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The Relationship Between Non-additivity Valuations, Cash Flows and Sales Growth

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Abstract
This study examines the effects of non-additivity valuations on cash flows and sales growth by using the Choquet fuzzy approach as a pooled non-additive integral. The study targets 62 parent companies with 322 subsidiaries in the Tehran stock market from 2011 to 2019. The study divides firms' assets into four categories (inventories, receivables, fixed, and long-term investments). It uses the Choquet integral's properties to determine the firms' total non-additivity values. The Choquet integral approach used in this study considers the synergy of a set's componential factors in different measurements by considering and implementing the weights and coefficients of the elements. The results indicate that while the market valuation of companies (based on non-additivity valuations) has no significant correlation with their operating cash flows, it has a positive correlation with their sales growth, which could be attributed to the synergy created by the business combination of the parent companies with their subsidiaries. Moreover, the findings show a significant correlation between sales growth and the market price-to-book ratio as a simple approach to measuring a company's performance. However, the results suggest that non-additivity valuations offer a better estimation in measuring the efficiency of companies than market price-to-book ratio-based approaches. Indeed, the valuations of companies are determined based on their abilities in using their resources compared to similar companies in the same industry when they use non-additivity valuations. These findings are expected to be very helpful for potential investors and shareholders.

Keywords Pricing · Non-additivity valuations · Cash flows · Sales growth · Performance

1 Introduction

This paper attempts to address theoretically and empirically an aggregation issue identified several decades ago (Miller, 1973) and discussed by Hodgson et al. (1993) and Gibbins and Willett (1997). In particular, Hodgson et al. (1993)

Extended author information available on the last page of the article address the aggregation issue through statistical transaction theory and consider the effect of interactions between assets. The current study extends this line of research, as it empirically captures the interactions between assets in place. The Choquet fuzzy Integral used in this paper estimates the average interactions (the Choquet capacities) for a sample of firms, which are applied to the individual firm's asset structure. A breakdown of enterprise value is facilitated by identifying and adding values for specific interactions (synergies or inhibitions) between subsets of assets to the existing assets' values. By applying the concept of Choquet capacities (Choquet, 1954), which are non-additive measures (Wu & Gonzalez, 1999), we can model the value of different combinations of assets (subsidiary companies) and estimate how much they contribute to enterprise value. The key parameters can be evaluated for samples, and the enterprise values can be predicted. The predicted enterprise values are based on 'Choquet capacities' for the industry.

Non-additive measures (Choquet capacities or fuzzy measures) and fuzzy integral theory are an evolution of classical measure theory. Non-additive measures and fuzzy integral theory consider the importance of criteria and interactions among them and have excellent potential for applications in different scientific fields. Non-additive measures and corresponding integrals have been studied and applied in diverse areas (Sabri et al., 2020).

One of the primary goals of financial reporting is to present helpful information for decision-making. Hence, accounting information systems play a significant role in providing necessary information for advancing the organisations' projects and the countries' economic environment. However, the valuation of a company has always been challenging for investors and financial analysts who are constantly seeking to identify the influential factors on the company's value to determine the company's actual value by controlling such factors (Gibbins & Willett, 1977).

Researchers have already studied valuation methods (Lazzati & Menichini, 2018). The influence of the values of companies on investment decisions also holds for the parent companies (holding subsidiary companies), as, in the viewpoint of such companies' investors, the more accurate the estimate of the company's value is, the more precise their decisions will be. However, the main question is which valuation method can determine and estimate the value of a company more accurately and realistically.

Accounting data-based valuation models test the relationship between accounting information and market values in capital market studies. The main challenge in applying the Olson model as one of the primary and widely used methods of valuing companies is using the details of other information that are not reported through financial accounting (Bergmann & Schultze, 2018).

The well-known accounting data-based valuation models are calculated based on the *additivity* feature by focusing on discounting future cash flows (FCF) or the residual income (RIM), failing to consider *synergy or inhibition* between the assets (Paugam et al., 2018). Additivity means that the value of assets is obtained through algebraic addition operations regardless of how the company combines them. Here, the sum of each asset item's fair value equals all assets' fair value (Vehmanen, 2013).

On the other hand, business units mainly use acquisitions and mergers as tools for business growth. Regardless of the main reasons companies undertake to combine business units, the first goal is to help business units create more value than the values of independent firms. This hypothesis introduces the meaning of synergy. When a merger occurs, this synergy is defined as a condition where the profitability of the companies merged into a single company outweighs the sum of each of those companies profits (Ramdas & Kumar, 2014, Campá García, 2019).

Due to the computational nature of additivity, traditional financial methods used to value companies face inaccuracy and uncertainty in the accounting data. On the other hand, as additivity has a simple structure, the additivity-based financial valuations methods do not consider the synergy between items of a coherent set, such as a company's assets (Casta & Bry, 1998). Moreover, as business units seek to create synergies between their assets and increase their profitability, the valuation only done by adding the individual value of each of their assets will not be desirable (Vehmanen, 2013).

To address the above issue, Bry and Casta (2003) consider the synergy between the company's assets and propose a model by applying the fuzzy integrals, arguing that as current financial valuation methods are based on the additivity feature, business units are not considered structured sets of assets. However, there is little evidence in the literature to show how the adoption of the fuzzy Integral can offer a better valuation for firms. Instead, the quantitative approach has been used in all studies conducted by classic researchers to measure value and profit (Willett, 1987).

To contribute to the literature on the above gap, this study investigates the application of the Choquet fuzzy Integral (Choquet, 1959), which is a non-additive pooled function, in valuing the holdings' subsidiary companies (parent companies) to find out if using this approach could be an appropriate alternative for valuing companies.

This paper further examines the relationship between the Non-Additivity Valuations (measured in this paper) and the firm's operating cash flow and sales growth efficiencies. Finally, it examines the relationship between the operational performance and the price-to-book ratio to evaluate the approach's ability to estimate efficiency compared to the ratio using OLS regression.

The remainder of this paper is structured as follows: Sect. 2 presents the literature and develops the theoretical framework and hypotheses. Section 3 describes the research method, and Sect. 4 presents the findings. Section 5 focuses on the discussion and conclusion and provides recommendations for future studies.

2 Theoretical Foundations and the Related Literature

Valuation of firms refers to estimating the business unit's economic value. In valuation methods, it is generally the market value of companies used. However, this value is merely based on the stock market value of companies and does not consider the structure of their assets, and thus, it is faced with some limitations. Therefore, a more comprehensive valuation of a company would consider the structure of the company's assets so that it could provide necessary information for the investors who intend to acquire or purchase the company (Ragas & Culp, 2021).

Ijiri (1975) argues that the fair value of a business unit lacks an additive feature and that the value does not equal the sum of the business unit's fair value of its assets. He also shows that goodwill indicates the significance of the merger function, as the total value of a holding does not necessarily equal the sum of its individual subsidiaries' value. In other words, each business unit is considered as non-additive by nature; a fact many researchers have addressed as it is one of the oldest topics in accounting (Yang, 1927, Canning, 1929, Paton & Littleton, 1970, Miller, 1973, Gynther, 1969).

Valuing companies through additive structures cannot correctly display the synergistic effects induced by organising the efficiency of resources. Therefore, a comprehensive approach seems to be needed. According to Basu and Waymire (2008), 'economic intangibles are cumulative, synergistic, and frequently inseparable from other tangible assets and/or economic intangibles'. So, it implies that sometimes we cannot estimate a separate accounting value for individual intangible assets. Therefore, it appears reasonable to consider the synergistic value between a company's current assets as internal goodwill.

It could, thus, be argued that when the componential elements of a unit are non-additive and exert mutual synergies on each other, their common synergistic effect can only be considered by fuzzy functions. Such nonlinear operative functions can consider the synergistic effect between the componential elements of a unit (Özdilek, 2020, Su et al., 2019). The Choquet integral is a fuzzy function used in multicriteria decision-making processes, control systems, and game theory (Khan et al., 2019).

To better organise business activities in today's world, investing is often done not by individuals but by legal entities, such as a parent or subsidiary companies. The parent or the leading company is the one that develops its business activities by creating subsidiary companies. Parent companies seek to establish specialised holdings by acquiring other companies, which often operate in similar industries and thus enjoy some benefits, the most important of which is the synergy resulting from this combination.

On the other hand, the valuation of companies is one of the most essential and complicated economic concepts in any country in such a way; that in developed countries with a firm and highly organised capital market, the valuation of companies is done by investment banks, investing counsellors, and the specific standards and rules of each industry. However, due to the low efficiency of the capital market and the minimal activity of newly established investment banks, the valuation of companies is performed inexpertly via trial and error.

NICOARĂ (2019) argues that the valuation of a business unit according to the current valuation methods is based on the theory that the value of each business unit is the sum of all items on the unit's balance sheet, including existing assets and tangible and intangible fixed assets. However, such methods do not consider the synergy of the assets, which bears a value unaccounted for in the corporates' balance sheets. According to Nicholls (2020), most of the approaches to integrate social and environmental accounts are carried out with additive financial statements, while these accounts are inherently non-addictive, and applying additive functions will lead to an exchange of value and adverse effects of one event with the value and or positive effects of another event. Lubberink and Wilet (2020) emphasise that the correlation between market value and accounting data must be measured through non-additive linear models.

Recognition and measurement are two essential concepts for valuation in accounting and preparing financial reports, which is regarded as the primary purpose of accounting (Bry & Casta, 2003). Mathematically, measurement refers to numerically delineating the related structure of the observed elements. Two schools of thought influence the application of measurement in accounting: The classical approach is also called the 'measure theory' and is directly based on *physical sciences*. It is concerned with those measurements and numerical values that are additive. The modern approach, also called the measurement theory, is related to social sciences and develops the measurement theory to determine the value of perceptual concepts, such as quantifying psychological characteristics (Stevens, 1958).

Addressing the criticisms against the traditional accounting models, many efforts have been made to integrate the qualitative approach into accounting theory (Abdel-Magid, 1979). However, these attempts failed to achieve their intended goals, and measurement remained a weak quantitative factor in financial accounting. The quantitative approach has been used in all studies conducted by classic researchers to measure value and profit (Willett, 1987).

The measurement seeks to offer some information concerning the subject being measured to reflect its economic reality. Let us assume that P is a unit of the primary and actual members, and S is a unit constituted of those members' substitutions (s), representing the members' values. The function m: ps is called the measurement process. Prescriptively, one of the features of the "m" function is that it shows the structure of the leading members through their substitutes. Therefore, it is crucially important to identify the nature of a measured member, as it helps determine all the detailed properties that a function needs to have to be considered a cumulative function (Grabisch et al., 2008). Maintaining the non-additive structure of economic realities requires the application of other characteristics of measures (or other features) which can be carried out by disregarding the additivity assumption, using the uniformity feature (uniform function), and finally, using the Choquet integral's capacities.

The Choquet integral is a powerful nonlinear aggregation operator that has been generalised and applied to many areas, such as image processing, multicriteria decision-making, skeletal age-at-death estimation in forensic anthropology, classification and pattern recognition. (Anderson et al., 2014). For example, Anzilli and Glove (2020) proposed a healthcare decision support system based on non-additive measures and the Choquet Integral. This methodology has been intensively applied in many real-world applications due to its capability to represent interactions among criteria and, thus, to model a wide range of preference structures. Given the above, Anzilli and Glove (2020) used the Choquet integral for a disease risk assessment by eliciting a non-additive measure (capacity), which was impossible with the previous method.

As another example, the application of non-additive measures and corresponding integrals for the problem of tourism management was discussed in Iraq. This problem was solved mathematically using the non-additive model (non-additive measures and corresponding integrals). Then, fuzzy integrals (Sugeno integral, Choquet integral and Shilkret Integral) were applied concerning λ-non additive measures to evaluate the grade of the gratification of the tourist of staying in a particular town for determining the best evaluation. (Sabri et al., 2020).

Torra (2014) reviewed the Choquet integral and several definitions of non-additive measures and some of their results. He briefly discussed some of the applications of the techniques and showed that non-additive measures and integrals solve some of the shortcomings of alternative models. His findings show that non-additive measures, when combined with Choquet integrals have more expressive capabilities than additive measures with the Lebesgue Integral. He concluded that in some practical cases, the best option is not always to select the model with better modelling capabilities because it can cause overfitting (Torra, 2014).

As Sarala and Jothi (2016) stated, the other tools for information of aggregation are the weighted average method and linear Integral. These methods assume that the information sources are non-interactive (or) independent; hence, their weighted effects are considered additivity. Still, in the real world, this approach is not realistic. They showed that For the human subjective evaluation processes, it would be better to apply the Choquet and Sugeno integrals model together with the definition of capacity Choquets, in which the additive property is unnecessary. They showed that using fuzzy integrals is a better tool for criteria aggregation in decision-making and evaluating medicine with illustrations of the hierarchical structure of capacity Choquets (Sarala & Jothi, 2016).

An essential feature of non–additive measures and fuzzy integrals is that they can represent the importance of individual information sources and interactions among them. There are many applications of non-additive measures and fuzzy integrals, such as image processing, multicriteria decision-making, information fusion, classification, and pattern recognition (Sabri et al., 2020).

A particular type of nonlinear Integral is a fuzzy integral to a non-additive measure. The fuzzy integrals are appropriate tools to represent criteria weights with non-additive characteristics as a non-additive measure. One of the most famous fuzzy integrals is the Choquet Integral. Choquet Integral has algebraic properties, making fuzzy Integral more suitable for multicriteria decision-making problems (Torra et al., 2020). This methodology has been intensively applied in many real-world applications to model a wide range of preference structures (Anzilli & Giove, 2020).

Non-additive measures (also known as fuzzy measures, capacities, and monotonic games) are a generalisation of additive measures in which the additivity axiom is replaced by monotonicity concerning set inclusion. So, the functions can be integrated concerning these measures. This paper considers the integration of functions for non-additive measures using the Choquet integral (Torra et al., 2020).

The redefinition of the concept of fuzzy measurement implies calling into question the definition of the integral about a measure (Sugeno, 1993, Choquet, 1954). Sugeno's Integral of a measurable function f: X ~ [0,1] relative to a measure m is defined as:

$$S(f) = \max(\min(\alpha, (\{xf(x) > \alpha\})))$$

Since it involves only operators max and min, this Integral is not appropriate for modelling synergy.

Choquet's Integral of a measurable function f: X ~ [0,l] relative to a measure μ is defined as:

$$C(f) = \int \mu(\{xf(x) > y\})d_y$$

For example, in the case of a finite set X = {× 1, × 2, … xn} with:

$$0 \leq f(x1)\cdots \leq f(xn) \leq 1Ai = \{\mathrm{x_i}, \ldots, \mathrm{x_n}\}$$

we have:

$$C_{(f)} = \sum_{i=1}^{n} [f(x_i) - f(x_{i-1})]\mu(A_i)$$

Moreover, 1(A = B) is the "indicator function", which takes value 1 if A = B and 0; otherwise, we can write:

$$C(f) = \int \left(\sum_{A \epsilon P(X)} \mu(A).1(A = \{xf(x) > y\}) \right) d_y$$

$$C(f) = \sum_{A \epsilon P(X)} \mu(A).\left(\int 1(A = \{xf(x) > y\}) \right) d_y$$

If we denote g_A {f) as the value of the expression J_l (A = {x I fix) > y}) dy, Choquet's integral may be expressed in the following manner:

$$C_{(f)} = \sum_{A \epsilon P(X)} \mu(A).g_A(f)$$

Choquet's Integral involves the sum and the usual product as operators. It reduces to Lebesgue's Integral when u is Lebesgue's measure and extends it to non-additive measures. As a result of monotonicity, it increasingly concerns the measure and the integrand. Hence, Choquet's Integral can be used as an aggregation operator (Bry & Casta, 2003).

The financial accounting system is based on the additivity concept. Accordingly, in asset valuation methods, the value of N set of assets is assumed to equal the sum of each unit's asset value (Casta et al., 2011).

$$V\left(\sum_{i=1} A_i\right) = \sum_{i=1} V(A_i)$$

The concept of "measurement" is used in the computational part of financial accounting. In the general sense, the concept is based on several properties, including the additivity:

$$m(\cup A_n) = \sum_n m(A_n)$$

If the set Ω consists of three separate members, A, B and C Ω = {A, B, C}.

Then its subsets will be as follows:

$$Z = \{\{A\}, \{B\}, \{C\}, \{A, B\}, \{B, C\}, \{A, C\}, \{A, B, C\}, \emptyset\}$$

The Z set includes a variety of combinations between its members. As {A}, {B}, and {C} are three separate members, therefore:

$$m(\cup A_n) = m(A \cup B \cup C) = m(\Omega) = m(A) + m(B) + m(C)$$

The above assumption is one of the essential assumptions of fuzzy integrals, as suggested by the Riemann Integral (1857) and the Lebesgue Integral (1928). However, as this assumption fails to consider the synergies between the business units' assets (as members of that unit), its application in valuing companies is limited. For instance, suppose a company comprises three assets: A, B, and C. In this company that is assumed to be characterised by an additivity feature, assets A, B, and C have no synergy. Therefore, the sum of each asset's value equals the sum of the total value of the assets. In other words, if one unit is added to one asset, the company's total value would increase by one unit. As the actual structure of economic realities is unknown, the most typical form of the company is considered in terms of a numerical example to show any potential synergy between the three assets.

Each network's connection points indicate the synergy between two or three assets whose measures are shown with the μ symbol. This "measure" offers different modes of synergies between the assets, including "synergy," "inhibition," or "lack of synergy." Instead of using the "m" measure, which is characterised by the additivity feature, we can use the Choquet capacities as a non-additive measure (μ) to determine synergy between the assets. Choquet capacities generalise the measurement concept through which non-additive accumulation can be measured.

To elaborate on Choquet integral's properties, Bry and Casta (2003) assumed that the set Ω is comprised of three separate members, A, B, and C, then we have:

$$\mu\left(\cup A_n\right) = \mu(A \cup B \cup C) = \mu(\Omega) \leq\geq \mu(A) + \mu(B) + \mu(C)$$

Therefore, the sum of the values of assets will not necessarily equal the sum of the values of each asset, making it possible to estimate the synergy (and inhibition) between the combination of assets. By definition, Choquet capacities should be used as all subsets of the set X; that is, the $2^n - 1$ correlation coefficient must be calculated.

Following the work of Choquet (1954) and Sugeno (1977), Bry and Casta (2003) have attempted to prepare the potential grounds for applying this method in accounting, to help the synergistic effects that are not clear in linear and classical accounting models to be analysed better and be used in financial valuation techniques. Therefore, trying to apply fuzzy measures and integrals (such as Sugeno and Choquet integrals) in financial valuations, Bry and Casta (2003) modelled the synergy between a company's assets, claiming that financial valuation methods are mainly based on the additivity feature and do not include business units as the structured sets of assets.

Financial synergy is defined by DePamphilis (2019) as the effect exerted on the final cost of purchasing or establishing a new company as a result of a merger or acquisition of shares. Aydin (2017) shows that mergers and combinations have not always been successful, leading to a negative synergy or, better to say, inhibition.

Seeking to use an approach based on accounting data in valuing companies in which the non-additivity of the assets is considered, Paugam et al. (2018) proposed a method that can measure the synergy between assets (Fig. 1). Using the Choquet integral's properties in their proposed approach, Paugam et al. (2018) claim that their approach can get a company's value to match the overall value of its assets.

Garcia (2019) argues that stock acquisition contracts are often made with the view that synergy occurs when the investment is made. The acquired firm has reliable resources that would become more valuable if combined with the acquirer's resources. Moreover, he articulates that those synergies can result from more efficient use of assets in merger transactions. It should be noted that synergy does not occur immediately when the merger is done, and synergy usually (is made of financial efficacy) appears gradually.

Conducting a study on the empirical relationship between stock market values and accounting figures, Lubberink and Willett (2020) report that the Multiplicative Power Law justifies the significant relationship between market and accounting values. Furthermore, they emphasise that various studies have so far been carried out on examining the relationship between stock market value and accounting values (Balachandran & Mohanram, 2011, Barth et al., 2021, Core et al., 2003, Holthausen & Watts, 2001, Song et al., 2010, Yu, 2013). However, it is not easy to interpret the models that test the efficacy of accounting values in estimating market values, and the results of those studies are mixed. Generally, the results of those studies indicate that linear additive models are not appropriate for investigating the relationship between accounting and market values.

Aggarwal and Garg (2020) state that the economic added value created through mergers is an essential parameter in assessing the impact of mergers on the overall performance of a company. Examining the creation of added value to a company through a merger, they showed that the value created in the acquired company had been growing for three years.

Considering that the capital of a company is related to the current and future financial and operational performance of the company and that the capital of the company is the knowledge that the company uses in combining its human and physical capitals to produce and present a satisfactory product, it seems that the efficiency estimated based on the proposed approach would have a significant positive correlation with the company's future operating performance and its capital (Lev et al., 2009). Therefore, the relationship between the measured performance of a company and its operating cash flow seems to be an indicator of operational performance. Accordingly, the first hypothesis is formulated as follows:

Hypothesis 1 A significant positive correlation exists between non-additive evaluation approach-induced efficiency and the operating cash flow in holdings.

However, as this criterion can only be assessed by the company after a sufficient time has passed (based on the business model and its life cycle), sales growth as an indicator of operational performance can be considered as another measure that can reveal a company's ability to generate future operating cash flow (Ahmed, 2015, David Mutua, 2014, Paugam et al., 2018, Paul et al., 2018, Walaa Wahid, 2017). So, the second hypothesis can be formulated as follows:

Hypothesis 2 There is a significant positive correlation between non-additive evaluation approach-induced efficiency and sales growth in holdings.

For a better understanding and evaluation of the suggested non-additive evaluation approaches in this study, we can compare them with the market price-to-book ratio-based method, which is a criterion for assessing the ability of a company's management to create value through its financial resources (Custodio, 2014). To this end, the third and fourth hypotheses can be proposed as follows:

Hypothesis 3 A significant positive correlation exists between the market price-to-book ratio and the operating cash flow.

Hypothesis 4 A significant positive correlation exists between the market price-to-book ratio and sales growth.

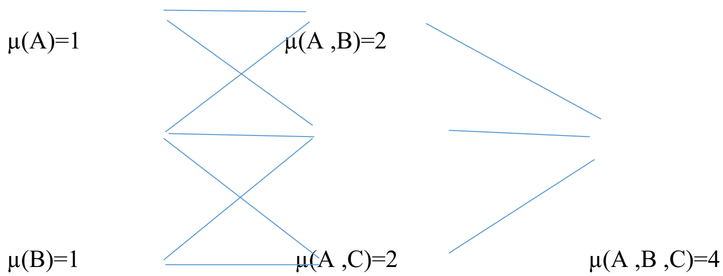


Fig. 1 The relationship between assets in a structured unit with a mutual synergistic effect

Finally, we can re-examine the first two hypotheses after controlling for the market price-to-book the relationship between the new approach-based estimated efficiency and the market price-to-book ratio and restate them as the fifth and sixth hypotheses as follows:

Hypothesis 5 There is a significant positive correlation between the non-additive evaluation approach-induced efficiency after controlling the market price-to-book ratio and the company's operational cash flow.

Hypothesis 6 A significant positive correlation exists between non-additive evaluation approach-induced efficiency after controlling the market price-to-book ratio and sales growth.

3 Research Methodology

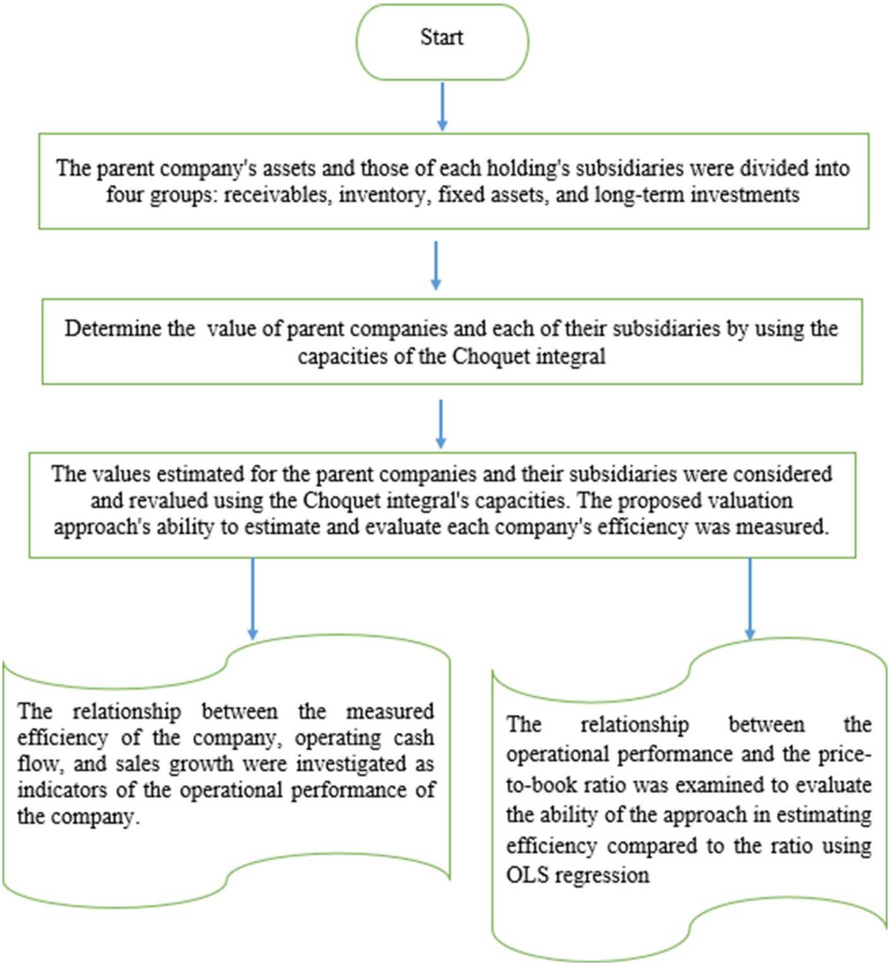
The statistical population of this study includes all companies listed on the Tehran Stock Exchange (TSE) that have been active in TSE from 2012 to 2020 and produced consolidated financial statements (parent companies). We collected 62 consolidated financial statements, including 322 subsidiary companies. Data are primarily based on the TSE's audited financial statements and board reports, a reliable source of information (Daryaei et al., 2022; Namakavarani et al., 2021; Nassirzadeh et al., 2022; Shandiz et al., 2022).

In the first stage, we divided the parent company's assets and their subsidiaries into four groups: account receivables, inventories, fixed assets, and long-term investments. Then we used business valuation methods to evaluate each firm's total value, taking the companies' synergies' assets into account ($\hat{E}V$).

In the second stage, to value each holding's subsidiary companies, tripartite, quadruple, quintuple, etc., combinations were considered according to the extent of the holding's subsidiaries, derived from the values obtained from applying Choquet integral capacities and a method based on a non-additivity assumption (Altekin & Bukchin, 2022, Beliakov, 2022, Gerami et al., 2022, Perederieieva et al., 2018). Then we valued each holding's subsidiary companies based on the approach proposed in the current study.

In the third stage, the ability of such a valuation approach to estimate and evaluate each company's efficiency (in terms of using its resources) was measured, followed by examining the relationship between the measured efficiency, the company's operating cash flow, and sales growth (as indicators of the company's operational performance). Finally, the relationship between the company's operating performance and the price-to-book ratio was examined to evaluate the approach's ability to estimate efficiency compared to the ratio using OLS regression.

The process can be presented as the following flowchart:



According to Casta et al. (2011), Choquet capacities are a generalisation of the measure concept as they allow non-additive aggregation. Grabish (2008) suggest that the Choquet integral is a generalised form of the Lebesgue Integral, which can be stated as following models:

Model No. 1:

$$L_{(f)} = \int_{\Omega} f dm = \sum_{k=0}^{N-1} (a_{k+1} - a_k) m(A_{k+1}) \text{ The Lebesgue integral}$$

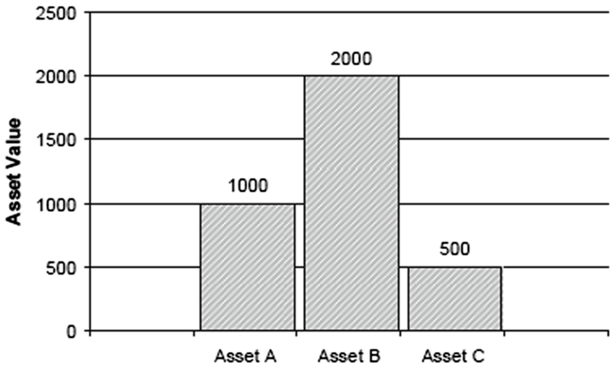


Fig. 2 Valuation of a set of structured assets based on the additivity feature

Model No. 2:

$$C_{(f)} = \int_{\Omega} f d\mu = \sum_{k=0}^{N-1} [a_{k+1} - a_k)]\mu(A_{k+1}) \text{ The Choquet integral}$$

If the two integrals have an additive capacity, the Choquet integral will equal the Lebesgue Integral. The Choquet integral can be used as a measurement, integrand function and a pooled operator. Therefore, this pooled non-additive Integral allows a set of assets to measure the assets' synergies. Murofushi and Sugeno (2000) describe the details of a company's valuation through the Choquet Integral as follows:

Consider a company with assets A, B, and C in the following way (assuming that the assets are additive)

$$a_1 = 1000, \quad a_2 = 2000, \quad a_3 = 500. \Omega;$$
$$f : \Omega \ _{\rightarrow}[0, +\infty] \, (f(A) = 1000, \ f(B) = 2000, \ f(C) = 500).$$

This approach requires calculating the area under the diagram (Abd-Elnaby & Aref, 2019) in Fig. 2. Model No.3 is the Lebesgue Integral used for such a calculation:

Model No. 3:

$$V_L = L_{(f)} = \sum_{(i=0)}^{(n-1)} \left(a_{i+1} - a_i\right) \times m\left(A_{i+1}\right)$$
$$= (500 - 0) \times m(A, B, C) + (1000 - 500) \times m(B, A) + (2000 - 1000) \times m(B)$$
$$= (500 - 0) \times 3 + (1000 - 500) \times 2 + (2000 - 1000) \times 1 = 3500$$

where $V_{.L.}$ represents the value of total assets based on the Lebesgue Integral, $A = \left\{x \in X : f(x) \geq a_{i+1}\right\}$ And $m \div (A_i)$ is the Lebesgue Integral's measurement derived from $A_{i,}$ which indicates the domain's length.

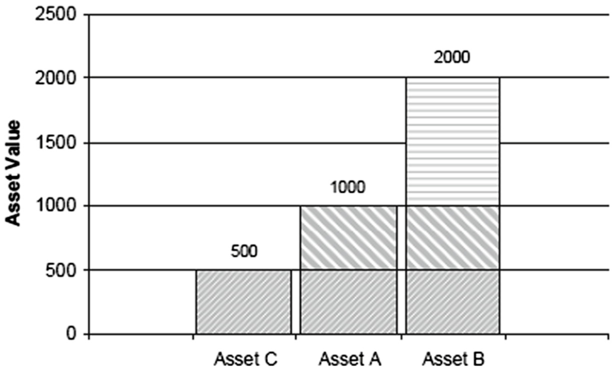


Fig. 3 The valuation of a set of structured assets based on Choquet integral

This value can also be calculated based on the Riemann Integral from the following equation:

$$V_R = R_{(f)} = \sum_{i=1}^{3} f\left(x_i\right) = 2000 + 1000 + 500 = 3500$$

where $V_{.R.}$ represents the value of total assets based on the Riemann Integral.

In the additivity approach, the sum of measures' values is assumed to be 1 as follows:

$$\mathrm{m(A) = m(B) = m(C) = 1}$$

Assuming non-additivity of the assets, the value of a company is, thus, calculated by considering the assets' synergies using the Choquet Integral. Contrary to the additivity approach, where the net sum of individual values equals the total value of the assets, the non-additivity approach also includes synergy and inhibition. In this assumption, additional information regarding the μ measurement allows for modelling different synergy and inhibition types.

Consider a company with three assets fairly valued in ascending order. Only three Choquet capacities will be used for its valuation for this company. In other words, if the value of the intended company's assets is f(A) < f(B) > f(C), then the Choquet capacities would include μ(C, A, B)μ ‹(A, B), and μ(B).[1] Let us assume that there is a synergy between assets A and B, and, therefore, the following equation exists:

$$\mu(\mathrm{A,B}) \geq \mu(\mathrm{A}) + \mu(\mathrm{B})$$

if

[1] Should the order of assets' value be different in another company, different Choquet capacities will be used. For instance, if their relationship is f(A) < f(B) < f(C), then the Choquet capacities would include μ (A, B, C), μ(B, C), and μ(B).

$$\mu(A, B, C) = 4; \mu(A, B) = 1.5; \mu(B) = 1$$

Now, we arrange the assets in order of their ascending fair value according to Fig. 3.

The total value calculated for the company is weighted based on the pooled non-additive function of each asset's combination. The company is valued based on Choquet's pooled function as follows (Model No. 4):

Model No. 4:

$$V_C = C_{(f)=} \sum_{i=0}^{n-1} (a_{i+1} - a_i) \times \mu\left(A_{i+1}\right)$$
$$= (500 - 0) \times \mu(A, B, C) + (1000 - 500) \times \mu(A, B) + (2000 - 1000) \times \mu(B)$$
$$= (500 - 0) \times 4 + (1000 - 500) \times 1.5 + (2000 - 1000) \times 1 = 3750$$

V.C.: Valuation based on Choquet's pooled function.

μ(Si): Choquet's capacity calculated for each set of assets.

3.1 Estimating the Choquet's Capacities

As Bry and Casta (2003) elaborated, using the Choquet integral for modelling requires the creation of a measurement that relates to the semantic part of the problem. Because the measurement is not theoretically divisible, it is necessary to define the value of 2^n-1 for μ(A) correlation coefficient where $A \in P(X)$. Casta et al. (2011) suggest a regression model-based indirect econometric method for estimating correlation coefficients. In cases where the synergic structure is approximately definable, the consolidated part of the problem could also be reduced by limiting the synergetic decomposition to the items existing in functional subsets. The current study applies an indirect estimation method to value companies using a sample of companies whose market values and all their assets' values are known.

Consider companies i whose market value is V and X set with j real variable (X^j), which indicates the value of each asset. f_i is a function that assigns the value of each X^j Variable to the company

$$\text{i}.f_i : x^j \rightarrow x_i^j$$

The purpose is to determine a set of Choquet capacities (μ) to approximate the following equation:

Model No. 5:

$$\forall i : C_{(f_i)} = EV_i$$

where EV_i is the value of company i, A is a member of P(X), and $g_A(f_i)$ represents the operator function A, which is defined for company i as follows:

Model No. 6:

$$i \rightarrow g_A\left(f_i\right) = \int 1_{\{x:f_i(x)>y\}}(A)d_y$$

To estimate Choquet capacities that are the intended measures, 2^n-1 correlation coefficients must be calculated via regression model No.1.

Therefore, Choquet and Lebesgue integrals are presented in terms of the following equations:

Model No. 7:

$$L_{(f)} = \sum_{A \in P(X)} g_A(f) \times m(A)$$

Model No. 8:

$$C_{(f)} = \sum_{A \in P(X)} g_A(f) \times \mu(A)$$

Thus, considering Model No.5 and No.8, the final model could be suggested as follows:

Model No. 9:

$$\forall i EV_j = \sum_{A \in P(X)} \mu(A) * g_A\left(f_i\right) + \varepsilon_j$$

The proposed model is a linear model with a 2^n-1 μ(A) coefficient (A) for all subsets A of P(X). The dependent variable is the company's value, and the independent variables are the operators belonging to the subsets of P(X). These coefficients are estimated through a regression model. The relevant operator function is calculated for each A member of P(X) based on Model 6. Individually, operator functions distinguish between the values of assets i + 1 and i.

It should be noted that the proposed model is linear for the operators but nonlinear towards the x^j variables. Moreover, the pooled function is of the Choquet type developed from the synergy of a company's assets. Accordingly, a set of Choquet's capacities is determined using a statistical method based on market estimates of the company's value.

Therefore, the relevant operators are first calculated (according to Model 6). Then, the value of each parent company and its subsidiaries (dependent variable) equal to their market value is fitted to the operator functions (independent variables) and each subset of assets. The estimated correlation coefficients are Choquet's capacities.

In a study conducted by Paugam et al. (2018) to value a company (concerning the synergy of assets), the company's total assets were categorised into current, fixed, and intangible assets, which in turn led to three binary combinations (subsets) and a tripartite combination (subset). However, our preliminary studies indicated that the value of intangible assets presented in the financial statements of our targeted populations was low compared to the value of other assets. Therefore, the tripartite-category-based model presented by Paugam et al. (2018) was considered inappropriate to offer an accurate estimate of the assets' synergy, as the model included only one binary combination and one tripartite combination, which couldn't accurately estimate the assets' synergies (considering the relatively low value of the intangible assets). Therefore, we revised the categorisation of the assets concerning the impact of their different combinations on the company's value.

Setiadharma and Machali (2017) argue that the difference in the structure of the assets affects the values of companies. Their study indicated that companies whose current assets outweighed their fixed assets were more willing to use debt to finance their activities. In contrast, the companies with more fixed assets tended to use capital to fund their projects. Moreover, according to a study by Nyamasege et al. (2014), companies that possess sufficient properties, machinery, and equipment to pledge as security are more capable of receiving loans from banks and credit institutions, which leads to an increase in their values.

Therefore, considering the different effects of the structure and combination of assets on a company's value, this study classifies the company's operational and non-operational assets into four groups: the receivables, inventories, fixed assets, and long-term investments so that a company could be valued based on a different combination of assets and their synergies.

3.2 Non-additive Valuation Model and Efficiency of Companies

The ability of the valuation approach to estimate and evaluate each company's efficiency (as a market evaluation of the company's relative efficiency) and its ability to combine assets to gain economic benefits (compared to the average capacity of companies in the relevant industry) was tested. Each company's value was predicted using the Choquet Integral. The efficiency of each company is calculated by considering the difference between the expected value based on the Choquet integral and the company's market value at the end of the year. Therefore, the efficiency would be above average when its market value is more significant than its predicted value. However, if the company's market value is less than expected, the efficiency would be lower than the average rate in the industry.

Model No. 10:

$$\text{Efficiency}_t = \text{EV}_{i,t} - \hat{\text{E}}\text{V}_{i,t}$$

$\text{EV}_{i,t}$ equals the company's market value at the end of the fiscal year. $\hat{\text{E}}\text{V}_{i,t}$ equals the company's predicted value based on Choquet Integral.

The above model can be used to measure the non-additive valuations of the firms for testing the hypotheses proposed in this paper. Firms' total sales are usually compared with their preceding years to calculate sales growth. Given the above, we used the operating cash flows and sales growth of the firms for two years after the valuation of the companies using Model 11 as follows:

Model No. 11:

$$\mathrm{OCF}_{\mathrm{i,\,t+k}} \text{ or } \Delta\mathrm{SALE}_{\mathrm{i,t+k}} = \mathrm{b}_0 + \mathrm{b}_1 \text{ Efficiency}_{\mathrm{I,t}} + \mathrm{b}_2 \text{ OCF}_{\mathrm{I,t}} \text{ or } \Delta\mathrm{SALE}_{\mathrm{i,t}} + \mathrm{b}_3 \text{ SIZE}_{\mathrm{i.t}} + \text{Year Fixed effects} + \varepsilon_{\mathrm{i}}$$

OCF: The Company's operating cash flow. ΔSALE: Sales growth, that equals the extent of changes in each year's sales compared to the previous year. MTB: The assets' market price-to-book ration. SIZE: The company's size, which equals the logarithm of its assets. Efficiency: The company's efficiency equals the difference between its market value at the end of the fiscal year and its predicted value based on the Choquet Integral. Year Fixed effects: The year's effects to control the company's business cycle.

If efficiency is measured correctly through the proposed non-additive approach, it is expected to correlate positively with the company's future performance (coefficient b1 is significantly positive).

To test the third and fourth hypotheses, we used Model 12 as follows:

Model No. 12:

$$OCF_{i,\,t+k} \ or \ \Delta SALE_{i,t+k} = b_0 + b_1 \ MTB_{I,t} + b_2 \ OCF_{I,t} \ or \ \Delta SALE_{i,t} + b_3 SIZE_{i.t} + Year\ Fixed\ effects + \varepsilon_{i,t}$$

The market price-to-book ratio is expected to correlate significantly with the company's operating cash flow and sales growth (the coefficient b1 is significantly positive).

Finally, to test the fifth and sixth hypotheses, we used Model 13 as follows:

Model No.13:

$$OCF_{i,\,t+k} \ or \ \Delta SALE_{i,t+k} = b_0 + b_1 \ Efficiency_{I,t} + b_2 \ MTB_{I,t} + b_3 \ OCF_{I,t} \ or \ \Delta SALE_{i,t} + b_4 SIZE_{i.t} + Year\ Fixed\ effects + \varepsilon_{i,t}$$

Should the non-additive approach have a greater ability to estimate the company's efficiency, the b1 coefficient of the above model must be significantly positive once the Market Price-to-Book ratio is controlled.

4 Findings

4.1 The Results of the Study's Models Fitting

Two fundamental developments have shaped the evolution of the reporting model: the ever-increasing complexity of business processes and the recent dramatic changes in information technology. At the same time, the development of modern social-economic societies has increased the resources needed, such as transportation systems and security resources. Such social assets require relevant, valid, reliable, and verifiable measurement forms to measure enterprises' wealth, income, and other attributes and develop equitable taxation systems (Romero et al., 2012).

Romero et al. (2012) discuss some problems inherent in the current accounting system. Standard setters and researchers have extensively addressed but not resolved these problems. They state that aggregation issues indicate that business organisations' boundaries are fuzzy, financial statements differ across industries, and the resulting measures often are not comparable or additive. But if it was the predictive value of accounting information that Bierman and others were hoping to improve, why did they give such a minor role to the theory of measurement?

This question is relevant because it seems that the only role given to modern measurement theory was to provide accounting with some new terminologies. It appears that no substantive insight was seriously looked for, that is, an insight that would change accounting methodology. This seems to have been the case until quite recently (Asay et al., 2022).

Perhaps the only recent publication trying to elaborate on the substantive potential of measurement theory in financial accounting is the book chapter by Vehmanen (2013), where the fundamental concepts of measurement theory are discussed and related to accounting valuation.

Communication of measurement uncertainty is a topic of enduring interest that has recently received prominent attention in potential changes to financial reporting. Significant uncertainty in financial statement estimates has drawn scrutiny from investors, regulators, auditors, and academics. Investors would like to be better apprised of such uncertainty (Majors, 2016).

As for the regression models of the study, first, an appropriate pattern for fitting the models was identified, and then the models were tested. To this end, first, the F-Limer and Hausmann tests were performed. The F-Limer test helps researchers choose the best option between simple and panel regression models (fixed effects models). To evaluate the results of fitting the study's regression models, items such as the model's overall significance, the significance of the coefficients of the model's variables, lack of autocorrelation of the model's remainders, and the model's determination coefficient were examined. Finally, the significance of each model was individually assessed. Table 1 shows the results of exploring an appropriate model fitting the data.

After an appropriate pattern was determined for fitting, the fits of the models were tested. Table 2 shows the comparative results of fitting the first model:

Table 1 The results of exploring an appropriate model for fitting the study's models

		F-Limer Test	Hausmann Test	According to the Pagan method	According to Godfrey/Durbin Watson	Dickey Fuller/ Kolmogorov Smirnov
Model 11						
OCF	t + 1	Simple regression model	Fixed effects	Variance homogeneity Generalised fixed effects model	Lack of autocorrelation	Abnormality
	t + 2	Panel model	Fixed effects	Variance homogeneity Generalised fixed effects model	Remainders' autocorrelation	Durability
SALE	t + 1	Simple regression	Fixed effects	Variance heterogeneity Generalised fixed effects model	Lack of autocorrelation	Abnormality
	t + 2	Simple regression	Fixed effects	Variance heterogeneity Generalised fixed effects model	Lack of autocorrelation	Abnormality
Model 12						
OCF	t + 1	Simple regression	Fixed effects	Variance heterogeneity Generalised fixed effects model	Auto-correlation	Abnormality
	t + 2	Simple regression	Fixed effects	Variance homogeneity Generalised fixed effects model	The Remainders' autocorrelation	Durability
SALE	t + 1	Simple regression	Fixed effects	Variance homogeneity Generalised fixed effects model	Auto-correlation	Abnormality
	t + 2	Simple regression	Fixed effects	Variance heterogeneity Generalised fixed effects model	Auto-correlation	Abnormality
Model 13						
OCF	t + 1	Simple regression	Fixed effects	Variance homogeneity Random fixed effects model	Lack of autocorrelation	Abnormality
	t + 2	Panel model	Fixed effects	Variance homogeneity Generalised fixed effects model	Remainders' autocorrelation	Durability

Table 1 (continued)

		F-Limer Test	Hausmann Test	According to the Pagan method	According to Godfrey/Durbin Watson	Dickey Fuller/ Kolmogorov Smirnov
SALE	t + 1	Simple regression	Fixed effects	Variance homogeneity	Lack of autocorrelation	Abnormality
				–		
	t + 2	Simple regression	Fixed effects	–	–	–
				–		

Table 2 The results of fitting the first model

	OCF_{t+1}		OCF_{t+2}		$\Delta SALE_{t+1}$		$\Delta SALE_{t+2}$	
	Coefficient	Significance level	Coefficient	Significance level	Coefficient	Significance	Coefficient	Significance
Efficiency	0.0026	0.3	0.0504	0.1639	0.0003	0.000***	1.747	0.002***
	(0.0025)		(1.392)		(3.76)		(2.525)	
OCF_t	0.0125	0.853	0.0728					
	(0.186)		(0.787)					
$\Delta SALE_t$					1.054	0.000***	1.282	0.000***
					(2.926)		(22.98)	
SIZE	0.1142	0.612	0.41	0.212	1.056	0.055	0.0419	0.474
	(−0.508)		(1.248)		(1.926)		(0.717)	
R^2	0.0068		0.014	0.215	0.68	0.001**	0.78	
Determination coefficient	(102.99)		(538.14)		(221.1)		(221.1)	

T-statistics are computed with firm-clustered standard errors and are presented in parentheses below each coefficient. ***, **, * Denote significance at the One percent, 5 percent, and 10 percent levels, respectively, using a two-tailed test

$$\mathbf{OCF_{i,\,t+k}\ or\ \Delta SALE_{i,t+k} = b_0 + b_1\ Efficiency_{I,t} + b_2\ OCF_{I,t}\ or\ \Delta SALE_{i,t} + b_3 SIZE_{i,t} + Year\ Fixed\ effects + \varepsilon_{i,t}}$$

The regression coefficients' t-statistic values are presented in Table 2. The findings presented in Table 2 help us to test our first and second proposed hypotheses as follows:

Hypothesis 1 A significant positive correlation exists between non-additive evaluation approach-induced efficiency and the operating cash flow in holdings.

Hypothesis 2 There is a significant positive correlation between non-additive evaluation approach-induced efficiency and sales growth in holdings.

The findings show no meaningful correlation between the market evaluation of a company and its operating cash flow. So, our first hypothesis is not supported.

The F statistic values of operating cash flows in the years t+1 (102.99) and t+2 (538/14) indicate that the linear relationships of the model are not significant. Moreover, as the model's determination coefficients (R2) for the above variable determination are 0.68% and 1.4% in the years t+1 and t+2, respectively, it could be argued that in the first regression model, only about 0.68% and 1.4% of the changes in the dependent variable of the surveyed companies are elaborated by the independent and control variables, suggesting a low correlation between the variables.

However, the results show a significant correlation between valuation and sales growth in the years t+1 and t+2 (0.000 and 0.002, respectively), supporting our second proposed hypothesis. The F statistic value for the sales growth in years t+1 and t+2 (221/1) indicates the significant linear relationships of the model. Furthermore, considering the determination coefficients of the model (R2) for the sales growth (in years t+1 and t+2) that are 68% and 78%, respectively, it can be concluded that the first regression model shows a significant correlation between valuation and sales growth.

Table 3 shows the comparative results of the fitting of the second model:

$$OCF_{i,\,t+k}\ or\ \Delta SALE_{i,t+k} = b_0 + b_1\ MTB_{I,t} + b_2\ OCF_{I,t}\ or\ \Delta SALE_{i,t} + b_3\ SIZE_{i,t} + Year\ Fixed\ effects + \varepsilon_{i,t}$$

The findings presented in Table 3 help us to test our third and fourth proposed hypotheses as follows:

Hypothesis 3 A significant positive correlation exists between the market price-to-book ratio and the operating cash flow.

Hypothesis 4 A significant positive correlation exists between the market price-to-book ratio and sales growth.

The findings show no meaningful correlation between the Market Price-to-Book ratio and the net operating cash flow in year t+1. However, the results suggest a strong

Table 3 Results of the fitting of the second model

	OCF_{t+1}		OCF_{t+2}		$\Delta SALE_{t+1}$		$\Delta SALE_{t+2}$	
	Coefficient	Significance level	Coefficient	Significance Level	Coefficient	The statistic's probability	Coefficient	Significance level
MTB	0.0002	0.9113	0.0005	0.0089*	2.735	0.007**	1.89	0.000***
	(0.112)		(2.615)		(2.781)		(4.29)	
OCF_t	0.0467	0.3228	0.0081	0.87				
	(0.99)		(0.164)					
$\Delta SALE_t$					0.0006	0.000***	1.297	0.000***
					(1.084)		(38.502)	
SIZE	0.3872	0.0904	0.0068	0.8022	0.0081	0.0258*	4.812	0.151
	(1.697)		(−0.251)		(0.108)		(1.438)	
R^2	0.001	0.3017	0.08	0.04*	0.85	0.000***	0.8	0.000***
	(−1.089)		−1.885		(0.079)		0.93	

T-statistics are computed with firm-clustered standard errors and are presented in parentheses below each coefficient. ***, **, * Denote significance at the One percent, 5 percent, and 10 percent levels, respectively, using a two-tailed test

correlation between the Market Price-to-Book ratio and the net operating cash flow in year t+2. So, the findings don't support our third hypothesis in a short period (within a year). Still, it supports the existence of a meaningful correlation between the Market Price-to-Book ratio and the net operating cash flow in the long term (in year t+2).

Nevertheless, the findings support our fourth short and long-term hypothesis. The results show a significant correlation between the Market Price-to-Book ratio and sales growth in t+1 and t+2 (0.000 and 0.007, respectively. The relevant t-statistic values for years t+1 and t+2 (2.781 and 4.290, respectively) are also greater than the t-statistic obtained from the table with the same degree of freedom (1.96), confirming a significant correlation between Market Price-to-Book ratio and sales growth in the years t+1 and t+2.

The F statistic value further confirms the above results. The F statistic value for the sales growth in the years t+1 and t+2 (949/4 and 0/561, respectively) indicates a significant linear relationship of the model. Furthermore, considering the determination coefficients of the model (R2) for the sales growth (in the years t+1 and t+2), which are 85% and 78%, respectively, it could be concluded that the second regression model shows a significant correlation between the dependent and independent variables.

Table 4 shows the comparative results of the fitting of the third model:

$$\boldsymbol{OCF_{i,t+k}\ or\ \Delta SALE_{i,t+k} = b_0 + b_1\ Efficiency_{I,t} + b_2\ \ MTB_{I,t} + b_3\ OCF_{i,t}\ or\ \Delta SALE_{i,t} + b_4\ SIZE_{i,t} + Year\ Fixed\ effects + \varepsilon_{i,t}}$$

The findings presented in Table 4 help us to test our fifth and sixth proposed hypotheses as follows:

Hypothesis 5 There is a significant positive correlation between the non-additive evaluation approach-induced efficiency after controlling the market price-to-book ratio and the company's operational cash flow.

Hypothesis 6 A significant positive correlation exists between non-additive evaluation approach-induced efficiency after controlling the market price-to-book ratio and sales growth.

After controlling the Market Price-to-Book ratio (MTB), the findings show no meaningful correlation between firms' valuations and the cash flow in the years t+1 and t+2 after controlling the Market Price-to-Book ratio (MTB). So, our fifth hypothesis is not supported. However, the results support our sixth hypothesis by showing a significant direct correlation between firms' valuations and sales growth in the years t+1 and t+2 after controlling the Market Price-to-Book ratio (MTB).

5 Conclusion

The majority of financial statements provided by companies do not present the actual values of the firms as they fail to consider the synergy between the company's assets (Bry & Casta, 2003). There is also little evidence in the literature to show how

Table 4 Results of the fitting of the third model

	OCF_{t+1}		OCF_{t+2}		$\Delta SALE_{t+1}$		$\Delta SALE_{t+2}$	
	Coefficient	The statistic's probability	Coefficient	The statistic's probability	Coefficient	The statistic's probability	Coefficient	The statistic's probability
Efficiency	−0.0023	0.365	−0.046	0.2024	0.145	0.000***	1.455	0.000***
	(−0.907)		(−1.275)		(5.896)		(0.613)	
MTB	−0.0222	0.171	−0.036	0.0927	1.403	0.000***	1.08	0.818
	(−1.374)		(−1.682)		(4.074)		(0.231)	
OCF_t	−0.0014	0.983	0.051	0.5857				
	(−0.021)		0.545					
$\Delta SALE_t$					28.75	0.000***	1.253	0.000***
					(0.34)		(19.17)	
SIZE	0.2763	0.277	0.6901	0.0595	2.016	0.0451*	86.67	0.351
	(−1.089)		−1.885		(0.079)		0.93	
R^2	0.0002	0.49	0.035	0.1194	0.83	0.00**	0.13	0.000***
	(0.84)		(875.34)		(277.6)		(491)	

T-statistics are computed with firm-clustered standard errors and are presented in parentheses below each coefficient. ***, **, * Denote significance at the One percent, 5 percent, and 10 percent levels, respectively, using a two-tailed test

the adoption of fuzzy Integral can offer a better valuation for firms. The quantitative approach has been used in almost all of the studies conducted by classic researchers to measure value and profit (Willett, 1987).

Addressing the above gap in the literature, this study considers the synergy between the company's assets and re-valuates the firms' assets using the Choquet fuzzy integral (Choquet, 1959), which is a non-additive pooled function. This paper further examines the relationship between the Non-Additivity Valuations (measured in this paper) and the firm's operating cash flow and sales growth efficiencies. Finally, it examines the relationship between the operational performance and the price-to-book ratio to evaluate the approach's ability to estimate efficiency compared to the ratio using OLS regression.

The findings show no meaningful correlation between firms' values and their operating cash flows in the short term (e.g., one year), regardless of whether you use the synergy between the company's assets. However, when using Non-Additivity Valuations, the findings show a meaningful correlation between firms' values and their operating cash flows in the long term (e.g., two years and more). At the same time, there is no meaningful correlation between firms' values and their operating cash flows when using additivity valuations.

The results further show a significant correlation between firms' values and sales growth in the short and long term based on additivity and non-additivity valuations. However, the findings suggest that using Non-Additivity Valuations (versus additivity valuations) can offer firms a more accurate estimation of sale growth. Indeed, Non-Additivity Valuations compare the market evaluation of a company's capabilities to those of similar companies in the same industry, providing a better valuation for decision-makers.

In today's world, holdings (being a parent company) are more engaged in increasing competition and rapid, unpredictable changes than single-business companies, trying to survive and maintain their status in such a competitive market by creating value in their subsidiaries and increasing synergy in their assets. As the primary task of a holding is to create value in its subsidiaries, it needs to use a proper evaluation method, having a complete understanding of its characteristics and those of its subsidiary businesses. The findings of this study offer a novel method for valuing companies and estimating the sales growths of firms as a result of expansion, merging and purchasing subsidiaries.

The method used in the paper is compared with only OLS regression. Due to the low amount of data and the limited number of companies in different industries, it was predicted that the artificial neural network method would not have a significant output. So, this can be considered a limitation in this study. Further studies are suggested to collect more extensive to use other methods, such as artificial neural networks.

Given the relatively high inflation rates (two digits) for the whole study period for the country where the targeted firms are located, using historical costs as a means of valuation could be considered a limitation for this study. Further studies are suggested to apply the method proposed in this study to different industries and compare the results to see if there is any meaningful difference in terms of capabilities of this valuation method in estimating sale growth for various industries.

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