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Fourier Like Systems, Frame of Translates and their Oblique Duals on LCA-groups

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Abstract

The theory of frames of translates has an essential role in many areas of mathematics and its applications such as wavelet theory and reconstruction of signals from sample values [1−4, 6, 11, 12, 13]. A lattice system of translates is a sequence in $L^2(\mathbb{R})$ that has the form $\mathcal{T}(g) = \{g(. - ak)\}_{k\in\mathbb{Z}}$ where $g \in L^2(\mathbb{R})$ and $a > 0$ are fixed. In the setting of $L^2(\mathbb{R})$, it is known that frames of translates can be characterized in terms of a 1-periodic function ([3, 6]). More precisely, for $g \in L^2(\mathbb{R})$, if we define $\Phi_g(\omega) = \sum_{k\in\mathbb{Z}} |\widehat{\phi}(\omega+k)|^2$, then Φ_g is a 1-periodic function which characterizes frames of translates as follows.

(a) $\mathcal{T}(g)$ is a frame sequence if and only if there exist $0 < A \le B < \infty$ such that $A \le \Phi_g \le B$, a.e. on the zero set of Φ_g.

(b) $\mathcal{T}(g)$ is a Riesz basis for the closure span of $\mathcal{T}(g)$ if and only if there exist $0 < A \le B < \infty$ such that $A \le \Phi_g \le B$, a.e.

(c) $\mathcal{T}(g)$ is an orthonormal basis for the closure span of $\mathcal{T}(g)$ if and only if $\Phi_g = 1$ a.e.

Our goal in this presentation is a generalization of frames of translates in the setting of locally compact abelian groups. Let G be a locally compact abelian (LCA) group and Γ be a uniform lattice in G (i.e. a discrete subgroup of G which is co-compact), with the annihilator Γ^* in $\widehat{G}$ (the dual group of G)[5, 7, 8, 10, 14 − 16]. For $g \in L^2(G)$, a system of translates generated by g via Γ, is defined as

$$\mathcal{T}(g) = \{g(. + \gamma)\}_{\gamma\in\Gamma}$$

We define a Γ^*-periodic function Φ_g on $\widehat{\Gamma}$ and investigate a characterization of translates of $g \in L^2(G)$ to have some properties. We achieve our goal by using an isometry from $L^2(G)$ into $L^2(\widehat{\Gamma})$, in such a way that the system of translates in $L^2(G)$ is transferred to a nice Fourier-like system in $L^2(\widehat{\Gamma})$. To do so, we consider a fix $\varphi \in L^2(\widehat{\Gamma})$ and define the Fourier-like system generated by φ as $\mathcal{E}(\varphi) = \{X_\gamma\varphi\}_{\gamma\in\Gamma}$, where X_γ is the corresponding character γ on $\widehat{\Gamma}$. We deduce the structure of the canonical dual frame of a frame sequence $\mathcal{T}(g)$. Using the fact that the frame operator of a frame of translates commutes with the translation operator, it is shown that the canonical dual frame of $\mathcal{T}(g)$ has the same form $\mathcal{T}(h)$ for some $h \in \overline{\text{span}}(\mathcal{T}(g))$. Some properties of Φ_g which are useful in the study of the translates sequence generated by g are

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investigated. In particular, it is shown that if Φ_g is continuous, then $\mathcal{T}(g)$ can not be a redundant frame.

Keywords: locally compact abelian group, Fourier-like system, Fourier-like frame, frame of translates, oblique dual.

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