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Investigating the Effect of Dependence of Fading Coefficients and its Modeling with Copula Theory in Non-Orthogonal Multiple Access (NOMA) Channels with Physical Layer Security

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Abstract. In wireless communication, in most mathematical modeling, it is assumed that the fading coefficients are independent of each other, if physically, there is a correlation between them. In this paper, the non-orthogonal multiple access (NOMA) Downlink with physical layer security and dependent fading coefficients is investigated.The average secrecy rate (ASR) for the NOMA channel in the presence of an eavesdropper has been investigated by modeling the dependence of extinction coefficients by copula functions. With mathematical calculations and numerical results, we compared the effect of correlation in the studied fading coefficients and independent fading coefficients to find out whether this modeling is useful or harmful.

1 Introduction

The Non-orthogonal multiple access scheme is known as one of the effective techniques in multiple access, which has the ability to improve spectral efficiency and fairness of users [1]-[3]. In wireless communication of the first to fourth generation, the idea of orthogonal use of available channel resources was proposed in order to prevent channel interference between each user, Therefore, the number of users who could use the channel's resources was limited. In order to separate the overlapping messages of different users, the technique of Successive Interference Cancellation (SIC) in the receivers is used [4].

Unlike OMA schemes, in NOMA, the superimposed messages of all multiple users are sent simultaneously on the entire channel, so there is a risk that an eavesdropper can hear these messages. Therefore, in NOMA, there is a necessity to secure confidential messages in case of illegal use [5]. A joint multivariate distribution function is used to investigate the performance of NOMA channels with dependent fading coefficients, so having an efficient and appropriate mathematical tool can help to investigate the dependence in NOMA channels. It is suggested to apply and use copula functions as an effective method to express the dependence between variables. Copula expresses joint distributions by applying marginal distribution functions, and these joint distributions, which have different types. Copula functions are used in many sciences including statistics, machine learning, image processing and many applications in engineering. [6,7]. The physical layer security has been studied by many researchers for non-orthogonal multiple access channels [8,9]

Our work. In this paper, we investigate the downlink NOMA with two legitimate users (one strong user and the other weak user) in the presence of an eavesdropper. It is assumed that there is a dependence between the fading coefficients, and we model this dependence using copula functions. The joint probability density function between fading coefficients is obtained with the help of copula functions, and the average secrecy rates of each user is calculated. Then the effect of dependence on the performance of each user is compared to the situation where the fading coefficients are independent.

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2 A Brief Review of Copula Theory

In order to model the dependence between channel fading coefficients, the copula function is used. Copula is a multivariate cumulative distribution function so that the distribution of marginal probabilities of each variable in the interval $[0,1]$ has a uniform distribution [6]. It is necessary to express the probability density function (PDF) of channel fading coefficients with the help of copula functions in order to calculate the average secrecy rate in NOMA systems. Suppose that $\mathcal{S} = (X_m, X_n)$ is a vector of two random variables with Cumulative Distribution Function (CDF) as $F(x_m, x_n)$ and marginal CDFs $F(x_m)$ and $F(x_n)$ respectively.

A copula is a function $C: [0,1]^2 \to [0,1]$ which satisfies following properties:

For each u and v in the interval $[0,1]$ we have:

$$c(u,0) = c(0,v) = 0\,, c(u,1) = u \text{ and } \quad c(1,v) = v \tag{1}$$

The 2-increasing property, for every u_1, u_2, v_1, v_2 in the interval $[0,1]$ such that $u_1 \leq u_2$ and $v_1 \leq v_2$:

$$C(u_2, v_2) - C(u_1, v_2) - C(u_2, v_1) + C(u_1, v_1) \geq 0 \tag{2}$$

Theorem 1 (Sklar's theorem): This theorem states that any multivariate CDF of a random variable can be expressed in terms of marginal functions. Let $F(x_m, x_n)$ as CDF of random variables with margins $F(x_i)$ for $i = m, n$. Then there exists a copula C such that for all x_m, x_n in $\bar{R}$ [5].

$$F(x_m, x_n) = C(F(x_m), F(x_n)) \tag{3}$$

Then, according to the Sklar's theorem, the joint PDF is obtained for the marginal functions $F(x_m)$ and $F(x_n)$ respectively.

$$f(x_m, x_n) = f(x_m) f(x_n) c(F(x_m), F(x_n)) \tag{4}$$

Where $c\big(F(x_\mathrm{m}), F(x_\mathrm{n})\big) = \frac{\partial^2 C(F(x_m), F(x_n))}{\partial F(x_m).\partial F(x_n)}$ is the Copula density function. Also $f(x_m)$ and $f(x_n)$ are marginal PDFs, respectively. There are many different copulas that can be used. In this paper, we use FGM copula to analyze the performance criteria of the proposed system from an empirical point of view, that the FGM copulas are the simplest mode for calculating the joint PDFs [6] and consider negative and positive correlations and independence situation. FGM copulas are defined as follows:

$$C(u,v) = uv(1 + \theta(1-u)(1-v)) \tag{5}$$

where $\theta \epsilon [-1,1]$ is defined as the dependence parameter and $u = F_{X_m}(x_m)$ and $v = F_{X_n}(x_n)$. Negative and positive values of θ indicate negative and positive dependence, respectively, and for zero value, we have independence.

3 Secrecy Rate Region of NOMA:

Theorem 2: The average secrecy rates (ASC) for users with dependent fading coefficients are obtained as follows:

$$R_{m,ave}^{Sec} \leq E_{\gamma_m, \gamma_e}[\log(1 + \alpha_m \gamma_m) - \log(1 + \alpha_m \gamma_e)]^+ \tag{6}$$

$$R_{n,ave}^{Sec} \le E_{\gamma_n,\gamma_e}[\log\left(1+\frac{\alpha_n\gamma_n}{\alpha_m\gamma_n+1}\right)-\log\left(1+\frac{\alpha_n\gamma_e}{\alpha_m\gamma_e+1}\right)]^+ \quad (7)$$

where α_i for $i=m,n$ expresses power allocation factors for each user, where $\alpha_m+\alpha_n=1$ and $0\le\alpha_m\le\alpha_n\le1$. In relations (6) and (7), the value of γ_i is equal to $\gamma_i=\frac{P|h_i|^2}{N_i}$ for $i=m,n,e$.

Lemma 1. The joint PDF of γ_i and γ_j $(f(\gamma_i,\gamma_j))$ based on Farlie-Gumbel-Morgenstern (FGM) Copula is determined as:

$$f(\gamma_i,\gamma_j)=\frac{e^{-\frac{\gamma_i}{\bar{\gamma}_i}-\frac{\gamma_j}{\bar{\gamma}_j}}}{\bar{\gamma}_i\bar{\gamma}_j}[1+\theta\left(1-2e^{-\frac{\gamma_i}{\bar{\gamma}_i}}\right)\left(1-2e^{-\frac{\gamma_j}{\bar{\gamma}_j}}\right)] \quad (8)$$

that the marginal probability density functions in relation (9) are defined as follows:

$$f_{\gamma_i}(\gamma_i)=\frac{1}{\bar{\gamma}_i}e^{-\frac{\gamma_i}{\bar{\gamma}_i}} \quad i=m,n,e \quad \gamma_i>0 \quad \bar{\gamma}_i=\frac{1}{2\sigma_{h_i}^2} \quad (9)$$

By simplifying and mathematical calculations, the ASC of the user m is calculated as follows:

$$\begin{aligned}R_{m,ave}^{sec} \le [&-e^{\frac{1}{\alpha_m\bar{\gamma}_m}}Ei\left(-\frac{1}{\alpha_m\bar{\gamma}_m}\right)+e^{\left(\frac{\bar{\gamma}_e+\bar{\gamma}_m}{\alpha_m\bar{\gamma}_e\bar{\gamma}_m}\right)}Ei\left(-\frac{\bar{\gamma}_e+\bar{\gamma}_m}{\alpha_m\bar{\gamma}_e\bar{\gamma}_m}\right)\\ &\theta(-e^{\left(\frac{\bar{\gamma}_e+\bar{\gamma}_m}{\alpha_m\bar{\gamma}_e\bar{\gamma}_m}\right)}Ei\left(-\frac{\bar{\gamma}_e+\bar{\gamma}_m}{\alpha_m\bar{\gamma}_e\bar{\gamma}_m}\right)+e^{\left(\frac{2\bar{\gamma}_e+\bar{\gamma}_m}{\alpha_m\bar{\gamma}_e\bar{\gamma}_m}\right)}Ei\left(-\frac{2\bar{\gamma}_e+\bar{\gamma}_m}{\alpha_m\bar{\gamma}_e\bar{\gamma}_m}\right)\\ &+e^{\left(\frac{\bar{\gamma}_e+2\bar{\gamma}_m}{\alpha_m\bar{\gamma}_e\bar{\gamma}_m}\right)}Ei\left(-\frac{\bar{\gamma}_e+2\bar{\gamma}_m}{\alpha_m\bar{\gamma}_e\bar{\gamma}_m}\right)-e^{\left(\frac{2\bar{\gamma}_e+2\bar{\gamma}_m}{\alpha_m\bar{\gamma}_e\bar{\gamma}_m}\right)}Ei\left(-\frac{2\bar{\gamma}_e+2\bar{\gamma}_m}{\alpha_m\bar{\gamma}_e\bar{\gamma}_m}\right))]^+\end{aligned} \quad (10)$$

The average secrecy rate of the user n is also calculated as follows:

$$\begin{aligned}R_{n,ave}^{Sec} \le [&e^{\frac{1}{\alpha_m\bar{\gamma}_n}}Ei\left(-\frac{1}{\alpha_m\bar{\gamma}_n}\right)-e^{\frac{1}{\bar{\gamma}_n}}Ei\left(-\frac{1}{\bar{\gamma}_n}\right)\\ &-e^{\left(\frac{\bar{\gamma}_e+\bar{\gamma}_n}{\alpha_m\bar{\gamma}_e\bar{\gamma}_n}\right)}Ei\left(-\frac{\bar{\gamma}_e+\bar{\gamma}_n}{\alpha_m\bar{\gamma}_e\bar{\gamma}_n}\right)+e^{\left(\frac{\bar{\gamma}_e+\bar{\gamma}_n}{\bar{\gamma}_e\bar{\gamma}_n}\right)}Ei\left(-\frac{\bar{\gamma}_e+\bar{\gamma}_n}{\bar{\gamma}_e\bar{\gamma}_n}\right)\\ &+\theta(e^{\left(\frac{\bar{\gamma}_e+\bar{\gamma}_n}{\alpha_m\bar{\gamma}_e\bar{\gamma}_n}\right)}Ei\left(-\frac{\bar{\gamma}_e+\bar{\gamma}_n}{\alpha_m\bar{\gamma}_e\bar{\gamma}_n}\right)-e^{\left(\frac{2\bar{\gamma}_e+\bar{\gamma}_n}{\alpha_m\bar{\gamma}_e\bar{\gamma}_n}\right)}Ei\left(-\frac{2\bar{\gamma}_e+\bar{\gamma}_n}{\alpha_m\bar{\gamma}_e\bar{\gamma}_n}\right)\\ &-e^{\left(\frac{\bar{\gamma}_e+2\bar{\gamma}_n}{\alpha_m\bar{\gamma}_e\bar{\gamma}_n}\right)}Ei\left(-\frac{\bar{\gamma}_e+2\bar{\gamma}_n}{\alpha_m\bar{\gamma}_e\bar{\gamma}_n}\right)+e^{\left(\frac{2\bar{\gamma}_e+2\bar{\gamma}_n}{\alpha_m\bar{\gamma}_e\bar{\gamma}_n}\right)}Ei\left(-\frac{2\bar{\gamma}_e+2\bar{\gamma}_n}{\alpha_m\bar{\gamma}_e\bar{\gamma}_n}\right)\\ &-e^{\left(\frac{\bar{\gamma}_e+\bar{\gamma}_n}{\bar{\gamma}_e\bar{\gamma}_n}\right)}Ei\left(-\frac{\bar{\gamma}_e+\bar{\gamma}_n}{\bar{\gamma}_e\bar{\gamma}_n}\right)+e^{\left(\frac{2\bar{\gamma}_e+\bar{\gamma}_n}{\bar{\gamma}_e\bar{\gamma}_n}\right)}Ei\left(-\frac{2\bar{\gamma}_e+\bar{\gamma}_n}{\bar{\gamma}_e\bar{\gamma}_n}\right)\\ &+e^{\left(\frac{\bar{\gamma}_e+2\bar{\gamma}_n}{\bar{\gamma}_e\bar{\gamma}_n}\right)}Ei\left(-\frac{\bar{\gamma}_e+2\bar{\gamma}_n}{\bar{\gamma}_e\bar{\gamma}_n}\right)-e^{\left(\frac{2\bar{\gamma}_e+2\bar{\gamma}_n}{\bar{\gamma}_e\bar{\gamma}_n}\right)}Ei\left(-\frac{2\bar{\gamma}_e+2\bar{\gamma}_n}{\bar{\gamma}_e\bar{\gamma}_n}\right))]^+\end{aligned} \quad (11)$$

4 NUMERICAL RESULS:

In this section, the ASR of each users per fixed eavesdropper channel gain SNR $\bar{\gamma}_e$, increases with the increase of each user's channel gains SNR $(\bar{\gamma}_m, \bar{\gamma}_n)$. The ASR of each users for different values of parameter θ is shown in Figure. 1 ,the ASR of user m for positive dependence has a better performance than the ASR of users with independent joint probability density function.

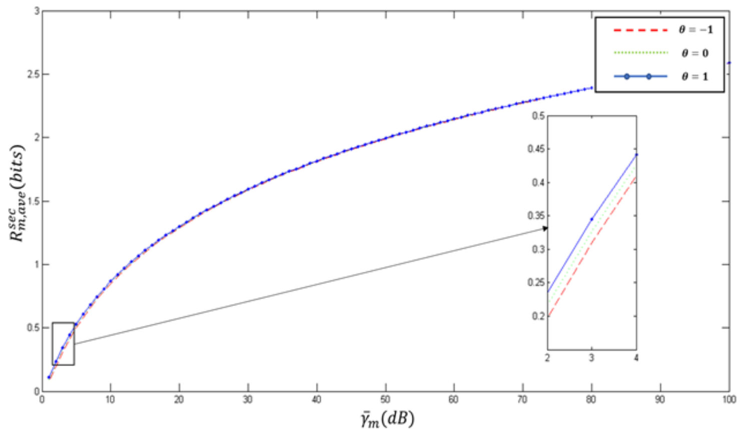

Figure. 1: The ASR of user m versus $\bar{\gamma}_m$, for different values of dependence parameter ϑ.

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