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Research Article

Total Product and Total Edge Product Cordial Labelings of Dragonfly Graph (Dg_n)

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In this paper, we study the total product and total edge product cordial labeling for dragonfly graph Dg_n. We also define generalized dragonfly graph and find product cordial and total product cordial labeling for this family of graphs.

1. Introduction

In this paper, all graphs $G = G(V, E)$ are simple and finite connected with order p and size q. We will give some definitions and other information, which are useful for this research. Terms that are not defined here, we refer to West [1]. Let function f be a vertex labeling of graph G and f^*, an edge labeling of graph G. Let $v_f(i)$ (respectively $e_{f^*}(i)$) denote the number of vertices (edges) labeled with $i = 0, 1$.

The cordial labeling was introduced in 1987 by Cahit [2], which he defines that a graph G is said to be cordial graph if there exists a vertex labeling $f: V \longrightarrow \{0, 1\}$ such that induces an edge labeling $f^*: E \longrightarrow \{0, 1\}$ defined by $f^*(uv) = |f(u) - f(v)|$ and satisfied $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$. In [2], Cahit proved many result for cordial labeling. Prime cordial labeling, A-cordial labeling, product cordial labeling, H-cordial labeling, etc. are some variations of labeling schemes introduced after cordial labeling. For product cordial labeling, it was introduced in 2004 by Sundaram, et al. [3], which $f^*(uv) = |f(u) - f(v)|$ on cordial labeling is replaced by $f^*(uv) = f(u)f(v)$. In this paper we investigate the total product and total edge product cordial labelings of dragonfly graph (Dg_n).

The product cordial labeling is defined in Definition 1.1.

Definition 1.1. A graph G is said to be the product cordial if there exists a vertex labeling $f: V \longrightarrow \{0, 1\}$ such that induces an edge labeling $f^*: E \longrightarrow \{0, 1\}$ defined by $f^*(uv) = |f(u) - f(v)|$ and satisfied $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$.

In [3], Sundaram et al. proved that unicycle graphs with odd order, trees, helms, triangular snakes, dragons, and unicon with two paths are product cordial. Furthermore, Vaidya and Barasara [4] discussed product cordial labeling of graph fans F_n, C_n with one chord, and C_n with two chord. Gao et al. [5] discussed product cordial labeling of graph $P_{n+1}^{(m)}$.

Motivated by definition of product cordial labeling, in [6], Sundaram et al. introduce a total product cordial labeling and investigate the total product cordial of some standard graphs. The total product cordial labeling is defined in Definition 1.2.

Definition 1.2. A graph G is said to be the total product cordial if there exists a vertex labeling $f: V \longrightarrow \{0, 1\}$ such that induces an edge labeling $f^*: E \longrightarrow \{0, 1\}$ defined by $f^*(uv) = |f(u) - f(v)|$ and satisfied $|(v_f(0) + e_{f^*}(0)) - (v_f(1) - e_{f^*}(1))| \leq 1$.

The total product cordial labeling of cycle C_7 is shown in Figure 1.

In [6, 7], Sundaram et al. proved that tree graph P_n, fans graph F_n, graph C_n, except $n = 4$, wheels graph w_n, helms

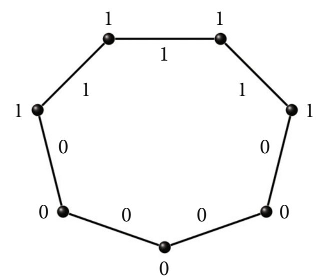

Figure 1: Total product cordial labeling of cycle C_7.

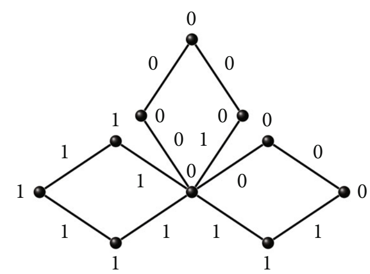

Figure 2: Total edge product cordial labeling of $C_4^{(3)}$.

graph H_n, and graph C_n with m edges appended at each vertex are total product cordial graph. They also proved that every product cordial graph G is a total product cordial if G has either even size and even order or odd order.

In [8], Vaidya and Barasara introduce the concept of edge product cordial labeling, which is defined in Definition 1.3.

Definition 1.3 (see [8]). A graph G is said to be edge product cordial if there exits an edge labeling $f : E \longrightarrow \{0, 1\}$ such that it induces a vertex labeling $f^* : V \longrightarrow \{0, 1\}$ defined by $f^*(v) = \Pi f(e_i)$ for $\{e_i | e_i \in E/e_i$ and e_i is incident to $v\}$ and satisfies $|e_f(0) - e_f(1)| \le 1$ and $|v_{f^*}(0) - v_{f^*}(1)| \le 1$.

In [8–10], Vaidya and Barasara have investigated several results related to edge product cordial labeling.

In, Vaidya and Barasara introduce the concept of total edge product cordial labeling, which is defined in Definition 1.4.

Definition 1.4 (see [10]). A graph G is said to be the total edge product cordial if there exits an edge labeling $f : E \longrightarrow \{0, 1\}$, such that it induces a vertex labeling $f^* : V \longrightarrow \{0, 1\}$ defined by $f^*(v) = \Pi f(e_i)$ for $\{e_i | e_i \in E/e_i$ and e_i is incident to $v\}$ and satisfies $|(e_f(0) + v_{f^*}(0)) - (e_f(1) + v_{f^*}(1))| \le 1$.

The total edge product cordial labeling of graph $C_4^{(3)}$ is shown in Figure 2.

In [4], Vaidya and Barasara have investigated total edge product cordial labeling in the context of various graph operations.

Proposition 1.5 (see [10]). *If every edge product cordial graph G has either even size or even order, then graph G is the total edge product cordial.*

In this paper, we determine the total product and total edge product cordial labelings of dragonfly graph, denoted by Dg_n, which is defined in Definition 1.6. Also, we generalized dragonfly graph, defined in Definition 3.1, and present two family of graphs in that, which are product and total product cordial graph.

Definition 1.6. For an integer n, the dragonfly graph Dg_n is the graph with vertex set:

$$V = \{u_i, v_j, w_k | i, j \in \{1, 2, \cdots, n+2\}, k \in \{1, 2, 3\}\}, \quad (1)$$

and edge set

$$\begin{aligned} E = \{&u_i u_{i+1}, i \in \{1, 2, \cdots, n+1\}\} \cup, \\ &\{u_i w_0 | i \in \{1, 2, \cdots, n+2\}\} \cup, \\ &\{v_i v_{i+1}, i \in \{1, 2, \cdots, n+1\}\} \cup, \\ &\{v_i w_0 | i \in \{1, 2, \cdots, n+2\}\} \cup, \\ &\{w_0 w_i | i \in \{1, 2\}\}. \end{aligned} \quad (2)$$

In Figure 3, we give a representation of our definition.

2. Main Results

Theorem 2.1. *The dragonfly Dg_n is product cordial graph.*

Proof. Let Dg_n is the dragonfly graph. Define the function $f : V(Dg_n) \longrightarrow \{0, 1\}$, we consider following two cases.

Case 1. Let n be even.

$$\begin{aligned} &f(w_0) = 1, \\ &f(w_i) = 0, 1 \le i \le 2, \\ &f(u_i) = 1, 1 \le i \le \frac{n+4}{2}, \\ &f(u_i) = 0, \frac{n+4}{2} + 1 \le i \le n+2, \\ &f(v_i) = 1, 1 \le i \le \frac{n+4}{2} - 1, \\ &f(v_i) = 0, \frac{n+4}{2} \le i \le n+2. \end{aligned} \quad (3)$$

By of the above labeling, we have $v_f(0) = 2.n + 4/2$ and $v_f(1) = 2.n + 4/2 - 1$. On the other hand, the edges of Dg_n with labels one are the following:

$$\begin{aligned} &f^*(u_i w_0) = 1, 1 \le i \le \frac{n+4}{2}, \\ &f^*(v_i w_0) = 1, 1 \le i \le \frac{n+2}{2}. \\ &f^*(u_i u_{i+1}) = 1, 1 \le i \le \frac{n+2}{2}, \\ &f^*(v_i v_{i+1}) = 1, 1 \le i \le \frac{n}{2}, \end{aligned} \quad (4)$$

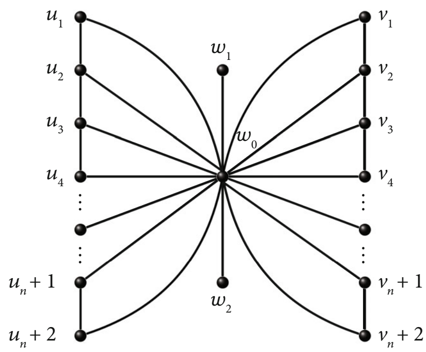

Figure 3: The dragonfly graph Dg_n.

and the edges of Dg_n with labels zero are the following:

$$\begin{aligned}
f^*(w_0w_i) &= 0, 1 \le i \le 2,\\
f^*(u_iw_0) &= 0, \frac{n+4}{2} + 1 \le i \le n+2.\\
f^*(v_iw_0) &= 0, \frac{n+2}{2} + 1 \le i \le n+2.\\
f^*(u_iu_{i+1}) &= 0, \frac{n+4}{2} \le i \le n+1,\\
f^*(v_iv_{i+1}) &= 0, \frac{n+4}{2} - 1 \le i \le n+1.
\end{aligned} \tag{5}$$

By of the above labeling, we have $e_{f^*}(0) = 2n+4$ and $e_{f^*}(1) = 2n+4$. Hence, $|v_f(0) - v_f(1)| = 1$ and $|e_{f^*}(0) - e_{f^*}(1)| = 0$. Thus, the graph Dg_n is product cordial labeling.

Case 2. Let n be odd.

$$\begin{aligned}
f(w_0) &= 1,\\
f(w_i) &= 0, 1 \le i \le 2,\\
f(u_i) &= 1, 1 \le i \le \left\lceil \frac{n+2}{2} \right\rceil,\\
f(u_i) &= 0, \left\lceil \frac{n+2}{2} \right\rceil + 1 \le i \le n+2,\\
f(v_i) &= 1, 1 \le i \le \left\lceil \frac{n+2}{2} \right\rceil,\\
f(v_i) &= 0, \left\lceil \frac{n+2}{2} \right\rceil + 1 \le i \le n+2.
\end{aligned} \tag{6}$$

By of the above labeling, we have $v_f(0) = 2.\lceil n+2/2 \rceil$ and $v_f(1) = 2.\lceil n+2/2 \rceil + 1$. On the other hand, the edges of Dg_n with labels one are the following:

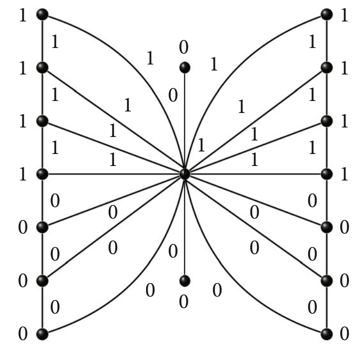

Figure 4: Total product cordial labeling of Dg_5.

$$\begin{aligned}
f^*(u_iw_0) &= 1, 1 \le i \le \left\lceil \frac{n+2}{2} \right\rceil,\\
f^*(v_iw_0) &= 1, 1 \le i \le \left\lceil \frac{n+2}{2} \right\rceil,\\
f^*(u_iu_{i+1}) &= 1, 1 \le i \le \left\lceil \frac{n+2}{2} \right\rceil - 1,\\
f^*(v_iv_{i+1}) &= 1, 1 \le i \le \left\lceil \frac{n+2}{2} \right\rceil,
\end{aligned} \tag{7}$$

and the edges of Dg_n with labels zero are the following:

$$\begin{aligned}
f^*(w_0w_i) &= 0, 1 \le i \le 2,\\
f^*(u_iw_0) &= 0, \left\lceil \frac{n+2}{2} \right\rceil + 1 \le i \le n+2,\\
f^*(v_iw_0) &= 0, \left\lceil \frac{n+2}{2} \right\rceil + 1 \le i \le n+2,\\
f^*(u_iu_{i+1}) &= 0, \left\lceil \frac{n+2}{2} \right\rceil \le i \le n+1,\\
f^*(v_iv_{i+1}) &= 0, \left\lceil \frac{n+2}{2} \right\rceil + 1 \le i \le n+1.
\end{aligned} \tag{8}$$

By of the above labeling, we have $e_{f^*}(0) = 2\lceil n+2/2 \rceil + 2$ and $e_{f^*}(1) = 2\lceil n+2/2 \rceil + 2$. Hence, $|v_f(0) - v_f(1)| = 1$ and $|e_{f^*}(0) - e_{f^*}(1)| = 1$. Thus, the graph Dg_n is product cordial labeling. Therefore, considering two cases above, we prove that graph Dg_n is product cordial graph. □

Theorem 2.2. *The dragonfly Dg_n is a total product cordial.*

Proof. By: Theorem 2.1, $|(e_f(0) + v_{f^*}(0)) - (e_f(1) + v_{f^*}(1))| \le 1$. Thus, the graph Dg_n is a total product cordial.

The total product cordial labeling of Dg_5 is shown in Figure 4. □

Theorem 2.3. *The dragonfly Dg_n is an edge product cordial.*

Proof. Let Dg_n is dragonfly graph. Define the function $f: E(Dg_n) \longrightarrow \{0, 1\}$, we consider following two cases.

Case 1. Let n be even.

$$\begin{aligned} f(w_0w_i) &= 1, 1 \le i \le 2,\\ f(w_0u_i) &= 0, 1 \le i \le \frac{n+4}{2},\\ f(w_0u_i) &= 1, \frac{n+4}{2} + 1 \le i \le n+2,\\ f(w_0v_i) &= 0, 1 \le i \le \frac{n+2}{2},\\ f(w_0v_i) &= 1, \frac{n+2}{2} + 1 \le i \le n+2,\\ f(u_iu_{i+1}) &= 0, 1 \le i \le \frac{n+2}{2},\\ f(u_iu_{i+1}) &= 1, \frac{n+2}{2} \le i \le n+1,\\ f(v_iv_{i+1}) &= 0, 1 \le i \le \frac{n}{2},\\ f(v_iv_{i+1}) &= 1, \frac{n}{2} \le i \le n+1. \end{aligned} \tag{9}$$

By of the above labeling, we have $e_f(0) = 4.n + 2/2$ and $e_f(1) = 4.n + 2/2$. On the other hand, the vertices of Dg_n with labels zero are the following:

$$\begin{aligned} f^*(w_0) &= 0,\\ f^*(u_i) &= 0, 1 \le i \le \frac{n+4}{2},\\ f^*(v_i) &= 0, 1 \le i \le \frac{n+2}{2}, \end{aligned} \tag{10}$$

and the vertices of Dg_n with labels one are the following:

$$\begin{aligned} f^*(w_i) &= 1, 1 \le i \le 2,\\ f^*(u_i) &= 1, \frac{n+4}{2} + 1 \le i \le n+2,\\ f^*(v_i) &= 1, \frac{n+2}{2} + 1 \le i \le n+2. \end{aligned} \tag{11}$$

By of the above labeling, we have $v_{f^*}(0) = n + 4$ and $v_{f^*}(1) = n + 3$. Hence, $|e_f(0) - e_f(1)| = 0$ and $|v_{f^*}(0) - v_{f^*}(1)| = 1$. Thus, the graph Dg_n is an edge product cordial labeling.

Case 2. Let n be odd.

$$\begin{aligned} f(w_0w_i) &= 1, 1 \le i \le 2,\\ f(w_0u_i) &= 0, 1 \le i \le \frac{n+3}{2},\\ f(w_0u_i) &= 1, \frac{n+3}{2} + 1 \le i \le n+2,\\ f(w_0v_i) &= 0, 1 \le i \le \frac{n+3}{2},\\ f(w_0v_i) &= 1, \frac{n+3}{2} + 1 \le i \le n+2,\\ f(u_iu_{i+1}) &= 0, 1 \le i \le \frac{n+1}{2},\\ f(u_iu_{i+1}) &= 1, \frac{n+1}{2} \le i \le n+1,\\ f(v_iv_{i+1}) &= 0, 1 \le i \le \frac{n+1}{2},\\ f(v_iv_{i+1}) &= 1, \frac{n+1}{2} \le i \le n+1. \end{aligned} \tag{12}$$

By of the above labeling, we have $e_f(0) = n + 2$ and $e_f(1) = n + 2$. On the other hand, the vertices of Dg_n with labels zero are the following:

$$\begin{aligned} f^*(w_0) &= 0,\\ f^*(u_i) &= 0, 1 \le i \le \frac{n+3}{2},\\ f^*(v_i) &= 0, 1 \le i \le \frac{n+3}{2}, \end{aligned} \tag{13}$$

and the vertices of Dg_n with labels one are the following:

$$\begin{aligned} f^*(w_i) &= 1, 1 \le i \le 2,\\ f^*(u_i) &= 1, \frac{n+3}{2} + 1 \le i \le n+2,\\ f^*(v_i) &= 1, \frac{n+3}{2} + 1 \le i \le n+2. \end{aligned} \tag{14}$$

By of the above labeling, we have $v_{f^*}(0) = n + 4$ and $v_{f^*}(1) = n + 3$. Hence, $|e_f(0) - e_f(1)| = 0$ and $|v_{f^*}(0) - v_{f^*}(1)| = 1$. Thus, the graph Dg_n is an edge product cordial labeling. Therefore, considering two cases above, we prove that graph Dg_n is edge product cordial. □

Corollary 2.4. *The dragonfly Dg_n is a total edge product cordial graph.*

Proof. Let Dg_n is dragonfly graph. Here, graph Dg_n has even size and in Theorem 2.3, Dg_n is edge product cordial. Then, by proposition 3.2, the result holds. □

The total edge product cordial labeling of Dg5 is shown in Figure 5.

3. The Generalized Dragonfly Graphs

In this section, we present a generalization of dragonfly graph and show that some of those graphs are total product cordial graphs.

Definition 3.1. For every $m \geq 2$ and $k \geq 1$, the generalized dragonfly graph, denoted by $\mathrm{Dg}_n^{(m,k)}$, is the graph with vertex set

$$V = \left\{v_i^1, v_i^2, \cdots, v_i^m, w_j | i \in \{1, 2, \cdots, n+2\}, j \in \{1, 2, \cdots, k\}\right\}, \tag{15}$$

and edge set

$$\begin{aligned} E = &\left\{v_i^\ell v_{i+1}^\ell, i \in \{1, 2, \cdots, n+1\},\right. \\ &\text{for } \ell \in \{1, 2, \cdots, m\}\} \cup, \\ &\left\{v_i^\ell w_0 | i \in \{1, 2, \cdots, n+2\},\right. \\ &\text{for } \ell \in \{1, 2, \cdots, m\}\} \cup, \\ &\left\{w_0 w_j | j \in \{1, 2, \cdots k\}\right\}. \end{aligned} \tag{16}$$

It is clear that $\mathrm{Dg}_n^{(2,1)} = \mathrm{Dg}_n$ (see Figure 6, for the case $m = k = 3$).

Theorem 3.2. *For $k \geq 1$, graph $Dg_{2k}^{(3,2)}$ is a product cordial graph.*

Proof. We define vertex labeling $f : V(\mathrm{Dg}_{2k}^{(3,2)}) \longrightarrow \{0, 1\}$, of vertices $\mathrm{Dg}_{2k}^{(3,2)}$ as follow.

$$\begin{aligned} f(w_0) &= 1, \\ f(w_i) &= 0, 1 \leq i \leq 2, \\ f\left(v_i^\ell\right) &= 1, 1 \leq i \leq k+1, 1 \leq \ell \leq 2, \\ f\left(v_i^\ell\right) &= 0, k+2 \leq i \leq 2k+2, 1 \leq \ell \leq 2, \\ f\left(v_i^3\right) &= 1, 1 \leq i \leq k+1, \\ f\left(v_i^3\right) &= 0, k+2 \leq i \leq 2k+2. \end{aligned} \tag{17}$$

By the above labeling, we have $v_f(0) = 3(k+1) + 1$ and $v_f(1) = 3(k+1) + 2$. On the other hand, the edges of $\mathrm{Dg}_{2k}^{(3,2)}$ with labels one are the following:

$$\begin{aligned} f^*\left(v_i^\ell w_0\right) &= 1, 1 \leq i \leq k+1, 1 \leq \ell \leq 2, \\ f^*\left(v_i^3 w_0\right) &= 1, 1 \leq i \leq k+2, \\ f^*\left(v_i^\ell v_{i+1}^\ell\right) &= 1, 1 \leq i \leq k, 1 \leq \ell \leq 2, \\ f^*\left(v_i^3 v_{i+1}^3\right) &= 1, 1 \leq i \leq k+1, \end{aligned} \tag{18}$$

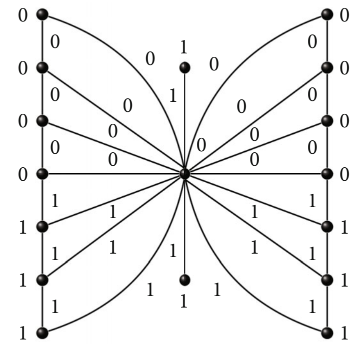

Figure 5: Total edge product cordial labeling of Dg_5.

and the edges of $\mathrm{Dg}_{2k}^{(3,2)}$ with labels zero are the following:

$$\begin{aligned} f^*(w_0 w_i) &= 0, 1 \leq i \leq 2, \\ f^*\left(v_i^\ell w_0\right) &= 0, k+2 \leq i \leq 2k+2, 1 \leq \ell \leq 2, \\ f^*\left(v_i^3 w_0\right) &= 0, k+3 \leq i \leq 2k+2, \\ f^*\left(v_i^\ell v_{i+1}^\ell\right) &= 0, k+1 \leq i \leq 2k+1, 1 \leq \ell \leq 2, \\ f^*\left(v_i^3 v_{i+1}^3\right) &= 0, k+2 \leq i \leq 2k+1. \end{aligned} \tag{19}$$

By the above labeling, we have $e_{f^*}(0) = 6(k+1)$ and $e_{f^*}(1) = 6(k+1) - 1$. Hence, $|v_f(0) - v_f(1)| = 1$ and $|e_{f^*}(0) - e_{f^*}(1)| = 1$. Thus, labeling f is a product cordial labeling for $\mathrm{Dg}_{2k}^{(3,2)}$, and the proof is completed. □

Corollary 3.3. *For $k \geq 1$, graph $Dg_{2k}^{(3,2)}$ is a total product cordial.*

Proof. By: Theorem 3.2, $|(e_f(0) + v_{f^*}(0)) - (e_f(1) + v_{f^*}(1))| \leq 1$. Therefore, the graph $\mathrm{Dg}_{2k}^{(3,2)}$ is a total product cordial. □

Theorem 3.4. *For $k \geq 1$, graph $Dg_{2k+1}^{(3,3)}$ is a product cordial graph.*

Proof. We define vertex labeling $f : V(Dg_{2k+1}^{(3,3)}) \longrightarrow \{0, 1\}$, of vertices $\mathrm{Dg}_{2k+1}^{(3,3)}$ as follow.

$$\begin{aligned} f(w_0) &= 1, \\ f(w_i) &= 0, 1 \leq i \leq 3, \\ f\left(v_i^\ell\right) &= 1, 1 \leq i \leq k+2, 1 \leq \ell \leq 3, \\ f\left(v_i^\ell\right) &= 0, k+3 \leq i \leq 2k+3, 1 \leq \ell \leq 3. \end{aligned} \tag{20}$$

By the above labeling, we have $v_f(0) = 3(k+2) + 1$ and $v_f(1) = 3(k+1) + 3$. On the other hand, the edges of D

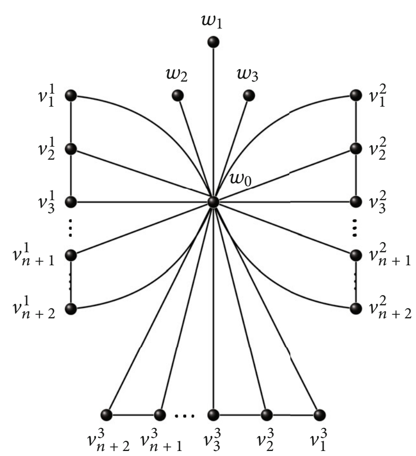

Figure 6: The generalize dragonfly graph $\mathrm{Dg}_n^{(3,3)}$.

$\mathrm{g}_{2k+1}^{(3,3)}$ with labels one are the following:

$$\begin{aligned} f^*\left(v_i^\ell w_0\right) &= 1, 1 \le i \le k+2, 1 \le \ell \le 3, \\ f^*\left(v_i^\ell v_{i+1}^\ell\right) &= 1, 1 \le i \le k+1, 1 \le \ell \le 3, \end{aligned} \tag{21}$$

and the edges of $\mathrm{Dg}_{2k+1}^{(3,3)}$ with labels zero are the following:

$$\begin{aligned} f^*\left(w_0 w_i\right) &= 0, 1 \le i \le 3, \\ f^*\left(v_i^\ell w_0\right) &= 0, k+3 \le i \le 2k+3, 1 \le \ell \le 3, \\ f^*\left(v_i^\ell v_{i+1}^\ell\right) &= 0, k+2 \le i \le 2k+2, 1 \le \ell \le 3, \end{aligned} \tag{22}$$

By the above labeling, we have $e_{f^*}(0) = 6(k+1)+3$ and $e_{f^*}(1) = 3(k+2)+3(k+1)$. Hence, $|v_f(0) - v_f(1)| = 1$ and $|e_{f^*}(0) - e_{f^*}(1)| = 0$. Thus, labeling f is a product cordial labeling for $\mathrm{Dg}_{2k+1}^{(3,3)}$, and the proof is completed. □

Corollary 3.5. *For $k \ge 1$, graph $Dg_{2k+1}^{(3,3)}$ is a total product cordial.*

It is interesting to find all values m, k, and n such that generalized dragonfly $\mathrm{Dg}_n^{(m,k)}$ is cordial product graph. We end the paper with the following question.

Question. Find all values m, k, and n, such that $\mathrm{Dg}_n^{(m,k)}$ is (edge) cordial product graph.

Data Availability

Data sharing is not applicable to this article as no data were collected or analyzed in this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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