Fault-Tolerant Sensor Reconciliation Schemes via LFT Unknown Input Observers

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Abstract: This papers presents a fault-tolerant sensor reconciliation design approach for over-sensed plants (see Fig. 1). The reconciliator is in charge of detecting, at each time instant, the possibly faulty physical sensors and generating a virtual output z (with $\dim y \geq \dim z$) where the corrupted measures coming from the pool of redundant physical output y are removed. In this way the virtual output is always healthy and usable for control purposes without requiring the reconfiguration of the nominal control law. The approach is based on the use of Unknown Input Observers (UIO) with Linear Fractional Transformation (LFT) parameter dependency and works together with an "ad-hoc" parameters estimator that is designed to estimate on-line at each time instant the sensor effectiveness matrix. The sensor faults here considered are limited to variation of sensors' gain and offset values. All main properties of the scheme are investigated and rigorously proved. A final simulation example is included to show the effectiveness of the proposed scheme.

Keywords: Fault detection, Sensor Reconciliation, Unknown Input Observer, Linear Fractional Transformation.

1. INTRODUCTION

The capability of control systems to detect faulty sensors and recover in turn uncorrupted data has progressively gained more relevance in the last two decades. Traditional control schemes are usually designed by assuming perfect working conditions of the sensors to be used. However, in practice, sensors are subject to fault occurrences and can provide wrong information about the system status, which could cause instability. Even in the fortuitous cases where stability is preserved, inaccurate sensor values may lead to poor regulation or tracking performance, which may be highly undesirable for many high precision control applications (Mirabadi et al. (2003); Romero et al. (2010); Djath et al. (2000)).

Such an issue is usually faced by designing a proper Sensor Reconciliation (SR) scheme (Vachhani et al. (2001)) in order to recover useful data from the pool of redundant sensors whenever unpredicatable fault events may eventually occur. Figure 1 depicts a quite general SR scheme. There, the SR block behaves as a *virtual sensor* (Steffen (2005)) in charge of translating measurements from the possibly faulty sensors y into the reliable virtual sensor vector z that can be trustfully used for control purposes.

A number of SR approaches based on Figure 1 are often proposed in the literature as part of a Fault Tolerant Control (FTC) scheme where traditional controllers are present. Such a choice avoids the usage of complex control reconfiguration strategies to accommodate sensor faults.

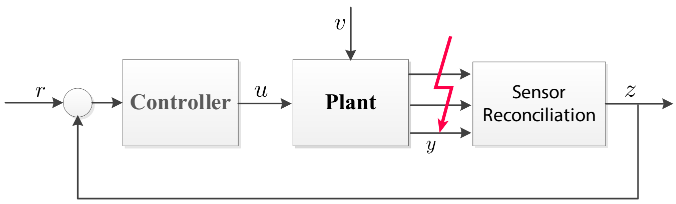

Fig. 1. Fault-tolerant sensor reconciliator basic scheme

In this respect, relevant contributions are Sun and Deng (2004); De Doná et al. (2009); Yetendje et al. (2011), where the sensor information are fused in a decentralized way by exploiting local estimators. A different class of SR FTC-based strategies is investigated in (Romero et al. (2010); Berbra et al. (2008)) where a switching mechanism is exploited involving sensors and related observers to implicitly detect the healthy components of the system. The estimates provided by the observers are compared at each sampling time by a switching logic that allows one to select the sensors-observer pair with the smallest estimation error.

All the above mentioned approaches are based on the execution of two basic steps: (i) identification of faults in the sensors, (ii) correction of sensor measurements. In this respect many effective methods have been developed for the estimation of either actuator or sensor faults He et al. (2013); Han et al. (2016); Alwi et al. (2011). See also Crowe (1996); Mah et al. (1976); Romagnoli and Stephanopoulos (1981) for relevant works in sensors rectification.

This paper is focused on a general SR method for linear discrete-time systems with redundant physical sensors possibly subject to loss of effectiveness (gain) and offset (bias) faults. The scheme differs from the above mentioned SR methods because it is based on the Unknown Input Observer (UIO) approach (Guan and Saif (1991)). In the present context, the UIO methodology has been widely investigated for the design of fault detection and isolation schemes for LTI continuous-time systems but limited to the detection of sensor bias (Chen et al. (1996); Chen and Saif (2006); Duan et al. (2002)).

Here the discrete-time system domain is considered by following the ideas of Zhou et al. (2013); Rodrigues et al. (2005), where Linear Matrix Inequality (LMI) procedures have been proposed to synthesize UIOs with constant observer gains. Such procedures have been here extended to address the more challenging case of jointly detecting both bias and gain sensor faults. To this end admissible ranges of current sensor gain estimates are treated as structural uncertainty in the the plant matrices. Such a choice leads to a non-convex uncertainty representation in the UIO equation. This paper complements the results of Behzad et al. (2016) for polytopic systems and solve some inherent difficulties. In particular, in Behzad et al. (2016) the non convexity of the uncertainty is overcome by embedding the uncertain matrices into a polytopic region and consequently by designing, via a specific LMI procedure, a polytopic Linear Parameter Varying (LPV) UIO observer. The resulting design procedure requires a very long computational time to compute the observer gains and can be impracticable in real applications. To cope with such a drawback, the uncertainty is modeled via the well-known Linear Fractional Transformation (LFT) approach (Cockburn and Morton (1997)). In this way, it is possible to build up a time-varying observer by solving LMI feasibility problems that are characterized by a lower complexity with respect to the general LPV case. As a result, the resulting computational burden to achieve the observer gain is comparable to that of the standard linear time-invariant case.

The proposed scheme consists of three interconnected modules: (i) a Parameter Estimator unit implemented via a constrained weighted least-squares batch method used within a windowing data processing approach to estimate the current gain sensor faults, (ii) a LFT UIO in charge of combining the corrupted information gathered by multiple sensors to reconstruct, on the basis of the output of the Parameter Estimator, the state of the system and estimating possible bias fault occurrences; (iii) a sensor reconciliation unit used to reconcile the sensor measures.

Properties of the proposed LFT-UIO scheme are formally proved and discussed and computational procedures are provided for the design of the UIO. A final numerical example is reported where comparisons with the approach presented in Behzad et al. (2016) are provided.

NOTATION

Let $\mathbb{R}$ denote the set of real numbers and $\mathbb{N}$ those of natural numbers. Let $v' \in \mathbb{R}^{1\times n}$ denote the transpose of a vector $v \in \mathbb{R}^n$ and $\|\cdot\|_2$ the weighted 2-norm of a vector (i.e. $|x|_2 = \sqrt{x'x}$). Given a matrix $M \in \mathbb{R}^{n\times m}$, the i-th row of M is denoted as $M^{(i)}$. For a matrix $M \in \mathbb{R}^{n\times m}$, the *Moore-Penrose Pseudoinverse* is denoted by $M^\dagger \in \mathbb{R}^{n\times m}$ and is computed as $M^\dagger := A'(A'A)^{-1}$. Linear Fractional Transformations (LFFs) are extensively used in the paper. For appropriately dimensioned matrices N and

$$M := \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

the lower LFT is defined as

$$LFT(M,N) := M_{11} + M_{12}N(I - M_{22}N)^{-1}M_{21}$$

2. PROBLEM FORMULATION

Let us consider a plant whose dynamics is described by the following discrete-time state-space representation

$$x_p(t+1) = Ax_p(t) + Bu(t) + Ev(t) \tag{1}$$

$$y(t) = \Delta\big(\gamma(t)\big)C_y x_p(t) + Fb(t) \tag{2}$$

$$z(t) = H_z C_y x_p(t) \tag{3}$$

where $x_p(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ is a known input while $v(t) \in \mathbb{R}^{n_v}$ is an unknown input. Moreover, $y(t) \in \mathbb{R}^m$ represents the *plant output* provided by physical redundant sensors possibly effected by both bias $b(t) \in \mathbb{R}^q$ and loss of effectiveness faults, the latter being modeled by the gain matrix $\Delta(\gamma) \in \mathbb{R}^{m\times m}$ that, for simplicity, we assume hereafter to have the following elementary structure:

$$\Delta(\gamma) = \begin{bmatrix} \gamma_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \gamma_m \end{bmatrix} \tag{4}$$

Finally, $z(t) \in \mathbb{R}^r$, with $r \le m$, is defined as the *virtual output* of the system and represents the healthy information we need to get from the plant for control purposes regardless of any fault possibly occurring on the physical sensors.

It is clear that in the absence of faults one would have $\Delta(\gamma) = I_m$ and $b(t) = 0_q$. However, in the more general case $b(t) \neq 0_q$ and $\Delta(\gamma) \neq I_m$, with γ confined in the generic polytope

$$\Gamma \subseteq \{\gamma : 0_m \le \gamma \le 1_m\} \tag{5}$$

For this reason, it is not convenient to evaluate the signal $z(t)$ as $z(t) = H_z y(t)$ because it would be affected by possibly corrupted information brought by $y(t)$. However, because the state $x_p(t)$ is assumed not directly measurable, $z(t)$ cannot be evaluated as simply as in (3), but a more sophisticated machinery is required. This aspect motivates the design of the *Sensor Reconciliator* (virtual sensor) unit depicted in Figure 1, which basically aims at addressing the following problem:

Sensor Reconciliaton Design Problem (SRDP-Problem) :
Given the system (1)-(3), compute, at each time $t \ge 0$ on the basis of the real output $y(t)$ measures, the best estimate $\hat{z}(t)$ of the virtual output $z(t) := H_z C_y x_p(t)$, despite the presence of both fault occurrences, corrupting the vectory(t), and disturbances $v(t)$.

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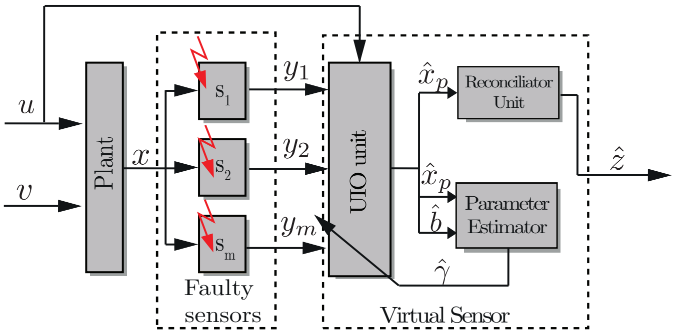

Fig. 2. Virtual Sensor Architecture

3. VIRTUAL SENSOR ARCHITECTURE

For the sake of generally, the **SRDP-Problem** solution here proposed is based on the estimation of $\hat{x}_p(t)$ of the state $x_p(t)$, that is therefore exploited to compute $\hat{z}(t)$ through the following equation

$$\hat{z}(t) = H_z C_y \hat{x}_p(t) \tag{6}$$

where H_z and C_y don't depend on the possible faults acting on the physical sensors. Such an approach require to face two crucial issues: 1) How to estimate the fault occurrences corrupting $y(t)$? 2) How to get a good estimation $\hat{x}_p(t)$ in presence of an unknown input $v(t)$ and time-varying sensor gains and biases?

The above mentioned questions are dealt with by introducing the *virtual sensor* architecture depicted in Fig. 2 that consists of three modules: an *Unknown Input Observer* (UIO) unit which is the core of this scheme and is designed not only to give an estimation of $x_p(t)$ but also to evaluate an approximation to the bias fault $b(t)$; a *Parameter Estimator* whose output is an estimate of effectiveness matrix (4) and a *Reconciliator Unit* that simply performs the computation indicated in (6).

3.1 Sensor Fault Augmented Model

In order to design the UIO, the following augmented state is considered including the bias fault $b(t)$ among its components

$$x(t) = \begin{bmatrix} x_p(t) \\ b(t) \end{bmatrix} \tag{7}$$

In this way, the related augmented model can be described as

$$\begin{aligned} x(t+1) &= \bar{A}x(t) + \bar{B}u(t) + \bar{E}v(t) + \bar{F}\Delta b(t) \\ y(t) &= \bar{C}_\gamma x(t) \end{aligned} \tag{8}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} E \\ 0 \end{bmatrix}, \bar{F} = \begin{bmatrix} 0 \\ I \end{bmatrix}$$
$$\bar{C}_\gamma = [\triangle(\gamma)C_y \;\; F], \; \Delta b(t) = b(t+1) - b(t) \tag{9}$$

Moreover, the following technical assumption is required

Assumption *1.*

$$\text{rank}\{\bar{C}_\gamma \bar{E}\} = \text{rank}(\bar{E}), \forall \gamma \in \Gamma \tag{10}$$

3.2 LFT Unknown Input Observer

In this section we describe the basic ingredients of the proposed UIO. Let us assume to be provided with an

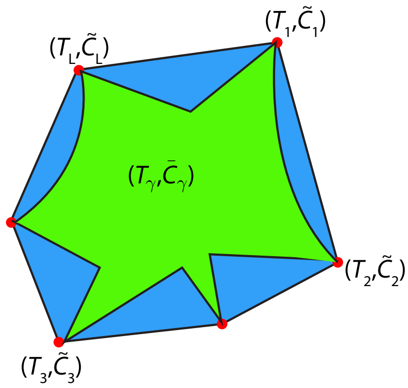

Fig. 3. Non convex representation (green region) and related polytopic embedding (blue region)

estimation $\hat{\gamma}(t)$ of $\gamma(t)$ at each time t. Then, a possible structure for an unknown input observer for the model (8) is given by

$$\begin{aligned} \hat{x}(t+1) = & T_{\hat{\gamma}(t)}\bar{A}\hat{x}(t) + T_{\hat{\gamma}(t)}\bar{B}u(t) + L_{\hat{\gamma}(t)}\big(y(t) - \hat{y}(t)\big) \\ & + Q_{\hat{\gamma}(t)}y(t+1) \end{aligned} \tag{11}$$

where $T_{\hat{\gamma}} \in \mathbb{R}^{(n+q)\times(n+q)}$, $L_{\hat{\gamma}} \in \mathbb{R}^{(n+q)\times m}$ and $Q_{\hat{\gamma}} \in \mathbb{R}^{(n+q)\times m}$ represent design parameters all depending on the effectiveness matrix (4). In particular, if $T_{\hat{\gamma}}$ were chosen to satisfy

$$T_{\hat{\gamma}} + Q_{\hat{\gamma}}\bar{C}_{\hat{\gamma}} = I_{n+q} \tag{12}$$

under the condition

$$T_{\hat{\gamma}}\bar{E} = 0, \forall \hat{\gamma} \in \Gamma \tag{13}$$

the system (8) could be represented as

$$\begin{aligned} x(t+1) = \;& T_{\hat{\gamma}(t)}\bar{A}x(t) + T_{\hat{\gamma}(t)}\bar{B}u(t) + T_{\hat{\gamma}(t)}\bar{E}\Delta b(t) \\ & + Q_{\hat{\gamma}(t)}y(t+1) \end{aligned} \tag{14}$$

Please notice that (13) is satisfied if $Q_{\hat{\gamma}}$ is chosen as

$$Q_{\hat{\gamma}} := \bar{E}(\bar{C}_{\hat{\gamma}}\bar{E})^{\dagger}, \forall \hat{\gamma} \in \Gamma \tag{15}$$

where the existence of the matrix $(\bar{C}_{\hat{\gamma}}\bar{E})^{\dagger}$ is guaranteed $\forall \hat{\gamma} \in \Gamma$ by **Assumption 1**.

As a consequence, the related uncertainty representation results non-convex and the system (14) cannot be considered a polytopic LPV form. For this reason, in order to take advantages of existing LMI based design techniques, in Behzad et al. (2016) a polytopic embedding approximation (see Figure 3 for a graphic idea) for matrices $T_{\hat{\gamma}}$ and $\bar{C}_{\hat{\gamma}}$ has been used. That approach leads to a design procedure characterized by a huge number of LMIs. In order to get a less computation demanding observer design, in this work we assume to be provided with a LFT representations of $T_{\hat{\gamma}} = LFT(T, \theta_T)$ and $\bar{C}_{\hat{\gamma}} = LFT(C, \theta_C)$ for certain matrices T and C respectively and

$$\theta_T(\gamma) := \text{diag}\{\theta_{T,1}(\gamma), ..., \theta_{T,n}(\gamma)\}, |\theta_{T,i}| \le 1, i = 1, ..., n$$
$$\theta_C(\gamma) := \text{diag}\{\theta_{C,1}(\gamma), ..., \theta_{C,m}(\gamma)\}, |\theta_{C,i}| \le 1, i = 1, ..., m$$

Such representations can be exploited to get (14) in the LFT form

$$
\begin{aligned}
x(t+1) &= T_{11}\bar{A}x(t) + T_{11}\bar{B}u(t) + T_{11}\bar{F}\Delta b(t) \\
&\quad + T_{12}p(t) + Q_{\hat{\gamma}}y(t+1) \\
y(t) &= C_{11}x(t) + C_{12}p(t) \\
q(t) &= C_{qx}x(t) + D_{qu}u(t) + D_{qb}\Delta b(t) + D_{qp}p(t) \\
p(t) &= \Theta(\hat{\gamma})q(t)
\end{aligned} \tag{16}
$$

where

$$
C_{qx} := \begin{bmatrix} T_{21}\bar{A} \\ C_{21} \end{bmatrix}, D_{qu} := \begin{bmatrix} T_{21}\bar{B} \\ 0 \end{bmatrix}, D_{qb} := \begin{bmatrix} T_{21}\bar{F} \\ 0 \end{bmatrix},
$$

$$
D_{qp} := \begin{bmatrix} C_{22} & 0 \\ 0 & T_{22} \end{bmatrix}
$$

with $\Theta(\gamma)$ being an uncertain parameter obeying the following structure

$$
\Theta(\gamma) := \begin{bmatrix} \theta_T(\gamma) & 0 \\ 0 & \theta_C(\gamma) \end{bmatrix} \tag{17}
$$

Then, a possible structure for an unknown input observer for the model (16) is given by

$$
\begin{aligned}
\hat{x}(t+1) &= T_{11}\bar{A}\hat{x}(t) + T_{11}\bar{B}u(t) + L\big(y(t) - \hat{y}(t)\big) \\
&\quad + T_{12}\hat{p}(t) + Q_{\hat{\gamma}(t)}y(t+1) \\
\hat{y}(t) &= C_{11}\hat{x}(t) + C_{12}\hat{p}(t) \\
\hat{q}(t) &= C_{qx}\hat{x}(t) + D_{qu}u(t) + D_{qp}\hat{p}(t) \\
\hat{p}(t) &= \Theta(\hat{\gamma})\hat{q}(t)
\end{aligned} \tag{18}
$$

As a consequence, the one-step ahead evolution of the state estimation error

$e(t) := x(t) - \hat{x}(t), \quad \tilde{p}(t) := p(t) - \hat{p}(t), \quad \tilde{q}(t) := q(t) - \hat{q}(t)$

would take the following form

$$
e(t+1) = Ne(t) + N_e\tilde{p}(t) + F_e w(t) \tag{19}
$$

$$
\tilde{q}(t) = C_{qx}e(t) + F_w w(t) + D_{qp}\tilde{p}(t) \tag{20}
$$

$$
\tilde{p}(t) = \Theta(\hat{\gamma})\tilde{q}(t) \tag{21}
$$

where

$$
N := T_{11}\bar{A} - LC_{11}, N_e := [T_{12} \;\; -LC_{12}], F_e := [T_{11}\bar{F}\, I],
$$

$$
F_w := \begin{bmatrix} T_{21}\bar{F} & 0 \\ 0 & 0 \end{bmatrix}, w(t) := \begin{bmatrix} \Delta b(t) \\ L(C_\gamma - C_{\hat{\gamma}})x(t) \end{bmatrix}
$$

We are interested at finding a gain L such that difference equation (19) is stable for any arbitrary time variation of the variables $\tilde{p}(t)$ and $\tilde{q}(t)$ and for any input $w(t) \in \ell_2$. As a consequence, the error $e(t)$ is bounded as

$$
\|e(.)\|_2 < \sigma \|w(.)\|_2 \tag{22}
$$

A convex optimization methodology to solve the above stated design problem is provided in the next Theorem 1.

Theorem 1. Assume that a symmetric positive matrix Q, a matrix S and positive scalars μ and λ exist such that the following optimization problem has a solution

$$
\min_{Q,S,\mu,\lambda} w_1\mu + w_2\lambda \tag{23}
$$

subject to:

$$
\begin{bmatrix} Q & QT_{11}\bar{A} - SC_{11} & \begin{bmatrix} QT_{12} & -SC_{12} \end{bmatrix} & QF_e \\ \star & Q - I - \lambda C_{qx}'C_{qx} & -\lambda C_{qx}'D_{qp} & -\lambda C_{qx}'F_w \\ \star & \star & \lambda I - \lambda D_{qp}'D_{qp} & -\lambda D_{qp}'F_w \\ \star & \star & \star & \mu I - \lambda F_w'F_w \end{bmatrix} > 0
$$

where $w_1, w_2 > 0$ are pre-specified weighting factors. Then, the boundedness of the observer estimation error as in (22) is ensured by choosing $L = Q^{-1}S$.

Proof: Consider the Lyapunov function

$$
V\big(e(t)\big) = e'(t)Qe(t) \tag{24}
$$

The related one-step ahead evolution of the above function on the observer error trajectory is given by

$$
V\big(e(t+1)\big) = e'(t+1)Qe(t+1) \tag{25}
$$

Using (19), one can recast (25) into

$$
\begin{aligned}
V\big(e(t+1)\big) = \big(Ne(t) + N_e\tilde{p}(t) + F_e w(t)\big)'Q\big(Ne(t) + N_e\tilde{p}(t) \\
+ F_e w(t)\big)
\end{aligned} \tag{26}
$$

Then, the Lyapunov function increment derived by (24) and (26) results to be given by

$$
\begin{aligned}
\Delta V(t) &= V\big(e(t+1)\big) - V\big(e(t)\big) \\
&= e'(t)\big(N'QN - Q\big)e(t) + 2\tilde{e}'(t)N'N_e\tilde{p}(t) \\
&\quad + 2\tilde{p}'(t)N_e'F_e w(t) + 2\tilde{e}'(t)N'F_e w(t) \\
&\quad + \tilde{p}'(t)N_e'QN_e\tilde{p}(t) + \tilde{w}'(t)F_e'QF_e\tilde{w}(t)
\end{aligned} \tag{27}
$$

It is well-known that the stability of system (19)-(21) with the $\mathcal{H}_\infty$ guaranteed performance (22) is ensured if

$$
\Delta V(t) \leq -e'(t)e(t) + \mu w'(t)w(t) \tag{28}
$$

for each $\tilde{q}(t)$, $\tilde{p}(t)$ satisfying

$$
\|\tilde{p}(t)\|_2 \leq \|\tilde{q}(t)\|_2 \tag{29}
$$

By replacing $\Delta V(e(t))$ with the expression (27), one is able to rewrite inequality (28) as

$$
\begin{bmatrix} e(t) \\ \tilde{p}(t) \\ w(t) \end{bmatrix}' \begin{bmatrix} N'QN - Q + I & N'QN_e & N'QF_e \\ N_e'QN & N_e'QN_e & N_e'QF_e \\ F_e'QN & F_e'QN_e & F_e'QF_e - \mu I \end{bmatrix} \begin{bmatrix} e(t) \\ \tilde{p}(t) \\ w(t) \end{bmatrix} < 0
$$

while inequality (29) can be recast as

$$
\begin{bmatrix} e(t) \\ \tilde{p}(t) \\ w(t) \end{bmatrix}' \begin{bmatrix} -C_{qx}'C_{qx} & -C_{qx}'D_{qp} & -C_{qx}'F_w \\ -D_{qp}'C_{qx} & I - D_{qp}'D_{qp} & -D_{qp}'F_w \\ -F_w'C_{qx} & -F_w'D_{qp} & -F_w'F_w \end{bmatrix} \begin{bmatrix} e(t) \\ \tilde{p}(t) \\ w(t) \end{bmatrix} < 0
$$

As a consequence, by means of the S-procedure, we can state that the above inequalities are true if and only if there exists a scalar λ such that

$$
\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ \star & U_{22} & U_{23} \\ \star & \star & U_{33} \end{bmatrix} < 0 \tag{30}
$$

$$
\begin{aligned}
U_{11} &:= N'QN - Q + I + \lambda C_{qx}'C_{qx} \\
U_{12} &:= N'QN_e + \lambda C_{qx}'D_{qp} \\
U_{13} &:= N'QF_e + \lambda C_{qx}'F_w \\
U_{22} &:= N_e'QN_e - \lambda I + D_{qp}'D_{qp} \\
U_{23} &:= N_e'QF_e + \lambda D_{qp}'F_w \\
U_{33} &:= F_e'QF_e - \mu I + \lambda F_w'F_w
\end{aligned}
$$

Notice that, by using Schur's complement lemma, (30) is equivalent to

$$\begin{bmatrix} Q & QN & QN_e & QF_e \\ \star & Q-I-\lambda C_{qx}'C_{qx} & -\lambda C_{qx}'D_{qp} & -\lambda C_{qx}'F_w \\ \star & \star & \lambda I-\lambda D_{qp}'D_{qp} & -\lambda D_{qp}'F_w \\ \star & \star & \star & \mu I-\lambda F_w'F_w \end{bmatrix} > 0$$

Finally, by taking the change of variable $QL = S$ into account, the inequality (24) results. □

3.3 Parameter Estimator

In this section the Parameter Estimator unit of Figure 2 is described. Its task consists in estimating the gain faults on the matrix $\Delta(\hat{\gamma})$ and is accomplished via a constrained batch least-mean-squares approach (Liew (1976)) used within a windowing data processing strategy. Such an approach is based on an algorithm that is able to detect constant or slowly-varying gain faults in systems of the form (2). The basic idea relies on finding a matrix $\Delta(\hat{\gamma}(t))$ that matches as much as possible the plant measured signals and the estimated state in the last N time instants, being N an arbitrarily chosen positive integer. In this respect the last N samples of both the *physical outputs* $y(t)$ and state estimation $\hat{x}(t)$ of the augmented system (8) are assumed to be provided at the generic time instant t. In this way, by considering $\hat{x}(t) = x(t)$ (*certainty equivalence hypothesis*), the following consistency equations can be imposed to the matrix $\Delta(\hat{\gamma}(t))$

$$y(t-i) = \Delta(\hat{\gamma}(t))C_y\hat{x}_p(t-i) + F\hat{b}(t-i),\ i = 0, ..., N-1 \quad (31)$$

that are equivalent to

$$y(t-i) - F\hat{b}(t-i) = X(t-i)\hat{\gamma}(t),\ i = 0, ..., N-1 \quad (32)$$

where

$$X(t-i) := \operatorname{diag}\left\{C_y^{(1)}\hat{x}_p(t-i), ..., C_y^{(m)}\hat{x}_p(t-i)\right\}$$

This allows one to recast the problem in the classical regressor form:

$$Y(t) = \varphi(t)\gamma(t) \quad (33)$$

where

$$Y(t) := \begin{bmatrix} y(t) - F\hat{b}(t) \\ \vdots \\ y(t-N+1) - F\hat{b}(t-N+1) \end{bmatrix}$$

are the measures and

$$\varphi(t) := [\,X(t), \ldots, X(t-N+1)\,]'$$

collects the linear regressors. Then, the variable $\hat{\gamma}(t)$ can be estimated through the resolution of the following quadratic program with linear constraints

$$\begin{aligned} \hat{\gamma}(t) := \arg\min_{\gamma}\ & \tfrac{1}{2}\|(Y(t) - \varphi(t)\gamma)\|_2^2 \\ \text{subject to}\ & \gamma \in \Gamma \end{aligned} \quad (34)$$

Under a constant $\gamma(t) = \gamma^\star$, it is possible to prove (Casavola and Garone (2010)) that a sufficient condition to guarantee convergence of $\hat{\gamma}(t)$ to $\gamma^\star$ for some $t^\star \gg N$ is

$$\operatorname{rank}\{\varphi(t^\star)\} = n \quad (35)$$

In particular, if $\bar{C}_y$ has not zero columns, a sufficient condition to ensure (35) is

$$\operatorname{rank}\{\hat{X}_p(t)\} = n \quad (36)$$

where matrix $\hat{X}_p(t)$ is defined as

$$\hat{X}_p(t) := [\hat{x}_p(t), \ldots, \hat{x}_p(t-N)]' \quad (37)$$

Such a property can be guaranteed if the state estimation $\hat{x}_p(t)$ problem is solved under a persistent excitation condition on the measures provided by the physical sensors or by a suitable artificial dither injected in the state estimation $\hat{x}_p(t)$ sent to the Parameter Estimator so as to force that signal to be persistently excited so as to make condition (36) hold true.

3.4 Reconciliation Algorithm

Finally, the proposed sensor reconciliation method can be summarized in the following algorithm

LFT-UIO based Sensor Reconciliation Algorithm (LFT-UIO-SR)

INITIALIZATION:

1: **compute** L, according to Theorem 1
2: **chose** horizon N for the Parameter Estimator;
3: **set** $\Delta(\hat{\gamma}(t)) = I_m$ and $\hat{b}(t) = 0$ for $t = 0, ..., N-1$;
4: **store** L, N, $\Delta(\hat{\gamma}(t))$ and $\hat{b}(t)$, $t = 0, ..., N-1$.

ON-LINE PHASE (generic time $t \geq N$):

1: **receive** $y(t)$ from the sensors;
2: **compute** $Q_{\hat{\gamma}(t-1)}$ as in (15);
3: **set** $T_{\hat{\gamma}(t-1)} := I_{n+q} - Q_{\hat{\gamma}(t-1)}\bar{C}_{\hat{\gamma}(t-1)}$;
4: **estimate** plant state and bias by evaluating

$$\begin{aligned}\hat{x}(t) =& T_{\hat{\gamma}(t-1)}\bar{A}\hat{x}(t-1) + T_{\hat{\gamma}(t-1)}\bar{B}u(t-1) \\ &+ L\big(y(t-1) - \hat{y}(t-1)\big) + Q_{\hat{\gamma}(t)}y(t)\end{aligned}$$

5: **estimate** $\hat{\gamma}(t)$ by solving (34)
6: **compute** the estimated *real output* as $\hat{y}(t) = \bar{C}_{\hat{\gamma}(t)}\hat{x}(t)$
7: **return** the *virtual output* $\hat{z}(t) = H_z\hat{y}(t)$
8: **set** $t := t + 1$
9: **go to** step 1

4. ILLUSTRATIVE EXAMPLE

In this section, the effectiveness of the proposed **LFT-UIO-SR** scheme is investigated by considering a linear stable model in form of (1)-(2) characterized by the following matrices

$$A = \begin{bmatrix} 0.98806 & 0.0096049 \\ -0.32754 & 0.93033 \end{bmatrix}, B = \begin{bmatrix} -0.0001 \\ -0.0921 \end{bmatrix},$$

$$C_y = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 10 & 1 \end{bmatrix}, E = 0.01 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}, F = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

with γ supposed to be confined within the polytope $\Gamma := \left\{\gamma : [\underline{\gamma}_1, \underline{\gamma}_2, \underline{\gamma}_3]' \leq \gamma \leq [\overline{\gamma}_1, \overline{\gamma}_2, \overline{\gamma}_3]'\right\}, \underline{\gamma}_i = 0.01, \overline{\gamma}_i = 1,\ i = 1, 2, 3.$

The goal of this simulation is to verify the capability of the proposed method of extracting the first component of the state $x_p(t)$ into the *virtual output* $z(t) = H_zC_yx_p(t)$ with the sensor reconciliation matrix given by $H_z = [0.5\ 0.5\ 0]$. Along the simulation, the known input $u(t)$ and the unknown input $v(t)$ are supposed to be those depicted in Figure 4.

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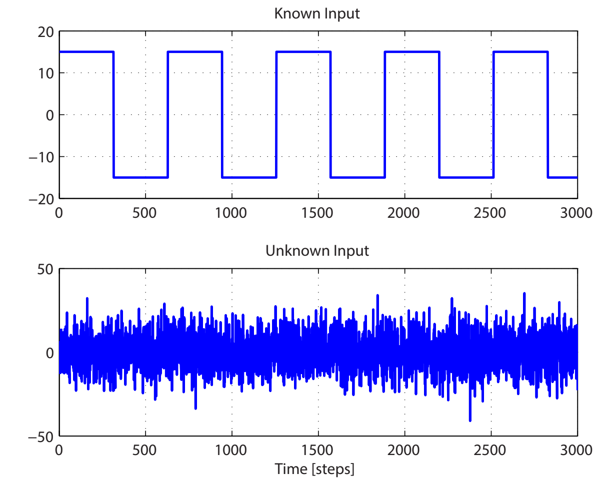

Fig. 4. Known Input (up) and Unknown Input (down)

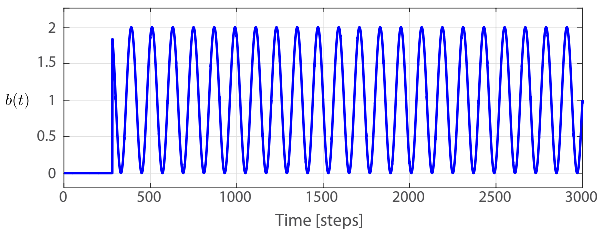

Fig. 5. Bias fault profile

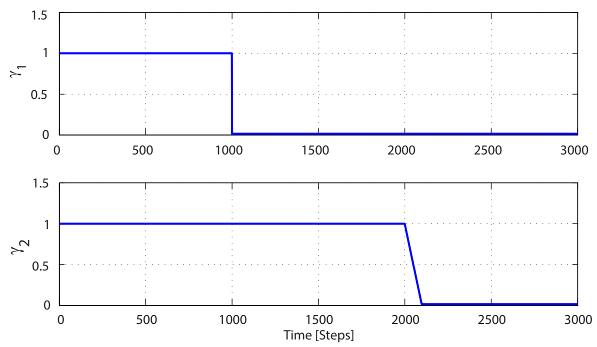

Fig. 6. Loss of effectiveness faults profiles

Moreover, the bias profile on the three available physical sensors changes along the simulations according to the profile depicted in Figure 5 and faults on the matrix effectiveness gain will affect the first two sensors as depicted in Figure 6. In this scenario, without any sensor reconciliator block the *virtual output* would result falsified, as depicted in Figure 10 (blue dashed line), because of faults occurrences on the physical sensors.

4.1 Setups

In order to exploit the **LFT-UIO-SR** with the LFT unknown input observer described in Section IV.B, the plant has to be recast in the augmented LFT form (16)-(17). In this respect, please notice that

$$\bar{C}_\gamma \bar{E} = \Delta(\gamma)G,\ G := [1,\ 1,\ 11]^T$$

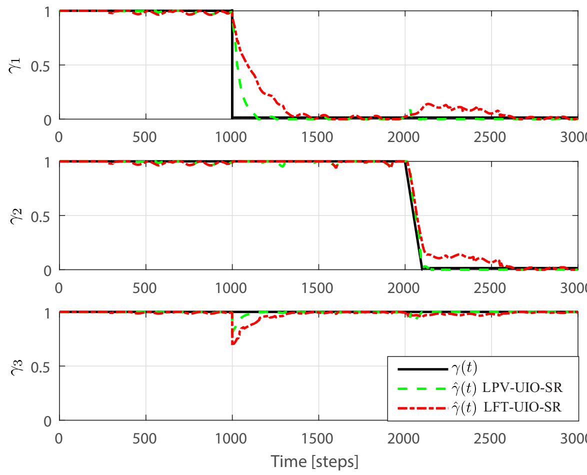

Fig. 7. Effectiveness Matrix Estimation

and

$$Q_\gamma = \bar{E}(\bar{C}_\gamma \bar{E})' \left((\bar{C}_\gamma \bar{E})'(\bar{C}_\gamma \bar{E})\right)^{-1} = \bar{E}G'(GG')^{-1}\Delta^{-1}(\gamma)$$

As a consequence, the matrix $T_{\hat{\gamma}}$ can be rewritten as

$$\begin{aligned}
T_{\hat{\gamma}} &= I_{n+q} - Q_{\hat{\gamma}}\bar{C}_{\hat{\gamma}} \\
&= I_{n+q} - \bar{E}G'(GG')^{-1}\Delta^{-1}(\hat{\gamma})\bar{C}_{\hat{\gamma}} \\
&= I_{n+q} - \bar{E}G'(GG')^{-1}\Delta^{-1}(\hat{\gamma})\,[\Delta(\gamma)C_y\ \ F] \\
&= I_{n+q} - \bar{E}G'(GG')^{-1}\Delta^{-1}(\hat{\gamma})\,(\Delta(\gamma)\,[C_y\ \ 0_3] + [0_{3\times 2}\ \ F]) \\
&= I_{n+q} - \bar{E}G'(GG')^{-1}\,[C_y\ \ 0_3] \\
&\quad - \bar{E}G'(GG')^{-1}\Delta^{-1}(\hat{\gamma})\,[0_{3\times 2}\ \ F]
\end{aligned} \tag{38}$$

A standard normalization for $\Delta^{-1}(\hat{\gamma})$ is required. It can be achieved e.g. by following the approach described in Cockburn (1998). As a result, one gets

$$\gamma_i^{-1} = a_i + b_i\delta_i, \quad |\delta_i| \le 1$$

with

$$a_1 = \frac{1}{2}(\overline{\gamma} + \underline{\gamma}) = 101.1, \quad a_2 = \frac{1}{2}(\overline{\gamma} - \underline{\gamma}) = 98.99$$

Then, a LFT representation for $T_{\hat{\gamma}}$ can be obtained as

$$\begin{aligned}
T_{\hat{\gamma}} &= I_{n+q} - \bar{E}G'(GG')^{-1}\,[C_y\ \ 0_3] \\
&\quad - \bar{E}G'(GG')^{-1}(\mathrm{diag}\{a_1, a_2, a_3\} \\
&\quad + \mathrm{diag}\{b_1\delta_1, b_2\delta_2, b_3\delta_3\})\,[0_{3\times 2}\ \ F] \\
&= I_{n+q} - \bar{E}G'(GG')^{-1}\,[C_y\ \ 0_3] \\
&\quad - \bar{E}G'(GG')^{-1}\mathrm{diag}\{a_1, a_2, a_3\}\,[0_{3\times 2}\ \ F] \\
&\quad - \bar{E}G'(GG')^{-1}\mathrm{diag}\{b_1\delta_1, b_2\delta_2, b_3\delta_3\}\,[0_{3\times 2}\ \ F] \\
&= LFT(T, \theta_T(\delta))
\end{aligned}$$

where

$$\begin{aligned}
T_{11} &:= I_{n+q} - \bar{E}G'(GG')^{-1}\,[C_y\ \ 0_3]\,, \\
T_{12} &:= -\bar{E}G'(GG')^{-1}, \\
T_{21} &:= [0_{3\times 2}\ \ F],\ T_{22} := [0_{3\times 3}] \\
\theta_T(\delta) &:= (\mathrm{diag}\{a_1, a_2, a_3\} + \mathrm{diag}\{b_1\delta_1, b_2\delta_2, b_3\delta_3\})
\end{aligned}$$

For the matrix $\bar{C}_\gamma$, the LFT representation is achieved in a simpler manner and it is given by

$$\bar{C}_\gamma = I_3 - (\Delta(\gamma) - I) = LFT(C, \theta_C(\gamma))$$

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where

$$C_{11} := C_{12} = C_{21} = I_3, \; C_{22} := 0_3, \; \theta_C(\gamma) := \Delta(\gamma) - I_3$$

By exploiting the above presented LFT representations, the following observer gain has been computed according to the LMI procedure of Theorem 1

$$L = \begin{bmatrix} 0.0046 & 0.0046 & -0.0001 \\ 0.0088 & 0.0088 & -0.1121 \\ 0.4934 & 0.4934 & -0.0976 \end{bmatrix}$$

Finally, a windowing horizon $N = 30$ has been chosen for the parameter estimator. All simulations have been performed by using the Yalmip interpreter Lofberg (2005) and the Sedumi solver; all under MATLAB 8.6 environment running on an Intel Core i5-3330 machine with 3.3 GHz and 8GB RAM.

4.2 Results and comparisons

A simulative comparison has been attempted between the presented **LFT-UIO-SR** scheme and the Sensor Reconciliating approach of Behzad et al. (2016), here referred to as **LPV-UIO-SR** and endowed with a LPV polytopic observer. In order to compute the observer's gain, the same embedding polytopic representation used in Behzad et al. (2016) has been here considered for the matrices T_γ and $\bar{C}_\gamma$, that consists in a polytope characterized by 64 vertices. The corresponding gain-scheduled observer's gain consists of the same number of elementary (64 gain matrices), jointly computed through an optimization procedure involving 262.144 LMIs. On the contrary, the **LFT-UIO-SR** scheme requires a procedure with a single LMI to derive the observer gain. This is not a minor aspect as it has a strong impact on the offline computational burden (see Table 1).

In Figures 7-10 these schemes have been compared. In particular, Figures 8,9 are related to the plant state and bias estimation respectively. Although both observers achieve good performance in recovering the true information related to $x_p^{(1)}$, as expected, the **LPV-UIO-SR** scheme exhibits a better behavior with respect to **LFT-UIO-SR**, both in estimating the state and the bias. This is mostly due to the fact the observer gain is time-varying scheduled in the **LPV-UIO-SR** scheme and it is able to "adapt" itself more quickly with respect to changes in the effectiveness matrix. Such an aspect translates in a better effectiveness parameter (gain matrix) estimation (Figure 7) and in a more accurate *virtual output* generation (Figure 10). Performance has been quantified by evaluating for quantities of interest the relative error

$$J_\xi := \frac{1}{T_{oss}} \sum_{t}^{T_{oss}} Err(\xi(t), \xi_{true}(t)), \qquad (39)$$

$$Err(\xi, \xi_{true}) := \frac{|\xi - \xi_{true}|}{\xi_{true}}$$

where T_{oss} denotes the simulation time interval. Results are reported in Table 1 along with the computational time required to get the solution in the offline phase. It is worth commenting that, although **LPV-UIO-SR** achieve a better performance, it involves a time-consuming design procedure that can be impracticable in the case of systems with a large number physical sensors to be monitored.

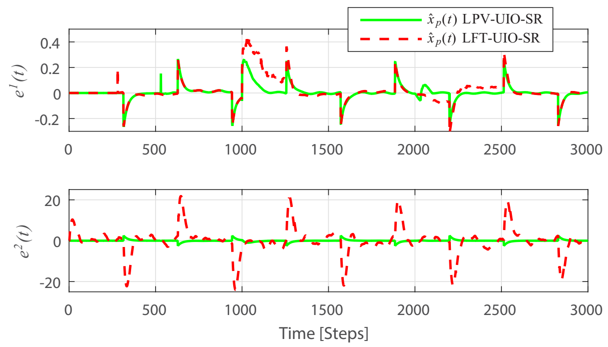

Fig. 8. State Estimation Error

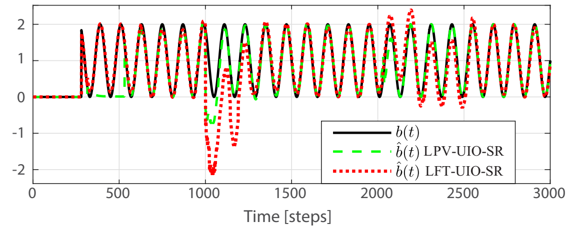

Fig. 9. Bias Estimation

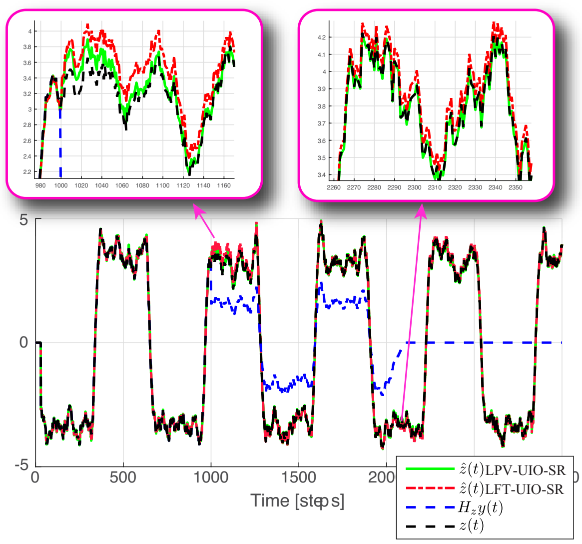

Fig. 10. Virtual Output

Table 1. Complete break-down of performance and CPU time for the different approaches

	Offline computation time	$J_{\gamma 1}$	$J_{\gamma 2}$	$J_{\gamma 3}$	J_β	J_z
LFT-UIO-SR	≈ 3min	6.51	2.15	0.025	6.64	0.012
LPV-UIO-SR	≈ 6h	1.59	0.04	0.005	4.53	0.004

5. CONCLUSIONS

In this paper LFT unknown input observers have been proposed to solve fault-tolerant sensor reconciliation design problems for linear discrete-time systems subject to possible faults on sensor gain and bias. The role of the observer relies on the estimation of both the state of the system and the current bias of the physical sensors whereas a least-squares batch algorithm provides estimates of the current effectiveness' matrix of the physical sensors. The resulting design procedure is quite simple and requires a low computational burden especially when compared with the analogous approach of Behzad et al. (2016), where an LPV UIO has been used to solve the same problem. The scheme has been fully described, its properties rigorously proved and, in the final simulation example, it has been shown able to achieve good performance in recovering useful data from the pool of redundant sensors.

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