Inventory management of manufacturers with yield uncertainty and lateral transshipment

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Abstract: This article deals with the issue of inventory management of one identical product in a manufacturers' network. Manufacturers use lateral transshipments between each other in response to uncertainties in yield and demand to maximise the total profit. The demand of each manufacturer is considered random as a non-identical continuous probability distribution and their corresponding yield follows some possible scenarios. The objective of our model is to determine the optimal production amount and lateral transshipments in order to maximise the total profit considering the proceeds from sale of goods and salvage of remaining product and the cost of production, lateral transshipments, and shortages. The problem is modelled as a nonlinear constrained programming and the optimal solution is obtained by Karush-Kuhn-Tucker approach. Sensitivity analysis of uncertainty parameters based on a numerical example showed that the utility of using lateral transshipment policy increases with increasing the uncertainty in production yield.

Keywords: inventory management; yield uncertainty; lateral transshipment.

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1 Introduction

In the real world, we are always faced with factors that are impossible to predict precisely. Therefore, decision makers have always sought solutions to minimise adverse effects of these uncertainties on the predetermined goals of enterprises. Manufacturing enterprises face high rate of uncertainties and cannot be sure about the demand of the next period or be confident in their production according to their planning. The demand of a manufacturing firm, depending on the purchasing power of customers and the competitive environment of the market can vary from one period to another. In addition, in these firms, two incidents of no production or production less than which has been planned may occur. Some reasons such as stop supplying of raw materials, non-acceptance of the material's quality delivery delay in addition to events such as flood, fire, earthquake and war can cause the manufacturing plant or fully cut off production. On the other hand, some factors such as machine failure and the variability of workers' performance can reduce production amount from which has been planned. For example, Alsobhi et al. (2018) investigated the issue of supply disruption due to transportation hazards and poor packaging and proposed a mathematical model to minimise total costs including damage, shipping and packaging costs and at the same time reducing the risk of supply disruption. In addition, Nikabadi et al. (2021) introduced a model of mix integer programming (MIP) in the issue of resilient supply portfolio selection under disruption risks by minimising the disturbance conditional value at risk is optimised and solved it with two meta-heuristic algorithms: harmony search and imperialist competition. The results showed that the value of the objective function is nearly same in the two algorithms. Saithong and Luong (2020) derived an optimal inventory policy for one retailer under a (r, S) replenishment structure when supply disruption may occur and demand is stochastic. In addition, Hu et al. (2020) analysed the coupling influence of multiple disruptions on the supply chain system by coupling utility model of multiple disruption which constructed based on the two functions of utility and degree of coupling and showed that the intensity coefficient of disruption is more significant than its frequency one.

In conventional models, when manufacturers face an unexpected inventory deficit, they only have two options: postpone customer demand, if possible, or lose the demand. However, there are several ways to cope on these uncertainties, including the immediate provision of goods from another source (Deiranlou et al., 2021), the reservation of goods from a certain source (Mohammadivojdan et al., 2021), and using lateral transshipment (Li et al., 2020) when there is a manufacturers' network with centralised decision-making structure. Lateral transshipment means sending products from manufacturer with surplus inventory in hand to other one which faces shortage in his inventory in order to maximise usage of system's inventory. In this paper, in the case of uncertainties in the production yield and demand of manufacturers with centralised decision-making structure, we utilise the lateral transshipment which can prevent payment of excessive purchases or reservations from other sources, and manufacturers can use their inventory to balance the demand and commodities in the entire network.

2 Literature review

The issue of using lateral transshipment in the face of uncertainty in yield and demand has been considered by researchers in recent years. In most of their work, the purpose of the research was to determine the optimal inventory and lateral transshipment policies in a way that minimises the total cost of system. From the perspective of the time of lateral transshipment, the literature can be divided into two subsets: proactive and reactive. Reactive lateral transshipment occurs after that uncertain parameters, such as demand are determined in order to cope with shortages, while proactive lateral transshipment occurs at the fixed determined points in the time to confront potential shortage in the system between firms of same echelon. In the context of proactive transshipment, Diks and De Kok (1996) presented consistent appropriate share rationing policy (CAS) for a system that has multiple retailers and legible lead time. In this policy, the goal is to balance the inventory of system by maintaining the share of each echelon from the total inventory planned for system. Banerjee et al. (2003) and Burton and Banerjee (2005) compared transshipment inventory equalisation policy (TIE) against transshipment based on availability policy (TBA). TIE a proactive policy that balances the number of supply and transshipment days for each retailer or distributor, while TBA policy is a reactive policy in which transshipments are initialised based on the shortage quantity of retailers. The results showed that TBA policy is merely more effective in preventing shortages, while TIE policy has a lower total cost. Tiacci and Saetta (2011) considered a system with two retailers and a central warehouse as a supplier in the presence of random demand. In this research, they used a heuristic method for determining the value and time of lateral transshipment, as well as the optimal parameters of the (s, S) replenishment policy. Dan et al. (2016) considered a supply chain with two retailers and a dominant manufacturer who is responsible for determining wholesale and transshipment prices before the start of the sales season. In this study, they assumed that demand is random and divided the sales season into two periods, at the beginning of the first period, the retailers independently determine the quantity of orders from the manufacturer and at the beginning of the second period, in a competitive environment, according to available inventory and realised demand, retailers determine the amounts of proactive transshipment between each other.

Recently, Meissner and Senicheva (2018) studied a centralised multi-retail location inventory system for one identical product under order-up-to-level replenishment policy in the case of some finite proactive transshipment opportunities in each order cycle. Every retailer faces with random demand and if they cannot satisfy that, there is no chance for backorders and demand is lost. The research objective was to find optimal transshipment policy in order to maximise the total profit of the network in a finite time horizon. They proposed a dynamic programming approach which could theoretically obtain optimal decisions by using Bellman's equation. However, an approach to optimal policy was practically found impossible for large size of the problem. So, they presented a forward approximate dynamic programming to find near-optimal decisions and they showed that it performs much better in comparison to pervious methods in the literature. Also, Feng et al. (2018) investigated the replenishment and lateral transshipment decisions in two-retailer inventory system in which two kind of lateral transshipment are allowed: emergency lateral transshipment (ELT) and preventive lateral transshipment (PLT). They proved that in both cases unique Nash equilibrium is existed and PLT solution converges to newsvendor one when transshipment price increases.

In the field of reactive transshipment, Robinson (1990) considered an inventory system with multiple retailers and periods given the assumption that transshipment time is negligible. The results of the research showed that the optimal solution can be obtained only for a system that has either multiple identical retailers or just two non-identical ones. Herer and Tzur (2001) studied the problem of an inventory system with two retailers when they are faced with a deterministic but dynamic demand over the time. They sought to determine optimal decisions for ordering and lateral transshipment in definite horizon time. They also investigated the key features of the system that conduct the framework for solving the problem in polynomial time. In the following, Herer and Tzur (2003) expanded the problem in the presence of several retailers. Hu et al. (2007) considered an inventory system with a decentralised decision-making structure which consists of two manufacturers who are faced with uncertainties in yield and demand and they can benefit from transshipment between each other. The researcher's goal was to determine the price of transshipment in such a way that persuades them to coordinate with each other for making overall optimal decisions. The results of the study showed that it is not possible to determine this kind of price in all cases. Hu et al. (2008) studied a system with two manufacturers in a multi-period condition, any manufacturer in each period is faced with uncertainty in yield and demand. To counteract this uncertainty, the authors presented an optimal inventory policy that included lateral transshipment after that demand and yield was exactly known. Olsson (2009) was looking for an optimal (R, Q) replenishment policy while considering possibility of transshipment and complete pooling assumption. One of Olson's interesting findings was that even in the presence of two retailers with similar characteristics, optimal policy is not symmetric. Özdemir et al. (2013) studied a supply chain involving several retailers receive their products from a capacitated supplier. The retailers can also use lateral transshipment between each other to satisfy more of their random demand. For ordering policy, it is assumed that retailers use order-up-to policy. They found that the behaviour of the system is dependent on the supplier's capacity. Lee and Park (2016) investigated two retailers and one uncertain supplier in the case of using transshipment and existence uncertainty in the supplier's capacity which can be limited or unlimited. They expanded the problem in both centralised and decentralised decision-making structures. In a decentralised structure, in the face of limited capacity, the supplier assigns the order of each retailer according to proportional allocation rule. This situation causes inflating retailers' order due to rationing game. They showed that in the optimal solution, if the supplier's limited capacity and transshipment prices between retailers are low, the order amounts will not be inflated. In the recent years, Shao (2018) proposed the optimal dynamic transshipment policies for a decentralised dual manufacturing system comprising of one disrupted and one another requested manufacturer which can produce up to its capacity over T periods. They investigated the problem for both independent and competitive market scenarios and characterised the optimal transshipment policies. The results indicated that the transshipment conditions in the competitive scenario are not as strict as those in the independent one. In the case of closed-loop supply chain network design, Jabbarzadeh et al. (2018) investigated the reactive lateral transshipment strategy with operational and disruption risks. Dehghani and Abbasi (2018) proposed a new transshipment policy for one echelon blood supply chain as a kind of perishable items with two service centres based on the age of the oldest item in the system and proved that it could be reduced the total inventory cost in comparison to actual case in Australian hospitals. Van Wijk et al. (2019) characterised the structure of lateral transshipment policy as a threshold type for two inventory location of spare part which face multiple demand classes by using stochastic dynamic programming.

In this paper, we consider a set of manufacturers in which they are uncertain in the amount of production and demand and use lateral transshipment between each other to deal with these uncertainties. Due to the existence of time-consuming process in production, the lateral transshipments are proactive and should be specified at the beginning of the period so that each manufacturer knows the planned quantities for production.

The main contribution of this paper is investigating lateral transshipment as an approach to deal with uncertainties in production yield and demand. Hence, we focus to contribute the utility of using lateral transshipment policy in the uncertainty environment. The objective is to determine the optimal production amount and lateral transshipments in order to maximise the total profit considering the proceeds from sale of goods and salvage of remaining product and the cost of production, lateral transshipments, and shortages. The problem is modelled as a nonlinear constrained programming. With regard to the concavity of objective function and convexity of solution space, the Karush-Kuhn-Tucker conditions (K-K-T) is used to obtaining the global optimal solution.

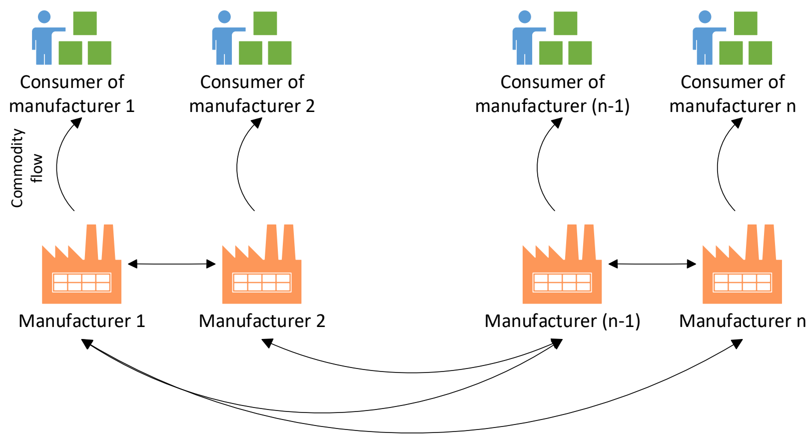
The paper is organised as follows. Following the introduction in Section 1, the problem definition and mathematical model are presented in Section 2. An exact method for solving the problem will be described in Section 3. The numerical results are presented in Section 4. Finally, conclusions and suggestions for future research are discussed in Section 5.

3 Problem definition and mathematical model

In this study, a network of manufacturers (Figure 1) with centralised decision-making structure in the presence of uncertainty in the yield of production and demand is considered. They produce one identical product in a single period and their demands are different. The production yield for each manufacturer is considered independently uncertain which means that the final production amount can be less than which has been planned due to some factors such as failure in supply of raw materials, non-acceptance of them and etc.

In this problem, some possible scenarios for production yield of each manufacturer are considered, which, from their combination, the yield scenario set for the manufacturers network is obtained, In other words, all manufacturers theoretically has n scenarios for their production yield which means that all yield scenarios for manufacturers network equal to 2^n. In order to cope with these uncertainties, manufacturers use lateral transshipment between each other to increase their overall profit. In fact, the goal is to determine the optimal production quantity and lateral transshipments in order to maximise the profit of entire network considering profits and cost including sales and salvage profits and the production, lateral transshipment and shortage costs.

Figure 1 Manufacturers network with lateral transshipments (see online version for colours)



Consider I as the set of manufacturers that can be counted by i index. We assume that the salvage price per unit (L_i) is lower than the production cost per unit (c_i) for each manufacturer, equation (1), So that manufacturers have incentives to meet their demand preferably from their inventory in hand. Also, for transshipment cost between manufacturers (s_{ij}), triangular inequality is established, which means that the cost of direct lateral transshipments is always less than the indirect one. In other words, no manufacturer is being used as an intermediate centre for lateral transshipment between the two others, equation (2).

$$L_i < c_i; \qquad \forall i \in I \tag{1}$$

$$s_{ij} < s_{ik} + s_{kj} \qquad \forall i, j, k \in I \tag{2}$$

In this article, it is also considered the logical assumptions of complete pooling policy for inventories presented by Tagaras (1989), equations (3)–(6). This means that each manufacturer with excess inventory uses its full capacity to cope with inventory shortages in other ones. In these inequalities, r_i and π_i represent the sales price and shortage cost per unit in i^{th} manufacturer, respectively. Equation (3) expresses that for all manufacturers production cost is less than supply costs from the others which implies that no manufacturer meets all its demand from the manufacturers. In equation (4), $s_{ij} + c_i - c_j$ represents net cost of lateral transshipment from i^{th} manufacturer to the j^{th} one and $r_j - c_j + \pi_j$ represents net shortage cost in manufacturer j. Obviously, to justify the lateral transshipment, its net cost must be less than the shortage cost. Equations (5) and (6), in the situation where two manufacturers at the same time face excess or shortage in their inventory, show that net lateral transshipment cost between them is greater than the differences in salvaging prices and shortage costs respectively. Therefore, the lateral transshipment between them is not profitable anymore.

$$c_j < c_i + s_{ij}; \qquad \forall i, j \in I \qquad (3)$$

$$s_{ij} + c_i - c_j - L_i < r_j - c_j + \pi_j; \qquad \forall i, j \in I \qquad (4)$$

$$L_j < s_{ij} + c_i - c_j + L_i; \qquad \forall i, j \in I \qquad (5)$$

$$r_j - c_j + \pi_j < s_{ij} + c_i - c_j + r_i - c_i + \pi_i; \qquad \forall i, j \in I \qquad (6)$$

To formulate the problem sets and indices, parameters, and variables will be defined as follows.

3.1 Sets and indices

I Set of manufacturers.

n Number of manufacturers.

i, j, k, l Index of manufacturer $i, j, k, l \in \{1, \ldots, n\}$.

$\overline{h}_i$ Number of exclusive scenarios for production yield of i^{th} manufacturer.

H_i Set of exclusive scenarios for production yield of i^{th} manufacturer.

h_i Index of exclusive scenarios for production yield of i^{th} manufacturer $h_i \in \{1, \ldots, \overline{h}_i\}$.

3.2 Parameters

c_i Production cost per unit in i^{th} manufacturer.

L_i Salvage price per unit in i^{th} manufacturer.

π_i Shortage cost per unit in i^{th} manufacturer.

r_i Sales price per unit in i^{th} manufacturer.

s_{ij} Lateral transshipment cost from i^{th} manufacturer to j^{th} one.

$p_{h_i}^{(i)}$ Probability of h_i^{th} scenario of production yield for i^{th} manufacturer.

$y_{h_i}^{(i)}$ Production yield of i^{th} manufacturer when h_i^{th} scenario for its production yield occurs.

$f_i(u_i)$ Probability density function of demand for i^{th} manufacturer.

$F_i(u_i)$ Cumulative distribution function of demand for i^{th} manufacturer.

σ_i Standard deviation of normal distribution for i^{th} manufacturer's demand.

μ_i Mean of the normal distribution for i^{th} manufacturer's demand.

σ_{y_i} Standard deviation of i^{th} manufacturer's production yield random variable.

μ_{y_i} Mean of i^{th} manufacturer's production yield random variable.

$$\mu_{y_i} = \sum_{h_i=1}^{\bar{h}_i} p_{h_i}^{(i)} y_{h_i}^{(i)}; \qquad \forall i \in I \tag{7}$$

3.3 Variables

Q_i Production quantity of i^{th} manufacturer (decision variable).

$X_{ijh_1\ldots h_n}$ Amount of lateral transshipment from i^{th} manufacturer to j^{th} one when h_1^{th} scenario for the first manufacturer up to h_n^{th} scenario for n^{th} one occur (decision variable).

Z Expected value of manufacturer network profit.

$$\begin{aligned}
Max\ Z = \sum_{h_1=1}^{\bar{h}_1} \ldots \sum_{h_n=1}^{\bar{h}_n} p_{h_1}^{(1)} \ldots p_{h_n}^{(n)} \Bigg(\sum_{i=1}^{n} r_i \Bigg[\int_0^{y_{h_i}^{(i)} Q_i - \sum_{j\neq i} X_{ijh_1\ldots h_n} + \sum_{j\neq 1} X_{jih_1\ldots h_n}} u_i f_i(u_i)\, du_i \\
+ \int_{y_{h_i}^{(i)} Q_i - \sum_{j\neq i} X_{ijh_1\ldots h_n} + \sum_{j\neq 1} X_{jih_1\ldots h_n}}^{\infty} \Bigg(y_{h_i}^{(i)} Q_i + \sum_{j\neq i} X_{jih_1\ldots h_n} \\
- \sum_{j\neq i} X_{ijh_1\ldots h_n} \Bigg) f_i(u_i)\, du_i \Bigg] \\
+ L_i \int_0^{y_{h_i}^{(i)} Q_i - \sum_{j\neq i} X_{ijh_1\ldots h_n} + \sum_{j\neq 1} X_{jih_1\ldots h_n}} \Bigg(y_{h_i}^{(i)} Q_i + \sum_{j\neq i} X_{jih_1\ldots h_n} \\
- \sum_{j\neq i} X_{ijh_1\ldots h_n} - u_i \Bigg) f_i(u_i)\, du_i \\
+ \pi_i \int_{y_{h_i}^{(i)} Q_i - \sum_{j\neq i} X_{ijh_1\ldots h_n} + \sum_{j\neq 1} X_{jih_1\ldots h_n}}^{\infty} \Bigg(u_i - y_{h_i}^{(i)} Q_i - \sum_{j\neq 1} X_{jih_1\ldots h_n} \\
+ \sum_{j\neq i} X_{ijh_1\ldots h_n} \Bigg) f_i(u_i)\, du_i - \sum_{j\neq i} s_{ji} X_{jih_1\ldots h_n} - c_i y_{h_i}^{(i)} Q_i \Bigg)
\end{aligned} \tag{8}$$

s.t.

$$\sum_{j\neq i} X_{ijh_1\ldots h_n} \leq y_{h_i}^{(i)} Q_i; \qquad \forall i \in I, \forall h_1 \in H_1, \ldots \forall h_n \in H_n \tag{9}$$

$$X_{ijh_1\ldots h_n} \geq 0; \qquad \forall i, j, \in I, \forall h_1 \in H_1, \ldots, \forall h_n \in H_n \tag{10}$$

$$Q_i \geq 0; \qquad \forall i \in I \tag{11}$$

Equation (8) is the objective function of the problem that shows the expected value of profit of manufacturer network. The first and second phrases state the revenue of each manufacturer when demand is more or less than stock in hand, respectively. The third and fourth phrases indicate the proceeds from salvaging non-sold goods and cost of shortage at the end of the period for each manufacturer, respectively. The fifth and sixth phrases express the lateral transshipment and production costs. Constraint (9) guarantees that the lateral transshipment of a manufacturer is not more than its production yield.

4 Solving procedure

We solve the problem based on derivative approach. First, the concavity of objective function is proven.

Preposition: The objective function (Z) is strictly concave.

Proof: Assume that $X = \{x_1, x_2, \ldots, x_n\}$ is a vector of n variables. $f(X)$, function of vector X, is strictly concave if and only if inequality (13) holds for all X vector values (Simon and Blume, 1994):

$$XH_X X^t < 0 \tag{12}$$

In which X^t and X_X are translation and Hessian matrix of vector X, respectively.

In our problem, variable vector is $X = [Q_1 \ldots Q_n \; X_{121\ldots1} \ldots X_{n(n-1)\bar{h}_1\ldots\bar{h}_n}]$. Based on inequality (12) the expected value of objective function (Z) is concave if and only if equation (13) is negative for all feasible values of decision variables.

$$\begin{bmatrix} Q_1 & \cdots & Q_n & X_{121\ldots1} & \cdots & X_{n(n-1)\bar{h}_1\ldots\bar{h}_n} \end{bmatrix}$$

$$\times \begin{bmatrix} \frac{\partial^2 Z}{\partial Q_1^2} & \cdots & \frac{\partial^2 Z}{\partial Q_1 \partial Q_n} & \frac{\partial^2 Z}{\partial Q_1 \partial Z_{121\ldots1}} & \cdots & \frac{\partial^2 Z}{\partial Q_1 \partial X_{n(n-1)\bar{h}_1\ldots\bar{h}_n}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 Z}{\partial q_n \partial Q_1} & \cdots & \frac{\partial^2 Z}{\partial Q_n^2} & \frac{\partial^2 Z}{\partial Q_n \partial X_{121\ldots1}^2} & \cdots & \frac{\partial^2 Z}{\partial Q_n \partial X_{n(n-1)\bar{h}_1\ldots\bar{h}_2}} \\ \frac{\partial^2 Z}{\partial X_{121\ldots1} \partial Q_1} & \cdots & \frac{\partial^2 Z}{\partial X_{121\ldots1} \partial Q_n} & \frac{\partial^2 Z}{\partial X_{121\ldots1}^2} & \cdots & \frac{\partial^2 Z}{\partial X_{121\ldots1} \partial X_{n(n-1)\bar{h}_1\ldots\bar{h}_n}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 Z}{\partial X_{n(n-1)\bar{h}\ldots\bar{h}_n} \partial Q_n} & \cdots & \frac{\partial^2 Z}{\partial_{n(n-1)\bar{h}_1\ldots\bar{h}_n} \partial Q_n} & \frac{\partial^2 Z}{\partial X_{n(n-1)\bar{h}_1\ldots\bar{h}_n} \partial X_{121\ldots1}} & \cdots & \frac{\partial^2 Z}{\partial X_{n(n-1)\bar{h}_1\ldots\bar{h}_n}^2} \end{bmatrix}$$

$$\times \begin{bmatrix} Q_1 \\ \vdots \\ Q_n \\ X_{121\ldots1} \\ \vdots \\ X_{n(n-1)\bar{h}_1\ldots\bar{h}_n} \end{bmatrix}$$

$$
\begin{aligned}
&= \sum_{i=1}^{n} Q_i^2 \frac{\partial^2 Z}{\partial Q_i^2} + \sum_{h_1}^{\bar{h}_1} \cdots \sum_{h_n=1}^{\bar{h}_n} \sum_{i=1}^{n} \sum_{j \neq i} 2 Q_i X_{ijh_1 \ldots h_n} \frac{\partial^2 Z}{\partial X_{ijh_1 \ldots h_n} \partial Q_i} \\
&+ \sum_{h_1=1}^{\bar{h}_1} \cdots \sum_{h_{n=1}}^{\bar{h}_n} \sum_{i=1}^{n} \sum_{j \neq i} 2 Q_i X_{jih_1 \ldots h_n} \frac{\partial^2 Z}{\partial X_{jih_1 \ldots h_n} \partial Q_i} \\
&+ \sum_{h_1=1}^{\bar{h}_1} \cdots \sum_{h_n=1}^{\bar{h}_n} \sum_{i=1}^{n} \sum_{j \neq 1} \sum_{k \neq j,i} 2 X_{ijh_1 \ldots h_2} X_{ikh_1 \ldots h_n} \frac{\partial^2 Z}{\partial X_{ijh_1 \ldots h_n} \partial X_{ikh_1 \ldots h_n}} \\
&+ \sum_{h_1=1}^{\bar{h}_1} \cdots \sum_{h_n=1}^{\bar{h}_n} \sum_{i=1}^{n} \sum_{j \neq 1} \sum_{k \neq j,i} 2 X_{ijh_1 \ldots h_n} X_{kjh_1 \ldots h_n} \frac{\partial^2 Z}{\partial X_{ijh_1 \ldots h_n} k d X_{kjh_1 \ldots h_n}} \\
&+ \sum_{h_1=1}^{\bar{h}_1} \cdots \sum_{h_n=1}^{\bar{h}_n} \sum_{i=1}^{n} \sum_{j \neq 1} 2 X_{ijh_1 \ldots h_n} X_{jih_1 \ldots h_n} \frac{\partial^2 Z}{\partial X_{ijh_1 \ldots h_n} \partial X_{kjh_1 \ldots h_n}} \\
&+ \sum_{h_1=1}^{\bar{h}_1} \cdots \sum_{h_n=1}^{\bar{h}_n} \sum_{i=1}^{n} \sum_{j \neq 1} \sum_{k \neq j,i} 2 X_{ijh_1 \ldots h_n} X_{kih_1 \ldots h_n} \frac{\partial^2 Z}{\partial X_{ijh_1 \ldots h_n} \partial X_{kih_1 \ldots h_n}} \\
&+ \sum_{h_1=1}^{\bar{h}_1} \cdots \sum_{h_n=1}^{\bar{h}_n} \sum_{i=1}^{n} \sum_{j \neq 1} \sum_{k \neq j,i} 2 X_{ijh_1 \ldots h_n} X_{jkh_1 \ldots h_n} \frac{\partial^2 Z}{\partial X_{ijh_1 \ldots h_n} \partial X_{jkh_1 \ldots h_n}} \\
&+ \sum_{h_1=1}^{\bar{h}_1} \cdots \sum_{h_n=1}^{\bar{h}_n} \sum_{i=1}^{n} \sum_{j \neq 1} X_{ijh_1 \ldots h_n}^2 \frac{\partial^2 Z}{\partial X_{ijh_1 \ldots h_n}^2} \\
&= \sum_{h_1=1}^{\bar{h}_1} \cdots \sum_{h_n=1}^{\bar{h}_n} \sum_{i=1}^{n} p_{h_1}^{(1)} \cdots p_{h_n}^{(n)} \left(y_{h_i}^{(i)} Q_i \sum_{j \neq i} X_{ijh_1 \ldots h_n} + \sum_{j \neq i} X_{jih_1 \ldots h_n} \right)^2 \\
&\times f_i \left(y_{h_i}^{(i)} Q_i \sum_{k \neq i} X_{ikh_1 \ldots h_n} + \sum_{k \neq i} X_{kih_1 \ldots h_n} \right) (-r_i - \pi_i + L_i)
\end{aligned}
\tag{13}
$$

All coefficients of equation (13) are positive for all feasible values of decision variables, except the last term, which always takes a negative value because in the logic of all markets, the selling price is always higher than the salvage one. The product of positive values in a negative one is always negative. Therefore, the concavity of objective function is proved.

With regard to the concavity of objective function and convexity of solution space, the K-K-T conditions are used to obtaining the global optimal solution. $\lambda_{ijh_1 \ldots h_n}$ is the Lagrange multipliers of constraints which set the limit of transshipment amount amongst manufacturers [set of constrains (9)]. In addition, λ_i and $\lambda_{ijh_1 \ldots h_n}$ are Lagrange multipliers of constraints related to non-negativity of production quantity [set of constrains (11)] and lateral transshipment decision variables [set of constrains (10)] respectively. Based on K-K-T approach, optimal production quantity, lateral transshipments and Lagrange coefficients can be easily achieved using GAMS software by solving the system of equations (14)–(24). In other words, the optimal solution is obtained by any feasible solution of relevant nonlinear programming structure in GAMS software. Although, as the number of manufacturer increases, the size of the problem increases exponentially, and hence, the problem cannot be solved within any acceptable time. The solving procedure time for a problem with four manufacturers is about four hours.

$$\sum_{h_1=1}^{\bar{h}_1}\cdots\sum_{h_n=1}^{\bar{h}_n} p_{h_1}^{(1)}\dots p_{h_n}^{(n)} y_{h_i}^{(i)}\Big[(L_i - r_i - \pi_i)F_i\Big(y_{h_i}^{(i)}Q_i - \sum_{j\neq i} X_{ijh_1\dots h_n} + \sum_{j\neq i} X_{jih_1\dots h_n}\Big) + (r_i + \pi_i - c_i)\Big] + \sum_{h_1=1}^{\bar{h}_1}\cdots\sum_{h_n=1}^{\bar{h}_n} \lambda_{ih_1\dots h_n} y_{h_i}^{(i)} + \lambda_i = 0; \quad \forall i \in I \tag{14}$$

$$p_{h_1}^{(1)}\dots p_{h_n}^{(n)}\Big(r_j + \pi_j - r_i - \pi_i - s_{ij} + (r_i - L_i + \pi_i)F_i\Big(y_{h_i}^{(i)}Q_i - \sum_{k\neq i} X_{ikh_1\dots h_n} + \sum_{k\neq i} X_{kih_1\dots h_n}\Big) - (r_j - L_j + \pi_j)F_j\Big(y_{h_i}^{(i)}Q_j - \sum_{k\neq j} X_{jkh_1\dots h_n} + \sum_{j\neq j} X_{kjh_1\dots h_n}\Big)\Big) - \lambda_{ih_1\dots h_n} + \lambda_{ijh_1\dots h_n} = 0; \quad \forall i, j, \in I, \forall h_1 \in H_1, \dots, \forall h_n \in H_n \tag{15}$$

$$Q_i\Big(\sum_{h_1=1}^{\bar{h}_1}\cdots\sum_{h_n=1}^{\bar{h}_n} p_{h_1}^{(1)}\dots p_{h_n}^{(n)} y_{h_i}^{(i)}(L_i - r_i - \pi_i)F_i\Big(y_{h_i}^{(i)}Q_i - \sum_{j\neq i} X_{ijh_1\dots h_n} - \sum_{j\neq i} X_{jih_1\dots h_n}\Big) + y_{h_i}^{i}\Big) = 0; \quad \forall i \in I \tag{16}$$

$$X_{ijh_1\dots h_n}\dots\Big(p_{h_1}^{(1)}\dots p_{h_n}^{(n)}\Big(r_j + \pi_j - r_i - \pi_i - s_{ij} + (r_i - L_i + \pi_i)F_i\Big(y_{h_i}^{(i)}Q_i - \sum_{k\neq i} X_{ikh_1\dots h_n} + \sum_{k\neq i} X_{kih_1\dots h_n}\Big) - (r_j - L_j + \pi_j)F_j\Big(y_{h_i}^{(i)}Q_j - \sum_{k\neq j} X_{jkh_1\dots h_n} + \sum_{k\neq j} X_{kjh_1\dots h_n}\Big)\Big) - \lambda_{ih_1\dots h_n}\Big) = 0; \quad \forall i, j, \in I, \forall h_1 \in H_1, \dots, \forall h_n \in H_n \tag{17}$$

$$\lambda_{ih_1\dots h_n}\Big(\sum_{j\neq i} X_{ijh_1} - y_{h_i}^{(i)}Q_i\Big) = 0 \quad \forall i, j, \in I, \forall h_1 \in H_1, \dots, \forall h_n \in H_n \tag{18}$$

$$\lambda_{ih_1\dots h_n} \geq 0; \quad \forall i, j, \in I, \forall h_1 \in H_1, \dots, \forall h_n \in H_n \tag{19}$$

$$\lambda_i \geq 0; \quad \forall i, j, \in I, \forall h_1 \in H_1, \dots, \forall h_n \in H_n \tag{20}$$

$$\lambda_{ijh_1\dots h_n} \geq 0; \quad \forall i, j, \in I, \forall h_1 \in H_1, \dots, \forall h_n \in H_n \tag{21}$$

$$\sum_{j\neq i} X_{ijh_1\dots h_n} \leq y_{h_i}^{(i)}Q_i \quad \forall i, j, \in I, \forall h_1 \in H_1, \dots, \forall h_n \in H_n \tag{22}$$

5 Numerical example

There are three manufacturers with normal distribution for demand and two scenarios including optimistic and pessimistic for production yield. For all manufacturers, the production yield in pessimistic and optimistic scenarios is 40% $(y_1^{(i)})$ and 80% $(y_2^{(i)})$ with occurrence probability of 0.3 $(p_1^{(i)})$ and 0.7 $(p_2^{(i)})$, respectively.

Table 1 Cost, income and demand distribution parameters

Manufacturer	*Production cost ($/unit)*	*Salvage price ($/unit)*	*Shortage cost ($/unit)*	*Sales price ($/unit)*	*Demand distribution parameters*	
					μ_i *(unit)*	σ_i *(unit)*
1	500	170	20	1,100	200	65
2	525	150	22	1,200	200	25
3	550	160	25	1,150	200	45

Table 2 Lateral transshipment cost between manufacturers ($/unit)

From \ *To*	*1*	*2*	*3*
1	0	177	183
2	177	0	173
3	183	173	0

Table 1 includes the cost and revenue parameters along with demand distribution parameters. The cost of lateral transshipment between manufacturers is shown in Table 2.

Table 3 Production yield scenarios of manufacturers network

Manufacturer	*Production yield scenarios*							
	*(0.027)** 1**	*(0.063) 2*	*(0.063) 3*	*(0.147) 4*	*(0.063) 5*	*(0.147) 6*	*(0.147) 7*	*(0.343) 8*
1	40%	40%	40%	40%	80%	80%	80%	80%
2	40%	40%	80%	80%	40%	40%	80%	80%
3	40%	80%	40%	80%	40%	80%	40%	80%

Notes: *Scenario number.
**Scenario probability.

The production yield scenarios for manufacturers network that derive from combination of production yield scenarios of each manufacturer is given in Table 3.

Table 4 Optimal production quantity and total profits of manufacturers for the proposed model

Z	Q_1	Q_2	Q_3
303,523.01	351.63	284.08	297.95

Table 5 Optimal production quantity and total profits of manufacturers with no lateral transshipment

Z	Q_1	Q_2	Q_3
287,429.04	309.16	275.14	286.45

Table 4 shows the optimal production quantity and expected value of manufacturer network profit. To illustrate the benefit of using lateral transshipments, we again solve the problem under the condition of no transshipments. Table 5 indicates the optimal solution of this case.

By comparison with the results of Tables 4 and 5, one can conclude that using lateral transshipment, total profit of manufacturers increases from 287,578.97 to 303,614.9, which indicates an increase of 5.6%. Also, production quantity in all manufacturers in the case of lateral transshipment is more than no-transshipment condition which can be interpreted due to lateral transshipments opportunity between manufacturers across the network.

In Tables 4 and 5, we can see $Q_1 > Q_3 > Q_2$, this refer to the standard deviation of demand probability distribution which is $\sigma_1 > \sigma_3 > \sigma_2$. In other word, more uncertainty in the demand of manufacturer results in more production quantity.

Table 6 Optimal quantity of transshipments between manufacturers

To manufacturer	*From manufacturer*	*Production yield scenarios*							
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*
2	1	27.87	0	0	0	67.73	61.59	0	0
3	1	0	0	0	0	41.07	0	59.2	0
1	2	0	0	0	19.45	0	0	0	0
3	2	0	0	36.35	0	0	0	17.84	0
1	3	0	0	0	24.94	0	0	0	0
2	3	0	62.08	0	0	0	23.45	0	0

Table 6 demonstrates the optimal quantity of lateral transshipment between manufacturers under different scenarios of production yield. In scenario 8, no manufacturer delivers any commodity to the others due to occurrence of optimistic scenario for production yield of all manufacturers. In other scenarios, the lateral transshipment is always accomplished only from one manufacturer that its production is matching with optimistic yield scenario to the one that its pessimistic yield scenario has occurred. In scenario 1, the total lateral transshipment quantities are the least in comparison to other scenarios in which transshipments are provided because in this scenario production yield of all manufacturers is in pessimistic status. Also, in scenario 1, the lateral transshipment is carried out on behalf of manufacturer 1, due to low production cost and high demand uncertainty in this manufacturer.

6 Sensitivity analysis

In this section, the sensitivity analysis of objective function and utility of using lateral transshipment policy with respect to the cost and income parameters, as well as to parameters related to uncertainty of production and demand are investigated.

The sensitivity analysis of the objective function with respect to the cost and income parameters is presented in Table 7. It can be concluded that the objective function is highly sensitive to the production cost and sales price and in contrast, it is low sensitive to the shortage and lateral transshipment costs and the salvaging price. In the other words, with similar lateral transshipment costs, objective function is insensitive by decreasing or increasing shortage cost or salvaging price. therefore, these factors are approximately neutral.

Table 7 Sensitivity analysis of the objective function

Percentage change \ Parameter	c_i	π_i	s_{ij}	r_i	L_i
–20%	22.76%	0.08%	0.53%	–40.88%	–0.9%
–10%	11.16%	0.04%	0.26%	–20.63%	–0.46%
10%	–10.76%	–0.04%	–0.26%	20.91%	0.47%
20%	–21.14%	–0.08%	–0.51%	42.02%	0.97%

Table 8 Sensitivity analysis of the objective function and utility of using lateral transshipments

Percentage change \ Percentage change in σ_i	–20%	–10%	10%	20%
Objective function	2.59%	1.33%	–1.37%	–2.79%
Utility of lateral transshipment policy	5.95%	5.78%	5.41%	5.22%

Table 8 shows sensitivity analysis of the objective function and utility of using lateral transshipment policy when standard deviation of the demand probability distribution for all manufacturers (σ_i) is changed. As can be seen, by increasing the standard deviation, the objective function will be reduced due to an increase of uncertainty in the manufacturers' demand and utility of using lateral transshipment remains constant which means that benefit of lateral transshipment policy is not sensitive to the demand uncertainty.

Table 9 Sensitivity analysis of the objective function

$y_1^{(i)}$	–20%			0%			20%		
$y_2^{(i)}$ \ $p_2^{(i)}$	–20%	0%	20%	–20%	0%	20%	–20%	0%	20%
–20%	2.9%	0.0%	–2.3%	4.5%	2.8%	1.6%	5.7%	4.9%	4.4%
0%	1.5%	–2.7%	–6.2%	2.9%	0.0%	–2.3%	4.2%	2.3%	0.9%
20%	0.4%	–4.8%	–9.2%	1.7%	–2.2%	–5.5%	2.9%	0.0%	–2.3%

Table 9 shows sensitivity analysis of the objective function due to changing in probability of all manufacturers' optimistic yield scenario $(p_2^{(i)})$ in addition to production yield in both pessimistic and optimistic scenarios for all manufacturers $(y_1^{(i)}, y_2^{(i)})$ which directly affects on production uncertainty base on equation (25).

$$\sigma_{y_i} = \sqrt{\sum_{h_i=1}^{\overline{h_i}} \left(y_{h_i}^{(i)} - \mu_{y_i}\right)^2 p_{h_i}^{(i)}}; \qquad \forall i \in I \tag{25}$$

Considering the results achieved in Table 9, the following conclusions can be mentioned:

- By decreasing the probability of optimistic yield scenario the objective function will be reduced due to increase of uncertainty in the production based on equation (25).
- With increasing production yield in pessimistic scenario, the objective function increases due to reduction in uncertainty of production quantity. On the contrary, with increasing the production yield in optimistic scenario, the objective function decreases due to an increase in production uncertainty based on equation (25).
- The objective function is not sensitive to the simultaneous changes in production yield in both optimistic and pessimistic scenarios, because in this condition, the coefficient of variation for production yield random variable for all manufacturers remains constant based on equations (7) and (25).
- The highest amount of objective function occurs in a situation where the production yield in optimistic scenario are at their lowest level and on the contrary, the production yield in pessimistic scenario and probability of optimistic scenario are at their highest level. Because in this situation production uncertainty is the least based on equation (19). The lowest amount of objective function occurs exactly when opposite of the above mentioned conditions are established.

Table 10 Sensitivity analysis of using lateral transshipment policy

$y_1^{(i)}$	–20%			0%			20%		
$y_2^{(i)}$ \ $p_2^{(i)}$	–20%	0%	20%	–20%	0%	20%	–20%	0%	20%
–20%	8.1%	5.3%	2.6%	4.3%	3.1%	1.6%	1.5%	1.2%	0.7%
0%	11.0%	7.0%	3.3%	8.1%	5.3%	2.6%	5.0%	3.6%	1.8%
20%	12.8%	7.9%	3.8%	10.5%	6.7%	3.2%	8.1%	5.3%	2.6%

The effects of changing in parameters associated with production uncertainty on the utility of using lateral transshipment are investigated in Table 10. In other words, the percentage increase in manufacturers' profit per different levels of production uncertainty parameters will be analysed. According to the results obtained in Table 10, the following conclusion can be deduced:

- The utility of using lateral transshipments increases with reduction in probability of optimistic yield scenario due to the increase in production uncertainty based on equation (25).
- With the increase in production yield of pessimistic scenario, the utility of using lateral transshipments decreases given the fact that production uncertainty is reduced based on equation (25). On the other hand, with the increase in production yield of optimistic scenario, the benefit of using lateral transshipments is increased, which can be explained by increasing the production uncertainty based on equation (25).
- The benefit of using lateral transshipments is not sensitive to simultaneous changes in production yield of both optimistic and pessimistic scenarios, meaning that it does not change with their simultaneous increment or decrement, because in this

condition, coefficient of variation for production yield random variable for all manufacturers remains constant based on equations (7) and (25).

- The greatest benefit of using lateral transshipment policy occurs in a situation where the probability of optimistic scenario for production yield and production yield in pessimistic scenario are at their lowest level and on the contrary, production yield in optimistic scenario is at its highest level. Because in this situation, production uncertainty is increased the most based on equation (25). The least benefit of using lateral transshipment policy occurs exactly when the opposite of the above mentioned conditions are established.

7 Conclusions and future studies

In this paper, considering the significant effect of uncertainty on the production space, a set of manufacturers managed by an enterprise facing with uncertainties in the production and demand was considered. The main contribution of the paper is investigating lateral transshipment as an approach to deal with uncertainties which can prevent payment of excessive purchases or reservations from other sources, and manufacturers can use their inventory to balance the demand and commodities in the entire network with centralised decision-making structure. The problem was modelled according to the continuity of the demand probability distribution function in the form of a nonlinear constrained programming. Then, based on the K-K-T conditions, optimal quantities of planned production and lateral transshipments were achieved by GAMS software. In the following, in order to validate the model, a numerical example consist of three manufacturers facing with normal distribution for their demand and with the assumption of existence two optimistic and pessimistic scenarios for production yield was studied. Finally, based on this example, the effects of parameter changes on manufacturers' profit and utility of using lateral transshipments were investigated. The results indicated a reduction in manufacturers' profit with increasing uncertainty in the production and demand amount. In addition, they showed that the benefit of lateral transshipment policy increases with respect to growth in production uncertainty, but does not change significantly by variation in demand uncertainty. Based on the results for more than four manufacturers, the problem cannot be solved in acceptable time. Hence, in future studies a heuristic algorithm can be developed to cope on this defect, Also, the problem can be extended by considering continuous form of probabilistic distribution for production yield and studying the problem in the presence of several periods for production. Also, considering the concept of risk aversion by adding the constraints of response and service levels for each manufacturer can increase the value of the problem.

References

Alsobhi, S.A., Krishnan, K.K., Gupta, D. and Almaktoom, A.T. (2018) 'Analysis of damage costs in supply chain systems', *International Journal of Industrial and Systems Engineering*, Vol. 28, No. 1, pp.70–98.

Banerjee, A., Burton, J. and Banerjee, S. (2003) 'A simulation study of lateral shipments in single supplier, multiple buyers supply chain networks', *International Journal of Production Economics*, Vol. 81, pp.103–114.

Burton, J. and Banerjee, A. (2005) 'Cost-parametric analysis of lateral transshipment policies in two-echelon supply chains', *International Journal of Production Economics*, Vol. 93, pp.169–178.

Dan, B., He, Q., Zheng, K. and Liu, R. (2016) 'Ordering and pricing model of retailers' preventive transshipment dominated by manufacturer with conditional return', *Computers & Industrial Engineering*, Vol. 100, pp.24–33.

Dehghani, M. and Abbasi, B. (2018) 'An age-based lateral-transshipment policy for perishable items', *International Journal of Production Economics*, Vol. 198, pp.93–103.

Deiranlou, M., Dehghanian, F. and Pirayesh, M. (2021) 'The effect of purchasing strategy on the economic production quantity model subject to random machine breakdown', *International Journal of Procurement Management*, Vol. 14, No. 3, pp.277–307.

Diks, E.B. and De Kok, A.G. (1996) 'Controlling a divergent 2-echelon network with transshipments using the consistent appropriate share rationing policy', *International Journal of Production Economics*, Vol. 45, Nos. 1–3, pp.369–379.

Feng, P., Wu, F., Fung, R.Y. and Jia, T. (2018) 'Evaluation of two transshipment policies in a two-location decentralized inventory system under partial backordering', *Transportation Research Part E: Logistics and Transportation Review*, Vol. 118, No. 3, pp.207–224.

Herer, Y.T. and Tzur, M. (2001) 'The dynamic transshipment problem', *Naval Research Logistics (NRL)*, Vol. 48, No. 5, pp.386–408.

Herer, Y.T. and Tzur, M. (2003) 'Optimal and heuristic algorithms for the multi-location dynamic transshipment problem with fixed transshipment costs', *IIE Transactions*, Vol. 35, No. 5, pp.419–432.

Hu, H., Ma, H., He, J., Li, B. and Gao, Y. (2020) 'Coupling influence of multiple disruptions in supply chain', *International Journal of Industrial and Systems Engineering*, Vol. 35, No. 2, pp.135–157.

Hu, X., Duenyas, I. and Kapuscinski, R. (2007) 'Existence of coordinating transshipment prices in a two-location inventory model', *Management Science*, Vol. 53, No. 8, pp.1289–1302.

Hu, X., Duenyas, I. and Kapuscinski, R. (2008) 'Optimal joint inventory and transshipment control under uncertain capacity', *Operations Research*, Vol. 56, No. 4, pp.881–897.

Jabbarzadeh, A., Haughton, M. and Khosrojerdi, A. (2018) 'Closed-loop supply chain network design under disruption risks: a robust approach with real world application', *Computers & Industrial Engineering*, Vol. 116, pp.178–191.

Lee, C. and Park, K.S. (2016) 'Inventory and transshipment decisions in the rationing game under capacity uncertainty', *Omega*, Vol. 65, pp.82–97.

Li, Y., Liao, Y., Hu, X. and Shen, W. (2020) 'Lateral transshipment with partial request and random switching', *Omega*, Vol. 92, pp.102–134.

Meissner, J. and Senicheva, O.V. (2018) 'Approximate dynamic programming for lateral transshipment problems in multi-location inventory systems', *European Journal of Operational Research*, Vol. 265, No. 1, pp.49–64.

Mohammadivojdan, R., Merzifonlouglu, Y. and Geunes, J. (2021) 'Procurement portfolio planning for a newsvendor with supplier delivery uncertainty', *European Journal of Operational Research*, Article in press, DOI: 10.1016/j.ejor.2021.05.026.

Nikabadi, M.S., Shambayati, H. and Ataei, N. (2021) 'Selection of resilient supply portfolio under disruption risks in supply chain', *International Journal of Industrial and Systems Engineering*, Vol. 37, No. 4, pp.432–462.

Olsson, F. (2009) 'Optimal policies for inventory systems with lateral transshipments', *International Journal of Production Economics*, Vol. 118, No. 1, pp.175–184.

Özdemir, D., Yücesan, E. and Herer, Y.T. (2013) 'Multi-location transshipment problem with capacitated production', *European Journal of Operational Research*, Vol. 226, No. 3, pp.425–435.

Robinson, L.W. (1990) 'Optimal and approximate policies in multiperiod, multilocation inventory models with transshipments', *Operations Research*, Vol. 38, No. 2, pp.278–295.

Saithong, C. and Luong, H.T. (2020) 'Effects of supply disruption on optimal inventory policy: the case of (r, S) continuous review inventory policy', *International Journal of Industrial and Systems Engineering*, Vol. 35, No. 4, pp.482–508.

Shao, X.F. (2018) 'Production disruption, compensation, and transshipment policies', *Omega*, Vol. 74, pp.37–49.

Simon, C.P. and Blume, L., (1994). *Mathematics for Economists*, Vol. 7, Norton, New York.

Tagaras, G. (1989) 'Effects of pooling on the optimization and service levels of two-location inventory systems', *IIE Transactions*, Vol. 21, No. 3, pp.250–257.

Tiacci, L. and Saetta, S. (2011) 'A heuristic for balancing the inventory level of different locations through lateral shipments', *International Journal of Production Economics*, Vol. 131, No. 1, pp.87–95.

Van Wijk, A.C.C., Adan, I.J. and Van Houtum, G.J. (2019) 'Optimal lateral transshipment policies for a two location inventory problem with multiple demand classes', *European Journal of Operational Research*, Vol. 272, No. 2, pp.481–495.