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ORIGINAL ARTICLE

Estimation of the groundwater recharge coefficient by minimizing the sum total error of a regional water balance

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Abstract

The sustainable use of groundwater resources requires that the components of a regional water balance are understood with a high degree of accuracy, especially in arid and semi-arid regions. Although groundwater recharge is one of the most important components of a groundwater balance, its rate is one of the most uncertain components. Despite the simplicity and widespread use of water balance method (WB) for estimating groundwater recharge in arid and semi-arid regions, the accuracy of the groundwater recharge estimation depends on the accuracy of the other components in this method, so a reduction in errors that are associated with measuring these can improve the accuracy of recharge estimation. Therefore, the main objective of the current paper was to present a method for estimating groundwater recharge based on minimization of the sum total error of the system water and groundwater balance equations simultaneously. A set of correction coefficients that reflect the error in estimation of each component of balance equations, were applied to different components in annual scale. Reasonable ranges, obtained from error analysis, were considered for the correction coefficients and the sum of absolute errors in the overall system water balance and groundwater balance equations was minimized for the period of study. The proposed method was used to estimate groundwater recharge in Mahvelat basin in Khorasan Razavi province of Iran, as a case study. The minimization process used in this research reduced the error of system water and groundwater balance equations by 55% and 65%, respectively. Moreover, as the results of optimization process on the correction coefficients, the recharge coefficients due to precipitation and irrigation return flow were estimated to be 2% and 16.5%, respectively.

Keywords Groundwater balance (budget) · System water balance · Groundwater recharge coefficient · Error minimization · Mahvelat

Introduction

Groundwater is one of the most important freshwater resources in the world which provides potable water for about 1.5 billion of the world's population. The demand for groundwater resources is continuing to increase due to rapid population growth, especially in arid and semi-arid regions. It is important that groundwater use does not exceed the natural recharge rate to ensure that these resources are sustainable. Consequently, it is particularly important that recharge rates are determined with a high level of certainty in regions where there is intensive groundwater use.

Although groundwater recharge is one of the most important groundwater studies, its rate is one of the most uncertain factors in groundwater studies. Recharge rates vary widely in time and space and their direct measurement is difficult (Healy and Scanlon 2010).

In general, groundwater recharge is estimated by one of the following methods (Bear 1979):

1. Estimation of the amount of recharge based on the groundwater balance of the region. In summary, the amount of recharge is achieved by the precipitation deducted by the accumulation of seepage, evapotranspiration, surface runoff and other components expressing the loss of precipitation.

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2. Estimation of the water penetration rate into soil based on precipitation and the permeability coefficient of soils, which is dependent on the soil texture and the degree of saturation.
3. Estimation of the amount of recharge through investigating the fluctuation of groundwater levels. This method is only suitable for short periods of time and where groundwater occurs at shallow depth.
4. Estimation of the recharge rate with natural and introduced chemical tracers and the use of heat as a thermal tracer.
5. Estimation of the recharge rate from inverse modelling and the calibration of groundwater flow models.

There are many ways to estimate the amount of groundwater recharge rate. However, it is difficult to choose the most appropriate method for a particular situation in terms of accuracy, ease, cost, and above all, adaptation to the physical and hydrological characteristics of the region. The purpose of the recharge estimation is of particular importance in choosing the appropriate method, because the purpose of the study has a significant effect on selecting the space/time scales (Scanlon et al. 2003).

According to the governing climate in most parts of Iran, including the study area, the methods used for arid and semi-arid regions are considered in the present study. Xu and Beekman (2019) investigated the methods of estimating groundwater recharge for arid and semiarid regions in a study on South Africa. The results of this study are summarized in Table 1. The methods in which tracers are not used use other physical methods.

In a study conducted in the Gaza Strip, the results obtained from the new GIS tool with the Thiessen polygon method showed that the Thiessen polygon method is not accurate in estimating groundwater recharge. The average annual groundwater recharge was estimated to be about 19.2% of the precipitation in this research (Mushtaha et al. 2019). Groundwater recharge rates were estimated using the Chaturvedhi empirical equations in parts of India by Mistry and Suryanarayana (2021). The maximum and minimum recharge rate were estimated to be 30.72% and 13.5% of annual precipitation respectively. Yimam et al. (2021) estimated recharge of groundwater in the Dangishta watershed in Ethiopian using the WTF method which indicated groundwater recharge was about 17–22% of annual precipitation. Some of other important recent research on groundwater recharge estimates in semi-arid regions is summarized in Table 2.

The physical conditions of some basins and of their groundwater hydrographs, including the case study in this study, do not allow the use of methods which are based on the relationship between groundwater level variations with the cumulative amount of precipitation (such as through the CRD method). However, in case of using water balance method and proper estimation of the region's water balance components, it is possible to estimate the approximate amount of recharge due to the precipitation and irrigation return flow in these basins. In the water balance method, all components of the balance equation which are measurable are calculated and then groundwater recharge is estimated as the unknown component in the balance equation. The accuracy of the recharge estimate in this method depends on the accuracy with which the other components in the water balance equation are measured and small errors in them can lead to an unreliable estimate of the recharge rate. So, if a proper estimation of the balance equation components is achieved, the recharge rate can be estimated using the regional water balance.

For example, Manghi et al. (2009) estimated groundwater recharge using the water balance method at the regional scale for the Hemet basin in western California. They used groundwater level information, groundwater extraction data and specific yield and provided groundwater balance of the basin. The annual average groundwater recharge in the region was then estimated to be 12.5 million m^3. Nyagwambo (2006) estimated groundwater recharge using WB (water balance methods), WTF and CMB in Nyundo basin in Zimbabwe. All three methods have estimated the amount of recharge to be between 8% and 15% of the annual average precipitation. The results of his sensitivity analysis have shown that the CMB, WTF and WB methods are sensitive to chloride concentration, specific yield, and spatial distribution of precipitation, respectively and their misestimating can produce uncertain results in the recharge estimation. Sun (2005) estimated groundwater recharge using water balance method for the Montagu region in South Africa. This study indicated that the recharge rate in the region varies from 0.1 to 38 mm per year. He has also reported that in the study area, the recharge rate would increase with increasing precipitation, but it would have a nonlinear relationship with precipitation. In the study conducted by Ketema and Broder (2009), a spatially distributed water balance model WetSpass was used to simulate long-term average recharge in Dire Dawa, a semiarid region of Ethiopia. The long-term temporal and spatial average annual precipitation of 626 mm was distributed as: surface runoff of 126 mm (20%), evapotranspiration of 468 mm (75%), and recharge of 28 mm (5%). This recharge corresponds to 817 l/s for the 920.12 km^2 study area. Soodegi (2018) have estimated the groundwater recharge using WB, CRD, CMB and isotopic methods in the Birjand basin in Iran. The annual average groundwater recharge in the region was estimated to be 9–11 million m^3. Recharge rates were calculated in the watershed located in the Brazilian Savannah region for the period from October 2009 to September 2011 using the methods of baseflow separation, water table fluctuation, and a sequential water

Table 1 The groundwater recharge estimation methods investigated in South Africa (from Xu and Beekman 2019)

Region	Approach	Method	Limitations	Basis
Surface water	Physical	HS (Hydrograph separation-base flow)	Ephemeral rivers	Stream hydrograph separation: outflow, evapotranspiration and abstraction balances recharge
		CWB (channel water budget)	Inaccurate flow measurements	Recharge derived from difference in flow upstream and downstream accounting for evapotranspiration, in and outflow and channel storage change
		WM (watershed modeling)	Ephemeral rivers	Numerical rainfall-runoff modelling; recharge estimated as a residual term
Unsaturated	Physical	Lysimeter	Surface runoff	Drainage proportional to moisture flux/recharge
		UFM (unsaturated flow modeling)	Poorly known relationship hydraulic conductivity–moisture content	Unsaturated flow simulation e.g. using numerical solutions to Richards equation
		ZFP (zero flux plane)	Subsurface heterogeneity; periods of high infiltration	Soil moisture storage changes below ZFP (zero vertical hydraulic gradient) proportional to moisture flux/recharge
	Tracer	CMB (chloride mass balance)	Long-term atmospheric deposition unknown	Chloride Mass Balance–Profiling: drainage inversely proportional to Cl in pore water
		Historical	Poorly known porosity; present 3H levels almost undetectable	Vertical distribution of tracer as a result of activities in the past (^{3}H)
Saturated–unsaturated	Physical	CRD (cumulative rainfall departure)	Deep (multi-layer) aquifer; sensitive to specific yield (Sy)	Water level response from recharge proportional to cumulative rainfall departure
		EARTH (extended model for aquifer recharge and moisture transport through unsaturated hard rock)	Poorly known Sy	Lumped distributed model simulating water level fluctuations by coupling climatic, soil moisture and groundwater level data
		WTF (water table fluctuation)	In/outflow and Sy usually unknown	Water level response proportional to recharge/discharge
	Tracer	CMB (chloride mass balance)	Long-term atmospheric deposition unknown	Amount of Cl into the system balanced by amount of Cl out of the system for negligible surface runoff/runon
Saturated	Physical	GM (groundwater modeling)	Time consuming; poorly known transmissivity; sensitive to boundary conditions S	Recharge inversely derived from numerical modeling groundwater flow and calibrating on hydraulic heads/groundwater ages
		SVF (saturated volume fluctuation)	Flow-through region; multilayered aquifers	Water balance over time based on averaged groundwater levels from monitoring boreholes
		EV-SF (equal volume-spring flow)	Confined aquifers	Water balance at catchment scale
	Tracer	GD (groundwater dating)	Known porosity/correction for dead carbon contribution	Age gradient derived from tracers, inversely proportional to recharge; Recharge unconfined aquifer based on vertical age gradient (^{3}H, CFCs, ^{3}H/^{3}He); Recharge confined aquifer based on horizontal age gradient (^{14}C)

Table 2 Recent estimates of groundwater recharge in semi-arid regions

Method	Case study	Year	Researchers
CMB, WB, WTF	Queensland (Australia)	2017	King et al.
Tracer (historical by Water isotopes: δ ^{18}O, δ ^{2}H, 3H)	Northwest aquifer of India	2018	Joshi et al.
GM (Watermark Numerical Computing)	Montana state (USA)	2019	Meredittha et al.
GM (Visual MODFLOW Flex)	Egypt	2019	Abdelhalim et al.
GM (MODFLOW)	Andimeshk (IRAN)	2020	Zeinali et al.
GM (MODFLOW)	Cachar (India)	2020	Kumar Singh et al.
WB, WTF	Jakarta	2021	Nugraha et al.

balance (BALSEQ) by Arnaldo and Lineu (2020). The recharge rates estimated by the baseflow separation, water table fluctuation and BALSEQ water balance methods were, respectively, 23.7%, 26.6% and 31.5% of the total precipitation of 1753.8 mm.

The objective of this study is to estimate the groundwater recharge as accurately as possible using water balance method. This was done by reducing errors in the components of water balance equations. This study was carried out using data from the Mahvelat basin in Khorasan Razavi province in Iran as a case study. According to the level of available information, the amount of groundwater recharge was estimated for the whole plain over average spatial and annual time scales.

The simplicity of the water balance method to estimate groundwater recharge leads to its widespread use, but the accuracy of estimation in this method depends on the accuracy of other balance components. Therefore, in this research, the error reduction method is also used in the water balance equations when using the balance method to reach an optimal solution for the estimation of recharge. One of the features of this research is that the error reduction process is performed on the system water balance and groundwater balance equations simultaneously. So, with regard to optimization of all water balance components, the results can be used in other groundwater studies.

Materials and methods

The concept of water balance and water balance in the study area

The water balance will examine all water exchanges in a study area. The equations are based on the principle of conservation of mass in the water cycle. Water balance evaluates all water entering a region and are consumed, stored or leaving in various forms in a given time period. All of these components would change over time if water balance is simply defined as “input − output = changes in storage”. Therefore, the water balance must be determined over a given time interval (Bredehoeft 2003).

Different time periods such as monthly, seasonal and/or annual periods can be adopted depending on the purpose of study and climate of the region. However, for arid and semiarid regions where groundwater levels are typically very deep, it is best to assume longer time period because of the lag time necessary for infiltrating water to reach groundwater (Manghi et al. 2009). So, in the present study, the annual time period has been considered and the water balance calculations were considered from the water year of 2001–2012.

Depending on the control volume, the water balance equation must consider three components: the surface water balance equation, the groundwater balance equation, and the water balance equation for the overall system (Todd and Mays 2005).

The system water balance equation for a catchment basin can be expressed by Eq. (1) as follows:

$$P + Q_{\mathrm{in}}^{\mathrm{sw}} + Q_{\mathrm{in}}^{\mathrm{gw}} - ET_{\mathrm{a}} - Q_{\mathrm{out}}^{\mathrm{sw}} - Q_{\mathrm{out}}^{\mathrm{gw}} = \Delta S^{\mathrm{sw}} + \Delta S^{\mathrm{gw}} \qquad (1)$$

where P is the precipitation, $Q_{\mathrm{in}}^{\mathrm{sw}}$ is the surface flow into the system, $Q_{\mathrm{in}}^{\mathrm{gw}}$ is the groundwater flow into the system ET_{a} is the evapotranspiration, $Q_{\mathrm{out}}^{\mathrm{sw}}$ is the surface flow out of the system, $Q_{\mathrm{out}}^{\mathrm{gw}}$ is the groundwater flow out of the system, ΔS^{sw} is the change in surface water storage and ΔS^{gw} is the change in groundwater storage. All components have a dimension of volume $[L^3]$.

The groundwater balance equation for an underground water reservoir can be expressed by Eq. (2) as follows:

$$\left(R_{\mathrm{ra}} + R_{\mathrm{fo}} + R_{\mathrm{ag}} + R_{\mathrm{ar}}\right) + Q_{\mathrm{in}}^{\mathrm{gw}} - ET^{\mathrm{gw}} - Q_{\mathrm{D}} - Q_{\mathrm{P}} - Q_{\mathrm{out}}^{\mathrm{gw}} = \Delta S^{\mathrm{gw}} \qquad (2)$$

where R_{ra} is the recharge from precipitation, R_{fo} is the recharge from rivers, lakes and ponds, R_{ag} is the recharge from irrigation return flow, R_{ar} is the artificial recharge, ET^{gw} is evapotranspiration from groundwater, Q_{D} is the natural discharge from the aquifer and Q_{P} is the groundwater withdrawal. All components have a dimension of volume $[L^3]$.

To estimate each component of water balance equation, the information of the first stage, which contains data points, should be interpolated and estimated as the spatial distribution layouts. For this purpose, the Geostatistical Analyst tool of ArcGIS software can be used. Regarding the information deficiencies, the estimation of each component would lead to complexities and problems. These issues need to be resolved by considering statistical factors groundwater hydraulics and the situation of the region simultaneously and also requires some innovations in this context.

Study area water balance

The Mahvelat basin, with an area of 2145 km^2, is located in the Central Desert catchment in the southwest of the Khorasan Razavi province in the northeastern part of Iran. This region lies between the longitudes of 57° 58′–59°03′ and latitudes of 34° 48′–35° 11′. The basin is bounded to the north by the Kashmar and Azghand plains, to the east by the Roshtkhar and Jangal plains, to the south by the Bejestan-Yunesi and to the west by the Kashmar plain. The surface water flows from the northern highlands of Azghand plain and, after recharging the Mahvelat unconfined alluvial aquifer, flows to the Bajestan saline desert. The hard formations in the study area are exposed in an east–west elongated outcrop. The geological map of the region shows that the oldest rock units with outcrops in the Mahvelat is the Kalshane Formation (with Cambrian age) which has an outcrop in the northwestern part of the basin. The Quaternary sediments of the Cenozoic era include alluvial terraces, alluvial cones and new alluvial terraces, river alluviums and desert sediments are widely spread in the central part of the study area. Smaller exposures of Neogene clayey sediments and conglomerates occur near the northern margin and in the eastern and southern parts of this study area. This area is located in the geological zone of Central Iran (Khorasan Razavi Regional Water Company 2011). The location of Mahvelat basin is shown in Fig. 1.

Fig. 1 Mahvelat basin location in Khorasan Razavi province, Iran

Hydrogeology of the study area The alluvial aquifer of Mahvelat Basin is of unconfined type and covers a large part of the center of this area with an approximate area of 767 square kilometers. The results of exploratory excavations and geophysical studies show that the aquifer of the Mahvelat is not the same in terms of type of sediments and thickness in different places. The maximum thickness of the alluvial aquifer in the central part is more than 200 m and the thickness of the aquifer decreases in all directions from this point. Generally, by moving away from the northern and northeastern highlands towards the south and southwest, the grain size of the alluvium becomes finer and the amount of evaporative deposits increases. A large part of the alluvial aquifer surface of the study area, especially the western parts, is made up of fine-grained desert sediments. The bedrock in the aquifer belongs to the Neogene formations and is composed of fine-grained clay and marl sediments.

According to the existing old maps (1974–1976), the direction of the groundwater flow was from northeast to southwest (desert). The sharp drop in the groundwater table (especially in the center of the plain) due to over-harvesting in recent years has caused the direction of the groundwater flow has changed ana is moving from the southwest (desert) towards the center of the plain. This change has caused the strong saltwater intrusion toward the Mahvelat aquifer. Due to this change in the direction of the groundwater flow, the aquifer does not have the outflow boundaries of the groundwater and only saline and brackish waters inflow to the aquifer from the western and southern boundaries (Khorasan Razavi Regional Water Company 2011, 2015).

Figure 2 shows the boundaries of the aquifer along with the direction of groundwater flow in plan and a cross section.

Water balance equations for the study area In the system water and groundwater balance equations for Mahvelat basin, some components are assumed to be minor or zero according to the characteristics of the basin and through scientific reports that have been published by the regional water company (Khorasan Razavi Regional Water Company 2011, 2015).

Due to the existence of two dams (Shahid Bahonar and Shahid Rajaee) in the elevated northern part of the study area, the incoming runoff to the basin is controlled. Therefore, the input surface flow component $Q_{\text{in}}^{\text{sw}}$ was neglected and eliminated from the system water balance. As mentioned in "Hydrogeology of the study area" section, due to the annual decline of groundwater in the region, as well as increasing salinity and declining groundwater quality of the aquifer, the existence of groundwater flow out of the aquifer has not been confirmed and the inflow of groundwater has

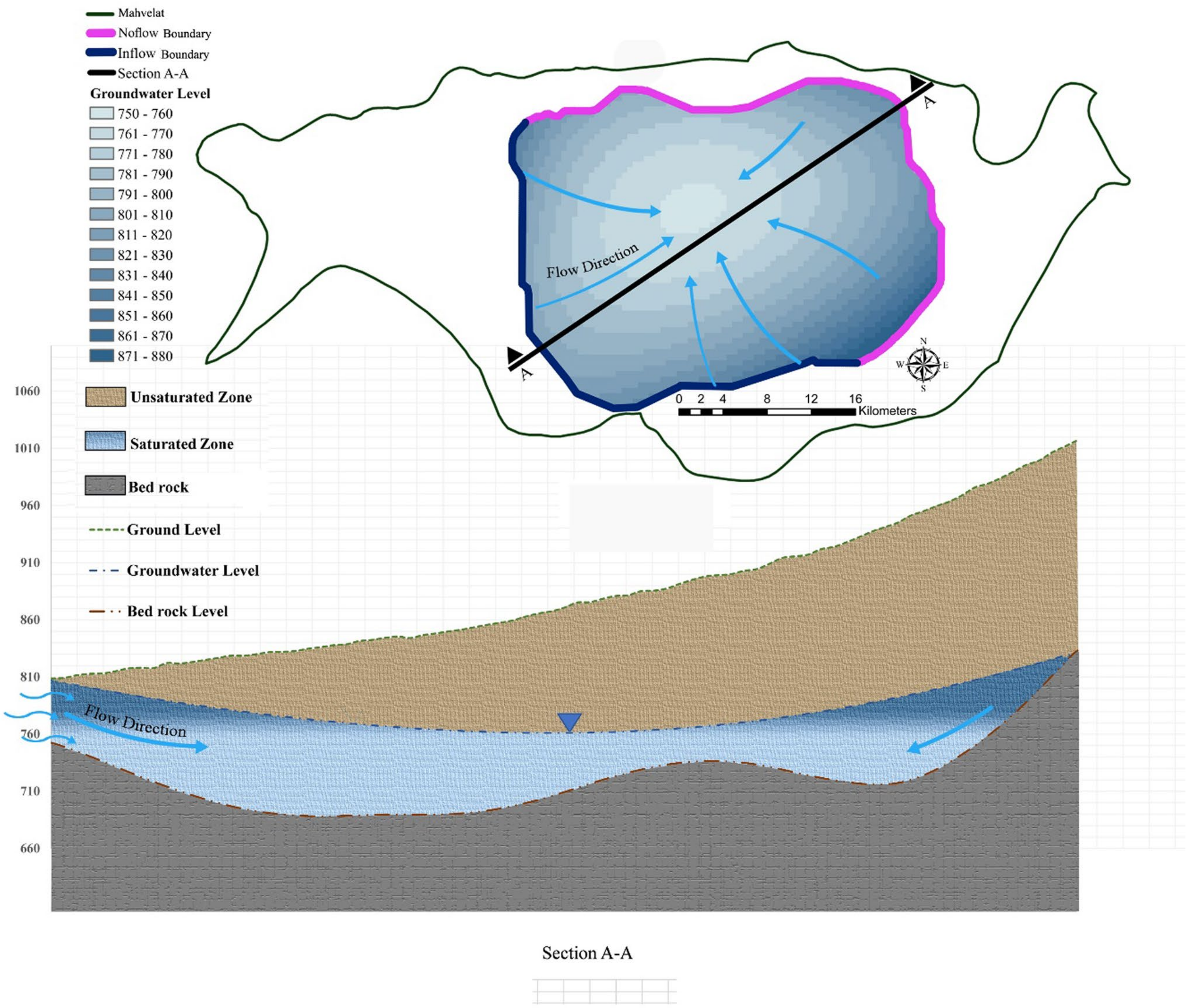

Fig. 2 Mahvelat aquifer boundaries and groundwater flow direction

been emphasized (Khorasan Razavi Regional Water Company 2011). So, the groundwater flow component running out of the aquifer $Q_{\mathrm{out}}^{\mathrm{gw}}$ was neglected. It is noteworthy that this issue was confirmed during the calculations process with regard to the hydraulic gradient and groundwater flow velocity vectors. The component of changes in surface water storage ΔS^{sw} was also eliminated due to the lack of permanent rivers and surface water resources. The unsaturated zone thickness in the Mahvelat basin is about 80 m. Thus, evapotranspiration from groundwater level ET^{gw} was considered to be zero (Khorasan Razavi Regional Water Company 2015). The absence of rivers and springs in the Mahvelat basin caused the natural discharge from the aquifer Q_{D} to be neglected. On the other hand, the main components of Mahvelat aquifer recharge are recharge due to precipitation R_{ra} and irrigation return flow R_{ag}. With these explanations, the system water and groundwater balance equations for Mahvelat basin are expressed by Eqs. (3) and (4).

$$P + Q_{\mathrm{in}}^{\mathrm{gw}} - ET_{\mathrm{a}} - Q_{\mathrm{out}}^{\mathrm{sw}} = \Delta S^{\mathrm{gw}} \tag{3}$$

$$\left(R_{\mathrm{ra}} + R_{\mathrm{ag}}\right) + Q_{\mathrm{in}}^{\mathrm{gw}} - Q_{\mathrm{P}} = \Delta S^{\mathrm{gw}} \tag{4}$$

Estimation of water balance components in the study area

In this study, ArcGIS has been used for hydrological information management and estimation of water balance components of Mahvelat basin. To determine the spatial distribution of point data in the study basin, the Geostatistical Analyst tool has been used in this software and

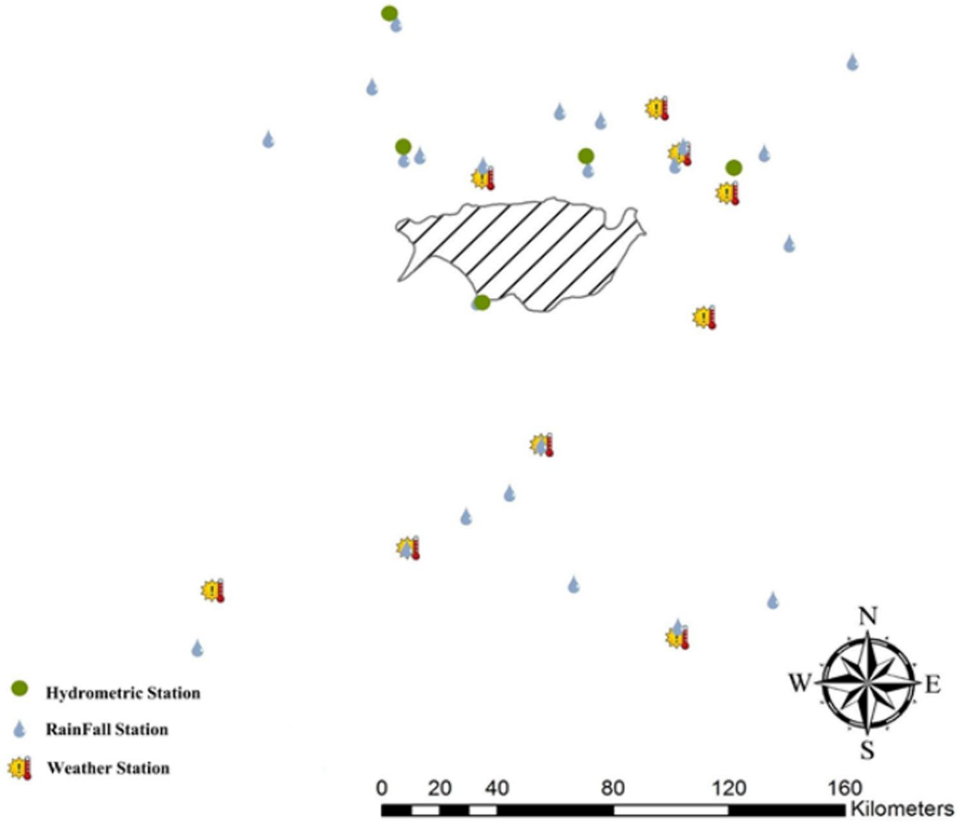

Fig. 3 The location of the stations used to estimate the precipitation of Mahvelat basin

the raster layouts required for the components of water balance equation were generated. It should be noted that for raster calculations, it is necessary to discrete the study area into cells. Based on the equations proposed by Hengl (2006), the dimensions of the cells up to the end of the calculations have been chosen as a square grid with a side of 500 m.

Precipitation In this study, rain-gage stations around the Mahvelat basin were used to estimate the precipitation within the basin (Fig. 3). The data on annual average precipitation of rain-gage stations were collected from Regional Water Authority of Razavi Khorasan. Then, the layer of data related to rain-gage stations and average precipitation of the stations in each year were provided in ArcGIS software. Several interpolation methods were used with Geostatistical Analyst to estimate rainfall for each year within the basin. Among the methods tested, the local polynomial with the Epanechnikov kernel function with power of 1 was chosen as the best method based on the RMSE criterion. Consequently, this method was selected to estimate the spatial distribution of rainfall within the basin. It is noteworthy that the average raster obtained for spatial distribution of precipitation was used as average annual precipitation.

Evapotranspiration Regarding the existent and available weather data available from the basin, the Turc relationship (Turc 1954) was used to estimate evapotranspiration. This equation is expressed by Eq. (5).

$$ET_{\mathrm{a}}=\frac{P}{\left[0.9+P/I_T{}^2\right]^{0.5}} \tag{5}$$

where ET_{a} is annual evapotranspiration in millimeters, P is precipitation in millimeters and I_T is the factor are related to the average annual temperature of the basin. I_T is calculated by Eq. (6).

$$I_T= 300 + 25T_a+0.05T_a^3 \tag{6}$$

where the T_{a} is average annual temperature in degrees Celsius.

To estimate the evapotranspiration using the Turc method, the mean annual temperature is needed. The data on recorded temperature at the evaporation stations were collected from the Regional Water Authorities of Razavi Khorasan province and South Khorasan province (Fig. 3). Afterwards, the data layout of the evaporation stations and their average temperature in each year were prepared using ArcGIS. A similar approach to precipitation was considered in choosing interpolation method and the one with

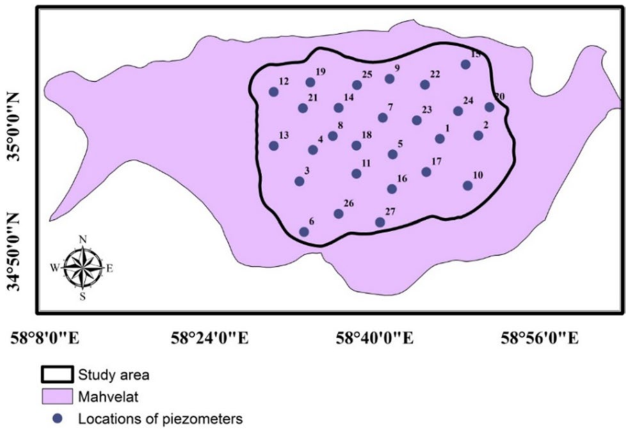

Fig. 4 Location of piezometric wells

least RMSE was selected as the best method for spatial distribution of temperature data. Kriging with a spherical semivariogram was selected as the best method based on the RMSE criterion and was selected as the method for determining the spatial distribution of temperature data.

In the following, raster calculations for the Turc equations were carried out with use of temperature and precipitation raster of the study area and with the help of raster calculator tool, an evapotranspiration raster was obtained for the study area.

The outlet surface runoff One of the common methods in computation of runoff is using the runoff coefficient. This coefficient is defined as the percentage of total precipitation that is converted to runoff.

To calculate the average runoff coefficient for the Mahvelat basin, the hydrometric stations around the study basin were used (Fig. 3). According to the obtained results and the values presented in the references based on the slope and coverage of the area, 0.14 was selected as the amount of runoff coefficient that is the runoff coefficient of upstream basin of hydrometric station of Soltanabad of Azghand. The outlet runoff in the study area was calculated according to the obtained runoff coefficient.

The groundwater inflow To calculate the inflow to the aquifer of Mahvelat basin, the Darcy equation can be written in the form of Eq. (7) to calculate the inflow to the aquifer.

$$Q_{\mathrm{in}}^{\mathrm{gw}} = T I^{\mathrm{gw}} l \tag{7}$$

where, T is aquifer transmissivity coefficient in square meter per day, I^{gw} is hydraulic head gradient of groundwater flow, l is the length of cell perpendicular to the flow direction and $Q_{\mathrm{in}}^{\mathrm{gw}}$ is the groundwater inflow to the aquifer in cubic meters per day. The transmissivity coefficient is itself the product of hydraulic conductivity and saturated thickness of the aquifer which is calculated using the Eq. (8):

$$T = Kb \tag{8}$$

where K is hydraulic conductivity and b is the saturated thickness in meter. To calculate the saturated thickness and hydraulic gradient of groundwater flow, the bed rock depth and water table elevation are required. The aquifer domain, input boundaries of groundwater flow, the flow direction in the boundary cells and the amount of hydraulic conductivity in these cells are also required to calculate groundwater inflow. In the following, it is explained how to estimate these parameters.

Water table elevation: The water table elevation data were collected in each of the piezometric wells in the study area from the Regional Water Authority of Razavi Khorasan (Fig. 4). Local polynomial interpolation was selected as the appropriate method for determining the spatial distribution of groundwater level data based on the minimum amount of RMSE. Then the layer of annual average water table elevation was estimated in ArcGIS. The average raster for water table elevation in each year was selected as the average water table elevation.

Bedrock depth: To estimate the bedrock depth, sounding points and deep wells were used. Afterwards, the most appropriate layer was estimated using the geostatistical analyst. The local polynomial interpolation method with polynomial function was determined to be the most appropriate method for estimating the bed rock depth.

Groundwater flow direction in boundaries: To determine the flow direction near the edges of the basin and to evaluate whether the groundwater flow is input or output in the boundary cells, the Darcy Flow tool, a subset of the Spatial Analyst toolbox of ArcGIS, was used. The Darcy Flow tool was used with the inputs: water table elevation, bed rock depth, transmissivity coefficient, effective porosity of the aquifer. This enabled the following outputs to be produced: seepage velocity, groundwater flow direction relative to geographic north and the volume balance residual of the target cell (Tauxe 1994).

Although the flow is unsteady on the boundaries, using the annual average data of the groundwater level and according to the presented descriptions, the flow direction at the boundary cells of the study area was determined. In most of the boundary cells, the flow direction was towards the inside of the study area. That is why the output flow from the study area was excluded (inflow boundaries and groundwater level at 2012 are shown in Fig. 2.).

Transmissivity coefficient: To calculate the transmissivity coefficient, values of hydraulic conductivity for the aquifer sediments are required. To estimate hydraulic conductivity in the study area, the following steps were taken:

1. The study area was meshed using the GIS tools (2.5 × 2.5 km) and the closest well to the center of each mesh was selected. 62 wells (total of 45 operational wells and 17 piezometers) with geological logs were selected.
2. The hydraulic conductivity of the different layers between the bed rock and the free surface of water of each well were estimated based on the materials that were described in the geological log for the well. Finally, the hydraulic conductivity of each log is calculated using the arithmetic average of hydraulic conductivity of different layers of the log.
3. The hydraulic conductivity layer was estimated based on the calculated hydraulic conductivity of each well in the second step using Geostatistical analysis. Based on the minimum value of the RMSE, the local polynomial interpolation method with exponential function was selected for estimating the raster layer of hydraulic conductivity.

Based on Eq. (6), the hydraulic conductivity raster was multiplied by the saturated thickness for each year to achieve the transmissivity coefficient raster of the desired year.

Calculation of groundwater inflow: According to the Eq. (7), the inflow to each cell at the input boundaries can be calculated and added together to calculate the volume of groundwater inflow to the study area. To do so, Eq. (9) was used.

$$Q_{\text{in}}^{\text{gw}} = \sum_{i=1}^{n} T_i I_i^{\text{gw}} l_i \tag{9}$$

where T_i is transmissivity coefficient in the cell in square meters per day, I_i^{gw} is the hydraulic gradient in the cell, l_i is a length of the cell perpendicular to the flow direction in meter and the index i represents the cell.

The magnitude of the hydraulic gradient was calculated using the water table elevation raster and the Slope tool of 3D Analysis toolbox. Knowing the flow direction with regard to north and the cell size, one can calculate a length of the cell which is perpendicular to the flow. Using the mentioned information and Eq. (9), the volume of groundwater flow and annual inflow to the study area was calculated.

Groundwater withdrawal To calculate the volume of withdrawals from groundwater resources in the 12-year study period, it is required to have operation time and the withdrawal discharge of every well to estimate the volume of withdrawals from groundwater resources by their production over a year. To determine the operation time of the wells, the information on the power consumption of each subscription was used, and incomplete and wrong information was completed and corrected.

The data on the agricultural well withdrawals were also collected from the Regional Water Authority and were used after defect resolution and data supplementation.

The discharge of each well was multiplied by its respective function hour to estimate the volume of a well's groundwater resource withdrawal over a year. By adding the withdrawal volumes, the total withdrawal volume was determined for a year.

Groundwater storage change To calculate the groundwater storage change, the Eq. (10) was used (Delleur 2006).

$$\Delta S^{\text{gw}} = S_y \cdot A \cdot \Delta H \tag{10}$$

where S_y is the specific yield of saturated layer at the water table elevation, A is the area of the study area in square meters, ΔH is the average annual changes in water table elevation in meters and ΔS^{gw} is groundwater storage changes in cubic meters.

Mahvelat aquifer is an unconfined aquifer and to determine the specific yield of saturated layer in the study area, a completely similar process to estimation of hydraulic conductivity is taken (Estimation of hydraulic conductivity requires analyzing the entire length of the log. However, specific yield only requires analyzing the geologic layer immediately below the water table). The specific yield of the each selected well was estimated based on the materials that were described in the geological log. Finally, the raster layer of specific yield of the study area was estimated using Geostatistical Analyst on the basis of RMSE minimum value, and using the local polynomial interpolation method with exponential basis function. According to the raster layer of specific yield, a value of 9.5% was determined to be the average specific yield of the Mahvelat aquifer.

To calculate the water table elevation annual changes, the difference between the water table elevation in the first moon (October) and the last moon (September) of every year was used.

Groundwater recharge: By summing the recharge resulting from the precipitation R_{ra} and the recharge resulting from the irrigation return flow R_{ag} in Eq. (11), the initial recharge value can be calculated by Eq. (11) as follows:

$$R_{\text{ra}} + R_{\text{ag}} = R_{\text{t}} = \alpha_{\text{ra}} \cdot (P \cdot A) + \alpha_{\text{ag}} \cdot Q_{\text{p}} \tag{11}$$

where P is precipitation in meters, A is the area of study area in square meters, Q_{p} is the groundwater withdrawal in cubic meters, α_{ra} is the recharge coefficient due to precipitation and α_{ag} is the recharge coefficient due to the irrigation return flow and R_{t} is the total groundwater recharge in cubic meters.

Choosing the initial recharge coefficient value can be done on base of previous studies. Scanlon et al. (2006) have found from their studies on 140 arid and semi-arid regions that the amount of recharge coefficient in the fields with agricultural irrigation that in addition to precipitation, the recharge coefficient is about 1–25% of total precipitation and agricultural irrigation water. Nyagwambo (2006) estimated groundwater recharge by three different methods in the Zimbabwean basin, and all three methods estimated the recharge rate to be between 8% and 15% of the average annual precipitation. Values of 11–13% of precipitation and 17–33% of agricultural irrigation water were estimated to be the share of groundwater recharge by Ahmadi (2013) in Neyshabour basin. Also, Omidifar (2019) estimated recharge coefficient due to precipitation and irrigation return flow in Azghand basin to be 10% and 20%, respectively.

The initial recharge coefficients due to precipitation and irrigation return flow, with regard to the above-mentioned previous studies and based on report of Khorasan Razavi Regional Water Company (2011), were chosen to be 5% and 10%, respectively.

Error in the water balance

Due to the several problems in measuring and independent estimation of the water balance equation components caused by human, device and computational errors, an error term is added to the balance equation which could have a considerable amount. Thus, the balance equation or Eq. (1) can be modified in the form of Eq. (12) (Sokolov and Chapman 1974):

$$I - O - \Delta S = \eta \tag{12}$$

where I is the volume of water inflow, O is the volume of water outflow, ΔS is the volume of water storage changes and η is error of the balance equation.

As such, an error term can be added to the system water balance equation of the study area and its groundwater balance. So, the Eqs. (3) and (4), which are related to the study area, could be modified to Eqs. (13) and (14).

$$P + Q_{\mathrm{in}}^{\mathrm{gw}} - ET_{\mathrm{a}} - Q_{\mathrm{out}}^{\mathrm{sw}} - \Delta S^{\mathrm{gw}} = \eta^{\mathrm{system}} \tag{13}$$

$$\left(R_{\mathrm{ra}} + R_{\mathrm{ag}}\right) + Q_{\mathrm{in}}^{\mathrm{gw}} - Q_{\mathrm{P}} - \Delta S^{\mathrm{gw}} = \eta^{\mathrm{gw}} \tag{14}$$

where η^{system} and η^{gw} are the system water balance and groundwater balance errors respectively in m^3. The values of the system water balance and groundwater balance components are presented in Tables 3 and 4, respectively as well as their error values in the study area of Mahvelat basin.

Estimation of recharge coefficient by minimizing the errors in components of the water balance

The main idea of this section is to introduce an optimization method which was implemented simultaneously on the system water balance and groundwater balance equations during the 12-year statistical period used in this study. This process was carried out to reduce the total error in the balance equations whilst possibly making the components values closer to their true values. It is evident that in case of success of this process, the recharge coefficient becomes also closer to its true value.

To reduce the total error in the balance equations and correct the values of the components, a correction coefficient is applied to each component. Then, these correction coefficients are estimated so that the error is minimized. Optimization constraints are in fact reasonable and justifiable range

Table 3 The values of the system water balance components and their errors in the study area (in millions of cubic meters)

Water year	Precipitation	Groundwater inflow	evapotranspiration	output runoff	Annual groundwater storage changes	Error of system water balance equation
2001	146.4	39.9	151.9	20.5	−137.7	151.6
2002	198.3	41.5	203.6	27.8	−122.6	131.1
2003	182.6	42.9	188.2	25.6	−133.2	144.9
2004	189	44.4	194.2	26.5	−153	165.8
2005	101.3	45.6	106.1	14.2	−129.6	156.4
2006	178.4	47.3	183.7	25	−131.8	148.8
2007	61.8	48.6	64.9	8.7	−105	141.8
2008	147.6	49.7	153	20.7	−81.6	105.3
2009	167.5	52.1	173	23.4	−83.3	106.4
2010	100.6	51.7	105.3	14.1	−85.4	118.4
2011	185.8	51.9	190.2	26	−50.1	71.6
2012	149.2	51.7	155	20.9	−43	68

Table 4 The values of groundwater balance components and their errors in the study area (in millions of cubic meters)

Water year	Groundwater recharge due to precipitation	Groundwater recharge due to irrigation return flow	Groundwater inflow	Groundwater withdrawal	Annual groundwater storage changes	Error of groundwater balance equation
2001	7.32	17.362	39.9	174.8	−137.7	25.1
2002	9.91	18.927	41.5	189.1	−122.6	1.4
2003	9.13	19.568	42.9	195.5	−133.2	6.7
2004	9.45	19.275	44.4	192.7	−153	30.9
2005	5.07	18.820	45.6	188.1	−129.6	8.1
2006	8.92	16.509	47.3	165	−131.8	37.4
2007	3.09	16.824	48.6	168.2	−105	2.6
2008	7.38	14.848	49.7	148.4	−81.6	3.2
2009	8.37	14.511	52.1	145.1	−83.3	11.4
2010	5.03	11.272	51.7	112.7	−85.4	39.2
2011	9.29	9.682	51.9	96.8	−50.1	23.3
2012	7.46	9.790	51.7	97.9	−43	13.0

of correction factors which are dictated to the optimization problem by the error in estimation of the components.

In the study directed by Daneshvar (2012), two error optimization models were used to reduce the error in the system water balance equation. He used the sum of absolute error and the sum of squared errors criteria to minimize the error in the balance equation. Daneshvar stated that optimization with sum of absolute error criterion has a higher capability in reducing the error in the balance equation. That is why in this study, the optimization method through sum of absolute error criterion has been used to reduce the error in the balance equation. The optimization model with sum of absolute error criterion is expressed as Eq. (15):

$$\text{Min} \sum_{i=1}^{n} |\eta_i| \tag{15}$$

where η_i is the error of the balance equation in the target year.

Combining the optimization model with Eq. (13) and applying the correction coefficients to each component of the system water balance equation, Eq. (16) is derived:

Combining the optimization model with Eq. (14) and applying the correction coefficients to each component of the groundwater balance equation, Eq. (17) is derived as follows:

$$\text{Min} \sum_{i=1}^{12} \Big| c_{R_{\text{ra}}}(R_{\text{ra}})_i + c_{R_{\text{ag}}}(R_{\text{ag}})_i - c_{Q_{\text{in}}^{\text{gw}}}(Q_{\text{in}}^{\text{gw}})_i \\ - c_{Q_{\text{P}}}(Q_{\text{P}})_i - c_{\Delta S^{\text{gw}}}(\Delta S^{\text{gw}})_i \Big| \tag{17}$$

where $c_{R_{\text{ra}}}$ is the recharge correction coefficient due to precipitation, $c_{R_{\text{ag}}}$ is the recharge correction coefficient due to the irrigation return flow and c_{Q_P} is the groundwater withdrawal correction coefficient.

An acceptable physical range must be considered for each of these coefficients. This range is determined for each basin based on the components estimation method for that basin (Daneshvar 2012). These ranges are applied as constraints to optimization model by Eqs. (18) and (19) for the system water balance equation (Eq. 16) and the groundwater balance equation (Eq. 17) respectively.

$$\text{Min} \sum_{i=1}^{12} \Big| c_{\text{P}}(P)_i + c_{Q_{\text{in}}^{\text{gw}}}(Q_{\text{in}}^{\text{gw}})_i - c_{ET_{\text{a}}}(ET_{\text{a}})_i - c_{Q_{\text{out}}^{\text{sw}}}\left(Q_{\text{out}}^{\text{sw}}\right)_i - c_{\Delta S^{\text{gw}}}(\Delta S^{\text{gw}})_i \Big| \tag{16}$$

where c_{P} is the precipitation correction coefficient, $c_{Q_{\text{in}}^{\text{gw}}}$ is the groundwater inflow correction coefficient, $c_{ET_{\text{a}}}$ is the evapotranspiration correction coefficient, $c_{Q_{\text{out}}^{\text{sw}}}$ is the surface outflow correction coefficient, $c_{\Delta S^{\text{gw}}}$ is the groundwater storage changes correction coefficient and i is the time index.

$$c_{P1} < c_P < c_{P2}; c_{Q_{\text{in}}^{\text{gw}}\,1} < c_{Q_{\text{in}}^{\text{gw}}} < c_{Q_{\text{in}}^{\text{gw}}\,2}; c_{\text{ET}_a 1} \\ < c_{\text{ET}_a} < c_{\text{ET}_a 2}; c_{Q_{\text{out}}^{\text{sw}}\,1} < c_{Q_{\text{out}}^{\text{sw}}} < c_{Q_{\text{out}}^{sw}\,2}; \\ c_{\Delta S^{\text{gw}}\,1} < c_{\Delta S^{\text{gw}}} < c_{\Delta S^{\text{gw}}\,2} \tag{18}$$

$$c_{R_{\mathrm{ra}}1} < c_{R_{\mathrm{ra}}} < c_{R_{\mathrm{ra}}2}; c_{R_{\mathrm{ag}}1} < c_{R_{\mathrm{ag}}} < c_{R_{\mathrm{ag}}2};$$
$$c_{Q^{\mathrm{gw}}_{\mathrm{in}}1} < c_{Q^{\mathrm{gw}}_{\mathrm{in}}} < c_{Q^{\mathrm{gw}}_{\mathrm{in}}2}; c_{Q_P1} < c_{Q_P} < c_{Q_P2};$$
$$c_{\Delta S^{\mathrm{gw}}1} < c_{\Delta S^{\mathrm{gw}}} < c_{\Delta S^{\mathrm{gw}}2} \tag{19}$$

After determining the range of correction coefficients, the correction coefficients are calculated using the process of minimizing the sum total error of the system water balance and groundwater balance equations. The minimization process is carried out using the General Reduced Gradient (GRG) method. It is obvious that the correction coefficient of the recharge coefficient is obtained as one of the outputs of this process which is in fact a recharge coefficient together with other components of the water balance in the region.

GRG method

One of the most popular methods for solving nonlinear optimization problems proposed by Lasdon et al. (1974) is The GRG method (Chapra and Canale 2011). Solving the nonlinear problem dealing with active inequalities is the main idea of this method. The reduced gradient is calculated to find the minimum in the search direction after dividing variables into a set of basic (dependent) variables and non-basic (independent) variables. This process is repeated until convergence is achieved (Venkataraman 2009). For more information, see the article by Lasdon et al. (1974), Entitled "Nonlinear optimization using the generalized reduced gradient method."

The range of correction coefficients

Regarding that the correction coefficients reflect the error in estimation of each component of balance equations, the error in estimating balance equations components was calculated and a particular amount of the error was considered as a criterion to determine the range of correction coefficients. Therefore, in this research, first the estimation error of each component was calculated and then the range of correction coefficients was determined based on the calculated errors.

Error estimation of balance equations components

In estimating the error of each of the balance equations components, different sources of error have been considered, including the error of measuring devices and computational errors. The interpolation error used to estimate the spatial distribution of point data is one of the main sources of computational error (Khazaei and Hosseini 2015). The interpolation errors were estimated by a cross validation procedure. In this procedure, an observation is removed from the data set and then the estimation model based on other data is used to predict the value of this removed observation. This process is repeated for each observation in the data set, and the errors obtained from this process can be used to find the validation error.

The mean absolute error is used for error estimation in this study as represented by Eq. (20).

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|z_{i,\mathrm{meas}} - z_{i,\mathrm{est}}\right| \tag{20}$$

where n is the number of points, $z_{i,\mathrm{meas}}$ is the amount of measured parameter in that point and $z_{i,\mathrm{est}}$ is and the estimated amount of the desired parameter in that point.

In cases that the component of interest is a function of several parameters, Eq. (21) was used to find the overall estimation error of each component in the balance equations in every year.

$$Error(f(x_1,x_2,x_3,\ldots,x_m,k)) = \sum_{j=1}^{m}\left(\left|\frac{\partial f}{\partial x_j}Error(x_j)\right|\right) \tag{21}$$

where f is the dependent function, $x_1, x_2, x_3, \ldots$ and x_m are independent parameters, k is the constant coefficient, $\frac{\partial f}{\partial x_j}$ represents partial derivative relative to the independent parameter x_j, $Error(x_j)$ is the error in the independent parameter x_j and j is the number of independent parameters.

According to Eq. (21), the errors associated with determining evapotranspiration, outlet surface runoff, groundwater inflow, groundwater withdrawal, groundwater storage change, groundwater recharge due to precipitation and groundwater recharge due to irrigation return flow components were calculated using the following equations, respectively.

$$Error(ET_{\mathrm{a}}) = \left|\frac{\partial ET_{\mathrm{a}}}{\partial T}\cdot Error(T)\right| + \left|\frac{\partial ET_{\mathrm{a}}}{\partial P}\cdot Error(P)\right| \tag{22}$$

$$Error(H^{\mathrm{sw}}_{\mathrm{out}}) = \left|\frac{\partial H^{\mathrm{sw}}_{\mathrm{out}}}{\partial C}\cdot Error(C)\right| + \left|\frac{\partial H^{\mathrm{sw}}_{\mathrm{out}}}{\partial P}\cdot Error(P)\right| \tag{23}$$

$$\begin{aligned}Error(Q^{\mathrm{gw}}_{\mathrm{in}}) = {} & \left|\frac{\partial Q^{\mathrm{gw}}_{\mathrm{in}}}{\partial K}\cdot Error(C)\right| + \left|\frac{\partial Q^{\mathrm{gw}}_{\mathrm{in}}}{\partial b}\cdot Error(b)\right| \\ & + \left|\frac{\partial Q^{\mathrm{gw}}_{\mathrm{in}}}{\partial I^{\mathrm{gw}}}\cdot Error(I^{\mathrm{gw}})\right| + \left|\frac{\partial Q^{\mathrm{gw}}_{\mathrm{in}}}{\partial l}\cdot Error(l)\right|\end{aligned} \tag{24}$$

$$Error(Q_P) = \left|\frac{\partial Q_P}{\partial Q_{well}}\cdot Error(Q_{well})\right| + \left|\frac{\partial Q_P}{\partial T_{well}}\cdot Error(T_{\mathrm{well}})\right| \tag{25}$$

$$Error(\Delta S^{\mathrm{gw}}) = \left| \frac{\partial \Delta S^{gw}}{\partial S_y} \cdot Error(S_y) \right| + \left| \frac{\partial \Delta S^{\mathrm{gw}}}{\partial \Delta H} \cdot Error(\Delta H) \right| \tag{26}$$

$$Error(R_{\mathrm{ra}}) = \left| \frac{\partial R_{\mathrm{ra}}}{\partial \alpha_{\mathrm{ra}}} \cdot Error(\alpha_{\mathrm{ra}}) \right| + \left| \frac{\partial R_{\mathrm{ra}}}{\partial P} \cdot Error(P) \right| \tag{27}$$

$$Error(R_{\mathrm{ag}}) = \left| \frac{\partial R_{\mathrm{ag}}}{\partial \alpha_{\mathrm{ag}}} \cdot Error(\alpha_{\mathrm{ag}}) \right| + \left| \frac{\partial R_{\mathrm{ra}}}{\partial Q_P} \cdot Error(Q_P) \right| \tag{28}$$

the details of the evapotranspiration error estimation are given as an example in Appendix 1.

Determining the range of correction coefficients

After calculating the estimation error of each component of the balance equations for each year, it is required to determine the range of correction coefficients for each component. So, Eq. (29) is used to determine the ranges of these coefficients.

$$\left(\overline{x}_j - a\sigma_j\right) \leq |x_j| \leq \left(\overline{x}_j + a\sigma_j\right) \tag{29}$$

where $\overline{x}_j$ is the average of the estimated error data of each of the balance equations components for a *j*-year statistical period, σ_j is their standard deviation and *a* is a constant coefficient. If the constant coefficient is considered to be 2, 95.45% of the study data would be in the above range. Considering the correction coefficient as 1 at the beginning and using Eq. (29), the target ranges were calculated for each correction coefficient. The results of calculation of the correction coefficients for each component of the system water balance and groundwater balance equations are presented below.

Constraints required to minimize the error of the system water balance equation:

$$0.91 < c_{\mathrm{p}} < 1.09,\ 0.81 < c_{\mathrm{ET}_a} < 1.19,\ 0.65 < c_{Q^{\mathrm{gw}}_{\mathrm{in}}} < 1.34,\ 0.62 < c_{Q^{\mathrm{sw}}_{\mathrm{out}}} < 1.38,\ 0.75 < c_{\Delta S^{\mathrm{gw}}} < 1.25 \tag{30}$$

Constraints required to minimize the error of the groundwater balance equation:

$$0.391 < c_{R_{\mathrm{ra}}} < 2.109,\ 0.318 < c_{R_{\mathrm{ag}}} < 2.182,\ 0.65 < c_{Q^{\mathrm{gw}}_{\mathrm{in}}} < 1.34,\ 0.824 < c_{Q_P} < 1.176,\ 0.75 < c_{\Delta S^{\mathrm{gw}}} < 1.25 \tag{31}$$

the details of calculating the range of the evapotranspiration correction coefficient are given as an example in Appendix.

Table 5 The correction coefficients

Component	Correction coefficient
Precipitation	1.09
Evapotranspiration	1.19
Outlet surface runoff	1.38
Groundwater storage changes	0.75
Groundwater inflow	0.65
Groundwater withdrawal	0.824
Groundwater recharge due to precipitation	0.391
Groundwater recharge due to irrigation return flow	1.647

Results and discussion

According to the model's constraints (Eqs. 30 and 31), the sum total absolute error of the system water balance and groundwater balance equations over the desired time period have been minimized. Due to the common precipitation component, groundwater inflow and groundwater storage changes between the two balance equations, the errors of the system and groundwater balance equations have been minimized simultaneously. The minimization process was done by selecting the GRG nonlinear engine solver method in Excel software and the correction coefficients were calculated. The obtained results are presented in Table 5.

Now, the correction coefficients obtained from the optimization process are applied in the balance equations components. The results of using the correction coefficients in the balance equations are presented in Tables 6 and 7.

As it can be seen from the results of Table 6, the sum of the absolute errors in the system water balance equation has decreased from 1510.1 to 520.6 million cubic meters. In other words, by applying correction coefficients, the total error of the system water balance equation has been reduced by about 65%. The groundwater balance equation error has also been reduced by about 50% using this method, from 202.3 to 100.3 million m^3. This percentage reduction of error indicates the capability of the method used in correcting the balance components and can be used as a tool in correcting the balance components, especially in basins where there is insufficient information to estimate the balance components.

Based on the results of Table 5, the correction coefficient of the percentage of recharge due to precipitation has been calculated to be 0.391. According to the initial value of 5% for the recharge due to precipitation coefficient, the modified recharge coefficient due to precipitation basin is 2%. In other words, only 2% of the annual precipitation in the basin has

Table 6 Variations of the error of the system water balance equation after application of correction coefficients (in million cubic meters)

Absolute error of system water balance equation after applying the correction coefficients	Absolute error of system water balance equation before applying the correction coefficients	Water year
2001	151.6	62.2
2002	131.1	32.1
2003	144.9	46.6
2004	165.8	60.9
2005	156.4	78.9
2006	148.8	50.8
2007	141.8	80.6
2008	105.3	26.2
2009	106.4	21.2
2010	118.4	50.0
2011	71.6	8.4
2012	68	2.8
Sum of absolute errors	1510.1	520.6

Table 7 Variations of the error of the groundwater balance equation after application of correction coefficients (in million cubic meters)

Water year	Absolute error of system water balance equation before applying the correction coefficients	Absolute error of system water balance equation after applying the correction coefficients
2001	25.1	12.8
2002	1.4	−7.1
2003	6.7	−3.0
2004	30.9	15.0
2005	8.1	−0.5
2006	37.4	19.7
2007	2.6	−4.2
2008	3.2	−5.5
2009	11.4	0.0
2010	39.2	22.2
2011	23.3	8.6
2012	13.0	1.6
Sum of absolute errors	202.3	100.3

caused groundwater recharge, which is consistent with previous studies in arid areas (Scanlon et al. 2006).

The recharge correction coefficient due to the irrigation return flow was also calculated according to Table 5, 1.647. As a result, the final modified recharge percentage due to annual irrigation return flow in the study area was estimated to be 16.5%.

It should be noted that the results obtained for the recharge coefficients are in agreement with the results obtained from the calibration of a numerical groundwater flow model for the Mahvelat aquifer, which was prepared at the request of Khorasan Razavi Regional Water Company (Khorasan Razavi Regional Water Company 2018).

Summary and conclusions

There are many methods to estimate the groundwater recharge. But choosing the right method in terms of accuracy, ease, cost and most importantly adaptation to the physical and hydrological characteristics of the area, is a difficult task. The purpose of estimating recharge is important in choosing the appropriate method, because the purpose will have a significant effect on the choice of temporal and spatial scales.

The method used in this paper is the water balance method for estimating groundwater recharge. In this method, all the components of the balance are estimated and then the groundwater recharge as the remaining component in the balance equation is obtained. The simplicity of this method has led to its widespread use, but the accuracy of the estimate in this method depends on the accuracy of the estimation of other components of the balance equation. Errors in estimating one or more components of the water balance can cause uncertain results in estimating recharge with this method. Therefore, in this paper an error minimization method was also used within the water balance to achieve the optimal answer in recharge estimation.

The Mahvelat Basin located in the Central Desert catchment of Iran in Khorasan Razavi Province was selected as a case study. The study was carried out using 12 consecutive years of data collected from 2001 to 2012. Due to the errors in estimating the components of the water balance equation that are caused by human, instrumental and computational error, an error phrase was added to the balance equations. To reduce the total error in the balance equations and correct the values of the components, a correction coefficient was applied to each component. The range of correction coefficients was determined based on the calculated errors in estimating of each component. Then, an error minimization process was applied simultaneously to the system water and groundwater balance equations during a 12-year statistical period. In this method, while reducing the overall error in the balance equations, components of the water balance, including recharge, should converge on their real values. It should be noted that in some situations, there may be more than one combination of values that minimizes the error, so professional judgement would also be needed to assess what is feasible. The main results of the research are as follows:

a. The minimization process used in this research reduced the error of the system water and groundwater balance equations about 55% and 65%, respectively, which shows the high performance in increasing the accuracy of balance components estimation. Therefore, considering the optimization of all components of water balance equations, the methodology could be used in other groundwater studies.
b. The recharge coefficient due to precipitation was estimated at 2%. Due to the large depth of the water table and the arid climate in the Mahvelat basin, the estimated recharge coefficient seems reasonable.
c. According to the correction coefficient obtained after the process of minimizing the balance equations error, the recharge coefficient due to the irrigation return flow was estimated to be 16.5%.

The results obtained for the recharge coefficients are consistent with the results of that were obtained by a numerical groundwater flow model that was developed for the Mahvelat aquifer by the regional water company. Applying the proposed method in the present study in basins where the data network allows the use of other methods of estimating recharge with high accuracy will help to validate this method and is suggested for future studies.

Appendix

As mentioned in "Evapotranspiration" section, the Turc equation was used to estimate evapotranspiration in the study area. To calculate the error of estimating evapotranspiration by the Turc method, the error of estimating precipitation and temperature must be combined with each other, according to the following equation (Eq. 20 in the text of the article).

$$Error(ET_{\mathrm{a}}) = \left|\frac{\partial ET_{\mathrm{a}}}{\partial T}\cdot Error(T)\right| + \left|\frac{\partial ET_{a}}{\partial P}\cdot Error(P)\right| \tag{32}$$

The mean absolute of temperature and precipitation interpolation error in each year was calculated by cross-validation procedure in Arc GIS geostatistical analyser. In addition to the interpolation error, the error of the measuring devices was also included in the calculation of the precipitation and temperature estimation error. Then, the temperature estimation error in each year was combined with the precipitation estimation error according to Eq. (32) using Microsoft Mathematics software.

According to Ekern and Chang (1985) and Gurney and Camillo (1984), there is a 10% error in estimating evapotranspiration using empirical relationships to direct measurement methods. Therefore, 10% should be added to the error calculated from the combination of temperature and precipitation error to calculate the error of estimating evapotranspiration using the Turc equation. The results of calculating the error of estimating the evapotranspiration are presented in the Table 8.

After calculating the estimation error in each year, the range of correction coefficient of evapotranspiration was calculated by Eq. (33) (Eq. (27) in the text of the article).

$$\left(\overline{x}_j - 2\sigma_j\right) \leq |x_j| \leq \left(\overline{x}_j + 2\sigma_j\right) \tag{33}$$

where $\overline{x}_j$ and σ_j are the average and standard deviation of the evapotranspiration estimation error for a j-year (12 year). The result of calculation of the range of correction coefficient of evapotranspiration is presented in below.

$$\begin{aligned} &1 - (12.54 + 2 \times 3.4)/100 < c_{ET_a} < 1 \\ &\quad + (12.54 + 2 \times 3.4)/100 \rightarrow 0.81 < c_{ET_a} < 1.19 \end{aligned} \tag{34}$$

Table 8 Evapotranspiration estimation error

Water year	Precipitation estimation error (Error (P)) (%)	temperature estimation error (Error (T)) (%)	$\frac{\partial ET_a}{\partial P}$ (%)	$\frac{\partial ET_a}{\partial T}$ (%)	$\left\lvert \frac{\partial ET_a}{\partial T} \cdot Error(T) \right\rvert + \left\lvert \frac{\partial ET_a}{\partial P} \cdot Error(P) \right\rvert$ (%)	Evapotranspiration estimation error + 10% (%)
2001	1.31	3.04	1.054071	0.000001	1.38	11.38
2002	1.04	3.11	1.054079	0.000001	1.10	11.10
2003	0.62	3.08	1.054088	0.000000	0.65	10.65
2004	1.6	3.19	1.054062	0.000002	1.69	11.69
2005	1.96	2.95	1.054045	0.000004	2.07	12.07
2006	1.69	3.13	1.054058	0.000003	1.78	11.78
2007	13.52	3.35	1.051937	0.001344	14.23	24.23
2008	1.2	3.10	1.054075	0.000001	1.26	11.26
2009	1.25	3.10	1.054073	0.000001	1.32	11.32
2010	1.89	3.56	1.054052	0.000004	1.99	11.99
2011	1.79	3.56	1.054056	0.000003	1.89	11.89
2012	1.55	2.89	1.054062	0.000002	1.63	11.63
Average						12.54
Standard deviation						3.4

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Availability of data and materials The authors confirm that data supporting the findings of this study are available in the article or its supplementary material.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest and certifies that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

References

Abdelhalim A, Sefelnasr A, Ismail E (2020) Response of the interaction between surface water and groundwater to climate change and proposed megastructure. J Afr Earth Sc 162:103723

Ahmadi T(2013) Groundwater recharge estimation using various methods and making comparison their results, case study: neishaboor plain. M.Sc. thesis, Ferdowsi University, Iran (in Persian)

Bear J (1979) Groundwater hydraulics. McGraw-Hill, New York

Bredehoeft JD (2003) From models to performance assessment: the conceptualization problem. Groundwater 41(5):571–577

Cambraia Neto AJ, Rodrigues LN (2020) Evaluation of groundwater recharge estimation methods in a watershed in the Brazilian Savannah. Environ Earth Sci. https://doi.org/10.1007/s12665-020-8884-x

Chapra SC, Canale RP (2011) Numerical methods for engineers, vol 1221. McGraw-hill, New York

Daneshvar A (2012) Provide an optimization process to reduce errors in the water balance equation. M.Sc. thesis, Ferdowsi University, Iran (in Persian)

Delleur JW (ed) (2006) The handbook of groundwater engineering. CRC Press, Boca Raton

Ekern PC, Chang JH (1985) Pan evaporation: State of Hawaii, 1894–1983. Department of Land and Natural Resources, Division of Water and Land Development, Honolulu, Hawaii, Rep. R74

Gurney RJ, Camillo PJ (1984) Modelling daily evapotranspiration using remotely sensed data. J Hydrol 69(1–4):305–324

Healy RW, Scanlon BR (2010) Estimating groundwater recharge. Cambridge University Press, Cambridge

Hengl T (2006) Finding the right pixel size. Comput Geosci 32(9):1283–1298

Joshi SK, Rai SP, Sinha R, Gupta S, Densmore AL, Rawat YS, Shekhar S (2018) Tracing groundwater recharge sources in the northwestern Indian alluvial aquifer using water isotopes (δ18O, δ2H and 3H). J Hydrol 559:835–847

Khazaei B, Hosseini SM (2015) Improving the performance of water balance equation using fuzzy logic approach. J Hydrol 524:538–548

Khorasan Razavi Regional Water Company (2011) Report of the consolidation of water resources studies in the desert basin, vol III. Analysis of statistics and information and water balance. Section V combined studies and water bill, Appendix No. 22, water balance in the Mahvelat studies area (in Persian)

Khorasan Razavi Regional Water Company (2015) Explanatory report on the ban on Feyzabad plain (in Persian)

Khorasan Razavi Regional Water Company (2018) Modeling saltwater intrusion into aquifers in the vicinity of deserts case study. Mahvelat Aquifer (in Persian)

King AC, Raiber M, Cox ME, Cendón DI (2017) Comparison of groundwater recharge estimation techniques in an alluvial aquifer system with an intermittent/ephemeral stream (Queensland, Australia). Hydrogeol J 25(6):1759–1777

Lasdon LS, Fox RL, Ratner MW (1974) Nonlinear optimization using the generalized reduced gradient method. Rev Franç D’automat Inf Rech Opér Rec Opér 8(3):73–103

Manghi F, Mortazavi B, Crother C, Hamdi MR (2009) Estimating regional groundwater recharge using a hydrological budget method. Water Resour Manage 23(12):2475–2489

Meredith E, Blais N (2019) Quantifying irrigation recharge sources using groundwater modeling. Agric Water Manage 214:9–16

Mistry P, Suryanarayana TM (2021) Study of groundwater recharge estimation using empirical equations in North Gujarat, India. SSRN J. https://doi.org/10.2139/ssrn.3806570

Mushtaha AM, Van Camp M, Walraevens K (2019) Quantification of recharge and runoff from rainfall using new gis tool: example of the Gaza Strip aquifer. Water 11(1):84

Nugraha GU, Lubis RF, Bakti H, Hartanto P (2021) Groundwater recharge estimation using water budget and water table fluctuation method in the Jakarta groundwater basin. Indonesian Assoc Geol J (IAGI j) 1(1):25–38

Nyagwambo NL (2006) Groundwater recharge estimation and water resources assessment in a tropical crystalline basement aquifer. A. A. Balkema, Florida

Omidifar H (2019) Investigation of the stability of Azghand aquifer in Khorasan Razavi province using a numerical model. M.Sc. thesis, Shahrood University of Technology, Iran (in Persian)

Scanlon BR, Healy RW, Cook PG (2003) Choosing appropriate techniques for quantifying groundwater recharge. Hydrogeol J 10(1):18–39

Scanlon BR, Keese KE, Flint AL, Flint LE, Gaye CB, Edmunds WM, Simmers I (2006) Global synthesis of groundwater recharge in semiarid and arid regions. Hydrol Process 20(15):3335–3370

Singh MK, Ghosh S (2021) Groundwater flow modeling for cachar, India using MODFLOW. ISH J Hydraul Eng 2:1–1

Sokolov AA, Chapman TG (1974) Methods for water balance computations: an international guide for research and practice, vol 17. Unesco Press, Paris

Soodegi A (2018) Birjand alluvial aquifer recharge estimation. M.Sc. thesis, Shahrood University, Iran (in Persian)

Sun X (2005) A water balance approach to groundwater recharge estimation in Montagu area of the Western Klein Karoo. University of the Western Cape, South Africa

Tauxe JD (1994) Porous medium advection–dispersion modeling in a geographic information system. Center for Research in Water Resources, Bureau of Engineering Research, University of Texas at Austin, Austin

Tilahun K, Merkel B (2009) Estimation of groundwater recharge using a GIS-based distributed water balance model in Dire Dawa, Ethiopia. Hydrogeol J 17:1443–1457

Todd KD, Mays LW (2005) Groundwater hydrology, 3rd edn. John Wiley & Sons, New Jersey

Turc L (1954) The water balance of soils relation between precipitation, evaporation and flow. Ann Agron 5:491–569

Venkataraman P (2009) Applied optimization with MATLAB programming, 2nd edn. John Wiley & Sons Inc., Hoboken, NJ, USA

Xu Y, Beekman HE (2019) Review: groundwater recharge estimation in arid and semi-arid southern Africa. Hydrogeol J 27:929–943

Yimam AY, Assefa TT, Sishu FK, Tilahun SA, Reyes MR, Prasad PV (2021) Estimating surface and groundwater irrigation potential under different conservation agricultural practices and irrigation systems in the Ethiopian highlands. Water 13(12):1645

Zeinali M, Azari A, Heidari MM (2020) Simulating unsaturated zone of soil for estimating the recharge rate and flow exchange between a river and an aquifer. Water Resour Manage 34(1):425–443