} := \left(1-\alpha_1^{(\cdot)}\right)\left(1-\alpha_2^{(\cdot)}\right)\alpha_3^{(\cdot)}, \\
&\rho_8^{(\cdot)} := \left(1-\alpha_1^{(\cdot)}\right)\left(1-\alpha_2^{(\cdot)}\right)\left(1-\alpha_3^{(\cdot)}\right)
\end{aligned}$$

Specifically

$$\alpha_i^1(\gamma) := \frac{\overline{\beta}_1(\gamma) - \beta_1(\gamma)}{\overline{\beta}_i(\gamma) - \underline{\beta}_i(\gamma)},\ \alpha_i^2(\gamma) := \frac{\overline{\gamma} - \gamma_i}{\overline{\gamma} - \underline{\gamma}}, \quad i = 1, 2, 3 \tag{37}$$

Secondly, it is required to compute the Cartesian product $\mathcal{P} := \mathcal{P}_T \times \mathcal{P}_C$. Finally, it is possible to get a polytopic embedding approximation for T_γ and $\bar{C}_\gamma$ as follows

$$T_\rho = \sum_{i=1}^{64} \rho_i(\gamma)\tilde{T}_i, \quad \bar{C}_\rho = \sum_{i=1}^{64} \rho_i(\gamma)\tilde{C}_i \tag{38}$$

where $\left(\tilde{T}_i, \tilde{C}_i\right) i = 1, ..., 64$ are the vertices of $\mathcal{P}$.

By exploiting representation (38) simulative analysis can be attempted and related results have been depicted in Figures 2-3 where the proposed approach shows a good behavior in faults estimation (Figg. 2,3). Consequence of this fact, as depicted in Fig. 4, is that the generated *virtual output* signal (green dashed line) actually corrects the corrupted measured output (blue dashed-dotted line).

V. CONCLUSIONS

An unknown input observer based scheme has been presented to solve fault-tolerant sensor reconciliation design problems for linear discrete-time systems subject to possible faults on sensor gain and bias. The observer has been used to both estimate the state of the system and the current bias of the sensors while a least-squares based algorithm has been used to estimate the current gains' matrix of the physical sensors. The approach has been fully investigated, its properties rigorously proved and the method shown to be able to provide good reconciliation performance in quite general situations.

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