۵۴ امین کنفرانس ریاضی ایران

دانشگاه زنجان، ۳-۱ شهریور ۱۴۰۲

54^{th} Annual Iranian Mathematics Conference

University of Zanjan, 23-25 August 2023

شماره: ۱۷۷۰۹۰-گ-ر

تاریخ: ۱۴۰۲/۶/۶

CERTIFICATION

This certificate is presented to

Paper code: aimc54-1237

Dr. HANIEH MIREBRAHIMI

for the short presentation entitled

Targeted robotic motion with obstacles

in **The 54th Annual Iranian Mathematics Conference**.

Authors: **HANIEH MIREBRAHIMI ، Seyyed Abolfazl Aghili**

دبیر کنفرانس

دکتر علی مرصعی

دبیر علمی کنفرانس

دکتر علی شکری

http://aimc54.znu.ac.ir

aimc54@znu.ac.ir

54th Annual Iranian Mathematics Conference
University of Zanjan
23-25 August 2023

TARGETED MOVEMENT OF THE ROBOT BY REMOVING OBSTACLES

HANIEH MIREBRAHIMI [1] * AND SEYYED ABOLFAZL AGHILI [2]

[1,2] *Department of Pure Mathematics, Ferdowsi University of Mashhad, Mashhad, Iran, P.O. Box 91775-1159.*
h-mirebrahimi@um.ac.ir
s.a.aghili@mail.um.ac.ir

Abstract. We express a new definition of complexity and call it targeted complexity. By removing the obstacles from the configuration space, we define a targeted movement for the robot. With the motivation to reduce the number of motion programs and obtain a more optimal value for complexity. Also we show that relative topological complexity of a pair $TC(X, B)$ is a special case of targeted complexity.

1. Introduction and preliminaries

Today, robots are an inseparable part of many industries and even human daily life. To use a given mechanical system as a robot, we need to program it to move. The space of all possible configurations of a mechanical system is called configuration space. Let X be the configuration space of a given mechanical system. The topological complexity introduced by M. Farber [2] is a number which measures discontinuity of the process of motion planning in the configuration space X. More precisely, it is the minimal number k such that there are $k + 1$ different motion planning rules, each defined on an open subset of $X \times X$, so that each rule is continuous in those configurations. Each rule is called a motion planning defined as a continuous map $s_i : U_i \to PX$

2010 *Mathematics Subject Classification.* Primary: 55M30; secondary: 53C23, 57N65.

Key words and phrases. topological complexity, motion planning, targeted motion.

* Speaker.

with $\pi s_i = id$ over U_i where $U_i \subseteq X \times X$. Here X is assumed path connected. Also PX denotes the space of all paths in X equipped with the compact-open topology, and let $\pi : PX \to X \times X$ is the natural path fibration mapping each path to its end points. If there is no such integer k, then $TC(X)$ is considered to be the infinity, $TC(X) := \infty$.

Definition 1.1 ([4])**.** Let $f : E \to B$ be a fibration. The Schwarz genus of f, denoted by $genus(f)$, is the smallest integer k such that there exists $\{U_i\}_{i=1}^k$, an open cover of B, along with sections $s_i : U_i \to E$ of f.

The relative topological complexity of a pair of spaces (X, Y), denoted by $TC(X, Y)$, was defined as the Schwarz genus of a natural path fibration map. The relative topological complexity of a pair of spaces is a lower bound for the classic topological complexity defined by Farber [2]. In fact, it is modified to count the number of rules presenting paths whose end points belong to some subset of configuration space considered as the target of the motion.

Definition 1.2 ([5])**.** Let $P_{X\times B} = \{\gamma \in PX | \gamma(0) \in X,\ \gamma(1) \in B\}$ and $B \subseteq X$. There is a natural fibration $\pi_B : P_{X\times B} \to X \times B$ with $\pi(\gamma) = (\gamma(0), \gamma(1))$. The relative topological complexity of the pair (X, B) is the Schwarz genus of π_B. That is, $TC(X, B) = genus(\pi_B)$.

2. Targeted motion

R. Short in [5] defined topological complexity of the pair (X, Y) for a pair of spaces X and Y such that $Y \subseteq X$ to count the number of algorithms of targeted motions. We intend to generalize this definition as follows: Let X be a path connected space and $B_1, B_2, \cdots, B_k$ be subsets of X. We define targeted complexity as follows; Let $k \in \mathbb{N}$ and X be a path connected space. Then

$$P_k(X, B) = \{(\gamma_1, \ldots, \gamma_k) \in P_{X\times B_1} \times PB_2 \times \ldots \times PB_k | \gamma_i(1) = \gamma_{i+1}(0)\},$$

And define $\pi_k : P_k(X, B) \to X \times B_k$ by the rule $\pi_k((\gamma_1, \gamma_2, \cdots, \gamma_k)) = (\gamma_1(0), \gamma_k(1))$.

Definition 2.1.
- A k-motion planner on open subset $U \subseteq X \times B_k$ is a section of π_k over U, i.e. a map $s : U \to P_k(X, B)$ by $s(x, b_k) = (\gamma_1, \gamma_2, \cdots, \gamma_k)$ where $\gamma_1(0) = x, \gamma_k(1) = b_k$ and $\gamma_i(1) = \gamma_{i+1}(0)$ for $1 \leq i \leq k-1$ such that $\pi_k s = id_U$.
- Targeted complexity denoted by $TC_k(X, B)$ is the least integer $l \geq 1$ such that there exists an open cover of $X \times B_k$ by l sets which admit k-motion planners.

It is clear that $TC_k(X,B) = Secat(\pi_k)$. Also for $k=1$ we have $TC_1(X,B) = TC(X,B_1)$ defined by R. Short in [5].

By this process, we only count and obtain targeted motion planners. In fact, we remove those parts of of the configuration space not needed or those parts in which robot is not able to move. Targeted complexity can be applied for the configuration space in which some obstacles appear. Since the given robot can not move over the obstacle sets $D_1, \ldots, D_k$, we remove the obstacles form the configuration space by setting target subsets $B_1 = X - D_1, \ldots, B_k = X - D_k$ and then present the rule of motion planners out of the obstacles.

Remark 2.2. In Definition 2.1 if every B_i is a singleton, then

$$TC_k(X,B) = cat(X).$$

Example 2.3. Consider $X = \mathbb{S}^2$ and $B_1 = \{N\}$ and $B_2 = \mathbb{S}^2 - \{S\}$. Then $TC_2(X,B) = 1$. For $(z_1, z_2) \in B_2$, there are two path $\lambda_1, \lambda_2 \in X$, such that

$$\lambda_1(s,t) = \begin{cases} (exp(s), exp(t)), & (t,s) \neq [1,1] \\ N, & (t,s) = [1,1]. \end{cases}$$

and

$$\lambda_2(s,t) = \begin{cases} (exp(s), exp(t)), & (t,s) \neq [0,0] \\ N, & (t,s) = [0,0]. \end{cases}$$

Therefore we define the section $s : \mathbb{S}^2 \times B_2 \to P_2(\mathbb{S}^2, B)$ of π_2 by $s(z,w) = (\lambda_1, \lambda_2)$.

One of our motivations of Definition 2.1 is to optimize the amount of complexity. In the following propositions, we show that by considering the targeted movement of the robot, the topological complexity is reduced; It means that less motion program is needed to move the robot. In other words if the number of target subsets increases, the topological complexity decreases. We also prove that $TC(X,B)$ is an upper bound for targeted complexity.

Proposition 2.4. *Let $k > 1$ and $B_i \subseteq X, 1 \leq i \leq k$. Then*

$$TC_k(X,B) \leq TC(X,B_i).$$

By the following corollary, it is clear that $TC_k(X,B) \leq TC(X)$.

Corollary 2.5. *Let $B_i \subseteq X$ and $k \in \mathbb{N}$. Then*

$$kTC(X,B) \leq TC(X,B_1) + TC(X,B_2) + \cdots + TC(X,B_k) \leq kTC(X).$$

Proposition 2.6. *If $k \leq j$, then $TC_j(X,B) \leq TC_k(X,B)$.*

In the following proposition, we prove that if the target set is smaller, we get closer to the target. That is, the number of motion planners is reduced.

Proposition 2.7. *Let $C_i \subseteq B_i \subseteq X$. Then $TC_k(X, C) \leq TC_k(X, B)$.*

In the following proposition, we compare the targeted complexity in the workspace and the configuration space.

Proposition 2.8. *Let $B_1, B_2, \cdots, B_k \subseteq X$ and $f : X \to Y$ be an injective map. Then*

$$TC_k(Y, f(B)) \leq TC_k(X, B).$$

In particular, if f is a homotopy equivalence, then $TC_k(X, B) = TC_k(Y, f(B))$.

References

[1] S. A. Aghili, H. Mirebrahimi and A. Babaee ,*On the targeted complexity of a map,* Hacettepe Journal of Mathematics and Statistics, 10.15672, 2022.

[2] M. Farber, *Topological complexity of motion planning,* Discrete and Computational Geometry, 29, 211-221, 2003.

[3] M. Is and I. Karaca, *Different types of topological complexity on higher homotopic distance,* arXiv:2203.02494 [math.AT], 2022

[4] Y. Rudyak, *On higher analogs of topological complexity,* Topology and its Applications, 157, 1118, 2010.

[5] R. Short. *Relative topological complexity of a pair,* Topology and its Applications, 248:7–23, 2018.