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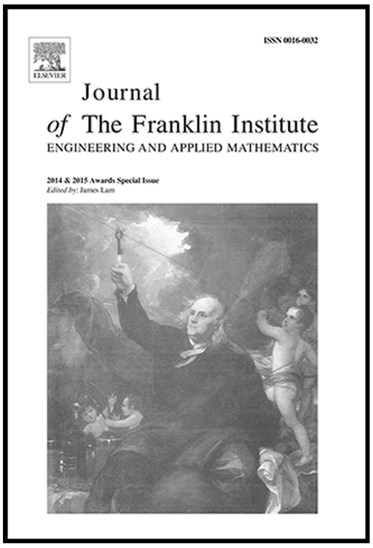

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Adaptive Fault Compensation Control for Nonlinear Uncertain Fractional-Order Systems: Static and Dynamic Event Generator Approaches

Milad Shahvali[a], Mohammad-Bagher Naghibi-Sistani[a,*] and Javad Askari[b]

[a]Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad 91775-1111, Iran
[b]Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran

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ABSTRACT

The tracking problem of the fractional-order nonlinear systems is assessed by extending new event-triggered control designs. The considered dynamics are accompanied by the uncertain strict-feedback form, actuator faults and unknown disturbances. By using the neural networks and the fault compensation method, two adaptive fault compensation event-triggered schemes are designed. Unlike the available control designs, two static and dynamic event-triggered strategies are proposed for the nonlinear fractional-order systems, in a sense that the minimum/average time-interval between two successive events can be prolonged in the dynamic event-triggered approach. Besides, it is proven that the Zeno phenomenon is strictly avoided. Finally, the simulation results prove the effectiveness of the presented control methods.

1. Introduction

In the past decades, many control strategies such as adaptive methodologies Regaya, Farhani, Zaafouri and Chaari (2019); He and Meng (2017), backstepping techniques Yu and Lin (2016); Swaroop, Hedrick, Yip and Gerdes (2000); Regaya, Zaafouri and Chaari (2013), neural networks (NNs) Lewis, Liu and Yesildirek (1995), fuzzy logic systems (FLSs) Sun, Mou, Qiu, Wang and Gao (2018); Regaya, Farhani, Zaafouri and Chaari (2017b) and optimal control Li, Chai, Lewis, Ding and Jiang (2018a), etc were of concern for a class of nonlinear systems with unknown dynamics. In these time-triggered control (TTC) solutions, the control signals are executed periodically. Although the TTC approach can facilitate the control design procedure and stability analysis, it wastes the limited network resources (e.g. bandwidth and energy consumption). Recently, the periodic sampling control based on fixed time interval has been presented to deal with this problem. Nevertheless, it is sometimes less preferable, because the sampling takes place periodically regardless of whether the current behaviors of the system states need or not. In order to eliminate the shortcomings of TTC and periodic sampling methods, an event-triggered control (ETC) is introduced in Mazo and Tabuada (2011).

In the ETC approaches, the data is transmitted in an aperiodic manner, which saves the network resources. The transmission data over networked systems depend on predetermined criterion which is named an event-triggering rule. Motivated by the real-world demands, there exist some studies run on the ETC approach for nonlinear systems with the strict-feedback dynamics Xing, Wen, Liu, Su and Cai (2016), non-affine forms Yang, Meng, Yue, Zhang and Liang (2020b), DoS attacks Zhao, Niu and Zhao (2019),

This paper attempts to design the adaptive event-triggered controllers for fractional-order strict-feedback systems.

*Corresponding author
m.shahvali@mail.um.ac.ir (M. Shahvali); mb-naghibi@um.ac.ir (M. Naghibi-Sistani); j-askari@cc.iut.ac.ir (J. Askari)
ORCID(s): 0000-0002-1460-5161 (M. Shahvali); 0000-0003-1414-8199 (M. Naghibi-Sistani); 0000-0002-3038-2391 (J. Askari)

discrete-time structures Wang, Wang, Chen and Sheng (2019), and finite-time performance Zhang and Yang (2019). By applying the dynamic surface technique, researchers in Zhang, Yao, Xu and Zhang (2020) presented an ETC algorithm for dynamic positioning of marine surface ships. In Zhang, Li, Sun and He (2018), a distributed fault-tolerant controller with the designed adaptive event-triggering rule was introduced for a class of uncertain nonlinear multi-agent systems. The mentioned ETC results are only limited to the classical integer-order systems, not applicable in the fractional-order systems.

As is known, many real applications like the signal processing, chaotic systems, electrical circuits, robotics and mechanical systems can be represented by the fractional-order systems, Kilbas, Srivastava and Trujillo (2006). Unlike the classical integer-order systems, the distinguished feature of the fractional-order systems is their infinite dimensions. This feature is generated by the non-locality property of the fractional-order derivatives. Hence, the pseudo-state concept is applied for the fractional-order systems. On the other hand, the controller design is more complicated for the fractional-order systems, in comparison with that of the integer-order ones, because some conventional rules for the integer-order calculations, like the Chain and Leibniz rules, are not well established for the fractional-order derivatives. Therefore, it is not trivial to extend the direct Lyapunov scheme as well as its associated control methods from the integer-order to the fractional-order systems. Recently, based on the serial-parallel estimation model and the dynamic surface approach, the authors in Yang, Yu, Lv, Zhu and Hayat (2020a) extended an adaptive output tracking control for a class of nonstrict-feedback fractional-order systems. Researchers in Song, Zhang, Song and Zhang (2020) provided an adaptive neuro-fuzzy tracking control of the fractional-order uncertain nonlinear systems with the input saturation. By applying the backstepping scheme, the presented NN adaptive control architecture in Shahvali, Azarbahram, Naghibi-Sistani and Askari (2020) for the fractional-order nonlinear multi-agent systems assures that distributed consensus errors are limited in a preset bound. In Li, Wang and Tong (2021), an adaptive fault-tolerant control scheme obtained for the fractional-order strict-feedback systems. A distributed adaptive fault compensation design studied in Gong, Lan and Han (2020) for the fractional-order multi-agent systems with the actuator failures. Notably, these approaches are all designed with respect to the TTC, where the limited network resources are not of major concern. To overcome this drawback, some results are provided to address the ETC problem for the linear fractional-order systems by Hu, He, Zhang and Zhong (2020); Ye, Su and Sun (2018); Ye and Su (2018). Unlike these results, systems with the nonlinear fractional-order dynamics and uncertain factors are more common, yet more challenging. There exists only one article Li, Liu and Chen (2018b), where the ETC problem for the nonlinear fractional-order systems is assessed. The scope of this ETC method is limited to the chaotic fractional-order systems with the single integrator dynamics that satisfied Lipschitz's condition. Besides, when some of actuators become faulty in the presented ETC approach for fractional-order systems, it may lead to the loss of control objectives. The demand for reliability and safety of a system to operate under actuator fault is a significant issue. A classic method with hardware redundancy is originally used to avoid failure. However, physical redundancy in real applications has some problems such as the limitation of space, weight and cost. Hence, the design of event-triggered fault compensation control for the uncertain nonlinear fractional-order systems such that can minimize the impact of faults, whilst reducing both energy consuming and extra costs owing to hardware redundancy is our theory and practical motivations.

All the previous static ETC designs, similar to proof Girard (2014) result in conservatism minimum inter-event time interval. Recently, Girard (2014) extended the available static ETC approaches by presenting a mechanism named the dynamic event-triggering rule for the linear systems. This method assures a larger minimum inter-event time than the corresponding static ETC schemes. Due to this advantage, the dynamic ETC is developed for the nonlinear systems Xu, Liu, Wang and Zhou (2020), stochastic systems Wang, Zheng and Zhang (2017b), and linear multi-agent systems Hu, Yang, Huang and Gui (2018). However, the dynamic ETC method is not discussed for the fractional-order systems.

The focus of this study is on the NN adaptive static and dynamic ETC designs for a class of the fractional-order nonlinear systems subject to unmatched dynamics, unknown functions, external disturbances, and actuator faults. There exist at least three main technical challenges associated with these proposed control problems.

First, two static and dynamic ETC methods are proposed, where the fractional-order strict-feedback systems execute their actuators only at sequences of time moments. In this context, how to design infinite dimension static and dynamic event-triggering rules to decide when the actuators are updated for the fractional-order systems is the first challenge.

Second, the methods of avoiding the Zeno phenomenon for the integer-order ETC schemes applied in Xing et al. (2016); Yang et al. (2020b); Zhao et al. (2019); Zhang and Yang (2019); Zhang et al. (2020, 2018) are no longer valid in the two proposed approaches, because the conventional Chain rule is not applicable for the fractional-order derivatives. To overcome this difficulty, a new method is proposed to approximate the fractional-order derivative of the measurement error signal, (i.e., see the proof of third statement of Theorem 2).

Third, a new relative dynamic event-triggering rule is proposed with respect to the control input in this article. Hence, this requires different stability analysis than the existing dynamic ETC methods. Additionally, as the actuators may encounter to loss of effectiveness, stuck, bias faults, and also the system functions are completely unknown, it is not easy to obtain the tracking of the desired trajectory and reduce the data transmission load in the controller to actuators' channels, simultaneously.

To face these challenges, the NN adaptive fault compensation control is combined with the design of static and dynamic ETC methods for the fractional-order systems. The main contributions of this article are as follows

1) Unlike all the available TTC schemes for the fractional-order systems, the static ETC protocol is offered for the fractional-order uncertain strict-feedback systems. According to the designed static event-triggering rule in the controller to actuators' channels, the transmission load and updating rate of the actuators are reduced, simultaneously.

2) A new dynamic event-triggering rule is proposed for the strict-feedback fractional-order (integer-order) systems, which contains the static ETC strategies in Hu et al. (2020); Ye et al. (2018); Ye and Su (2018); Li et al. (2018b) (Xing et al. (2016); Yang et al. (2020b); Zhao et al. (2019); Zhang and Yang (2019); Wang et al. (2019); Zhang et al. (2020, 2018)) as its special form. Also, a new internal dynamic variable is applied in the triggering rule which assures a larger inter-event time interval than the existing static ETC results. On the other hand, all the existing dynamic ETC methods are mainly presented for the systems without unknown dynamics and actuator faults. Hence, the problem of unknown nonlinear dynamics with faulty actuators in the dynamic ETC design is assessed in this article.

3) Given that the actuators of the fractional-order systems are faulty, the static and dynamic event-triggered fault compensation controls are proposed by introducing infinite dimension fault compensator variables. The previous works in Gong et al. (2020); Li et al. (2021) investigate the fault compensation issue for the fractional-order systems, however they are in the TTC manner. Besides, the employed fractional-order differentiator (FOD) in this article not only removes the explosion of complexity problem in Shahvali et al. (2020); Li et al. (2021) but also can assure the global stability in contrast with the dynamic surface control methods in Yang et al. (2020a); Song et al. (2020)

4) The presented ETC designs in Yang et al. (2020b); Wang et al. (2019); Zhang et al. (2020, 2018) can only assure the semi-global ultimate bound stability due to applying the conventional NNs and FLSs. According to the static and dynamic event-triggered approaches proposed in this article, all the yield closed-loop signals of the networked systems are globally converged to their ultimate bounds.

This study is structured as follows. The preliminaries are introduced in Section 2; the problem is formulated in Section 3; the controller designs and related stability analysis are discussed in Section 4; the simulation results are shown in Section 5, and the article is concluded in Section 6.

2. Preliminaries

Notations: In this article, the set of real scalars and natural numbers are symbolized by $\mathbb{R}$ and $\mathbb{N}$, respectively. $\mathbb{R}^{\geq 0}$ ($\mathbb{R}^{>0}$) defines the set of non-negative (strict positive) real scalars. To describe the set of non-negative integers, $\mathbb{Z}^{\geq 0}$ is applied. For the complex numbers $z \in \mathbb{C}$, symbols $\arg(z)$ and $\mathrm{Re}(z)$ are the argument and the real part of z, respectively. The notation $\mathbb{R}^n$ is the n-dimensional Euclidean space. For $x \in \mathbb{R}$, its absolute value is expressed by $|x|$. The Euclidean norm of the column vector $\bar{x} \in \mathbb{R}^n$ is represented by $||\bar{x}||$. Let I_n be the identity matrix with n-dimension. The notations $\lambda_{\max}(A) := \max\{\lambda(A)\}$ and $\lambda_{\min}(A) := \min\{\lambda(A)\}$ are defined, where $\lambda(A)$ is the set of all the eigenvalues of square real matrix $A \in \mathbb{R}^{n\times n}$. $A > 0$, representing the positive definite property for $A \in \mathbb{R}^{n\times n}$, and $\mathrm{diag}[a_1, \dots, a_n]$ is the real diagonal matrix of their argument with n-dimension. For the signal $x(t) \in \mathbb{R}$, notation $|x(t)|_\infty$ is the ∞-norm. In this article, superscripts (s) and (d) denote static and dynamic ETC approaches, respectively.

Fractional-order calculation: The two common fractional-order derivatives are Riemann-Liouville and Caputo. The popularity of the Caputo is because only the integer-order derivatives of its initial states appear in its Laplace transform (LT); consequently, it is applied to represent the fractional-order model in this article.

For $q \in (\ell - 1, \ell)$ with $\ell \in \mathbb{N}$, the q-order Caputo derivative of function $f(t)$ is defined as (Podlubny (1999))

$$ {}_{t_0}^{C}D_t^q f(t) = \frac{1}{\Gamma(\ell - q)} \int_{t_0}^{t} \frac{f^{(\ell)}(p)}{(t-p)^{q+1-\ell}} \mathrm{d}p, \tag{1} $$

where $f^{(\ell)}(t)$ is the ℓ-th time derivative of $f(t)$ and $\Gamma(1-q) = \int_0^{+\infty} p^{-q}\exp(-p)\mathrm{d}p$. Hereafter, it is assumed that $\ell = 1$. The LT of the q-order Caputo derivative is expressed as (Podlubny (1999))

$$ \mathcal{L}\left\{ {}_{t_0}^{C}D_t^q f(t) \right\} = s^q F(s) - s^{q-1} f(t_0), \tag{2} $$

where $\mathcal{L}$ is the LT operator and $F(s)$ is the LT of $f(t)$.

Definition 1 (Podlubny (1999)): The Mittag-Leffler function is defined as

$$ M_{(q,\gamma)}(z) = \sum_{m=0}^{+\infty} \frac{z^m}{\Gamma(qm+\gamma)}, \tag{3} $$

with $z \in \mathbb{C}$ and $\gamma \in \mathbb{R}^{>0}$, whose the LT of the Mittag-Leffler function with $\eta \in \mathbb{R}$ is expressed as

$$ \mathcal{L}\left\{ t^{\gamma-1} M_{(q,\gamma)}(-\eta t^q) \right\} = \frac{s^{q-\gamma}}{s^q + \eta}, \quad \mathrm{Re}(s) > |\eta|^{-q}. \tag{4} $$

Lemma 1 (Podlubny (1999)): If $\gamma \in \mathbb{R}^{>0}$, $\phi \in \mathbb{R}^{>0}$, and $q \in \mathbb{R}^{>0}$ are satisfied by $\frac{\pi q}{2} < \phi < \pi q$ and $q \in (0,1)$, then there exists $\Xi \in \mathbb{R}^{>0}$, in a sense that the Mittag-Leffler function is bounded by

$$ |M_{(q,\gamma)}(z)| \leq \frac{\Xi}{1+|z|}, \quad \phi \leq |\arg(z)| \leq \pi. \tag{5} $$

Lemma 2 (Podlubny (1999)): If $\bar{x}(t) = [x_1(t), \dots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the smooth vector function, then the following relations are satisfied for any $t \geq t_0$

$$ {}_{t_0}^{C}D_t^q \left(\bar{x}^{\mathrm{T}}(t)\bar{x}(t) \right) = \sum_{j=0}^{+\infty} \binom{q}{j} {}_{t_0}^{C}D_t^j \bar{x}^{\mathrm{T}}(t) {}_{t_0}^{C}D_t^{\alpha-j} \bar{x}(t), \tag{6} $$

$$ {}_{t_0}^{C}D_t^q\Big(\bar{x}^{\mathrm{T}}(t)\bar{x}(t)\Big) \le 2\bar{x}^{\mathrm{T}}(t){}_{t_0}^{C}D_t^q\bar{x}(t). \tag{7} $$

Lemma 3 (Shahvali et al. (2020)): For any given continuous and positive definite function $V(\bar{x}(t)) : \mathbb{R}^n \to \mathbb{R}^{\ge 0}$, if there exist the two bounded parameters $\beta > 0$ and $\varrho \ge 0$ in a sense that ${}_{t_0}^{C}D_t^q V(\bar{x}(t)) \le -\beta V(\bar{x}(t)) + \varrho$, $\forall t \ge t_0$, then this fractional-order differential inequality has an unique solution for every bounded initial condition, and yields

$$ V(\bar{x}(t)) \le V(\bar{x}(t_0))M_{(q,1)}\Big(-\beta(t-t_0)^q\Big) + \frac{\varrho d}{\beta}, \tag{8} $$

where $d := \max\{1, \Xi\}$ which Ξ is defined in (5).

Remark 1: The stability analysis of the fractional-order nonlinear systems is different from the integer-order ones. This is because the fractional-order derivative of the standard quadratic Lyapunov functions generates some infinite series according to Eq. (6). To overcome this difficulty, the fractional-order Lyapunov stability (i.e., Lemma 1, Eq. (7), and Lemma 3) are applied in this article.

Neural networks: In general, the radial basis function neural networks (RBFNNs) have been widely employed in extending the adaptive control strategies to handle parametric uncertainties in structured dynamics, Lewis et al. (1995); Gao, Song and Wen (2016). According to Gao et al. (2016), for any continuous function $f(X(t)) : \mathbb{R}^n \to \mathbb{R}$, there exists a RBFNN such that

$$ f(X(t)) = w^{*\mathrm{T}}\mu(X(t)) + \rho(X(t)), \tag{9} $$

$$ \mu_s(X(t)) = \exp\left[-\frac{(X(t)-c_s)^{\mathrm{T}}(X(t)-c_s)}{\chi_s}\right], \quad s = 1,\dots,r, \tag{10} $$

where $X(t) \in \mathbb{R}^n$ is the input vector of the NN, r is the NN node number, $w^* = [w_1^*,\dots,w_r^*]^{\mathrm{T}} \in \mathbb{R}^r$ is the ideal constant NN weight vector which minimizes the modeling error $\rho(X(t))$, $\mu(X(t)) = [\mu_1(X(t)),\dots,\mu_r(X(t))]^{\mathrm{T}} \in \mathbb{R}^r$ is the activation vector of the NN, and its members are the Gaussian basis functions defined in Eq. (10). $\chi_s \in \mathbb{R}^{>0}$ and $c_s = [c_{s,1},\dots,c_{s,n}]^{\mathrm{T}} \in \mathbb{R}^n$ are the standard deviation and the center vector of the Gaussian function, respectively. The ideal weight vector of the NNs is bounded, in a sense that $||w^*|| \le w$, where w is an unknown scalar.

The following Lemmas are very important in the subsequent control designs and their stability analysis.

Lemma 4 (Xing et al. (2016)): For any real positive bounded parameter ϵ and $\forall x(t) \in \mathbb{R}$, the following inequality holds true

$$ 0 \le |x(t)| - x(t)\tanh(x(t)/\epsilon) \le \upsilon\epsilon, \tag{11} $$

where $\upsilon := \sup_{t>0}\{\frac{t}{1+\exp(t)}\} = 0.2785$.

Lemma 5 (Khalil and Grizzle (2002)): For any $(\bar{y}_1(t), \bar{y}_2(t)) \in \mathbb{R}^n$ and $\Gamma > 0$, the following inequalities hold true

- $\bar{y}_1^{\mathrm{T}}(t)\bar{y}_2(t) \le \frac{\epsilon^{\kappa_1}}{\kappa_1}||\bar{y}_1(t)||^{\kappa_1} + \frac{1}{\kappa_2\epsilon^{\kappa_2}}||\bar{y}_2(t)||^{\kappa_2}$,
- $\lambda_{\min}(\Gamma)||\bar{y}_1(t)||^2 \le \bar{y}_1^{\mathrm{T}}(t)\Gamma\bar{y}_1(t) \le \lambda_{\max}(\Gamma)||\bar{y}_1(t)||^2$,

where $\epsilon \in \mathbb{R}^{>0}$, κ_1 and κ_2 are the positive parameters satisfying $\kappa_1 + \kappa_2 = \kappa_1\kappa_2$.

3. Problem Formulation

The focus of this article is on a class of the fractional-order uncertain strict-feedback systems Song et al. (2020); Shahvali et al. (2020); Li et al. (2021), the dynamics of which are described through the following equations

$$\begin{cases} {}_{t_0}^{C}D_t^q x_k(t) = x_{k+1}(t) + f_k(\bar{x}_k(t)) + d_k(t), \\ {}_{t_0}^{C}D_t^q x_n(t) = (B(\bar{x}_n(t)))^{\mathrm{T}} u(t) + f_n(\bar{x}_n(t)) + d_n(t), \\ y(t) = x_1(t), \quad k = 1, 2, \ldots, n-1, \end{cases} \tag{12}$$

where ${}_{t_0}^{C}D_t^q x_m(t)$ is the Caputo fractional derivative of the pseudo state $x_m(t) \in \mathbb{R}$ for $m \in \{1, \ldots, n\}$, $0 < q < 1$ is the order of the fractional derivative which is known, $u(t) = [u^1(t), \ldots, u^{\mathcal{S}}(t)]^{\mathrm{T}} \in \mathbb{R}^{\mathcal{S}}$ and $y \in \mathbb{R}$ are the actuators' output vector and the system output, respectively, and $\mathcal{S}$ represents the number of actuators. $\bar{x}_m(t) = [x_1(t), \ldots, x_m(t)]^{\mathrm{T}} \in \mathbb{R}^m$ is the pseudo state vector, $f_m(\bar{x}_m(t)) \in \mathbb{R}$ is the smooth unknown function with $f_m(0, \ldots, 0) = 0$, $d_m(t) \in \mathbb{R}$ is the unknown and bounded external disturbance, and $B(\bar{x}_n(t)) = [B^1(\bar{x}_n(t)), \ldots, B^{\mathcal{S}}(\bar{x}_n(t))]^{\mathrm{T}}$ is the known and bounded vector function with $B^i(\bar{x}_n(t)) \in \mathbb{R}^{>0}$ for $i \in \{1, \ldots, \mathcal{S}\}$.

Assume that the actuators' output for the fractional-order system in Eq. (12) is modeled through

$$u(t) = (\mathrm{I}_{\mathcal{S}} - \varsigma)\vartheta\tau(t) + \varsigma\bar{u}(t) + \Theta\Delta(t), \tag{13}$$

where $\varsigma = \mathrm{diag}[\varsigma^1, \ldots, \varsigma^{\mathcal{S}}]$, the value of ς^i is 0 or 1, $\vartheta = \mathrm{diag}[\vartheta^1, \ldots, \vartheta^{\mathcal{S}}]$, $0 < \vartheta^i \leq 1$ is an unknown efficient constant factor which representing the proportion of effectiveness of the control input, and $\bar{u}(t) = [\bar{u}^1(t), \ldots, \bar{u}^{\mathcal{S}}(t)]^{\mathrm{T}}$ is an unknown time-varying vector function which is bounded. $\Theta = \mathrm{diag}[(\theta^1)^{\mathrm{T}}, \ldots, (\theta^{\mathcal{S}})^{\mathrm{T}}]$ is an actuator constant uncertain matrix with $\theta^i = [p_1^i, \ldots, p_b^i]^{\mathrm{T}}$, $\Delta(t) = [\delta^1(t), \ldots, \delta^s(t)]^{\mathrm{T}}$ is a bounded and unknown time-varying vector with $\delta^i(t) = [v_1^i(t), \ldots, v_b^i(t)]^{\mathrm{T}}$, and $\tau(t) = [\tau^1(t), \ldots, \tau^{\mathcal{S}}(t)]^{\mathrm{T}}$ is a control input vector. The time occurrence of fault for the i-th actuator (t_i^f), pattern and its value are assumed unknown.

The considered actuator fault model in Eq. (13) includes the following normal case and the typical actuator faults

1. $\varsigma^i = 0$, $\theta^i = 0$, and $\vartheta^i = 1$ imply that the i-th actuator operates normally.
2. $\varsigma^i = 0$, $\vartheta^i = 1$, and $\theta^i \neq 0$ indicate the bias fault for the i-th actuator.
3. $\varsigma^i = 1$, $\theta^i = 0$ and $\vartheta^i = 1$ imply that the i-th actuator works under stuck.
4. $\varsigma^i = 0$, $\theta^i = 0$, and $0 < \vartheta^i < 1$ means the i-th actuator undergoes the partial loss of effectiveness.

Remark 2: System in Eq. (12) represents the fractional-order nonlinear systems with the strict-feedback form consisting of some power and electrical applications, like machine-infinite bus power systems Song et al. (2020), doubly-fed induction generators Aounallah, Essounbouli, Hamzaoui and Bouchafaa (2018), hydro-turbine governing systems Xu, Chen, Zhang and Wang (2015), and Chua's and Duffing systems Podlubny (1999).

Control problem: The control problem of this article is to design the static and dynamic event-triggered fault compensation methods for the fractional-order system (12) with the actuator fault (13) such that

1. All the signals in the closed-loop system are globally uniformly ultimately bounded, while the tracking error (i.e., $\pi_1(t) = x_1(t) - x_d(t)$) is ultimately restricted to a compact set with adjustable size $a > 0$ as follows

$$\Pi := \left\{ \pi_1(t) \in \mathbb{R} \middle| \mathrm{Lim}_{t\to+\infty} |\pi_1(t)| \leq a \right\},$$

where x_d is a desired trajectory.

2. There exist two positive constants $t^{*(s)}$ and $t^{*(d)}$ such that

$$t_{j+1}^{(s)} - t_j^{(s)} \geq t^{*(s)}, \quad t_{j+1}^{(d)} - t_j^{(d)} \geq t^{*(d)}, \quad \forall j \in \mathbb{N},$$

namely, the Zeno behavior does not happen.

To solve the aforesaid control problem, first, a global NN fault compensation TTC is designed. Next, by combining the new static ETC technique with the NN fault compensation, the static event-triggered fault compensation control is devised which saves energy resources than the proposed TTC. To overcome the conservatism minimum inter-event time interval in the proposed static ETC, an internal dynamic variable is incorporated into the static ETC so that the practical dynamic ETC is introduced.

As to this control problem, the following Assumptions are made.

Assumption 1 (Li et al. (2021)): For the fractional-order system in Eq. (12), if any up to $\mathcal{S}$-1 actuators stuck, and the others lose effectiveness, the closed-loop system can still be driven to obtain the control objective.

Assumption 2: The desired trajectory $x_d(t) \in \mathbb{R}$ and its q-order fractional derivative ${}_{t_0}^{C}D_t^q x_d(t) \in \mathbb{R}$ are exist, continuous and bounded. However, ${}_{t_0}^{C}D_t^q x_d(t)$ is unknown for control design.

Assumption 3 (Gao et al. (2016)): The NN modeling error satisfies the following inequality

$$|\rho(X(t))| \leq \bar{\varphi}^* + \psi^* \phi(X(t)), \tag{14}$$

where $\bar{\varphi}^*$ and ψ^* are the unknown positive scalars, and $\phi(X(t))$ is a known positive function.

Remark 3: Assumption 1 is necessary and standard for the controllability of the fractional-order and also the integer-order systems in presence of actuator faults. Assumption 2 reduces conservatism in Yang et al. (2020a); Song et al. (2020); Shahvali et al. (2020); Gong et al. (2020), where x_d, ${}_{t_0}^{C}D_t^q x_d(t)$ and ${}_{t_0}^{C}D_t^{2q} x_d(t)$ should be known and available for control design. In the most adaptive NN control results, the modeling error is assumed to be bounded by a constant over a compact set. However, it cannot be ensured before the stability of the closed-loop system is established. In this article, Assumption 3 similar to Gao et al. (2016) is more general form of assumption on modeling error in Shahvali et al. (2020); Wang et al. (2019); Zhang et al. (2020, 2018) specifies that the modeling error is bounded by a state-dependent function. This enables us to provide the closed-loop stability analysis over the whole state space, not just over a compact set, and thus removes the unreasonable assumption in the conventional adaptive NN control approaches. As a result, Assumptions 1-3 do not pose a strong limitation from a practical viewpoint upon the considered class of the fractional-order systems and also the proposed control strategy.

4. Main results

After proposing the NN fault compensation TTC procedure for the fractional-order systems, two static and dynamic ETC methods are devised. Hereafter, symbol t is removed from relations when is unnecessary.

4.1. Time-triggered fault compensation approach

The backstepping strategy is adopted to design the fault compensation time-triggered controller for the fractional-order strict-feedback systems. Using this control approach, the actual control input can be designed as $\tau^i = \frac{\hat{\wp}^i \varpi_{n+1}}{B^i(\bar{x}_n)}$, for all $i = 1, \ldots, \mathcal{S}$, where $\hat{\wp}^i$ and ϖ_{n+1} are given letter. To this end, the coordinate transformation is presented as (Khalil and Grizzle (2002); Regaya, Farhani, Zaafouri and Chaari (2017a, 2018))

$$\pi_m = x_m - \varpi_m, \quad m = 1, \ldots, n, \tag{15}$$

where π_m is the error surface, $\varpi_1 := x_d$, and ϖ_m for $m = 2, \dots, n$ is the virtual control law which is given later.

Now, the design procedure is began.

• **First step:** From (12), (15), and with respect to the universal approximation of RBFNNs (9),(i.e., $f_1(\bar{x}_1) = w_1^{*\mathrm{T}}\mu_1(\bar{x}_1) + \rho_1(\bar{x}_1)$), one can follow that

$$\begin{aligned} {}_{t_0}^{C}D_t^q \pi_1 &= {}_{t_0}^{C}D_t^q x_1 - {}_{t_0}^{C}D_t^q x_d, \\ &= x_2 + f_1(\bar{x}_1) + d_1 - {}_{t_0}^{C}D_t^q x_d, \\ &= x_2 + w_1^{*\mathrm{T}}\mu_1(\bar{x}_1) + \rho_1(\bar{x}_1) + d_1 - {}_{t_0}^{C}D_t^q x_d. \end{aligned} \tag{16}$$

In contrast to the common assumption for the integer-order systems in Zhao et al. (2019); Zhang and Yang (2019); Wang et al. (2019); Zhang et al. (2020, 2018), where the time derivatives of the desired trajectory are known, the q-order fractional derivative of the desired trajectory is often unknown (Shahvali et al. (2020)). To overcome this difficulty, a fractional-order differentiator (FOD) with input x_d, and two pseudo state variables η_{1_1} and η_{1_2} is used to estimate ${}_{t_0}^{C}D_t^q x_d$ as follows (Sheng, Wei, Cheng and Shuai (2017))

$$\begin{cases} {}_{t_0}^{C}D_t^q \eta_{1_1} = \eta_{1_2}, \\ {}_{t_0}^{C}D_t^q \eta_{1_2} = -a_1 \tanh\left(\eta_{1_1} - x_d + \frac{\eta_{1_2}|\eta_{1_2}|}{a_1}\left(1 - \frac{\Gamma^2(q+1)}{\Gamma(2q+1)}\right)\right), \end{cases} \tag{17}$$

where $a_1 > 0$ is a design factor, the pseudo states η_{1_1} and η_{1_2} are converged into x_d and ${}_{t_0}^{C}D_t^q x_d$ with bounded estimation errors as $\varepsilon_{1_1} = x_d - \eta_{1_1}$ and $\varepsilon_{1_2} = {}_{t_0}^{C}D_t^q x_d - \eta_{1_2}$, respectively. Then, the following is yielded

$${}_{t_0}^{C}D_t^q \pi_1 = x_2 + w_1^{*\mathrm{T}}\mu_1(\bar{x}_1) + \rho_1(\bar{x}_1) + d_1 - \eta_{1_2} - \varepsilon_{1_2}. \tag{18}$$

The virtual control and the adaptive law are designed as

$$\varpi_2 = -c_1\pi_1 - \hat{w}_1^{\mathrm{T}}\mu_1(\bar{x}_1) - \hat{\varphi}_1 \tanh\left(\frac{\pi_1}{\epsilon}\right) - \hat{\psi}_1\phi_1(\bar{x}_1)\tanh\left(\frac{\pi_1\phi_1(\bar{x}_1)}{\epsilon}\right) + \eta_{1_2}, \tag{19}$$

$${}_{t_0}^{C}D_t^q \hat{w}_1 = \Gamma_{w_1}\left(\pi_1\mu_1(\bar{x}_1) - r_{w_1}\hat{w}_1\right), \tag{20}$$

$${}_{t_0}^{C}D_t^q \hat{\varphi}_1 = \gamma_{\varphi_1}\left(\pi_1 \tanh\left(\frac{\pi_1}{\epsilon}\right) - r_{\varphi_1}\hat{\varphi}_1\right), \tag{21}$$

$${}_{t_0}^{C}D_t^q \hat{\psi}_1 = \gamma_{\psi_1}\left(\pi_1\phi_1(\bar{x}_1)\tanh\left(\frac{\pi_1\phi_1(\bar{x}_1)}{\epsilon}\right) - r_{\psi_1}\hat{\psi}_1\right), \tag{22}$$

where $c_1 > 0$ and $\epsilon > 0$ are the design parameters, $\Gamma_{w_1} > 0$, $\gamma_{\varphi_1} > 0$ and $\gamma_{\psi_1} > 0$ are the adaptive gains, and $r_{w_1} > 0$, $r_{\varphi_1} > 0$ and $r_{\psi_1} > 0$ are the sigma modification factors.

Choose the Lyapunov function candidate as

$$V_1 = \frac{1}{2}\pi_1^2 + \frac{1}{2}\tilde{w}_1^{\mathrm{T}}\Gamma_{w_1}^{-1}\tilde{w}_1 + \frac{1}{2\gamma_{\varphi_1}}\tilde{\varphi}_1^2 + \frac{1}{2\gamma_{\psi_1}}\tilde{\psi}_1^2, \tag{23}$$

where $\tilde{w}_1 = w_1^* - \hat{w}_1$, $\tilde{\varphi}_1 = \varphi_1^* - \hat{\varphi}_1$, and $\tilde{\psi}_1 = \psi_1^* - \hat{\psi}_1$.

Applying (7) together with (23) and (18) results in

$${}_{t_0}^{C}D_t^q V_1 \leq \pi_1\left(x_2 + w_1^{*\mathrm{T}}\mu_1(\bar{x}_1) + \rho_1(\bar{x}_1) + d_1 - \varepsilon_{1_2} - \eta_{1_2}\right) - \tilde{w}_1^{\mathrm{T}}\Gamma_{w_1}^{-1}\,{}_{t_0}^{C}D_t^q \hat{w}_1 - \frac{1}{\gamma_{\varphi_1}}\tilde{\varphi}_1\,{}_{t_0}^{C}D_t^q \hat{\varphi}_1 - \frac{1}{\gamma_{\psi_1}}\tilde{\psi}_1\,{}_{t_0}^{C}D_t^q \hat{\psi}_1. \tag{24}$$

Through Assumption 3 and Lemma 4, the following inequality is obtained

$$\pi_1\left(\rho_1(\bar{x}_1) + d_1 - \varepsilon_{1_2}\right) \leq \varphi_1^* \pi_1 \tanh\left(\frac{\pi_1}{\epsilon}\right) + \psi_1^* \pi_1 \phi_1(\bar{x}_1) \tanh\left(\frac{\pi_1 \phi_1(\bar{x}_1)}{\epsilon}\right) + p_1, \tag{25}$$

where $|\varepsilon_{1_2}| \leq \varepsilon_{1_2}^*$, $|d_1| \leq d_1^*$, $\varphi_1^* = \varepsilon_{1_2}^* + \bar{\varphi}_1^* + d_1^*$, and $p_1 = \upsilon\epsilon(\varphi_1^* + \psi_1^*)$. Substituting (25) in (24) yields

$$\begin{aligned} {}_{t_0}^{C}D_t^q V_1 \leq & \pi_1\left(x_2 + w_1^{*\mathrm{T}}\mu_1(\bar{x}_1) + \varphi_1^* \tanh\left(\frac{\pi_1}{\epsilon}\right) + \psi_1^* \phi_1(\bar{x}_1) \tanh\left(\frac{\pi_1 \phi_1(\bar{x}_1)}{\epsilon}\right) - \eta_{1_2}\right) - \tilde{w}_1^{\mathrm{T}} \Gamma_{w_1}^{-1} {}_{t_0}^{C}D_t^q \hat{w}_1 \\ & - \frac{1}{\gamma_{\varphi_1}} \tilde{\varphi}_1 {}_{t_0}^{C}D_t^q \hat{\varphi}_1 - \frac{1}{\gamma_{\psi_1}} \tilde{\psi}_1 {}_{t_0}^{C}D_t^q \hat{\psi}_1 + p_1. \end{aligned} \tag{26}$$

Combining (15), (19)-(22) with (26) gives

$${}_{t_0}^{C}D_t^q V_1 \leq -c_1 \pi_1^2 + \pi_1 \pi_2 + r_{w_1} \tilde{w}_1^{\mathrm{T}} \hat{w}_1 + r_{\varphi_1} \tilde{\varphi}_1 \hat{\varphi}_1 + r_{\psi_1} \tilde{\psi}_1 \hat{\psi}_1 + p_1. \tag{27}$$

• **Second step** ($2 \leq k \leq n-1$): From (12), (15), and universal approximation of NNs (9), Eq. (28) is yielded

$${}_{t_0}^{C}D_t^q \pi_k = x_{k+1} + w_k^{*\mathrm{T}} \mu_k(\bar{x}_k) + \rho_k(\bar{x}_k) + d_k - {}_{t_0}^{C}D_t^q \varpi_k. \tag{28}$$

In the proposed backstepping control design procedure, the repeated fractional-order derivative of the virtual control should be calculated, (i.e., ${}_{t_0}^{C}D_t^q \varpi_k$). This calculation cannot be derived exactly, because the Chain rule is not well established for the fractional-order derivative. To overcome this drawback, a FOD with a design factor $a_k > 0$, input ϖ_k, and two pseudo states η_{k_1} and η_{k_2} is used to estimate ${}_{t_0}^{C}D_t^q \varpi_k$ as follows

$$\begin{cases} {}_{t_0}^{C}D_t^q \eta_{k_1} = \eta_{k_2}, \\ {}_{t_0}^{C}D_t^q \eta_{k_2} = -a_k \tanh\left(\eta_{k_1} - \varpi_k + \frac{\eta_{k_2}|\eta_{k_2}|}{a_k}\left(1 - \frac{\Gamma^2(q+1)}{\Gamma(2q+1)}\right)\right), \end{cases} \tag{29}$$

where η_{k_1} and η_{k_2} are converged into ϖ_k and ${}_{t_0}^{C}D_t^q \varpi_k$ with bounded estimation errors as $\varepsilon_{k_1} = \varpi_k - \eta_{k_1}$ and $\varepsilon_{k_2} = {}_{t_0}^{C}D_t^q \varpi_k - \eta_{k_2}$, respectively. It follows that

$${}_{t_0}^{C}D_t^q \pi_k = x_{k+1} + w_k^{*\mathrm{T}} \mu_k(\bar{x}_k) + \rho_k(\bar{x}_k) + d_k - \eta_{k_2} - \varepsilon_{k_2}. \tag{30}$$

The virtual control and the adaptive update law are designed as

$$\varpi_{k+1} = -c_k \pi_k - \hat{w}_k^{\mathrm{T}} \mu_k(\bar{x}_k) - \hat{\varphi}_k \tanh\left(\frac{\pi_k}{\epsilon}\right) - \hat{\psi}_k \phi_k(\bar{x}_k) \tanh\left(\frac{\pi_k \phi_k(\bar{x}_k)}{\epsilon}\right) + \eta_{k_2}, \tag{31}$$

$${}_{t_0}^{C}D_t^q \hat{w}_k = \Gamma_{w_k}\left(\pi_k \mu_k(\bar{x}_k) - r_{w_k} \hat{w}_k\right), \tag{32}$$

$${}_{t_0}^{C}D_t^q \hat{\varphi}_k = \gamma_{\varphi_k}\left(\pi_k \tanh\left(\frac{\pi_k}{\epsilon}\right) - r_{\varphi_k} \hat{\varphi}_k\right), \tag{33}$$

$${}_{t_0}^{C}D_t^q \hat{\psi}_k = \gamma_{\psi_k}\left(\pi_k \phi_k(\bar{x}_k) \tanh\left(\frac{\pi_k \phi_k(\bar{x}_k)}{\epsilon}\right) - r_{\psi_k} \hat{\psi}_k\right), \tag{34}$$

where $c_k > 0$ and $\epsilon > 0$ are the design parameters, $\Gamma_{w_k} > 0$, $\gamma_{\varphi_k} > 0$ and $\gamma_{\psi_k} > 0$ are the adaptive gains, and $r_{w_k} > 0$, $r_{\varphi_k} > 0$ and $r_{\psi_k} > 0$ are the sigma modification factors.

Define the Lyapunov function candidate as

$$V_k = \frac{1}{2}\pi_k^2 + \frac{1}{2}\tilde{w}_k^{\mathrm{T}} \Gamma_{w_k}^{-1} \tilde{w}_k + \frac{1}{2\gamma_{\varphi_k}} \tilde{\varphi}_k^2 + \frac{1}{2\gamma_{\psi_k}} \tilde{\psi}_k^2, \tag{35}$$

where $\tilde{w}_k = w_k^* - \hat{w}_k$, $\tilde{\varphi}_k = \varphi_k^* - \hat{\varphi}_k$, and $\tilde{\psi}_k = \psi_k^* - \hat{\psi}_k$.

According to (30), (35) and Lemma 2 one has

$$ {}_{t_0}^{C}D_t^q V_k \le \pi_k \left(x_{k+1} + w_k^{*\mathrm{T}} \mu_k(\bar{x}_k) + \rho_k(\bar{x}_k) + d_k - \varepsilon_{k_2} - \eta_{k_2} \right) - \tilde{w}_k^{\mathrm{T}} \Gamma_{w_k}^{-1} {}_{t_0}^{C}D_t^q \hat{w}_k - \frac{1}{\gamma_{\varphi_k}} \tilde{\varphi}_k {}_{t_0}^{C}D_t^q \hat{\varphi}_k - \frac{1}{\gamma_{\psi_k}} \tilde{\psi}_k {}_{t_0}^{C}D_t^q \hat{\psi}_k. \tag{36} $$

Being similar to the same case of Eq. (25), the following is obtained

$$ \pi_k \big(\rho_k(\bar{x}_k) + d_k - \varepsilon_{k_2} \big) \le \varphi_k^* \pi_k \tanh\left(\frac{\pi_k}{\epsilon}\right) + \psi_k^* \pi_k \phi_k(\bar{x}_k) \tanh\left(\frac{\pi_k \phi_k(\bar{x}_k)}{\epsilon}\right) + p_k, \tag{37} $$

where $|\varepsilon_{k_2}| \le \varepsilon_{k_2}^*$, $|d_k| \le d_k^*$, $\varphi_k^* = \varepsilon_{k_2}^* + d_k^* + \bar{\varphi}_k^*$, and $p_k = \upsilon\epsilon(\varphi_k^* + \psi_k^*)$. As to (37), ${}_{t_0}^{C}D_t^q V_k$ is reexpressed as

$$ {}_{t_0}^{C}D_t^q V_k \le \pi_k \left(x_{k+1} + w_k^{*\mathrm{T}} \mu_k(\bar{x}_k) + \varphi_k^* \tanh\left(\frac{\pi_k}{\epsilon}\right) + \psi_k^* \phi_k(\bar{x}_k) \tanh\left(\frac{\pi_k \phi_k(\bar{x}_k)}{\epsilon}\right) - \eta_{k_2} \right) - \tilde{w}_k^{\mathrm{T}} \Gamma_{w_k}^{-1} {}_{t_0}^{C}D_t^q \hat{w}_k - \frac{1}{\gamma_{\varphi_k}} \tilde{\varphi}_k {}_{t_0}^{C}D_t^q \hat{\varphi}_k - \frac{1}{\gamma_{\psi_k}} \tilde{\psi}_k {}_{t_0}^{C}D_t^q \hat{\psi}_k + p_k. \tag{38} $$

Substituting (31)-(34) into (38), the following holds

$$ {}_{t_0}^{C}D_t^q V_k \le -c_k \pi_k^2 + \pi_k \pi_{k+1} + r_{w_k} \tilde{w}_k^{\mathrm{T}} \hat{w}_k + r_{\varphi_k} \tilde{\varphi}_k \hat{\varphi}_k + r_{\psi_k} \tilde{\psi}_k \hat{\psi}_k + p_k. \tag{39} $$

• **Final step:** By considering (12), (13) and (15), the q-order fractional derivative of π_n is computed as follows

$$ {}_{t_0}^{C}D_t^q \pi_n = \sum_{i=1}^{\mathcal{S}} B^i(\bar{x}_n) \Big((1-\varsigma^i)\vartheta^i \tau^i + \varsigma^i \bar{u}^i + (\theta^i)^{\mathrm{T}} \delta^i \Big) + w_n^{*\mathrm{T}} \mu_n(\bar{x}_n) + \rho_n(\bar{x}_n) + d_n - {}_{t_0}^{C}D_t^q \varpi_n, \tag{40} $$

The following FOD with a design factor $a_n > 0$ is applied to estimate ${}_{t_0}^{C}D_t^q \varpi_n$ as

$$ \begin{cases} {}_{t_0}^{C}D_t^q \eta_{n_1} = \eta_{n_2}, \\ {}_{t_0}^{C}D_t^q \eta_{n_2} = -a_n \tanh\left(\eta_{n_1} - \varpi_n + \frac{\eta_{n_2}|\eta_{n_2}|}{a_n}\left(1 - \frac{\Gamma^2(q+1)}{\Gamma(2q+1)}\right)\right), \end{cases} \tag{41} $$

where η_{n_1} and η_{n_2} are converged into ϖ_n and ${}_{t_0}^{C}D_t^q \varpi_n$ with the bounded estimation errors as $\varepsilon_{n_1} = \varpi_n - \eta_{n_1}$ and $\varepsilon_{n_2} = {}_{t_0}^{C}D_t^q \varpi_n - \eta_{n_2}$, respectively. Then, (40) can be rewritten as

$$ {}_{t_0}^{C}D_t^q \pi_n = \sum_{i=1}^{s} B^i(\bar{x}_n) \Big((1-\varsigma^i)\vartheta^i \tau^i + \varsigma^i \bar{u}^i + (\theta^i)^{\mathrm{T}} \delta^i \Big) + w_n^{*\mathrm{T}} \mu_n(\bar{x}_n) + \rho_n(\bar{x}_n) + d_n - \eta_{n_2} - \varepsilon_{n_2}. \tag{42} $$

For real applications without prior knowledge of the actuator faults, the actual TTC law is designed as

$$ \tau^i = \frac{\hat{\wp}^i \varpi_{n+1}}{B^i(\bar{x}_n)}, \quad i = 1, \dots, \mathcal{S}, \tag{43} $$

where ϖ_{n+1} is the virtual control, besides according to the definition of ϑ^i, there mathematically exists unknown constant parameter (i.e., $\wp^{i*}$), like that $\sum_{i=1}^{s} \wp^{i*}(1-\varsigma^i)\vartheta^i = 1$, in which $\wp^{i*}$ is approximated by $\hat{\wp}^i$.

The virtual control and the adaptive update law are

$$\varpi_{n+1} = - c_n\pi_n - \hat{w}_n^{\mathrm{T}}\mu_n(\bar{x}_n) - \hat{\varphi}_n \tanh\left(\frac{\pi_n}{\epsilon}\right) - \hat{\psi}_n\phi_n(\bar{x}_n)\tanh\left(\frac{\pi_n\phi_n(\bar{x}_n)}{\epsilon}\right) + \eta_{n_2}, \tag{44}$$

$${}_{t_0}^{C}D_t^q\hat{w}_n = \Gamma_{w_n}\left(\pi_n\mu_n(\bar{x}_n) - r_{w_n}\hat{w}_n\right), \tag{45}$$

$${}_{t_0}^{C}D_t^q\hat{\varphi}_n = \gamma_{\varphi_n}\left(\pi_n\tanh\left(\frac{\pi_n}{\epsilon}\right) - r_{\varphi_n}\hat{\varphi}_n\right), \tag{46}$$

$${}_{t_0}^{C}D_t^q\hat{\psi}_n = \gamma_{\psi_n}\left(\pi_n\phi_n(\bar{x}_n)\tanh\left(\frac{\pi_n\phi_n(\bar{x}_n)}{\epsilon}\right) - r_{\psi_n}\hat{\psi}_n\right), \tag{47}$$

$${}_{t_0}^{C}D_t^q\hat{\wp}^i = -\gamma_{\wp}^i\left(\pi_n\varpi_{n+1} + r_{\wp}^i\hat{\wp}^i\right), \quad i = 1, \dots, \mathcal{S}, \tag{48}$$

where $c_n > 0$ and $\epsilon > 0$ are the design parameters, $\Gamma_{w_n} > 0$, $\gamma_{\varphi_n} > 0$, $\gamma_{\psi_n} > 0$ and $\gamma_{\wp}^i > 0$ are the adaptive gains, and $r_{w_n} > 0$, $r_{\varphi_n} > 0$, $r_{\psi_n} > 0$ and $r_{\wp}^i > 0$ are the sigma modification factors.

The following Lyapunov candidate function is of concern

$$V_n = \frac{1}{2}\pi_n^2 + \frac{1}{2}\tilde{w}_n^{\mathrm{T}}\Gamma_{w_n}^{-1}\tilde{w}_n + \frac{1}{2\gamma_{\varphi_n}}\tilde{\varphi}_n^2 + \frac{1}{2\gamma_{\psi_n}}\tilde{\psi}_n^2 + \sum_{i=1}^{\mathcal{S}}\frac{\vartheta^i}{2\gamma_{\wp}^i}(1-\varsigma^i)(\tilde{\wp}^i)^2, \tag{49}$$

where $\tilde{w}_n = w_n^* - \hat{w}_n$, $\tilde{\varphi}_n = \varphi_n^* - \hat{\varphi}_n$, $\tilde{\psi}_n = \psi_n^* - \hat{\psi}_n$, and $\tilde{\wp}^i = \wp^{i*} - \hat{\wp}^i$.

Based on the (42), (43), and Lemma 2, the q-order fractional derivative of V_n is obtained as

$$\begin{aligned}{}_{t_0}^{C}D_t^qV_n \leq& \pi_n\left(\varpi_{n+1} - \sum_{i=1}^{s}\vartheta^i\tilde{\wp}^i(1-\varsigma^i)\varpi_{n+1} + w_n^{*\mathrm{T}}\mu_n(\bar{x}_n) + \hbar + \rho_n(\bar{x}_n) + d_n - \varepsilon_{n_2} - \eta_{n_2}\right) - \tilde{w}_n^{\mathrm{T}}\Gamma_{w_n}^{-1}\,{}_{t_0}^{C}D_t^q\hat{w}_n \\ &- \frac{1}{\gamma_{\varphi_n}}\tilde{\varphi}_n\,{}_{t_0}^{C}D_t^q\hat{\varphi}_n - \frac{1}{\gamma_{\psi_n}}\tilde{\psi}_n\,{}_{t_0}^{C}D_t^q\hat{\psi}_n - \sum_{i=1}^{\mathcal{S}}\frac{\vartheta^i}{\gamma_{\wp}^i}(1-\varsigma^i)\tilde{\wp}^i\,{}_{t_0}^{C}D_t^q\hat{\wp}^i,\end{aligned} \tag{50}$$

where $\hbar = \sum_{i=1}^{\mathcal{S}} B^i(\bar{x}_n)\big(\varsigma^i\bar{u}^i + (\theta^i)^{\mathrm{T}}\delta^i\big)$. Similar to Eqs. (25) and (37) in previous steps, the following is evident

$$\pi_n\big(\rho_n(\bar{x}_n) + d_n + \hbar - \varepsilon_{n_2}\big) \leq \varphi_n^*\pi_n\tanh\left(\frac{\pi_n}{\epsilon}\right) + \psi_n^*\pi_n\phi_n(\bar{x}_n)\tanh\left(\frac{\pi_n\phi_n(\bar{x}_n)}{\epsilon}\right) + p_n, \tag{51}$$

where $|\varepsilon_{n_2}| \leq \varepsilon_{n_2}^*$, $|d_n| \leq d_n^*$, $|\hbar| \leq \hbar^*$, $\varphi_n^* = \varepsilon_{n_2}^* + d_n^* + \bar{\varphi}_n^* + \hbar^*$ and $p_n = \upsilon\epsilon(\varphi_n^* + \psi_n^*)$.

Substituting (51) into (50), we have

$$\begin{aligned}{}_{t_0}^{C}D_t^qV_n \leq& \pi_n\left(\varpi_{n+1} - \sum_{i=1}^{\mathcal{S}}\vartheta^i\tilde{\wp}^i(1-\varsigma^i)\varpi_{n+1} + w_n^{*\mathrm{T}}\mu_n(\bar{x}_n) + \varphi_n^*\tanh\left(\frac{\pi_n}{\epsilon}\right) + \psi_n^*\phi_n(\bar{x}_n)\tanh\left(\frac{\pi_n\phi_n(\bar{x}_n)}{\epsilon}\right) - \eta_{n_2}\right) \\ &- \tilde{w}_n^{\mathrm{T}}\Gamma_{w_n}^{-1}\,{}_{t_0}^{C}D_t^q\hat{w}_n - \frac{1}{\gamma_{\varphi_n}}\tilde{\varphi}_n\,{}_{t_0}^{C}D_t^q\hat{\varphi}_n - \frac{1}{\gamma_{\psi_n}}\tilde{\psi}_n\,{}_{t_0}^{C}D_t^q\hat{\psi}_n - \sum_{i=1}^{\mathcal{S}}\frac{\vartheta^i}{\gamma_{\wp}^i}(1-\varsigma^i)\tilde{\wp}^i\,{}_{t_0}^{C}D_t^q\hat{\wp}^i + p_n.\end{aligned} \tag{52}$$

From Eqs. (44)-(48) and (52), the following inequality is obtained

$${}_{t_0}^{C}D_t^qV_n \leq -c_n\pi_n^2 + r_{w_n}\tilde{w}_n^{\mathrm{T}}\hat{w}_n + r_{\varphi_n}\tilde{\varphi}_n\hat{\varphi}_n + r_{\psi_n}\tilde{\psi}_n\hat{\psi}_n + \sum_{i=1}^{\mathcal{S}}\vartheta^i(1-\varsigma^i)r_{\wp}^i\tilde{\wp}^i\hat{\wp}^i + p_n. \tag{53}$$

4.1.1. Stability analysis of TTC

The results as mentioned earlier are briefed in the following theorem.

Theorem 1: Let Assumptions 1-3 are satisfied, then by considering the closed-loop networked system including the fractional-order nonlinear dynamics (12) with the actuator faults (13), the actual control law (43), the virtual laws (19), (31) and (44), the FOD (17), (29) and (41), the following hold

- All signals of the closed-loop networked system are globally uniformly ultimately bounded.
- The tracking error ultimately converges into the neighborhood of the origin with adjustable size as follows

$$\Pi := \left\{ \pi_1(t) \in \mathbb{R} \middle| \mathrm{Lim}_{t\to+\infty} |\pi_1(t)|^2 \le 2\frac{\varrho d}{\beta} \right\}.$$

Proof: The total Lyapunov function candidate is selected as $V = \sum_{m=1}^{n} V_m$. Note to (27), (39), (53), and applying Lemma 5, the q-order fractional derivative of the proposed V results in

$$\begin{aligned} {}_{t_0}^{C}D_t^q V \le & -(c_1-0.5)\pi_1^2 - \sum_{m=2}^{n-1}(c_m-1)\pi_m^2 - (c_n-0.5)\pi_n^2 - \frac{1}{2}\sum_{m=1}^{n}\left(r_{w_m}||\tilde{w}_m||^2 + r_{\varphi_m}\tilde{\varphi}_m^2 + r_{\psi_m}\tilde{\psi}_m^2 \right) \\ & - \frac{1}{2}\sum_{i=1}^{S}\vartheta^i(1-\varsigma^i)r_{\wp}^i(\tilde{\wp}^i)^2 + \frac{1}{2}\sum_{m=1}^{n}\left(r_{w_m}||w_m^*||^2 + r_{\varphi_m}\varphi_m^{*2} + r_{\psi_m}\psi_m^{*2} \right) \\ & + \frac{1}{2}\sum_{i=1}^{S}\vartheta^i(1-\varsigma^i)r_{\wp}^i(\wp^{i*})^2 + \sum_{m=1}^{n} p_m. \end{aligned} \tag{54}$$

From (54), one has

$${}_{t_0}^{C}D_t^q V \le -\beta V + \varrho, \tag{55}$$

where $\beta = \min_{\substack{m=1,\dots,n \\ i=1,\dots,S}} \left\{ 2(c_m-1), \lambda_{\min}(\Gamma_{w_m})r_{w_m}, \gamma_{\varphi_m} r_{\varphi_m}, \gamma_{\psi_m} r_{\psi_m}, \gamma_{\wp}^i r_{\wp}^i \right\}$ and $\varrho = 0.5\sum_{m=1}^{n}\Big(r_{w_m}||w_m^*||^2 +$ $r_{\varphi_m}\varphi_m^{*2}$
$+ r_{\psi_m}\psi_m^{*2} + 2p_m \Big) + 0.5\sum_{i=1}^{S}\vartheta^i(1-\varsigma^i)r_{\wp}^i(\wp^{i*})^2$.

From (55), Lemmas 3 and 1, we have $V(t) \le V(t_0)M_{(q,1)}(-\beta(t-t_0)^q) + \frac{\varrho d}{\beta}$, and $V(t) \le \frac{V(t_0)\Xi}{1+|\beta(t-t_0)^q|} + \frac{\varrho d}{\beta}$. Therefore, it can be deduced that the solutions of the resulting closed-loop networked system are globally uniformly ultimately bounded; thus, the first part of the Theorem 1 is proved. Moreover, $\frac{1}{2}\pi_1^2(t) \le V(t)$ implies that $\Pi := \left\{ \pi_1(t) \in \mathbb{R} \middle| \mathrm{Lim}_{t\to+\infty}|\pi_1(t)|^2 \le 2\frac{\varrho d}{\beta} \right\}$. This completes the proof of the second part of Theorem 1. ■

Remark 4: The dynamic surface control approaches in Yang et al. (2020a); Song et al. (2020) remove the explosion of complexity drawback by introducing the first-order fractional filter (FOFF) as follows

$$\emptyset_k {}_{t_0}^{C}D_t^q \xi_k + \xi_k = \varpi_k, \quad k = 2, \dots, n. \tag{56}$$

where $0 < \emptyset_k < 1$ is a design parameter. Similar to Swaroop et al. (2000), the presented filter in Yang et al. (2020a); Song et al. (2020) is highly sensitive to $\emptyset_k$ and only semi-global stability is ensured. However, in the proposed approach, FOD is applied to eliminate the explosion of complexity drawback. Compared to the FOFF, the main characteristics of the applied FOD is fast convergence and good filtering precision. Therefore, in this article the global tracking stability with high accuracy is assured than Yang et al. (2020a); Song et al. (2020).

4.2. Static event-triggered fault compensation approach

The proposed fault compensation structure with a static ETC strategy is combined to avoid unnecessary energy consumption caused by updating the actuators and transmitting the signals in continuous time. For simplicity, because all the virtual control laws are updated in the time-trigger manner, only the actual control design in the final step is discussed.

To design the fault compensation static ETC, the following two auxiliary signals are proposed as

$$\tau_e^{i(s)} = \frac{\hat{\wp}^i \varpi_{n+1}}{B^i(\bar{x}_n)}, \quad i = 1, \dots, \mathcal{S}, \tag{57}$$

$$\breve{\tau}^{i(s)} = -(1+\aleph^i)\Big(\tau_e^{i(s)} \tanh\Big(\frac{\tau_e^{i(s)} B^i(\bar{x}_n)\pi_n}{\sigma^i}\Big) + \zeta_1^i \tanh\Big(\frac{\zeta_1^i B^i(\bar{x}_n)\pi_n}{\sigma^i}\Big)\Big), \quad i = 1, \dots, \mathcal{S}, \tag{58}$$

where $0 < \aleph^i < 1$, $\sigma^i > 0$, and $\zeta_1^i > 0$ are the design parameters, π_n, ϖ_{n+1}, and $\hat{\wp}^i$ are given in Eqs. (15), (44), and (48), respectively.

For the i-th actuator, a control input and static event-triggering rule are respectively designed as

$$\tau^{i(s)} = \breve{\tau}^{i(s)}(t_j^{i(s)}), \quad \forall t \in [t_j^{i(s)}, t_{j+1}^{i(s)}), \tag{59}$$

$$t_{j+1}^{i(s)} = \inf\Big\{t \in \mathbb{R}^{>0} \,\Big|\, |\alpha^{i(s)}| - \aleph^i |\tau^{i(s)}| - \zeta_2^i \geq 0\Big\}, \tag{60}$$

where $\alpha^{i(s)} = \breve{\tau}^{i(s)} - \tau^{i(s)}$ is the measurement error, $\tau^{i(s)}$ is the control input, and inequality $0 < \zeta_2^i < \zeta_1^i(1-\aleph^i)$ for design parameter ζ_2^i is satisfied. $t_j^{i(s)}$ ($j \in \mathbb{Z}^{\geq 0}$ and $t_0^{i(s)} = 0$) is an event-triggered time moment for i-th actuator in the proposed static ETC method which satisfies $t_0^{i(s)} < t_1^{i(s)} < t_2^{i(s)} < \dots < t_j^{i(s)} < \dots$, and $\lim_{j\to+\infty} t_j^{i(s)} = +\infty$.

Remark 5: It is important to note that the i-th actuator output ($\tau^{i(s)}$) is updated according to the current sampled auxiliary signal ($\breve{\tau}^{i(s)}$) whenever its static event-triggering rule in Eq. (60) is satisfied which is applicable for digital implementation. From the measurement error signal, it can be deduced that for every triggering time moment ($t = t_j^{i(s)}$), this signal is equal zero, and for the time interval $t \in [t_j^{i(s)}, t_{j+1}^{i(s)})$, each actuator output holds a last transmitted control signal as a piecewise constant, (i.e., $\tau^{i(s)} = \breve{\tau}^{i(s)}(t_j^{i(s)})$). Thus, the transmission load between the controller and the actuators (also the actuators' execute) are effectively reduced.

4.2.1. *Stability analysis of static ETC*

The main results of the static ETC for the fractional-order networked systems are presented as follows.

Theorem 2: Let Assumptions 1-3 are satisfied. Consider the closed-loop fractional-order networked system (12) subject to the actuator fault (13) with the actual control input (59), the virtual laws (19), (31), and (44); then, under the proposed static event-triggering rule (60) with the design parameters $0 < \aleph^i < 1$, $\zeta_1^i > 0$, and $0 < \zeta_2^i < \zeta_1^i(1-\aleph^i)$ the following hold

- All signals of the closed-loop networked system are globally uniformly ultimately bounded.
- The tracking error ultimately converges into the neighborhood of the origin with adjustable size as follows

$$\Pi^s = \Big\{\pi_1(t) \in \mathbb{R} \Big| \mathrm{Lim}_{t\to+\infty} |\pi_1(t)|^2 \leq 2\frac{\breve{\varrho} d}{\beta}\Big\}.$$

- The Zeno phenomenon is strictly excluded, (i.e., $t_{j+1}^{(s)} - t_j^{(s)} \geq t^{*(s)} > 0, \ \forall j \in \mathbb{N}$).

Proof: Considering $V = \sum_{m=1}^{n} V_m$ together with (27), (39), (42), and applying Lemma 5, the following is obtained

$$\begin{aligned}{}_{t_0}^{C}D_t^q V \leq& \pi_n\left(\sum_{i=1}^{S} B^i(\bar{x}_n)(1-\varsigma^i)\vartheta^i\tau^{i(s)} + \hbar + w_n^{*\mathrm{T}}\mu_n(\bar{x}_n) + \rho_n(\bar{x}_n) + d_n - \eta_{n_2} - \varepsilon_{n_2}\right) - (c_1-0.5)\pi_1^2 - \sum_{m=2}^{n-1}(c_m-1)\pi_m^2 \\ &+ 0.5\pi_n^2 - 0.5\sum_{m=1}^{n-1}\left(r_{w_m}||\tilde{w}_m||^2 + r_{\varphi_m}\tilde{\varphi}_m^2 + r_{\psi_m}\tilde{\psi}_m^2\right) + 0.5\sum_{m=1}^{n-1}\left(r_{w_m}||w_m^*||^2 + r_{\varphi_m}\varphi_m^{*2} + r_{\psi_m}\psi_m^{*2}\right) \\ &+ \sum_{m=1}^{n-1} p_m - \tilde{w}_n^{\mathrm{T}}\Gamma_{w_n}^{-1}\, {}_{0}^{C}D_t^q\hat{w}_n - \frac{1}{\gamma_{\varphi_n}}\tilde{\varphi}_n\, {}_{t_0}^{C}D_t^q\hat{\varphi}_n - \frac{1}{\gamma_{\psi_n}}\tilde{\psi}_n\, {}_{t_0}^{C}D_t^q\hat{\psi}_n - \sum_{i=1}^{S}\frac{\vartheta^i}{\gamma_{\wp}^i}(1-\varsigma^i)\tilde{\wp}^i\, {}_{t_0}^{C}D_t^q\hat{\wp}^i.\end{aligned} \quad (61)$$

According to (Xing et al. (2016)) and Eq. (60), for any $t \in [t_j^{i(s)}, t_{j+1}^{i(s)})$, it can be deduced that there exists $|\lambda_p^i| \leq 1$ for $p = 1, 2$ such that $\breve{\tau}^{i(s)} = (1 + \aleph^i\lambda_1^i)\tau^{i(s)} + \lambda_2^i\zeta_2^i$. Therefore, it can easily be obtained that $\tau^{i(s)} = \frac{\breve{\tau}^{i(s)}}{1+\aleph^i\lambda_1^i} - \frac{\lambda_2^i\zeta_2^i}{1+\aleph^i\lambda_1^i}$. Hence, it results in from Eq. (58) that

$$\tau^{i(s)} = -\frac{(1+\aleph^i)}{1+\aleph^i\lambda_1^i}\left(\tau_e^{i(s)}\tanh\left(\frac{\tau_e^{i(s)}B^i(\bar{x}_n)\pi_n}{\sigma^i}\right) + \zeta_1^i\tanh\left(\frac{\zeta_1^i B^i(\bar{x}_n)\pi_n}{\sigma^i}\right)\right) - \frac{\lambda_2^i\zeta_2^i}{1+\aleph^i\lambda_1^i}. \quad (62)$$

Then, Eq. (63) is yielded from (62) as follows

$$B^i(\bar{x}_n)\pi_n\vartheta^i\tau^{i(s)} \leq -B^i(\bar{x}_n)\pi_n\vartheta^i\left(\tau_e^{i(s)}\tanh\left(\frac{\tau_e^{i(s)}B^i(\bar{x}_n)\pi_n}{\sigma^i}\right) + \zeta_1^i\tanh\left(\frac{\zeta_1^i B^i(\bar{x}_n)\pi_n}{\sigma^i}\right)\right) + B^i(\bar{x}_n)\vartheta^i|\pi_n|\frac{\zeta_2^i}{1-\aleph^i}. \quad (63)$$

According to Lemma 4, the following inequalities hold true

$$-\pi_n B^i(\bar{x}_n)\vartheta^i\zeta_1^i\tanh\left(\frac{\zeta_1^i B^i(\bar{x}_n)\pi_n}{\sigma^i}\right) \leq -B^i(\bar{x}_n)\vartheta^i\zeta_1^i|\pi_n| + \upsilon\vartheta^i\sigma^i, \quad (64)$$

$$-\pi_n B^i(\bar{x}_n)\vartheta^i\tau_e^{i(s)}\tanh\left(\frac{\tau_e^{i(s)}B^i(\bar{x}_n)\pi_n}{\sigma^i}\right) \leq B^i(\bar{x}_n)\vartheta^i\pi_n\tau_e^{i(s)} + \upsilon\vartheta^i\sigma^i. \quad (65)$$

Substituting Eqs. (64) and (65) into (63), with respect to (57), and recalling definition $\hat{\wp}^i$, the following inequality is yielded

$$\pi_n\sum_{i=1}^{S} B^i(\bar{x}_n)(1-\varsigma^i)\vartheta^i\tau^{i(s)} \leq \pi_n\varpi_{n+1} - \pi_n\varpi_{n+1}\sum_{i=1}^{S}\vartheta^i(1-\varsigma^i)\tilde{\wp}^i + 2\upsilon\sum_{i=1}^{S}(1-\varsigma^i)\vartheta^i\sigma^i. \quad (66)$$

Combining Eqs. (61), (66), (44)-(48) and after a simple calculation similar to (51), Eq. (67) is obtained

$$\begin{aligned}{}_{t_0}^{C}D_t^q V \leq& -(c_1-0.5)\pi_1^2 - \sum_{m=2}^{n-1}(c_m-1)\pi_m^2 - (c_n-0.5)\pi_n^2 - \frac{1}{2}\sum_{m=1}^{n}\left(r_{w_m}||\tilde{w}_m||^2 + r_{\varphi_m}\tilde{\varphi}_m^2 + r_{\psi_m}\tilde{\psi}_m^2\right) \\ &- \frac{1}{2}\sum_{i=1}^{S} r_{\wp}^i(1-\varsigma^i)\vartheta^i(\tilde{\wp}^i)^2 + \frac{1}{2}\sum_{i=1}^{S} r_{\wp}^i(1-\varsigma^i)\vartheta^i(\wp^{i*})^2 + \frac{1}{2}\sum_{m=1}^{n}\left(r_{w_m}||w_m^*||^2 + r_{\varphi_m}\varphi_m^{*2} + r_{\psi_m}\psi_m^{*2}\right) \\ &+ \sum_{m=1}^{n} p_m + 2\upsilon\sum_{i=1}^{S}(1-\varsigma^i)\vartheta^i\sigma^i.\end{aligned} \quad (67)$$

According to Eq. (67), the following inequality holds true

$$ {}_{t_0}^{C}D_t^{q} V \leq -\beta V + \breve{\varrho}, \tag{68} $$

where $\breve{\varrho} = \varrho + 2\upsilon \sum_{i=1}^{s}(1-\varsigma^i)\vartheta^i\sigma^i$, ϱ and β are defined under Eq. (55). From (68), Lemmas 1 and 3, one can obtain $V(t) \leq \frac{V(t_0)\Xi}{1+|\beta(t-t_0)^q|} + \frac{\breve{\varrho}d}{\beta}$. Hence, the first statement of the Theorem 2 directly follows from Eq. (68). According to the definition of the Lyapunov function, $\Pi^s = \left\{ \pi_1(t) \in \mathbb{R} \middle| \mathrm{Lim}_{t\to+\infty}|\pi_1(t)|^2 \leq 2\frac{\breve{\varrho}d}{\beta} \right\}$ is obtained. Hence, the tracking error remains in the adjustable compact set including origin. The second statement of the Theorem 2 is now assured.

For $\forall t \in [t_j^{i(s)}, t_{j+1}^{i(s)})$, the $|\alpha^{i(s)}| = |\breve{\tau}^{i(s)} - \breve{\tau}^{i(s)}(t_j^{i(s)})| = |{}_{t_j^{i(s)}}^{C}D_t^{-q}\,{}_{t_j^{i(s)}}^{C}D_t^{q}\breve{\tau}^{i(s)}|$ can be obtained. Thus, one has

$$ |\alpha^{i(s)}| \leq \frac{1}{\Gamma(q)} \int_{t_j^{i(s)}}^{t} (t-p)^{q-1} |{}_{t_j^{i(s)}}^{C}D_t^{q}\breve{\tau}^{i(s)}(p)| \mathrm{d}p. \tag{69} $$

According to (Wang, Wen, Gou, Ye and Chen (2017a)), ${}_{t_j^{i(s)}}^{C}D_s^{q} g(h(s))$ can be approximated by $\frac{\partial}{\partial h} g(h)\,{}_{t_j^{i(s)}}^{C}D_s^{q} h(s)$ with the error approximation U which is bounded by $\bar{U}$. Thus, ${}_{t_j^{i(s)}}^{C}D_t^{q}\breve{\tau}^{i(s)} = A + U$, where $A = \sum_{a=1}^{n}\Big((\partial\breve{\tau}^{i(s)}/\partial x_a)\,{}_{t_j^{i(s)}}^{C}D_t^{q} x_a + (\partial\breve{\tau}^{i(s)}/\partial \eta_{a_2})\,{}_{t_j^{i(s)}}^{C}D_t^{q}\eta_{a_2} + (\partial\breve{\tau}^{i(s)}/\partial\hat{\varphi}_a)\,{}_{t_j^{i(s)}}^{C}D_t^{q}\hat{\varphi}_a + (\partial\breve{\tau}^{i(s)}/\partial\hat{\psi}_a)\,{}_{t_j^{i(s)}}^{C}D_t^{q}\hat{\psi}_a + (\partial\breve{\tau}^{i(s)}/\partial\hat{w}_a)\,{}_{t_j^{i(s)}}^{C}D_t^{q}\hat{w}_a \Big) + (\partial\breve{\tau}^{i(s)}/\partial\hat{\wp}^i)\,{}_{t_j^{i(s)}}^{C}D_t^{q}\hat{\wp}^i + (\partial\breve{\tau}^{i(s)}/\partial x_d)\,{}_{t_j^{i(s)}}^{C}D_t^{q} x_d$. Because A and U are bounded signals, the $|\,{}_{t_j^{i(s)}}^{C}D_t^{q}\breve{\tau}^{i(s)}\,| \leq \Im^i$ is satisfied, where $\Im^i$ is an unknown parameter. Thus, from (69) the following inequality for $i = 1, \ldots, \mathcal{S}$ is satisfied

$$ |\alpha^{i(s)}| \leq \frac{\Im^i}{\Gamma(q+1)} (t - t_j^{i(s)})^q, \quad \forall t \in [t_j^{i(s)}, t_{j+1}^{i(s)}). \tag{70} $$

Based on the event-triggering rule (60), the next event cannot occur before the $t_{j+1}^{i(s)}$ moment, which satisfies $|\alpha^{i(s)}(t_{j+1}^{i(s)})| = \aleph^i|\tau^{i(s)}(t_{j+1}^{i(s)})| + \zeta_2^i$. By considering this fact and Eq. (70), the following is yielded

$$ \aleph^i|\tau^{i(s)}(t_{j+1}^{i(s)})| + \zeta_2^i \leq \frac{\Im^i}{\Gamma(q+1)} (t_{j+1}^{i(s)} - t_j^{i(s)})^q. \tag{71} $$

From Eq. (71), $t_{j+1}^{i(s)} - t_j^{i(s)} \geq \left(\frac{\Gamma(q+1)(\zeta_2^i + \aleph^i|\tau^{i(s)}(t_{j+1}^{i(s)})|)}{\Im^i} \right)^{-q} > 0$ is obtained, and the Zeno behavior is strictly alleviated. Following this, the third statement of the Theorem 2 is established, and finally this Theorem is completely proved. ■

4.3. Dynamic event-triggered fault compensation approach

In this subsection, a dynamic ETC strategy in the framework of adaptive fault compensation is assessed for the networked fractional-order systems. Hereafter for simplicity, only the actual dynamic ETC design and the closed-loop stability are of concern.

It is observed in Eq. (60) that the proposed static event-triggering rule includes some fix design parameters, which should be adjusted in an off-line manner by an expert designer. The tuning of these parameters may lead to undesirable performance on the static ETC, (e.g., some unnecessary data transmission from the controller to the actuators and updates of the actuators), particularly whenever the tracking performance is approaching. To overcome this drawback, the dynamic ETC mechanism with an internal dynamical variable is introduced.

For designing the fault compensation dynamic ETC, the following are proposed

$$\tau_e^{i(d)} = \frac{\hat{\wp}^i \varpi_{n+1}}{B^i(\bar{x}_n)}, \quad i = 1, \ldots, \mathcal{S} \tag{72}$$

$$\breve{\tau}^{i(d)} = -(1+\aleph^i)\Big(\tau_e^{i(d)} \tanh\Big(\frac{\tau_e^{i(d)} B^i(\bar{x}_n)\pi_n}{\sigma^i}\Big) + \zeta_1^i \tanh\Big(\frac{\zeta_1^i B^i(\bar{x}_n)\pi_n}{\sigma^i}\Big) + \zeta_3^i \eta^i \tanh\Big(\frac{\zeta_3^i B^i(\bar{x}_n)\eta^i \pi_n}{\sigma^i}\Big)\Big), \tag{73}$$

where ζ_3^i is a positive design parameter, ϖ_{n+1}, $\hat{\wp}^i$, and $\aleph^i$, ζ_1^i, σ^i are given in Eqs. (44), (48), and (58), respectively.

At this stage, the control input and dynamic event-triggering rule for i-th actuator are respectively proposed as

$$\tau^{i(d)} = \breve{\tau}^{i(d)}(t_j^{i(d)}), \ \ \forall t \in [t_j^{i(d)}, t_{j+1}^{i(d)}), \tag{74}$$

$$t_{j+1}^{i(d)} = \inf\Big\{ t \in \mathbb{R}^{>0} \ \Big| \ |\alpha^{i(d)}| - \aleph^i |\tau^{i(d)}| - \zeta_2^i - \zeta_4^i \eta^i \geq 0 \Big\}, \tag{75}$$

where $\alpha^{i(d)} = \breve{\tau}^{i(d)} - \tau^{i(d)}$ is the measurement error, $\tau^{i(d)}$ is the control input, and inequalities $0 < \zeta_2^i < \zeta_1^i(1-\aleph^i)$ and $0 < \zeta_4^i < \zeta_3^i(1-\aleph^i)$ for design parameters ζ_2^i and ζ_4^i are respectively satisfied. $t_j^{i(d)}$ ($j \in \mathbb{Z}^{\geq 0}$ and $t_0^{i(d)} = 0$) is an event-triggered time moment for i-th actuator in the proposed dynamic ETC which satisfies $t_0^{i(d)} < t_1^{i(d)} < t_2^{i(d)} < \ldots < t_j^{i(d)} < \ldots$, and $\lim_{j\to+\infty} t_j^{i(d)} = +\infty$.

Choose an internal dynamic variable η^i as follows

$${}_{t_0}^{C}D_t^q \eta^i = \omega^i\Big(\aleph^i|\tau^{i(d)}| + \zeta_2^i - |\alpha^{i(d)}|\Big) - \xi^i \eta^i, \qquad \eta^i(t_0) \geq 0, \tag{76}$$

where ω^i and ξ^i are the two positive design parameters.

Remark 6: The internal dynamic variable in Eq. (75) is regulated by the proposed fractional-order differential equation in Eq. (76) which contains the actuator output, the measurement error, some design parameters, and negative self-feedback. The applied negative self-feedback is contributive to assuring the boundedness of the dynamical variable (for more detail, see Lemma 6). Unlike the proposed static event-triggering rule in (60), using η^i in (75) has a significant role on tuning the threshold dynamically. Note that the count of actuators' updates is effectively reduced by applying η^i in the proposed ETC method.

Remark 7: In Eq. (76), if $\omega^i = 0$ is selected, the result will be $\eta^i = \eta^i(t_0)M^i_{(q,1)}(-\xi^i t^q)$. Then, the proposed event-triggering rule (75) is turned into the static ETC case, similar to Li et al. (2018b). In Eq. (75), if ζ_4^i is selected as zero, then the proposed static ETC rule in (60) is obtained; consequently, the static even-triggering rules in Eq. (60) and Li et al. (2018b) can be observed as limit cases of this proposed dynamic even-triggering rule.

Definition 2: The system (76) is input-to-state practically stable (ISpS) with respect to the control input $\tau^{i(d)}$, if for $\eta^i(t_0) \in L_\infty$ and $\tau^{i(d)} \in L_\infty$ these exist $\gamma^i(\cdot) \in \mathcal{K}_\infty$, $\beta^i(\cdot) \in \mathcal{KL}$, and $m^i > 0$ such that

$$|\eta^i| \leq \beta(|\eta^i(t_0)|, t - t_0) + \gamma^i(|\tau^{i(d)}|_\infty) + m^i. \tag{77}$$

The definitions $\mathcal{K}_\infty$ and $\mathcal{KL}$ functions are given in Khalil and Grizzle (2002).

Assumption 4: The dynamical variable is ISpS; i.e., the system (76) has an ISpS Lyapunov function $V^i(\eta^i)$ which satisfies

$$\grave{a}^i(|\eta^i)|) \leq V^i(\eta^i) \leq \acute{a}^i(|\eta^i|), \tag{78}$$

$$ {}_{t_0}^{C}D_t^q V^i(\eta^i) \leq -a^i V^i(\eta^i) + \gamma^i(|\tau^{i(d)}|) + m^i, \tag{79}$$

where $\acute{a}^i(\cdot)$, $\grave{a}^i(\cdot)$, and $\gamma^i(\cdot)$ are $\mathcal{K}_\infty$ functions, $a^i > 0$ and $m^i > 0$ are known parameters.

Remark 8: The dynamical variable system in Eq. (76) is assumed to be ISpS with the input $\tau^{i(d)}$ in Assumption 4. Although Eq. (79) in Assumption 4 may be a restrictive condition, it is satisfied in many real applications due to control inputs should have reasonable bounded amplitudes. The similar assumptions can be seen in Girard (2014); Sahoo, Xu and Jagannathan (2015); Szanto, Narayanan and Jagannathan (2016); Postoyan, Tabuada, Nešić and Anta (2014).

Lemma 6: For the fractional-order differential equation (76) with $\eta^i(t_0) \geq 0$, the following holds true

$$\Omega_{\eta^i} = \left\{ \eta^i(t) \in \mathbb{R}^{\geq 0} \,\middle|\, \eta^i(t) \leq \frac{\omega^i(\zeta_2^i + \aleph^i \sup_{t_0\leq\iota\leq t}|\tau^{i(d)}(\iota)|) + \eta^i(t_0)}{\xi^i} \right\}. \tag{80}$$

Proof: According to Eq. (75), for all $t \in (t_j^{i(d)}, t_{j+1}^{i(d)})$, the inequality $|\alpha^{i(d)}| - \aleph^i|\tau^{i(d)}| - \zeta_2^i - \zeta_4^i\eta^i < 0$ is obtained, which implies that

$$ {}_{t_0}^{C}D_t^q \eta^i > -(\omega^i\zeta_4^i + \xi^i)\eta^i. \tag{81}$$

Moreover, at $\forall t = t_{j+1}^{i(d)}$, the relation $|\alpha^{i(d)}| - \aleph^i|\tau^{i(d)}| - \zeta_2^i - \zeta_4^i\eta^i = 0$ is obtained, therefore, Eq. (82) is yielded

$$ {}_{t_0}^{C}D_t^q \eta^i = -(\omega^i\zeta_4^i + \xi^i)\eta^i. \tag{82}$$

According to definition of the Mittag-Leffler function in (3), and (81) and (82), the $\eta^i \geq 0$ on $t \in [t_0, +\infty)$ is obtained.

On the other hand, let $\eta^i(t) > \frac{\omega^i(\zeta_2^i + \aleph^i \sup_{t_0\leq\iota\leq t}|\tau^{i(d)}(\iota)|) + \eta^i(t_0)}{\xi^i}$. Therefore, the following is obtained

$$ {}_{t_0}^{C}D_t^q \eta < -\omega^i\left(|\alpha^{i(d)}| - \aleph^i|\tau^{i(d)}| + \aleph^i \sup_{t_0\leq\iota\leq t}|\tau^{i(d)}(\iota)|\right) - \eta^i(t_0), \tag{83}$$

and as a result $\eta^i < 0$, which contracts the fact that $\eta^i > 0$. Hence, the above-mentioned supposition is not satisfied, and it would lead to $0 \leq \eta^i(t) \leq \frac{\omega^i(\zeta_2^i + \aleph^i \sup_{t_0\leq\iota\leq t}|\tau^{i(d)}(\iota)|)}{\xi^i}$ on $t \in [t_0, +\infty)$. Thus, the $\eta^i(t)$ in Eq. (76) is an ISpS variable with respect to the $\tau^{i(d)}(t)$. The proof is completed.

4.3.1. *Stability analysis of dynamic ETC*

The results concerning the dynamic ETC of the fractional-order networked systems are addressed as follows.

Theorem 3: Let Assumptions 1-4 are satisfied. Consider the fractional-order system (12) subject to the actuator fault (13) with the virtual control laws (19), (31), (44), and the internal dynamic variable (76). Then, under the dynamic event-triggering rule (75) and the actual control input (74) with design parameters $0 < \aleph^i < 1$, $\omega^i > 0$, $\xi^i > 0$, $\sigma^i > 0$, $0 < \zeta_2^i < \zeta_1^i(1 - \aleph^i)$, and $0 < \zeta_4^i < \zeta_3^i(1 - \aleph^i)$, the following hold

- The overall closed-loop networked system is globally uniformly ultimately bounded.
- The tracking error ultimately converges into the neighborhood of the origin with adjustable size as follows

$$\Pi^d = \left\{ \pi_1(t) \in \mathbb{R} \middle| \mathrm{Lim}_{t\to+\infty}|\pi_1(t)|^2 \leq 2\frac{\bar{\varrho}d}{\beta} \right\}.$$

- The Zeno behavior is strictly excluded, (i.e., $t_{j+1}^{(d)} - t_j^{(d)} \geq t^{*(d)} > 0, \forall j \in \mathbb{N}$).

Proof: For any $t \in [t_j^{i(d)}, t_{j+1}^{i(d)})$, similar to (Xing et al. (2016)), it can be deduced from Eq. (75) that there exists $|\lambda_p^i| \leq 1$ for $p = 1, 2, 3$ such that $\breve{\tau}^{i(d)} = (1 + \aleph^i \lambda_1^i)\tau^{i(d)} + \lambda_2^i \zeta_2^i + \zeta_4^i \lambda_3^i \eta^i$. Therefore, by applying (73), Eq. (84) is yielded

$$\tau^{i(d)} = -\frac{(1+\aleph^i)}{1+\aleph^i\lambda_1^i}\Big(\tau_e^{i(d)}\tanh\Big(\frac{\tau_e^{i(d)}B^i(\bar{x}_n)\pi_n}{\sigma^i}\Big) + \zeta_1^i\tanh\Big(\frac{\zeta_1^iB^i(\bar{x}_n)\pi_n}{\sigma^i}\Big) + \zeta_3^i\eta^i\tanh\Big(\frac{\zeta_3^iB^i(\bar{x}_n)\eta^i\pi_n}{\sigma^i}\Big)\Big) - \frac{\zeta_2^i\lambda_2^i}{1+\aleph^i\lambda_1^i} - \frac{\zeta_4^i\lambda_3^i\eta^i}{1+\aleph^i\lambda_1^i}. \tag{84}$$

Similar to (63), the following is obtained

$$B^i(\bar{x}_n)\pi_n\vartheta^i\tau^{i(d)} \leq -B^i(\bar{x}_n)\pi_n\vartheta^i\Big(\tau_e^{i(d)}\tanh\Big(\frac{\tau_e^{i(d)}B^i(\bar{x}_n)\pi_n}{\sigma^i}\Big) + \zeta_1^i\tanh\Big(\frac{\zeta_1^iB^i(\bar{x}_n)\pi_n}{\sigma^i}\Big) + \zeta_3^i\eta^i\tanh\Big(\frac{\zeta_3^i\eta^iB^i(\bar{x}_n)\pi_n}{\sigma^i}\Big)\Big) + B^i(\bar{x}_n)\vartheta^i|\pi_n|\frac{\zeta_2^i}{1-\aleph^i} + B^i(\bar{x}_n)\vartheta^i\eta^i|\pi_n|\frac{\zeta_4^i}{1-\aleph^i}. \tag{85}$$

From Lemma 4, the following inequalities are yielded

$$-\pi_nB^i(\bar{x}_n)\vartheta^i\zeta_1^i\tanh\Big(\frac{\zeta_1^iB^i(\bar{x}_n)\pi_n^i}{\sigma^i}\Big) \leq -B^i(\bar{x}_n)\vartheta^i\zeta_1^i|\pi_n| + \upsilon\vartheta^i\sigma^i, \tag{86}$$

$$-\pi_nB^i(\bar{x}_n)\vartheta^i\tau_e^{i(d)}\tanh\Big(\frac{\tau_e^{i(d)}B^i(\bar{x}_n)\pi_n}{\sigma^i}\Big) \leq B^i(\bar{x}_n)\vartheta^i\pi_n\tau_e^{i(d)} + \upsilon\vartheta^i\sigma^i, \tag{87}$$

$$-\pi_nB^i(\bar{x}_n)\vartheta^i\zeta_3^i\eta^i\tanh\Big(\frac{\zeta_3^iB^i(\bar{x}_n)\eta^i\pi_n}{\sigma^i}\Big) \leq -B^i(\bar{x}_n)\vartheta^i\zeta_3^i\eta^i|\pi_n| + \upsilon\vartheta^i\sigma^i. \tag{88}$$

From (85)-(88), (72), with respect to the Theorem 3, and recalling definition $\hat{\wp}$, the following inequality is obtained

$$\pi_n\sum_{i=1}^{\mathcal{S}}B^i(\bar{x}_n)(1-\varsigma^i)\vartheta^i\tau^{i(d)} \leq \pi_n\varpi_{n+1} - \pi_n\varpi_{n+1}\sum_{i=1}^{\mathcal{S}}(1-\varsigma^i)\vartheta^i\tilde{\wp}^i + 3\upsilon\sum_{i=1}^{\mathcal{S}}(1-\varsigma^i)\vartheta^i\sigma^i. \tag{89}$$

Combining Eqs. (61), (89), (44)-(48), and after a simple calculation similar to Eq. (51), the following is yielded

$$\begin{aligned}{}_{t_0}^{C}D_t^qV \leq& -(c_1-0.5)\pi_1^2 - \sum_{m=2}^{n-1}(c_m-1)\pi_m^2 - (c_n-0.5)\pi_n^2 - \frac{1}{2}\sum_{m=1}^{n}\Big(r_{w_m}||\tilde{w}_m||^2 + r_{\varphi_m}\tilde{\varphi}_m^2 + r_{\psi_m}\tilde{\psi}_m^2\Big)\\ &- \frac{1}{2}\sum_{i=1}^{\mathcal{S}}r_{\wp}^i(1-\varsigma^i)\vartheta^i(\tilde{\wp}^i)^2 + \frac{1}{2}\sum_{m=1}^{n}\Big(r_{w_m}||w_m^*||^2 + r_{\varphi_m}\varphi_m^{*2} + r_{\psi_m}\psi_m^{*2}\Big)\\ &+ \frac{1}{2}\sum_{i=1}^{\mathcal{S}}r_{\wp}^i(1-\varsigma^i)\vartheta^i(\wp^{*i})^2 + \sum_{m=1}^{n}p_m + 3\upsilon\sum_{i=1}^{\mathcal{S}}(1-\varsigma^i)\vartheta^i\sigma^i.\end{aligned} \tag{90}$$

From Eq. (90), the following holds true

$$ {}_{t_0}^{C}D_t^q V \leq -\beta V + \bar{\varrho}, \tag{91} $$

where $\bar{\varrho} = \breve{\varrho} + \upsilon \sum_{i=1}^{S}(1-\varsigma^i)\vartheta^i\sigma^i$, β and $\breve{\varrho}$ are defined under Eq. (68). From Eq. (91), Lemmas 1 and 3, the $V(t) \leq \frac{V(t_0)\Xi}{1+|\beta(t-t_0)^q|} + \frac{\bar{\varrho}d}{\beta}$ is obtained. Hence, the first objective of the Theorem 3 is established. According to the obtained upper bound of the Lyapunov function, it can be deduced that the tracking error ultimately converges into a compact set $\Pi^d = \left\{ \pi_1(t) \in \mathbb{R} \middle| \mathrm{Lim}_{t\to+\infty} |\pi_1(t)|^2 \leq 2\frac{\bar{\varrho}d}{\beta} \right\}$. Now, the second objective of the Theorem 3 is proved.

To assure that the proposed dynamic ETC is feasible, one has $t_{j+1}^{i(d)} - t_j^{i(d)} \geq \left(\frac{\Gamma(q+1)(\zeta_2^i + \aleph^i|\tau^{i(d)}(t_{j+1}^{i(d)})|) + \zeta_4^i \eta^i(t_{j+1}^{i(d)}))}{\mathfrak{I}^i} \right)^{-q} > 0$ in a similar way of the static ETC as in the previous subsection. The details of this proof are omitted here. Hence, the proof of the Theorem 3 is wholly given. ■

Remark 9: The sizes of the ultimate bounds for the tracking errors subject to the proposed static and dynamic ETC methods (Π^s and Π^d, respectively) can be reduced by increasing the values of $\aleph^i$, ζ_1^i, ζ_3^i, control gains c_m, adaptive gains $\lambda_{\min}(\Gamma_{w_m}), \gamma_{\psi_m}, \gamma_{\varphi_m}$, and decreasing σ^i and sigma-modification factors $r_{w_m}, r_{\psi_m}, r_{\varphi_m}$. Higher tracking accuracy may require large control amplitude and short period of control updates, which would increase number of actuator executes; consequently, there should be a trade-off between the tracking accuracy, control input amplitude, and the number of actuator executes.

Remark 10: In comparison with the available results, the main additive value and conservatism of these proposed methods are as follows

Additive value: The Zeno-free static and dynamic event-triggered fault compensation control approaches of the networked fractional-order nonlinear uncertain systems are proposed for the first time, in a sense that for the closed-loop networked system these methods assure the global stability and save the energy resources.

Conservatism: In this article, all the pseudo state variables of the fractional-order systems are measurable, and the uniform ultimate bound stability is obtained, while, in many real applications, they are not measurable, and also tracking with high accuracy is required. As one of the future studies, assessing the asymptotic observer-based event-triggered tracking control for the fractional-order nonlinear systems would be an interesting subject.

The implementation of the dynamic ETC fault compensation algorithm for the fractional-order systems is briefed as Algorithm 1. Note, if $\zeta_3^i = 0$ and $\zeta_4^i = 0$ are set in this algorithm, it would be converted into the static ETC fault compensation algorithm.

5. Simulation results

In this section, the control performance and efficiency of the proposed static and dynamic event-triggered fault

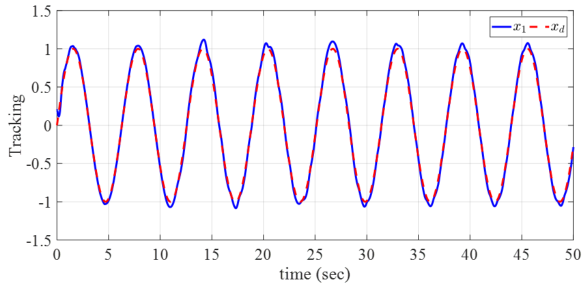

Figure 1: System output (solid) and desired signal (dash).

Algorithm 1. Control algorithm of fractional-order systems via dynamic event-triggered strategy

Require:

Set $t_0 = 0$, $j^{i(d)} = 0$ and $\alpha^{i(d)}(t_0) = 0$, $i = 1, \dots, \mathcal{S}$.
Initialize all design constants, such ω^i, ξ^i, ζ_1^i, ζ_2^i, $\aleph^i$, and so on.
Set initial states, such $u(t_0)$, $x_m(t_0)$, $m = 1, \dots, n$, $\eta^i(t_0)$ and so on.

Ensure:

for t **do** $t_0^{\dagger} + dt : dt : t_{\text{final}}$
  Update desired trajectory $x_d(t)$.
  for m **do** $1, \dots, n$
    Compute error surface $\pi_m(t)$ by (15).
    Compute virtual and adaptive laws $\varpi_m(t)$ and $\hat{\omega}_m(t)$, $\hat{\varphi}_m(t)$, $\hat{\psi}_m(t)$ by (19), (31), (44) and (20)-(22), (32)-(34), (45)-(47), respectively.
      for i **do** $1 : 1 : \mathcal{S}$
        Compute fault compensator law $\wp^i(t)$ and internal dynamic variable $\eta^i(t)$ by (48) and (76).
        Compute $\tau_e^{i(d)}(t)$ and $\breve{\tau}^{i(d)}(t)$ by (72) and (73).
        Compute $\alpha^{i(d)}(t)$ and relative threshold.
        if $\alpha^{i(d)}(t)$ exceeds the relative threshold **then**
          Update $\tau^{i(d)}(t)$ by (74).
          Update $\breve{\tau}^{i(d)}(t_j^{i(d)}) = \tau^{i(d)}(t)$, $j^{i(d)} = j^{i+1(d)}$, and $\alpha^{i(d)}(t_j^{i(d)}) = 0$
          Update $x(t)$ at $t_{j+1}^{i(d)}$ by $\tau^{i(d)}(t)$.
        else
          Update $x(t)$ at $t_{j+1}^{i(d)}$ by $\tau^{i(d)}(t)$ which is got by zero order hold.
        end if
      end for
  end for
end for

[†] for $t = t_0$, system states and actuators are updated by $u^i(t_0)$.

compensation control approaches are demonstrated by the following simulation example.

The model of fractional-order uncertain strict-feedback system suffering from the actuator faults is

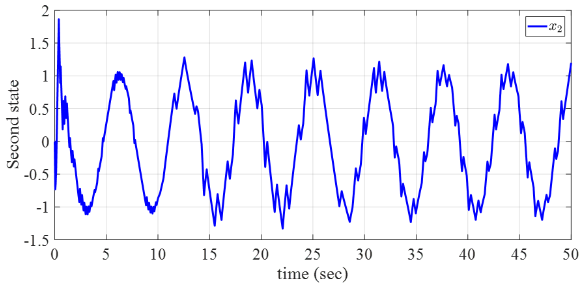

Figure 2: Pseudo state.

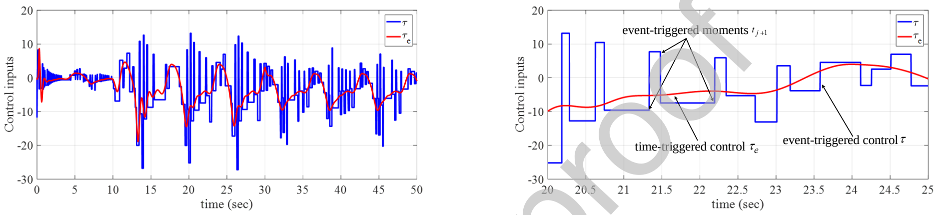

Figure 3: Control input.

Table 1. Tracking performance for different strategies.

Control Schemes	RMS	IAE	TA
TTC (Li et al. (2021))	1.731	6.590	0.138
Static ETC	1.565	6.442	0.092
Dynamic ETC	1.490	6.190	0.095

described as follows

$$\begin{cases} {}_0^C D_t^{0.95} x_1 = x_2 + f_1(\bar{x}_1) + d_1, \\ {}_0^C D_t^{0.95} x_2 = u + f_2(\bar{x}_2) + d_2, \\ y = x_1, \end{cases} \tag{92}$$

where x_1 and x_2 are pseudo state variables, d_1 and d_2 are external disturbances, u is the output of the actuator, and the desired signal is selected as $x_d = \sin(t)$.

To illustrate the validity of the proposed ETC schemes, the following dynamics are selected $f_1(\bar{x}_1) = 0.1x_1$, $f_2(\bar{x}_2) = x_1 x_2 \exp(x_1 x_2)$, and $d_1 = d_2 = 0.1\sin(t)$.

For simulation, the Gaussian basis function for introducing the activation vector of the RBFNNs are selected as $\mu_r(X_i) = \exp(-0.25||X_i + 2 - 0.5(r-1)||^2)$, for $r = 1, \ldots, 7$, and $i = 1, 2$, where $X_1 = x_1$ and $X_2 = [x_1, x_2]^{\mathrm{T}}$.

In this study, the actuator failure patterns are simulated as: the actuator undergoes a loss of effectiveness pattern at 80% from $t = 10s$ and encounters with bias $\Theta = 0.6$, $\Delta(t) = 1$ for $t \geq 30$.

The initial values for this simulation are all set as $x_1(0) = 0.2$, $x_2(0) = 0$, $\eta_{1_1}(0) = 0.1$, $\eta_{1_2}(0) = 0.2$, $\eta_{2_1}(0) = 0.1$, $\eta_{2_2}(0) = 0.3$, $\hat{w}_1 = [0, \ldots, 0] \in \mathbb{R}^7$, $\hat{w}_2 = [0, \ldots, 0] \in \mathbb{R}^7$, $\hat{\varphi}_1(0) = 0.2$, $\hat{\varphi}_2(0) = 0.1$,

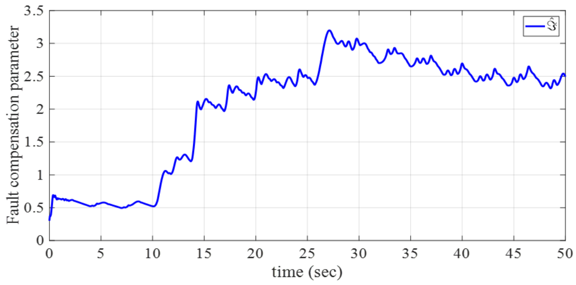

Figure 4: Fault compensator parameter.

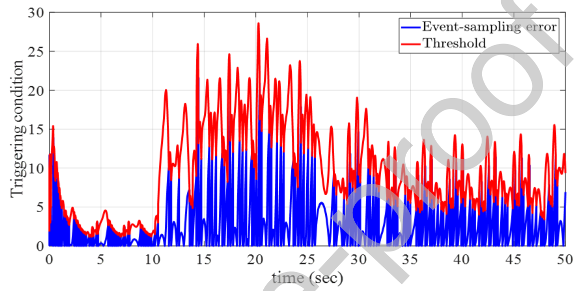

Figure 5: Event-triggered condition.

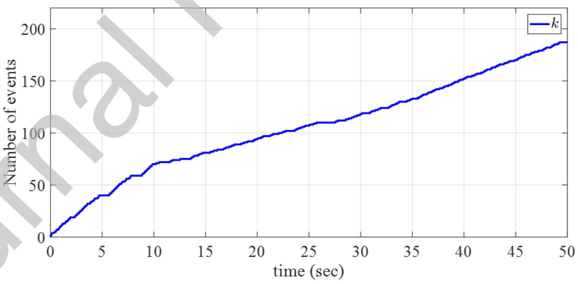

Figure 6: Number of events.

$\hat{\psi}_1(0)=0$, $\hat{\psi}_2(0)=0.5$, $\hat{\wp}(0)=0.1$ and $\eta(0)=0.2$.

The design positive parameters for these proposed schemes are as $c_1=c_2=5$, $a_1=a_2=20$, $\gamma_{\varphi_1}=\gamma_{\varphi_2}=3$, $\Gamma_{w_1}=\Gamma_{w_2}=3\mathrm{diag}\{1,1,1,1,1,1,1\}$, $\gamma_{\psi_1}=\gamma_{\psi_2}=2$, $\gamma_{\wp}^1=1$, $r_{\varphi_1}=r_{\varphi_2}=0.1$, $r_{w_1}=r_{w_2}=0.2$, $r_{\psi_1}=r_{\psi_2}=0.2$, $r_{\wp}^1=0.5$, $\aleph^1=0.7$, $\epsilon=5$, $\zeta_1^1=10$, $\zeta_2^1=0.1$, $\zeta_3^1=30$, $\zeta_4^1=8$, $\omega^1=1$, $\xi^1=0.3$ and $\sigma^1=10$.

For the fractional-order system (92), these proposed ETC schemes can be applied in addressing the

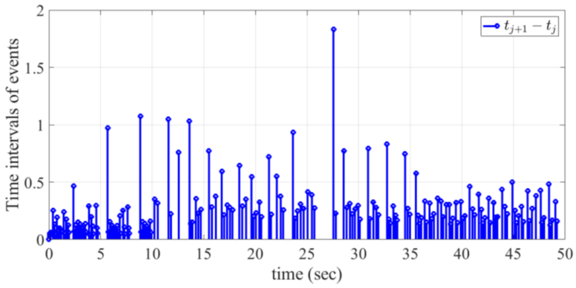

Figure 7: Time intervals of triggering events.

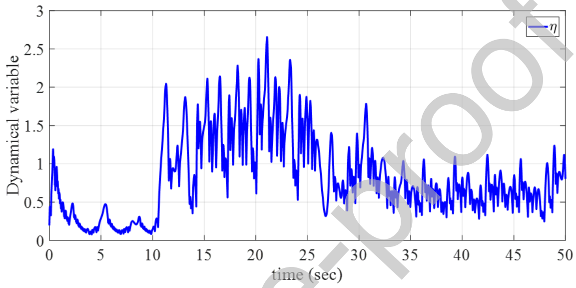

Figure 8: Dynamical variable.

Table 2. Computational resource for different strategies.

Control schemes	Simulation time (s)	$\max\{t_{j+1} - t_j\}$ (s)	$\min\{t_{j+1} - t_j\}$ (s)	$\text{ave}\{t_{j+1} - t_j\}$ (s)	Control updates
TTC (Li et al. (2021))	0-50	0.001	0.001	0.001	5000
Static ETC	0-50	1.58	0.002	0.1369	365
Dynamic ETC	0-50	1.43	0.012	0.2244	185

fault compensation problem subject to the following two cases.

Case 1 (Dynamic event-triggered): By applying the dynamic ETC method proposed in Theorem 3 for (92), the simulation results are shown in Figs. 1-8. As observed in Fig. 1, the system's output can follow the desired trajectory with bounded error. The bounded trajectory of the second pseudo state is shown in Fig. 2. The outputs of the dynamic event-triggered controller ($\tau^{1(d)}$) and the virtual control ($\tau_e^{1(1)}$) are displayed in Fig. 3 as bounded signals. From the partial enlarged landscape in Fig. 3, it can be obviously seen that the proposed dynamic ETC mitigates the update times of the controller, and as a result, reduce the load of communication channel between controller and actuator. Besides, it can be derived that the proposed approach decreases the amount of the controller updates by about 14 % in $[20(s), 25(s)]$ than the static ETC strategy under the same conditions. The boundedness of the fault compensation signal trajectory is revealed in Fig. 4. Fig. 5 demonstrates that the absolute value of the measurement error signal ($|\alpha|$) and the dynamic relative threshold ($\aleph|\tau| + \zeta_2 + \zeta_4\eta$) are bounded, where the measurement error is increased in 10(s) due to lose of effectiveness fault, and then, held in a small area. As observed in Fig. 6, the count of events is 185 in the period [0(s), 50(s)], which is less than both the control updates required in the TTC and the static ETC strategies. Therefore, it can be found that the introduction of the internal dynamic variable (η) in the event-triggering rule (75) is applicable in reducing the count of actuator updates. The corresponding time intervals of sequential events in dynamic ETC are shown in Fig. 7, where the Zeno phenomenon of the closed-loop system does not occurs. In Fig. 8, the boundedness of the non-negative internal dynamical variable is depicted.

These results indicate that this proposed dynamic ETC method can assure the stability of the faulty system. Besides, a proper dynamic event-triggered threshold can be selected to decrease the frequency of the controller updates.

Case 2 (Comparative results): To further illustrate the benefits of this proposed dynamic ETC method, simulation of the time-triggered fault compensation control in Li et al. (2021) and the proposed static event-triggered fault compensation control method are run for the system Eq. (92). For a precise evaluation and quantitative comparisons, two tables to are provided when the TTC, the static ETC, and the dynamic ETC strategies are applied.

Some indices of tracking performance, like mean square of tracking error (MSE), $\sqrt{\frac{1}{50}\int_0^{50}|\pi_1^2|dt)}$, integral of absolute tracking error (IAE), $\int_0^{50}|\pi_1|dt$, and tracking accuracy (TA) in [0(s), 5(s)], $\max_{0\leq t\leq 5}|\pi_1(t)|$ for different triggering schemes are tabulated in Table 1. As observed in Table 2, by tuning appropriate design parameters the tracking error by the dynamic ETC control is smaller than the other triggered methods.

For the computational complexity, different indices are evaluated for the proposed algorithms in Table 2. According to the minimum, maximum, and average time interval of events, as to saving in communication and computation resources, the triggering count is reduced according to the dynamic triggering design; thus, it can be deduced that this dynamic ETC strategy is more effective than its counterparts.

6. Conclusions

Two new static and dynamic ETC methods are proposed to address the tracking problem for a class of the nonlinear systems. The assessed class of nonlinear systems is equipped with the fractional-order structures and have unmatched uncertainties with both external disturbances and actuator faults. The conventional drawback that actuators of the fractional-order systems need to update in the periodic time intervals is eliminated in this article, which, in turn, saves resource consumption. In this attempt, first, by defining the measurement error and the relative threshold signals, the fault compensation static event-triggering rule is proposed. Next, by defining the novel interval dynamical signal, the new fault compensation dynamic event-triggered is extended. It is revealed that in both the proposed event-triggered schemes, the Zeno behavior is not happening. In this article, the strict-feedback fractional-order system is considered. A nonstrict-feedback case Shahvali and Askari (2016); Shahvali, Naghibi-Sistani and Askari (2018) is an interesting topic that needs further study.

CRediT authorship contribution statement

Milad Shahvali: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Writing - Original Draft, Writing - Review, Editing. **Mohammad-Bagher Naghibi-Sistani:** Supervision, Project administration. **Javad Askari:** Supervision, Project administration.

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Conflict of interest
The authors declare that they have no conflict of interest.