Robot identification using fractional subspace method

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Robot Identification Using Fractional Subspace Method

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***Abstract*—This paper is concerned with fractional identification of state space model of continuous time MIMO systems. The methodology used in this paper involves a continuous-time fractional operator allowing to find fractional derivatives of the stochastic input – output data which are treated in time domain and identifying the state space matrices of the system using QR factorization.. There are many advantages in describing a physical system using fractional CT models in that the dynamic behavior of the system is, in actuality, inherently fractional. The efficacy of the approach is examined by comparing with other approaches using integer identification.**

I. Introduction

Although fractional calculus was first introduced in 1695 by Leibniz and L'Hospital, the first systematic studies seems to have been made at the beginning and middle of the nineteenth century by Liouville, Riemann, and Holmgren[1,2].

In the field of system identification using fractional orders some research has been done. (Oustaloup[3]; Trigeassou et al[4]; Malti et al[5]). Thomassin et al (2009)[6] had a thorough review of the old ways. Most of the researchers were concentrated on rational transfer function. The present paper, however, considers identification of a continuous-time fractional system in its state-space form.Cois etal (2001)[7] and Poinot etal (2004)[8] conducted a study on system identification employing fractional state-space representation. Their methods, however, are based on minimization of an output error criterion by nonlinear programming techniques. That is, as the number of parameters to estimate becomes large in a MIMO system, these methods are considered more suitable for SISO systems and are difficult to apply in the MIMO case.

In this paper we concentrate on subspace methods which is an extension of Ohsumi etal(2001)[9] method for rational systems. This method offers a novel approach to identifying the continuous-time state-space model using input-output data. The method is based on higher derivatives of input and output in the presence of both system and observation noises.

Finally to verify the algorithm, the method was tested on a robot manipulator and then compared to similar integer order method.

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II. Mathmatical Background of Fractional Systems

Fractional differintegration is developed from integer differentiation and integration.

Riemann-Liouville's definition of fractional diffeintegrals in fractional calculus is defined by[1]:

$$ {}_aD_t^{\alpha} f(t) = D^m I^{m-\alpha} f(t) = \frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-m+1}}dt\right] \quad (1)$$

where α is the real positive integration order. $\Gamma(n)$ is the Euler Gamma function[1]:

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt \quad (2)$$

Second definition is given by Grunwald -Letnikov:

$$D^n f(x) = \lim_{h\to 0} \frac{1}{h^\alpha} \sum_{m=0}^{\frac{x-a}{h}} (-1)^n \frac{\Gamma(\alpha+1)}{m!\,\Gamma(\alpha-m+1)} f(x-mh) \quad (3)$$

The fractional LTI state-space is presented In MIMO case as[6]:

$$\begin{cases} D^\alpha x(t) = Ax(t) + Bu(t) \\ \quad y = Cx(t) + Du(t) \end{cases} \quad (4)$$

Where α is the order of the system and $u \in R^m$, $y \in R^p, x \in R^n$ are input, output and state vectors, respectively. System matrices A, B, C and D have appropriate dimensions.

Here initial conditions are considered to be zero $(x(t) = 0 \; for \; t \le 0)$.

The fractional system is stable if[6]:

$$0 < \alpha < 2 \;\; and \;\; |arg(\lambda_k)| > \alpha\frac{\pi}{2} \quad \forall k = 1, \dots, n$$

Where λ_k is the k^{th}-eigenvalue of A and $-\pi < arg\,(\lambda_k) \le \pi$graphic.

III. Generalized Random Function

Let (Ω,F,P_α) be the probability triple where P_α is fractional probability and is defined as[10,11] :

$0 < \alpha < 1, P_\alpha : F \to [0,1]$

1. $P_\alpha(A) \ge 0$ for all $A \epsilon F$
2. $P_\alpha(\Omega) = 1$
3. ***for all*** $A_i \in F$ ***if*** A_i***s are pairwise disjoint, then***

$$P_\alpha(\cup_{i=1} A_i) \le \sum_{i=1} P_\alpha(A_i) \quad (5)$$

and let D denote the space of the (real-valued nonrandom) scalar C^∞ functions $\{\varphi(t)\}$defined on R. This function has a compact support and is called the test function in the distribution theory[9].

Here Ω represents sample space, F represents σ-fields and P is probability measure.

Ito (1953)[12] and Gel'fand and Vilenkin(1964)[13,14], maintained that continuous linear random functional defined on D is called a random distribution or a generalized random function (process); and the totality of them will be denoted by D' *(dual space). In other words, a random distribution function F is a measurable map from a probability space (Ω,F,*P_α*) to the space Δ of distribution functions on the closed unit interval I, where A is endowed with its natural Borel σ-field, that is, the smallest σ-field containing the customary weak topology[15].*

The random distribution $y(\varphi)$ *defined by:*

$$y(\varphi) = \int_{-\infty}^{\infty} y(t)\varphi(t)dt \qquad (\varphi \in \mathfrak{D}) \tag{6}$$

Where $\{y(t,w), -\infty < t < \infty, w \in \Omega\}$ *is a (real) vector continuous stochastic process.*

using fractional integration:

$$y(\varphi) = {}_{-\infty}D^{-\alpha}\big(y(t)\varphi(t)\big) \quad (\varphi \epsilon D) \tag{7}$$

The random distribution $y(\varphi)$ *is called a Gaussian process if for any function* $\varphi(t)\epsilon D$ *the random variable* $y(\varphi)$ *is Gaussian[9].*

For $\varphi\epsilon D$ *the first and second derivatives of the stochastic process y(t), regarding to distribution, are calculated by using integration by parts as:*

$$D^{\alpha}y(\varphi) = {}_{-\infty}D_{\infty}^{-\alpha}\big(D^{\alpha}(y(t)).\varphi(t)\big) = -{}_{-\infty}D_{\infty}^{-\alpha}\big(y(t).D^{\alpha}\varphi(t)\big) = -y\big(D^{\alpha}\varphi(t)\big) \tag{8}$$

$$D^{2\alpha}y(\varphi) = {}_{-\infty}D_{\infty}^{-\alpha}\big(D^{2\alpha}(y(t)).\varphi(t)\big) = -{}_{-\infty}D_{\infty}^{-\alpha}(D^{\alpha}(y(t)).D^{\alpha}(\varphi(t))) = {}_{-\infty}D_{\infty}^{-\alpha}\big(y(t).D^{2\alpha}\varphi(t)\big) = y\big(D^{2\alpha}\varphi(t)\big) \tag{9}$$

In general, the kth derivative of the stochastic process y(t) is defined by:

$$D^{k\alpha}y(\varphi) = {}_{-\infty}D_{\infty}^{-\alpha}\left(D^{k\alpha}(y(t)).\varphi(t)\right) = (-1)^k{}_{-\infty}D_{\infty}^{-\alpha}\left(y(t).D^{k\alpha}(\varphi(t))\right) = (-1)^k y\left(D^{k\alpha}\varphi(t)\right) \tag{10}$$

IV. *SUBSPACE ALGORITHM FOR FRACTIONAL TIME DOMAIN IDENTIFICATION*

Use Consider the following continuous-time fractional stochastic linear systems:

$$D^{\alpha}x(t) = Ax(t) + Bu(t) + w(t)$$
$$y(t) = Cx(t) + Du(t) + v(t) \tag{11}$$

Where $y(t) \in R^l, x(t) \in R^n, u(t) \in R^m$ *are the output, the input and the state vector, and* $v(t) \in R^l$*,* $w(t) \in R^n$ *are system and observation noises, respectively. The noises {w(t)} and {v(t)} are both assumed to be stationary white Gaussian processes which has a zero-mean. The covariance matrix of the noises is:*

$$E_p\left\{\begin{bmatrix} w(t) \\ v(t)\end{bmatrix}\begin{bmatrix} w^T(t) & v^T(t)\end{bmatrix}\right\} = \begin{bmatrix} Q & S \\ S^T & R\end{bmatrix}\delta(t-s) \tag{12}$$

In the relation above, δ *denotes the Dirac-delta function and* E_p *represents mathematical expectation. We assume that input {u(t)} is independent of system noises and our objective is to find system order (n), system differentiation order (α) and matrices (A,B,C,D) using continuous stationary random input and output data, {u(t)} and {y(t)},* $(-\infty < t < +\infty)$*. States of the system can be estimated using kalman-filter.*

According to Thomassin etal method(2009), system differentiation order (α∈(0,2))can be estimated, by minimizing a quadratic criterion:

$$\hat{\alpha} = \arg\min \frac{1}{2}\|\hat{y}_c(\alpha) - y_c\|_2^2 \tag{13}$$

Consider first the case where the noise and disturbance is zero.

$$D^{\alpha}x(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \tag{14}$$

According to subspace algorithm quadruple (A,B,C,D) can be calculate using fractional derivatives of input and output at least up to (i-1)th derivative. Though we have the input-output algebraic (matrix) relationship:

$$Y_i(t)(t_j) = \Gamma_i X_i(t)(t_j) + H_i U_i(t)(t_j) \tag{15}$$

$$\Gamma_i = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1}\end{bmatrix}$$

$$H_i = \begin{bmatrix} D & 0 & \dots & 0 \\ CB & D & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & D\end{bmatrix}$$

$$Y_i(t) = \begin{bmatrix} y(t_1) & y(t_2) & \dots & y(t_N) \\ D^{\alpha}y(t_1) & D^{\alpha}y(t_2) & \dots & D^{\alpha}y(t_N) \\ \vdots & \vdots & & \vdots \\ D^{(i-1)\alpha}y(t_1) & D^{(i-1)\alpha}y(t_2) & \cdots & D^{(i-1)\alpha}y(t_N)\end{bmatrix}$$

Now matrices Γ_i *and* H_i *can be calculated using least square method and finally the quadruple (A,B,C,D) will be known. But it should be noted that the system output is stained with noise and since derivatives of noisy output can not be calculated, we used the test function and distribution theory. Considering this theory, input, output and states of the system are expressed as follow:*

$$y(\varphi) = {}_{-\infty}D_{\infty}^{-\alpha}\big(y(t)\varphi(t)\big) \quad (\varphi\epsilon D)$$
$$u(\varphi) = {}_{-\infty}D_{\infty}^{-\alpha}\big(u(t)\varphi(t)\big) \quad (\varphi\epsilon D)$$
$$x(\varphi) = {}_{-\infty}D_{\infty}^{-\alpha}\big(x(t)\varphi(t)\big) \quad (\varphi\epsilon D)$$

Now we write the output and its derivatives:

$$ {}_{-\infty}D_{\infty}^{-\alpha}(D^{\alpha}(y(t))\varphi(t^{\cdot},t_j)) = C_{-\infty}D_{\infty}^{-\alpha}(D^{\alpha}(x(t))\varphi(t^{\cdot},t_j)) + D_{-\infty}D_{\infty}^{-\alpha}\left(D^{\alpha}(u(t))\varphi(t^{\cdot},t_j)\right) + {}_{-\infty}D_{\infty}^{-\alpha}\left(D^{\alpha}(v(t))\varphi(t^{\cdot},t_j)\right) \tag{16}$$

Using relation (10) and (11):

$$-y(D^{\alpha}\varphi)(t_j) = CAx(\varphi)(t_j) + CBu(\varphi)(t_j) - Du(D^{\alpha}\varphi)(t_j) + Cw(\varphi)(t_j) - v(D^{\alpha}\varphi)(t_j) \tag{17}$$

And through the same, 2αth derivative is calculated as follow:

$$y(D^{2\alpha}\varphi)(t_j) = CA^2x(\varphi)(t_j) + CABu(\varphi)(t_j)$$
$$-CBu(D^{\alpha}\varphi)(t_j) + Du(D^{2\alpha}\varphi)(t_j)$$
$$+CAw(\varphi)(t_j) - Cw(D^{\alpha}\varphi)(t_j) + v(D^{2\alpha}\varphi)(t_j) \qquad (18)$$

Repeating this with α(i-1) times , α(i-1)th derivative is calculated as follows:

$$-(1)^{i-1}y(D^{(i-1)\alpha}\varphi)(t_j)$$
$$= CA^{i-1}x(\varphi)(t_j) + CA^{i-2}Bu(\varphi)(t_j)$$
$$-CA^{i-3}Bu(D^{\alpha}\varphi)(t_j) + \cdots$$
$$+ (-1)^{i-1}Du(D^{(i-1)\alpha}\varphi)(t_j) \qquad (19)$$
$$+CA^{i-2}w(\varphi)(t_j)$$
$$-CA^{i-3}w(D^{\alpha}\varphi)(t_j) + \cdots + (-1)^{i-2}Cw(D^{(i-2)\alpha}\varphi)(t_j) +$$
$$(-1)^{i-1}v(D^{(i-1)\alpha}\varphi)(t_j)$$

And finally input-output algebraic relationship can be calculated:

$$y(\varphi)(t_j) = \Gamma_i x(\varphi)(t_j) + H_i u(\varphi)(t_j) + \Sigma_i w(\varphi)(t_j) + v(\varphi)(t_j) \qquad (20)$$

Which $y(\varphi)(t_j)$ ***is as follow:***

$$y(\varphi)(t_j) = [y^T(\varphi)(t_j)\ , y^T(D^{\alpha}\varphi)(t_j), \ldots, (-1)^{i-1}y^T(D^{(i-1)\alpha}\varphi)(t_j)]^T \qquad (21)$$

Structure of $u(\varphi)(t_j)$ و $v(\varphi)(t_j)$ و $w(\varphi)(t_j)$ ***is similar to*** $y(\varphi)(t_j)$ ***and the structure of*** Γ_i ***,*** H_i ***and*** $\Sigma_i \epsilon R^{i\ell\times in}$ ***is as follow:***

$$\Sigma_i = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ C & 0 & \ldots & 0 \\ CA & C & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2} & CA^{i-3} & \ldots & 0 \end{bmatrix} \qquad (22)$$

$$\Gamma_i = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{bmatrix} \qquad (23)$$

$$H_i = \begin{bmatrix} D & 0 & \ldots & 0 \\ CB & D & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & D \end{bmatrix} \qquad (24)$$

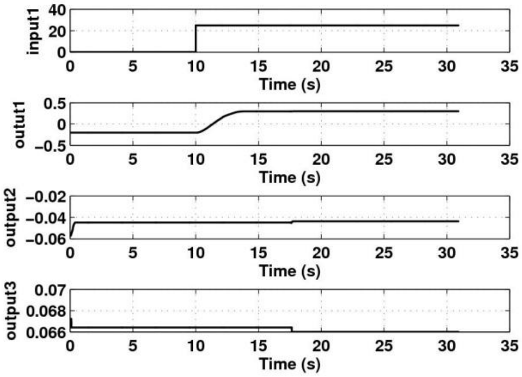
Therefore, by arranging the output column vector (20) in a row (j=1 to N) we have the input-output algebraic relationship:

$$y_i(\varphi) = \Gamma_i X_i(\varphi) + H_i U_i(\varphi) + \Sigma_i W_i(\varphi) + V_i(\varphi) \qquad (\varphi \in \mathfrak{D}) \quad (25)$$

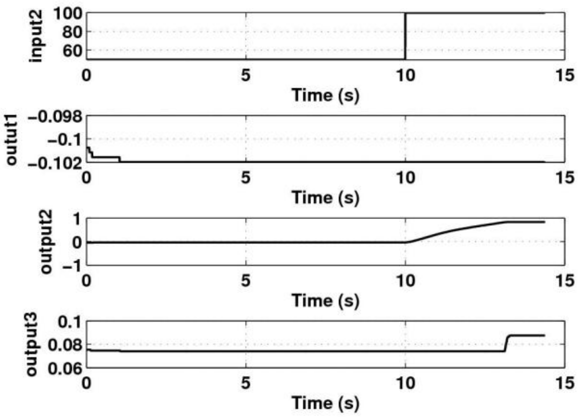
Where state and output matrices are as follow:

$$X_i(\varphi) = [x(\varphi)(t_1), x(\varphi)(t_2), \ldots, x(\varphi)(t_N)] \quad (n \times N)$$
$$Y_i(\varphi) = [y(\varphi)(t_1), y(\varphi)(t_2), \ldots, y(\varphi)(t_N)] \quad (i\ell \times N),$$

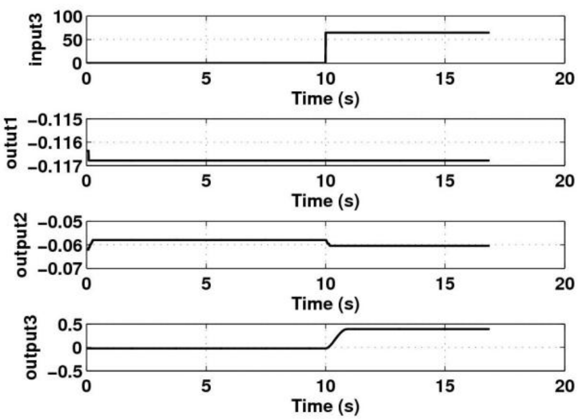
And $U_i(\varphi)\epsilon R^{im\times N}, W_i(\varphi)\epsilon R^{in\times N}$ ***and*** $V_i(\varphi)\epsilon R^{i\ell\times N}$ ***will be founded similarly to*** $Y_i(\varphi)$***.***



(a)



(b)



(c)

Fig.1. step response for robot manipulator (a) make an step to the first input and measure the outputs (b) make an step to the second input and measure the outputs (c) make an step to the third input and measure the outputs.

V. *SYSTEM IDENTIFICATION*

The In this section we'll review how to remove noise and obtain quadruple (A,B,C,D) using the algorithm developed by Ohsumi etal to fractional order.

Theory 1) Assume that {u(t)},{v(t)} and {w(t)} be independent zero-mean stochastic processes. Pick $\varphi, \psi \in \mathfrak{D}$ ***and assuming that*** $U_h(\varphi)\epsilon R^{hm\times N}$ ***is a matrix with random***

distribution that has a structure similar to $U_i(\varphi)\epsilon R^{im\times N}$***instead of the test function*** $\varphi(t,t_j)$ ***and the number of block rows i, then:***

$$\frac{1}{N}W_i(\varphi)U_h^T(\psi)\to 0,\frac{1}{N}V_i(\varphi)U_h^T(\psi)\to 0 \text{ as } N\to\infty \qquad (26)$$

Proof:

According to

$$W_i(\varphi)=\left[(-1)^k w(D^{k\alpha}\varphi(t_j))\right]_{k=0,1,\dots,i-1:j=1,2,\dots,N} \qquad (27)$$

$$U_h(\psi)=\left[(-1)^p u(D^{p\alpha}\psi\ \ (t_j))\right]_{p=0,1,\dots,h-1:j=1,2,\dots,N} \qquad (28)$$

We have for the (k, p)-element of the matrix $\frac{1}{N}W_i(\varphi)U_h^T(\psi)$ ***:***

$$\frac{1}{N}\left[W_i(\varphi)U_h^T(\psi)\right]_{kp}=$$
$$\frac{1}{N}\left[(-1)^k w(D^{k\alpha}\varphi)(t_1),\dots,(-1)^k w(D^{k\alpha}\varphi)(t_N)\right]$$
$$\times\left[(-1)^p u(D^{p\alpha}\psi)(t_1),\dots,(-1)^p u(D^{p\alpha}\psi)(t_N)\right]^T$$
$$=\ (-1)^{k+p}\frac{1}{N}\sum_{j=1}^{N} w(D^{k\alpha}\varphi)(t_j)u^T(D^{p\alpha}\psi)(t_j) \qquad (29)$$

And for fixed k and p:

$$Z(t_j)=w(D^{k\alpha}\varphi)(t_j)u^T(D^{p\alpha}\psi)(t_j) \qquad (30)$$

This is a stationary stochastic sequence.

Therefore prove summarized to show the following two cases:

i) ***the ergodicity is hold for*** $Z(t_j)$

ii) $E\{Z(t_j)\}=0$

To prove (i) it's sufficient to find the covariance function$R_z(\tau)$ ***:***

$$R_z(\tau)=E\left\{\left[Z(t_j+\tau)-E\{Z(t_j+\tau)\}\right]\left[Z(t_j)-\right.\right.$$
$$EZtjT \qquad (31)$$

And verify that it tends to zero as $|\tau|$ ***tend to infinity. So average is obtained as follow:***

$$E\{Z(t_j)\}=E\{w(D^{k\alpha}\varphi)(t_j)u^T(D^{p\alpha}\psi)(t_j)\}$$
$$=E\left\{{}_{-\infty}D_{\infty}^{-\alpha}\left(w(t)D^{k\alpha}\varphi(t^{\cdot},t_j)\right){}_{-\infty}D_{\infty}^{-\alpha}(u^T(t)D^{p\alpha}\psi(s^{\cdot},t_j))\right\}$$
$${}_{-\infty}D_{\infty}^{-\alpha}{}_{-\infty}D_{\infty}^{-\alpha}E\{w(t)u^T(s)\}D^{k\alpha}\varphi(t^{\cdot},t_j)D^{p\alpha}\psi(s^{\cdot},t_j)=0$$
(32)

Where {w(t)} and {u(t)} are independent zero-mean random processes. hence:

$$R_z(\tau)=E\{Z(t_j+\tau)Z^T(t_j)\}$$
$$=E\{\left[w(D^{k\alpha}\varphi)(t_j+\tau)u^T(D^{p\alpha}\psi)(t_j+\tau)\right]$$
$$\times\left[w(D^{k\alpha}\varphi)(t_j)u^T(D^{p\alpha}\psi)(t_j)\right]^T\}$$
$$=\left[{}_{-\infty}D_{\infty}^{-\alpha}{}_{-\infty}D_{\infty}^{-\alpha}(r_u(s_1-s_2)\right.$$
$$\left.\times D^{p\alpha}\psi(s_1^{\cdot},t_j+\tau)D^{p\alpha}\psi(s_2^{\cdot},t_j))\right]$$
$$\times Q\ {}_{-\infty}D_{\infty}^{-\alpha}(D^{k\alpha}\varphi(s_3^{\cdot},t_j+\tau)D^{k\alpha}\varphi(s_3^{\cdot},t_j)) \qquad (33)$$

Where $r_u(s_1-s_2)=E\{u^T(s_1)u(u_2)\}$.

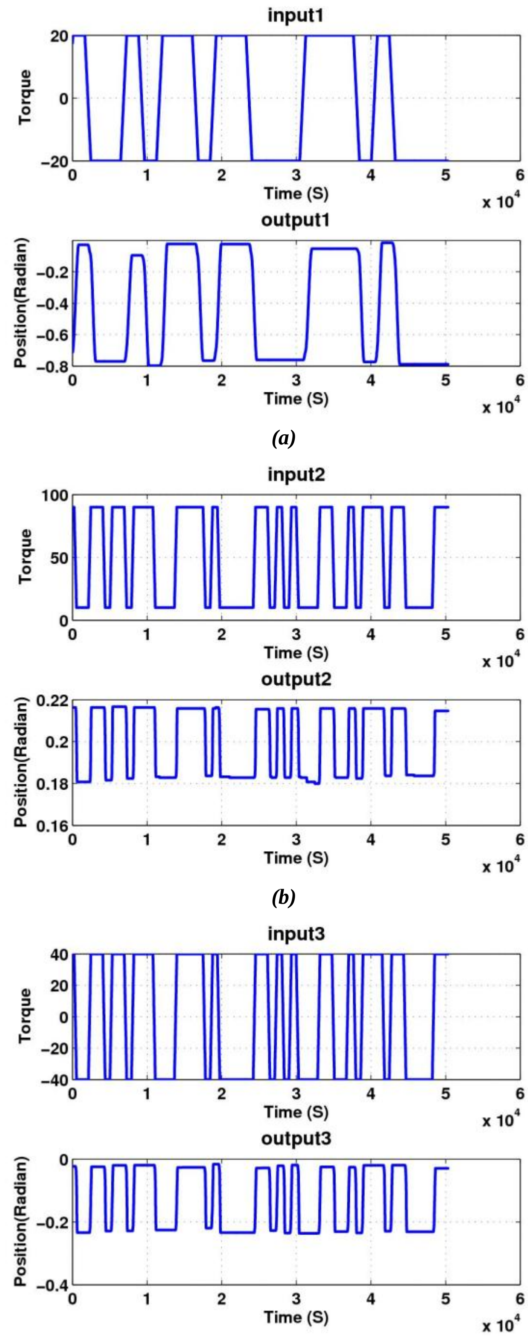


Fig.2. input-output data (a)first manipulator (b) second manipulator (c)third manipulator

as regards $|r_u(\tau)|\le c_1$ ***for*** $-\infty<\tau<\infty$ ***،we have for the bracketed term in the last equality that:***

$$\left|{}_{-\infty}D_{\infty}^{-\alpha}{}_{-\infty}D_{\infty}^{-\alpha}(r_u(s_1-s_2)D^{p\alpha}\psi(s_1^{\cdot},t_j+\tau)D^{p\alpha}\psi(s_2^{\cdot},t_j))\right|$$
$$\le {}_{-\infty}D_{\infty}^{-\alpha}{}_{-\infty}D_{\infty}^{-\alpha}\left|r_u(s_1-s_2)D^{p\alpha}\psi(s_1^{\cdot},t_j+\tau)\right|$$
$$\times\left|D^{p\alpha}\psi(s_2^{\cdot},t_j)\right|$$
$$\le$$
$$c_1\left[{}_{-\infty}D_{\infty}^{-\alpha}\left|D^{p\alpha}\psi(s_1^{\cdot},t_j+\tau)\right|\right]\times\left[{}_{-\infty}D_{\infty}^{-\alpha}\left|D^{p\alpha}\psi(s_2^{\cdot},t_j)\right|\right]\le$$
$$c_2(\text{const}) \qquad (34)$$

On the other hand the last integral in equation (31) is as follow:

$${}_{-\infty}D_{\infty}^{-\alpha}\left(D^{k\alpha}(\varphi(s_{3'},t_j+\tau))D^{k\alpha}\left(\varphi(s_{3'},t_j)\right)\right) \to 0 \text{ as } |\tau| \to \infty \qquad (35)$$

Since $D^{k\alpha}\varphi(.;.)$ ***has compact support, and according to (34) and (35) we have:***

$$R_z(\tau) \to 0 \text{ as } |\tau| \to \infty \qquad (36)$$

And consequently the ergodicity:

$$\frac{1}{N}\sum_{j=1}^{N} Z(t_j) \to E\{Z(t_j)\} \quad (N \to \infty) \qquad (37)$$

Holds for the random sequence defined by (30). This implies that :

$$\frac{1}{N}\left[w_i(\varphi)U_h^T(\psi)\right]_{kp} = (-1)^{k+p}\frac{1}{N}\sum_{j=1}^{N} Z(t_j) \underset{N\to\infty}{\rightarrow} (-1)^{k+p}E\{Z(t_j)\} \qquad (38)$$

The second part is hold according to (32)and consequently the first assertion in (26) follows. Similarly, the second assertion also follows.

Theory 2) pick $\varphi,\psi \in \mathfrak{D}$ ***and assume that*** $U_h(\psi)$ ***is as equation of theory (1). perform QR factorization as (39):***

$$\begin{bmatrix} U_i(\varphi) \\ U_h(\psi) \\ Y_i(\varphi) \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} Q_{L1}^T \\ Q_{L2}^T \\ Q_{L3}^T \end{bmatrix} \qquad (39)$$

Then the following relations are given for large N:

$$\frac{1}{\sqrt{N}}L_{31} = \Gamma_i\left\{\frac{1}{\sqrt{N}}X_i(\varphi)Q_{L1}\right\} + H_i\left\{\frac{1}{\sqrt{N}}L_{11}\right\} \qquad (40)$$

$$\frac{1}{\sqrt{N}}L_{32} = \Gamma_i\left\{\frac{1}{\sqrt{N}}X_i(\varphi)Q_{L2}\right\} \qquad (41)$$

Where L_{32} ***is noiseless matrix and contains*** $\text{span}_{col}\{\Gamma_i\}$.

Proof:

postmultiplying both sides of equation (25) by $\frac{1}{\sqrt{N}}Q_{L1}$ ***and*** $\frac{1}{\sqrt{N}}Q_{L2}$ ***respectively we have:***

$$\frac{1}{\sqrt{N}}Y_i(\varphi)Q_{L1} = \Gamma_i\left\{\frac{1}{\sqrt{N}}X_i(\varphi)Q_{L1}\right\} + H_i\left\{\frac{1}{\sqrt{N}}U_i(\varphi)Q_{L1}\right\} = \Sigma_i\left\{\frac{1}{\sqrt{N}}W_i(\varphi)Q_{L1}\right\} + \frac{1}{\sqrt{N}}v_i(\varphi)Q_{L1} \qquad (42)$$

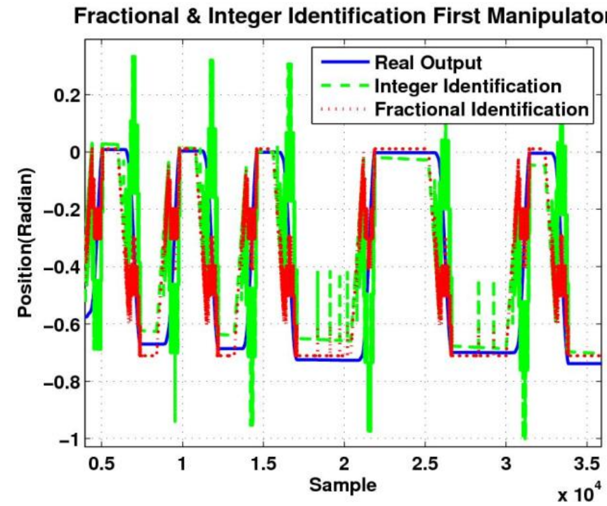
$$\frac{1}{\sqrt{N}}Y_i(\varphi)Q_{L2} = \Gamma_i\left\{\frac{1}{\sqrt{N}}X_i(\varphi)Q_{L1}\right\} + H_i\left\{\frac{1}{\sqrt{N}}U_i(\varphi)Q_{L2}\right\} = \Sigma_i\left\{\frac{1}{\sqrt{N}}W_i(\varphi)Q_{L2}\right\} + \frac{1}{\sqrt{N}}v_i(\varphi)Q_{L2} \qquad (43)$$

Combining these into an equation, we obtain:

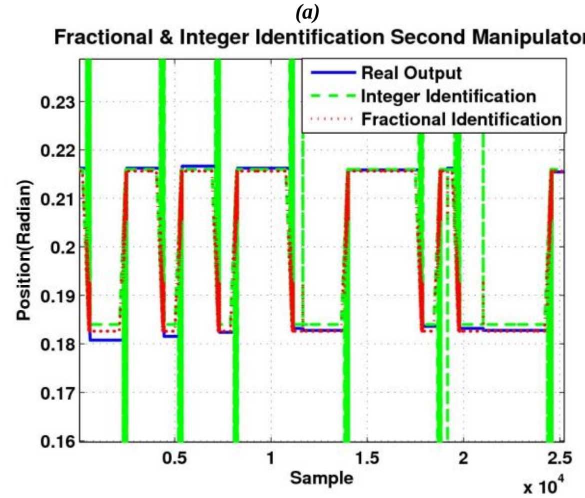
$$\begin{aligned}\frac{1}{\sqrt{N}}Y_i(\varphi)[Q_{L1},Q_{L2}] &= \Gamma_i\left\{\frac{1}{\sqrt{N}}X_i(\varphi)[Q_{L1},Q_{L2}]\right\} \\ &+ H_i\left\{\frac{1}{\sqrt{N}}U_i(\varphi)[Q_{L1},Q_{L2}]\right\} \\ &+\Sigma_i\left\{\frac{1}{\sqrt{N}}W_i(\varphi)[Q_{L1},Q_{L2}]\right\} + \frac{1}{\sqrt{N}}v_i(\varphi)[Q_{L1},Q_{L2}] \qquad (44)\end{aligned}$$

Now according to (39):

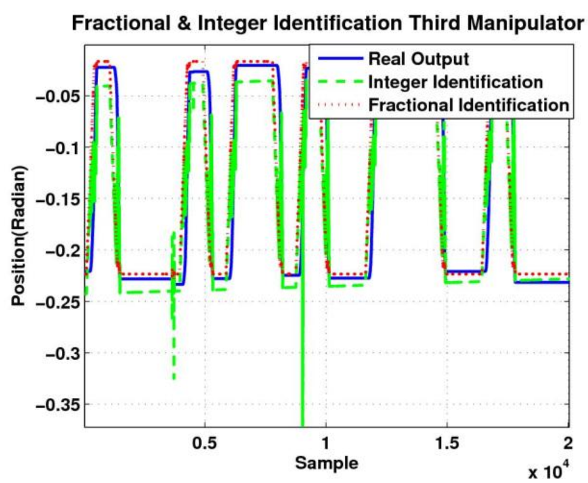
$$[Q_{L1},Q_{L2}] = U_h^T(\psi)\begin{bmatrix} L_{21}^T \\ L_{22}^T \end{bmatrix}^{+} \qquad (45)$$



(a)



(b)



(c)

Fig.3. comparing real, fractional estimated and integer estimated output (a)first manipulator, (b) second manipulator, (c)third manipulator

Where $+$ ***denotes the Moor-Penrose pseudo-inverse. The third term in the right part of equation (44) becomes as follow:***

$$\Sigma_i\left\{\frac{1}{\sqrt{N}}W_i(\varphi)[Q_{L1},Q_{L2}]\right\} =$$

$$\sqrt{N}\Sigma_i\left\{\frac{1}{N}W_i(\varphi)U_h^T(\psi)\right\}\begin{bmatrix}L_{21}^T\\ L_{22}^T\end{bmatrix}^{+} \begin{matrix}\to 0\\ N\to\infty\end{matrix} \qquad (46)$$

in which the convergence to zero as $N\to\infty$ yields from Theorem 1. Similarly, the method is established for the fourth term in the RHS of (44).

Hence for large N, equation (44) becomes as follow:

$$\frac{1}{\sqrt{N}}Y_i(\varphi)[Q_{L1},Q_{L2}] = \frac{1}{\sqrt{N}}\Gamma_i X_i(\varphi)[Q_{L1},Q_{L2}] + \frac{1}{\sqrt{N}}H_i U_i(\varphi)[Q_{L1},Q_{L2}] \qquad (47)$$

Now according to (39) the left part of the equation is expressed as:

$$\frac{1}{\sqrt{N}}Y_i(\varphi)[Q_{L1},Q_{L2}] = \frac{1}{\sqrt{N}}\left(L_{31}\,Q_{L1}^T + L_{32}\,Q_{L2}^T + L_{33}\,Q_{L3}^T\right)[Q_{L1},Q_{L2}]\frac{1}{\sqrt{N}}[L_{31},L_{32}] \qquad (48)$$

And for the second right part of the equation we obtain:

$$\frac{1}{\sqrt{N}}H_iU_i(\varphi)[Q_{L1},Q_{L2}] = \frac{1}{\sqrt{N}}H_iL_{11}Q_{L1}^T[Q_{L1},Q_{L2}] = \frac{1}{\sqrt{N}}H_i[L_{11},0] \qquad (49)$$

Hence equation (47)becomes as follow:

$$\frac{1}{\sqrt{N}}[L_{31},L_{32}] = \frac{1}{\sqrt{N}}[\Gamma_iX_i(\varphi)Q_{L1},\Gamma_iX_i(\varphi)Q_{L2}] + \frac{1}{\sqrt{N}}[H_iL_{11},0] \qquad (50)$$

By extracting two block matrices, we obtain (40) and (41).

VI. SIMULATION

In order to test the algorithms, an experiment was conducted. In this experiment, quadruple (A,B,C,D) and fractional order of system (α) were identified.

The algorithm is able to find optimum $\epsilon\alpha(0,2)$ by minimizing equation (13).

In the following experiments, the Gaussian test function with σ=1/4 was selected as follow:

$$\varphi(t;t_j) = \exp\left\{-\frac{(t-t_j)^2}{2\sigma}\right\} \qquad (\sigma>0) \qquad (51)$$

The number of state variables (n) is assumed to be known. In all simulation studies, the continuous-time processes were discretized with time-partition Δt, and the sampling instants $t_j(j=1,2,\dots,N)$ were taken with equal distance and constant integer M as:

$$t_{j+1} = t_j + M\Delta t \qquad (52)$$

In this test we applied the fractional algorithm to a robot with three manipulators named Phantom and compared the results with integer one. In the experiment, we considered sampling interval MΔt=0.625s and time interval Δt=0.00625s.

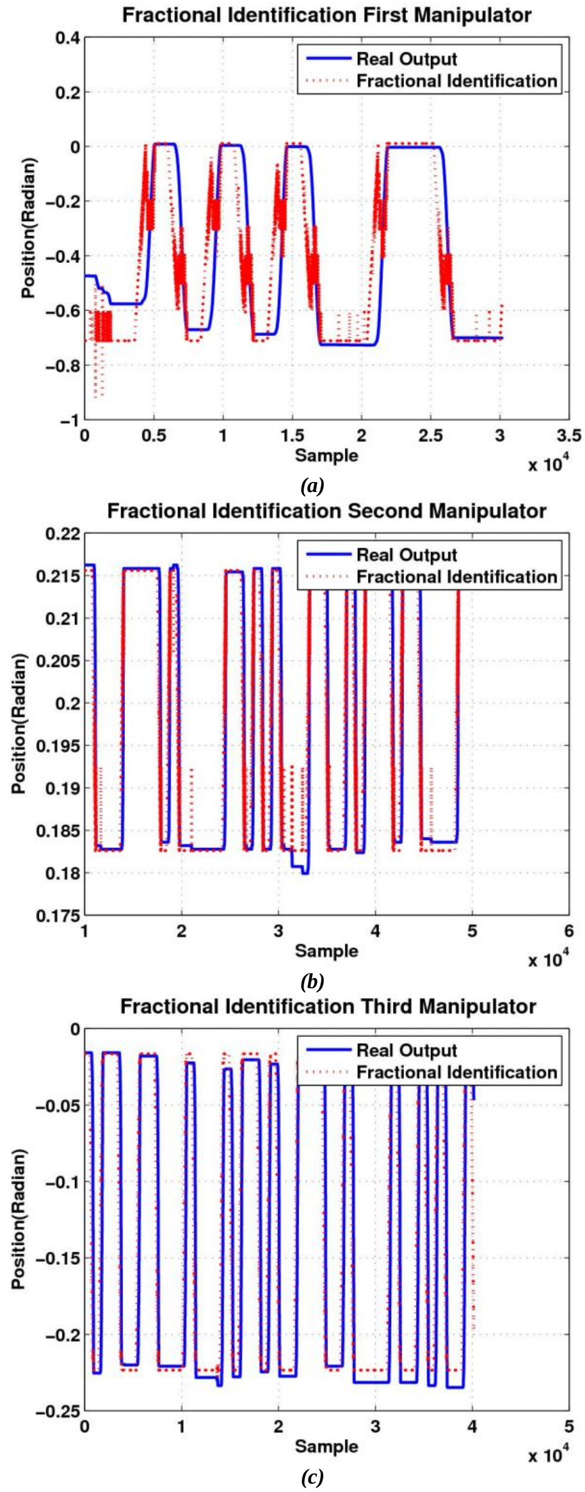


Fig.4. investigating simulation using test data (a)first manipulator, (b) second manipulator, (c)third manipulator

robot is in open loop manner and without controller. Experiment result shows that the response rate is τ=2. 9s for the first manipulator and 0.7s ,0.5s for the second and third one respectively. As the outputs are almost independent, it is reasonable to decompose the system to three SISO independent manipulator.

Persistent exciting input in the next step was adopted in *such a way that it can stimulate all the various modes of the system. The input signal is a pseudo-random binary sequence (PRBS).*

After sampling, data were examined to be pre-filtered

in case they are false or destroyed information or signs of nonlinear effect are observed in the data. For complete information refer to Fig(1). To investigate amount of output sensor noise, a pseudo-random binary sequence (PRBS) with K= 3 periods each of length M = 63 for the first manipulator and M=127 for second and third, are exactly measured. Then, a record of N = KM =189 for manipulator one and N=KM=381 data points for second and third is collected. The data set over the 2nd periods is displayed in Figure (2).

As we know, robots are nonlinear while state-space model is linear, so the measurement data are made around the operating point.

Choosing the same rank (α)for all inputs in such a way that output error being minimized is a major problem of MIMO systems. For coupled systems there is a compromise between the same rank (α) and minimum output, but as shown in Fig(1) this robot has a decouple manipulator and consequently amount of α in each of manipulator is independent of the other. Finally an input is applied to each of the manipulators and the effect is measured in output.

Fig(3) compares actual output, fractional estimated output and integer estimated output. As shown in this figure, a more accurate response was received from a fractional identification experiment. Fractional state-space matrices of robot are as follow:

$$\begin{bmatrix} d^{0.1}x/dt^{0.1} \\ d^{0.4}x/dt^{0.4} \\ d^{0.5}x/dt^{0.5} \end{bmatrix} = \begin{bmatrix} -1.4982 & 0 & 0 \\ 0 & -4.5101 & 0 \\ 0 & 0 & -6.3623 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 9.433 \times 10^{-5} & 0 & 0 \\ 0 & 0.0037 & 0 \\ 0 & 0 & 0.1544 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -10.8342 & 0 & 0 \\ 0 & -3.1104 & 0 \\ 0 & 0 & -2.0509 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

To verify the simulation, we investigate our simulation using test data. Results are depicted in Fig(4).

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