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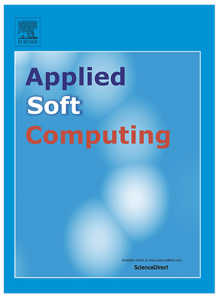

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PII: S1568-4946(23)00874-8
DOI: https://doi.org/10.1016/j.asoc.2023.110856
Reference: ASOC 110856

To appear in: *Applied Soft Computing*

Received date : 29 April 2023
Revised date : 2 September 2023
Accepted date : 8 September 2023

Please cite this article as: N. Goldani, M. Kazemi, Z. Naji-Azimi et al., An interval type-2 fuzzy best-worst method and likelihood-based multi-criteria method in group decision-making, *Applied Soft Computing* (2023), doi: https://doi.org/10.1016/j.asoc.2023.110856.

An interval type-2 fuzzy best-worst method and likelihood-based multi-criteria method in group decision-making

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Abstract

In real-world decision-making problems, a group of decision-makers (DMs) with different levels of expertise gathers to investigate the problem from various perspectives. Making the best choice from an analogous shortlist of competitors is challenging, even for an expert group. To address these types of problems, a new multi-criteria group decision-making (MCGDM) methodology under the interval type-2 trapezoidal fuzzy (IT2TrF) environment is proposed. The proposed IT2TrF MCGDM method comprises three phases. In the first phase, the IT2TrF cognitive best-worst method (IT2TrF-CBWM) is introduced to calculate the initial weights of criteria using the interval scale. Subsequently, a new consistency index (CI) and consistency ratio (CR) are presented. In the second phase, an optimization model is proposed to determine the final IT2TrF weights of criteria. Finally, a likelihood-based method for solving MCGDM problems is introduced in the third phase. The validity of the proposed method is illustrated by addressing a healthcare waste (HCW) treatment technology selection problem during the COVID-19 pandemic. Sensitivity and comparative analyses highlight the superiority and effectiveness of the presented MCGDM method.

Keywords Multi-criteria group decision-making method, Interval type-2 fuzzy, Interval scale, Best-worst method, Likelihood, Healthcare waste treatment technology, COVID-19.

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Abbreviations

CI	Consistency index
CR	Consistency ratio
DSP	Distinguishing power
AHP	Analytic hierarchy process
BWM	Best-worst method
DM	DM
HCW	Healthcare waste
IT2F	Interval type-2 fuzzy
IT2FS	Interval type-2 fuzzy set
IT2TrF	Interval type-2 trapezoidal fuzzy
IT2TrF-CBWM	IT2TrF cognitive best-worst method
IT2TrFN	Interval type-2 trapezoidal fuzzy number
IT2TrFS	Interval type-2 trapezoidal fuzzy set
LMF	Lower membership function
MCDM	Multi-criteria decision-making
MCGDM	Multi-criteria group decision-making
MF	Membership function
POM	Pairwise opposite matrix
SDM	Senior DM
T1FS	Type-1 fuzzy set
T2FS	Type-2 fuzzy set
TF	Triangular fuzzy
TOPSIS	Technique for order preference by similarity to an ideal solution
UMF	Upper membership function
VIKOR	Visekriterijumska optimizacija i kom-promisno resenje

1. Introduction

In complex decision-making environments, a single decision-maker (DM) might not possess the capability to analyze every facet of a problem comprehensively [1]. This realization has driven the evolution of multi-criteria group decision-making (MCGDM) methods, which integrate the insights and talents of different experts [2]. Typically in MCGDM cases, a group of DMs with unique expertise unites to investigate the issue at hand. The structure and roles within this group are often preset or resistant to alteration. For instance, a senior DM (SDM) might assemble a group of DMs to evaluate staff performance or deliberate on technology acquisitions.

This paper categorizes such group decision-making scenarios as autocratic-democratic problems. Two distinct styles of autocratic-democratic decision-making exist [1, 2]: the first style manifests when the SDM prefers democratic decision-making and values the opinions of the DMs. In contrast, the second style emerges where the SDM is reluctant to involve other experts' opinions in the final decision. Each approach carries its merits and drawbacks. While the democratic style encourages collaboration and idea-sharing among team members, it can be time-intensive and might not be suited for every situation. On the other hand, the autocratic style proves beneficial during time-sensitive emergencies, but it may result in the loss of expert insights and support.

In practical decision-making problems, DMs often express their subjective judgments using linguistic terms. Given that these terms must be encoded and converted into numerical variables for utilization in mathematical models, various methods have been proposed to effect this transformation [3]. Given the inherent fuzziness of terms, different types of fuzzy sets have been formulated [4]. Nonetheless, the task of modeling a specific linguistic term using a particular membership function is intricate [4], and most research neglects to consider that a linguistic expression might hold different meanings for different individuals [3]. In response to these limitations, the introduction of the type-2 fuzzy set (T2FS) has emerged as a method for encoding linguistic terms.

T2FS, an extension of the type-1 fuzzy set (T1FS) initially proposed by Zadeh [5], aims to enhance the modeling of fuzzy information [2]. The defining characteristic of T2FS lies in its membership function (MF), the primary and secondary MFs, thereby offering more extensive flexibility in modeling linguistic terms than T1FS alone [6]. Indeed, within this context, more imprecise information can be covered by the T2FS theory [3, 6]. Nevertheless, the high computational complexity associated with T2FS limits its applicability. To address this concern, the interval type-2 fuzzy (IT2F) set (IT2FS) was introduced as a modified version of T2FS [2, 7]. IT2FS retains the advantages of T2FS while boasting simpler calculability, making it more appealing for encoding linguistic terms [3].

Previous research has identified two distinct types of uncertainty associated with words: interpersonal and intrapersonal [8–11]. Interpersonal uncertainty relates to an individual's perception and comprehension of a word, while intrapersonal uncertainty pertains to a group's perception of a linguistic term [11]. The IT2FS is established based on the footprint of uncertainty- the area between the lower and upper MFs- and can appropriately model these uncertainties [10, 11]. Consequently, the utilization of IT2FS for encoding linguistic terms can compensate for humans' restricted cognitive capacity, thereby yielding a more precise representation of linguistic terms within decision-making scenarios.

In addition to the encoding of words, it is essential to investigate the multi-criteria decision-making (MCDM) problem from different angles and consider its characteristics distinctively. Specifically, in terms of scale, Yeun [12] demonstrated that the ratio scale defined in the analytic hierarchy process (AHP) yields exaggerated results and may not be a suitable scale for homogeneous and similar alternatives. For instance, consider a scenario where A is 160 cm, and B measures 161 cm; here, it is reasonable to perceive B as only slightly taller than A. By employing the fuzzy ratio scale, slightly taller translate to values of (1,2,3), leading to the height of B being (160,320,480), which significantly diverges from human cognitive expectations [13]. Figure 1 visually exemplifies this exaggeration, and the utilization of the interval type-2 trapezoidal fuzzy (IT2TrF) ratio scale accentuates this effect even further. To avoid this issue, Yuen [14] introduced the concept of the interval scale. In this scale, DM preferences are ideally equal to differences between utilities, not weight ratios [15]. In this case, one indicates the slightly taller, so B's height is correctly calculated at 161 cm or based on the fuzzy interval scale (160, 161,162).

This research paper introduces a novel method to MCGDM within the context of the IT2TrF environment. This method is applied to the selection of a healthcare waste (HCW) treatment technology during the COVID-19 pandemic in Iran. IT2TrFSs represent a distinctive category of fuzzy sets that can be reduced into T1FSs (trapezoidal and triangular fuzzy sets), interval numbers, and crisp numbers. This characteristic enables the adaptability of our proposed method to other mathematical environment. Given the limited resources, the technology is selected from a short list of analogous alternatives, which can be viewed as an application of the interval scale. The outlined IT2TrF MCGDM methodology consists of three phases driving its proposed application.

In the initial phase, a new pairwise comparison method based on the interval scale is introduced, which is employed to establish the initial criteria weights. We extend the best-worst method (BWM) to derive these initial criteria weights. The structure of the BWM centers on the pairwise comparisons of all criteria across two opposing reference criteria. This specific arrangement significantly contributes to mitigating inconsistencies in experts' judgments and helps to alleviate the expert's anchors toward a particular criterion [16]. Consequently, we present a new BWM within the IT2TrF environment based on the interval scale, aiming to generate more reliable weights.

Moving to the second phase, we present an IT2TrF optimization model. This model serves the purpose of computing the final criteria weights. Our optimization model tries to calculate the criteria weights, employing both the initial criteria weights and the reliability of DM's judgments obtained from the first phase. Particularly beneficial for scenarios featuring an extensive array of criteria, simplifying the aggregation process and enhancing the reliability of the results by considering the

DM's judgment reliability.

In the final phase, we propose an MCGDM method called IT2TrF-likelihood-based MCGDM. This technique ranks alternatives based on the IT2TrF likelihoods of preference relations obtained from evaluating alternatives across criteria. We extend the likelihood-based MCDA method initially proposed by Chen [17], this method is recognized for its simplicity and straightforwardness in ranking alternatives. These features as well as flexibility stands as a vital aspect of our proposed method as we aim to purpose a model that can address different structures of decision-making problems. This adaptable MCGDM approach can effectively tackle both emergent and routine problems, encompassing hierarchical or flat decision-making structures. From a mathematical perspective, aiming to preserve information as much as possible all calculations are conducted within the IT2TrF environment. The contributions of this paper can be outlined as follows:

- We extend the BWM within the IT2TrF environment and the interval scale. Our BWM's mathematical optimization model is grounded on the concept of robust optimization to minimize the probability of losing IT2TrF information. The mathematical optimization model yields normalized IT2TrF criteria weights regarding the DM's pairwise comparisons. Subsequently, we present a new consistency index (CI) and consistency ratio (CR) to evaluate the consistency of IT2TrF pairwise comparisons.

- The inclusion of an IT2TrF optimization model within the domain of group decision-making is our next contribution. This optimization model serves to aggregate the initial IT2TrF criteria weights, obtaining the final normalized IT2TrF criteria weights. The model takes into account both the reliability of DMs' judgments and their hierarchical positions within the decision-making group. This approach efficiently addressing an array of decision-making scenarios, be they emergent or routine. The seamless integration of the IT2TrF-CBWM method and the IT2TrF-likelihood-based MCGDM method forms the bedrock of this optimization model, facilitating the adept fusion of weighting and ranking methods.

- We extend the likelihood-based MCDA method to rank alternatives within the MCGDM framework. The proposed extension effectively addresses group decision-making problems of different structures, whether they are flat or hierarchical. This enhancement boosts the method's capability to manage group decision-making processes involving multiple DMs.

- We present a novel MCGDM technique that seamlessly integrates the IT2TrF-CBWM and IT2TrF optimization model and IT2TrF-likelihood-based MCGDM. The unified MCGDM method delivers reliable results, providing a pragmatic approach to autocratic-democratic decision-making problems.

- The provided MCGDM method is employed to address the problem of selecting HCW treatment technologies during the COVID-19 pandemic. By leveraging our IT2TrF MCGDM method, we can examine decision-making problems from a fresh angle. This approach allows for an exploration of the issue through different decision-making styles.

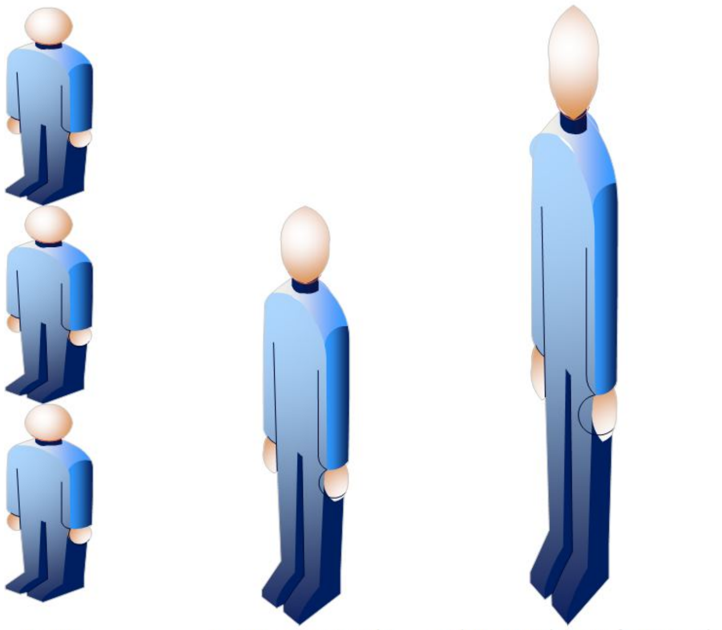


Figure 1: Estimation of B's height using ratio and fuzzy ratio scale

The remainder of this paper is structured as follows. Section 2 presents a concise overview of the relevant literature. Section 3 presents preliminaries focusing on the basic concepts of IT2TrFSs, and IT2TrF comparison relations for interval scale. We introduce the proposed BWM alongside the new CI, and the corresponding CR in Section 4. Section 5 presents the comprehensive development of the MCGDM method. In this section, the complete integration of the IT2TrF-CBWM, the IT2TrF optimization model, and the IT2TrF-likelihood-based MCGDM method are introduced. We discuss a case study on selecting an HCW treatment technology during the COVID-19 pandemic in Iran and present details of data collection in Section 6. Section 7 thoroughly discusses the implementation of different phases of the proposed IT2TrF MCGDM method. Section 8 includes some sensitivity and comparative analyses to indicate the validity of the proposed method. Section 9 provides detailed discussions about the theoretical, methodological, and managerial implications of the study. Finally, we provide our conclusions and future research directions in Section 10.

2. Literature review

This section briefly reviews research on IT2F MCDM methods and BWM.

2.1. Review on IT2F MCDM/MCGDM

T2FS was introduced by Zadeh [5] as an extension of T1FS. In contrast to T1FS, T2FS is characterized by two membership functions: primary and secondary [3, 6]. However, the heavy computational workload associated with T2FS limits the theory's applicability [6]. As a result, IT2FS was introduced as a simplified version of T2FS, with a secondary membership of one [18]. Scholars widely adopt IT2FS due to its simplicity in calculations and effectiveness in capturing vague and imprecise information [19], especially in the MCDM/MCGDM methods and problems. Since DMs can more conveniently express their preferences using linguistic terms, IT2FS is frequently employed to encode these terms [3]. For instance, in recent times, a variety of MCDM/MCGDM problems, such as healthcare [7, 20–22], supplier selection [23], and location selection [24–27], have been addressed using IT2FS. Theoretically, MCDM/MCGDM methods have been extended within the IT2F environment. This includes methods like the likelihood-based decision-making [17, 23], technique for order preference by similarity to an ideal solution (TOPSIS) [28, 29], visekriterijumska optimizacija i kom-promisno resenje (VIKOR) [30**?** , 31], combined compromise solution [19], Preference Ranking Organization Method for Enrichment Evaluations II (PROMETHEE II) [32], organísation, rangement et Synthèse de données relarionnelles (ORESTE) [33], BWM [2, 34, 35], and AHP [36]. Celik et al. [37] conducted a comprehensive research review on MCDM/MCGDM methods within the IT2F environment, paving the way for future research. This paper proposes a new IT2TrF MCGDM method that extends and integrates the BWM and a likelihood-based MCDA method as a novel approach to address the problem of HCW treatment technology selection.

2.2. Review on BWM

BWM is one of the latest MCDM methods constructed based on the pairwise comparison method to calculate the criteria weights. Since its first proposal by Rezaei [38], it has captured scholars' attention. Compared to AHP, it is a more robust method in terms of the number of pairwise comparisons [39]. Till now, prominent research has been conducted to encode imprecise evaluations using different fuzzy set theories combined with BWM. Guo and Zhao. [40] extended BWM using triangular fuzzy numbers. Ali and Rashid [41] considered hesitancy within the information received from DMs and introduced hesitant fuzzy BWM. Aboutorab et al. [42] combined Z-number with the triangular fuzzy BWM to add reliability to the model. Wu et al. [30] developed a linear BWM under the IT2F environment for solving MCGDM problems. Wan, Dong, and Chen [39] developed a goal-programming model for BWM based on generalized interval-valued trapezoidal fuzzy numbers. Tavana et al. [19] proposed a novel combined BWM and CoCoSo within the IT2F environment. The method has also been used in many applications area. Gulum Tas [43] provided an overview of the recent applications of BWM in health. The author divided the related studies into six categories and listed some of the contributions of BWM to HCW management. Huang et al. [44] proposed a framework to select an appropriate HCW treatment technology based on the BWM. Yazdani et al. [45] integrated the BWM with a new dombi-bonferroni means to select a location for healthcare waste disposal. Pamu˘car et al. [46] utilized the combination of BWM and multi-attributive border approximation area comparison method for selecting an HCW treatment technology. Torkayesh et al. [47] presented a scenario-based BWM, stratified BWM, for selecting a waste disposal technology. This paper explores the BWM based on the interval scale within the IT2F environment to propose a new variant of the method.

2.3. Research gap

In this section, we clarify the gaps in the relevant literature.

Although the BWM exhibits unique characteristics that make it well-suited for application across different scenarios and mathematical environments [39], the challenge of extending BWM to incorporate vague or imprecise information using the interval scale remains significant. Exploring the method based on the interval scale, as discussed in the introduction section, becomes crucial for dealing with a similar and homogeneous list of alternatives. Addressing this challenge is one of our research aims as we investigate the extension of BWM within the IT2F environment based on the interval scale. This endeavor necessitates the development of a novel optimization model and the establishment of a CI and CR specifically tailored to the characteristics of the interval scale.

Furthermore, while the likelihood-based MCDM method proposed by Chen [17] provides a straightforward means of ranking alternatives, its applicability to MCGDM problems remains unexplored, to the best of the authors' knowledge. Given the inherent hierarchical structure of DMs, a thorough investigation of the likelihood-based MCDA method becomes imperative. Our study aims to fill this gap by evaluating the efficacy of this method within the MCGDM context.

The two extended methodologies are integrated using a novel mathematical optimization model, aiming to determine the final and collective criteria weights. Introducing a model that integrates weighting and ranking methods is crucial to flexibly consider different decision-making scenarios. Ultimately, this paper seeks to enhance the practicality and precision of these methods in complex group decision-making scenarios with varying structures, whether hierarchical or flat.

3. Preliminaries

This section reviews some basic concepts of IT2TrFS and IT2TrF pairwise comparisons for the interval scale.

3.1. Interval type-2 trapezoidal fuzzy set (IT2TrFS)

Definition 1 ([18]). Let $\hat{A}$ be a T2FS in a universe of discourse X. It is indicated by a type-2 membership function $\mu_{\hat{A}}(x, u)$, such that:

$$\hat{A} = \{((x, u), \mu_{\hat{A}}(x, u)) \mid x \in X, u \in [0, 1]\} \tag{1}$$

Definition 2 ([18]). Let $\hat{A}$ be a T2FS in a universe of discourse X. If all $\mu_{\hat{A}}(x, u) = 1$, then the T2FS is IT2FS and is presented as follows:

$$I_x = \{u \in [0, 1] \mid \mu_{\hat{A}}(x, u) = 1\} \tag{2}$$

Definition 3 ([20]). An IT2FN $\hat{A} = [A^L, A^U] = [(a_1^L, a_2^L, a_3^L, a_4^L; H(A^L)), (a_1^U, a_2^U, a_3^U, a_4^U; H(A^U))]$ is denoted as IT2TrF numbers (IT2TrFN) and is recognized by two MFs as follows:

$$\mu_{\hat{A_x^L}}(x) = \begin{cases} \frac{x-a_1^L}{a_2^L-a_1^L} H(A^L), & \text{if } a_1^L \leq x \leq a_2^L \\ H(A^L), & \text{if } a_2^L \leq x \leq a_3^L \\ \frac{a_4^L-x}{a_4^L-a_3^L} H(A^L), & \text{if } a_3^L \leq x \leq a_4^L \end{cases} \tag{3}$$

and

$$\mu_{\hat{A_x^U}}(x) = \begin{cases} \frac{x-a_1^U}{a_2^U-a_1^U} H(A^U), & \text{if } a_1^U \leq x \leq a_2^U \\ H(A^U), & \text{if } a_2^U \leq x \leq a_3^U \\ \frac{a_4^U-x}{a_4^U-a_3^U} H(A^U), & \text{if } a_3^U \leq x \leq a_4^U \end{cases} \tag{4}$$

Where $\mu_{\hat{A_x^L}}(x)$ and $\mu_{\hat{A_x^U}}(x)$ are the lower MF (LMF) and upper MF (UMF), respectively, each element of $\hat{A}$ are non-negative real values satisfies $a_1^L \leq a_2^L \leq a_3^L \leq a_4^L$, $a_1^U \leq a_2^U \leq a_3^U \leq a_4^U$, $a_1^U \leq a_1^L$, and $a_4^L \leq a_4^U$. Also, $H(A^L)$ and $H(A^U)$ represent the heights of A^L and A^U, respectively, and satisfy $0 \leq H(A^L) \leq H(A^U) \leq 1$. The geometrical interpretation of an IT2TrFN is depicted in Figure 2.

Definition 4 ([2]). Let $\hat{A}$ be an IT2TrFS. The UMF must completely cover the LMF. The following constraints provide the condition.

$$a_2^L \geq \frac{H(A^L)(a_2^U - a_1^U))}{H(A^U)} + a_1^U \tag{5}$$

$$a_3^L \leq \frac{H(A^L)(a_3^U - a_4^U))}{H(A^U)} + a_4^U \tag{6}$$

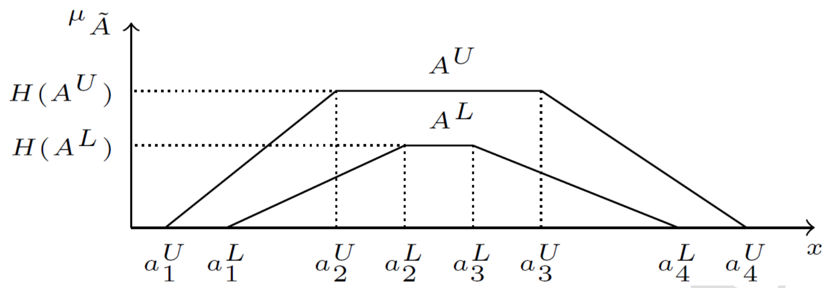


Figure 2: Interval type-2 trapezoidal fuzzy number

Definition 5 ([30]). Let $\hat{A} = [(a_1^L, a_2^L, a_3^L, a_4^L; H(A^L)), (a_1^U, a_2^U, a_3^U, a_4^U; H(A^U))]$ and $\hat{B} = [(b_1^L, b_2^L, b_3^L, b_4^L; H(B^L)), (b_1^U, b_2^U, b_3^U, b_4^U; H(B^U))]$ be two non-negative IT2TrFNs, the arithmetic operations between $\hat{A}$ and $\hat{B}$ are as follows:

$$\begin{aligned}\hat{A} + \hat{B} = [&(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; min\{H(A^L), H(B^L)\}),\\ &(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; min\{H(A^U), H(B^U)\})]\end{aligned} \tag{7}$$

$$\begin{aligned}\hat{A} - \hat{B} = [&(a_1^L - b_1^L, a_2^L - b_2^L, a_3^L - b_3^L, a_4^L - b_4^L; min\{H(A^L), H(B^L)\}),\\ &(a_1^U - b_1^U, a_2^U - b_2^U, a_3^U - b_3^U, a_4^U - b_4^U; min\{H(A^U), H(B^U)\})]\end{aligned} \tag{8}$$

$$\begin{aligned}\hat{A}\hat{B} = [&(a_1^L b_1^L, a_2^L b_2^L, a_3^L b_3^L, a_4^L b_4^L; min\{H(A^L), H(B^L)\}),\\ &a_1^U b_1^U, a_2^U b_2^U, a_3^U \times b_3^U, a_4^U b_4^U; min\{H(A^U), H(B^U)\})]\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\hat{A}}{\hat{B}} = [&(\frac{a_1^L}{b_1^L}, \frac{a_2^L}{b_2^L}, \frac{a_3^L}{b_3^L}, \frac{a_4^L}{b_4^L}; min\{H(A^L), H(B^L)\}),\\ &(\frac{a_1^U}{b_1^U}, \frac{a_2^U}{b_2^U}, \frac{a_3^U}{b_3^U}, \frac{a_4^U}{b_4^U}; min\{H(A^U), H(B^U)\})]\end{aligned} \tag{10}$$

$$k \times \hat{A} = [(ka_1^L, ka_2^L, ka_3^L, ka_4^L; H(A^L)), (ka_1^U, ka_2^U, ka_3^U, ka_4^U; H(A^U))] \tag{11}$$

Definition 6 ([7]). The distance between two IT2TrFNs $\hat{A}$ and $\hat{B}$ can be defined by the following equation.

$$D(\hat{A}, \hat{B}) = \frac{1}{8}\sqrt{\sum_{k=1}^{4}(a_k^L - b_k^L)^2 + (a_k^U - b_k^U)^2} \tag{12}$$

Definition 7 ([34]). Defuzzification of $\hat{A}$ which is conducted by the centroid approach, $COA(\hat{A})$ is:

$$COA(\hat{A}) = \frac{1}{2}[\frac{(a_4^L - a_1^L + H(A^L) \times a_2^L - a_1^L + H(A^L) \times a_3^L - a_1^L)}{4} + a_1^L] + \frac{1}{2}[\frac{(a_4^U - a_1^U + H(a^U) \times a_2^U - a_1^U + H(A^U) \times a_3^U - a_1^U)}{4} + a_1^U] \quad (13)$$

Definition 8 ([17]). The likelihood of an IT2TrF preference relation $\hat{a}_{ij} \geq \hat{a}_{i'j}$ is:

$$L(\hat{a}_{ij} \geq \hat{a}_{i'j}) = \frac{L^-(\hat{a}_{ij} \geq \hat{a}_{i'j}) + L^+(\hat{a}_{ij} \geq \hat{a}_{i'j})}{2} \quad (14)$$

Where $L^-(\hat{a}_{ij} \geq \hat{a}_{i'j})$ and $L^+(\hat{a}_{ij} \geq \hat{a}_{i'j})$ represent the lower and upper likelihood of an IT2TrF preference relation $\hat{a}_{ij} \geq \hat{a}_{i'j}$. The value of $L(\hat{a}_{ij} \geq \hat{a}_{i'j})$ indicates the possibility that $\hat{a}_{i'j}$ is not greater than $\hat{a}_{ij}$. In what follows, the related definitions, and formulations are explained.

Definition 9 ([17]). Let $\hat{a}_{ij}$ and $\hat{a}_{i'j}$ be two non-negative IT2TrFN. If at least one of $H(a_{ij}^L) \neq H(a_{i'j}^U)$, $a_{ij1}^L \neq a_{ij4}^L$, $a_{i'j1}^U \neq a_{i'j4}^U$, and $a_{ijk}^L \neq a_{i'jk}^U$ for $k = 1, 2, 3, 4$ holds true, the lower likelihood $L^-(\hat{a}_{ij} \geq \hat{a}_{i'j})$ of an IT2TrF preference relation $\hat{a}_{ij} \geq \hat{a}_{i'j}$ is:

$$L^-(\hat{a}_{ij} \geq \hat{a}_{i'j}) = max\{1 - max\{\frac{\sum_{k=1}^4 max\{(a_{ijk}^U - a_{i'jk}^L), 0\} + (a_{ij4}^U - a_{i'j1}^L) + 2 \times max\{H(A_{ij}^U) - H(A_{ij}^L), 0\}}{\sum_{k=1}^4 |a_{ijk}^U - a_{i'jk}^L| + (a_{ij4}^L - a_{i'j1}^L) + (a_{ij4}^U - a_{i'j1}^U) + 2|H(A_{ij}^U) - H(A_{ij}^L)|}\}, 0\} \quad (15)$$

Definition 10 ([17]). Let $\hat{a}_{ij}$ and $\hat{a}_{i'j}$ be two non-negative IT2TrFN. If at least one of $H(a_{ij}^U) \neq H(a_{i'j}^L)$, $a_{ij1}^U \neq a_{ij4}^U$, $a_{i'j1}^L \neq a_{i'j4}^L$, and $a_{ijk}^U \neq a_{i'jk}^L$ for $k = 1, 2, 3, 4$ holds true, the upper likelihood $L^+(\hat{a}_{ij} \geq \hat{a}_{i'j})$ of an IT2TrF preference relation $\hat{a}_{ij} \geq \hat{a}_{i'j}$ is:

$$L^+(\hat{a}_{ij} \geq \hat{a}_{i'j}) = max\{1 - max\{\frac{\sum_{k=1}^4 max\{(a_{ijk}^L - a_{i'jk}^U), 0\} + (a_{ij4}^L - a_{i'j1}^U) + 2 \times max\{H(A_{ij}^L) - H(A_{ij}^U), 0\}}{\sum_{k=1}^4 |a_{ijk}^L - a_{i'jk}^U| + (a_{ij4}^L - a_{i'j1}^L) + (a_{ij4}^U - a_{i'j1}^U) + 2|H(A_{ij}^U) - H(A_{ij}^L)|}\}, 0\} \quad (16)$$

3.2. IT2TrF pairwise comparisons for interval scale

Individuals can subjectively convey the degree of preference for criterion i over criterion j and construct a pairwise opposite matrix (POM). In this context, the term "Equal" is represented by zero, indicating that there is no difference between the importance of criterion i and j [12, 14].

An IT2TrF POM, $\tilde{B} = [\tilde{b}_{ij}]_{n\times n}$, is established to interpret the individual utilities of criteria. Let $\tilde{V} = (\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_n)$ be an ideal IT2TrF utility set and the comparison score is $\tilde{b}_{ij} \cong \tilde{v}_i - \tilde{v}_j$, the ideal IT2TrF POM is $\tilde{\hat{B}} = [\tilde{v}_i - \tilde{v}_j]_{n\times n}$. The $\tilde{\hat{B}}$ is determined by $\tilde{B}$ as follows:

$$\tilde{\hat{B}} = \begin{bmatrix} \tilde{0} & \tilde{v}_1 - \tilde{v}_2 & \cdots & \tilde{v}_1 - \tilde{v}_n \\ \tilde{v}_2 - \tilde{v}_1 & \tilde{0} & \cdots & \tilde{v}_2 - \tilde{v}_n \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{v}_n - \tilde{v}_1 & \cdots & \tilde{v}_n - \tilde{v}_{n-1} & \tilde{0} \end{bmatrix} \cong \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \cdots & \tilde{b}_{1n} \\ \tilde{b}_{21} & \tilde{b}_{22} & \cdots & \tilde{b}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{b}_{n1} & \tilde{b}_{n2} & \cdots & \tilde{b}_{nn} \end{bmatrix} = \tilde{B} \tag{17}$$

Where $\tilde{b}_{ij} + \tilde{b}_{ji} = \tilde{0}$ for all $i \neq j = 1, 2, 3, ..., n$ and when $i = j$ then $\tilde{b}_{ij} = \tilde{v}_i - \tilde{v}_j = \tilde{0}$. $\tilde{B}$ is perfectly consistent if $\tilde{b}_{ij} = \tilde{b}_{ik} + \tilde{b}_{kj}$ for all $i, j, k = 1, 2, 3, ..., n$.

Definition 11. Let $\tilde{V} = (\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_n)$ be the IT2TrF utility vector, for all $j = 1, 2, \ldots, n$, and $\tilde{k}$ be the maximum IT2TrF utility. The normalized/rescaled form of the vector $\tilde{V}$ can be obtained as follow:

$$\begin{aligned} \tilde{W} = \{[(w_{j1}^L, w_{j2}^L, w_{j3}^L, w_{j4}^L; H(w_j^L)), (w_{j1}^U, w_{j2}^U, w_{j3}^U, w_{j4}^U; H(w_j^U))] | \\ [(\frac{v_{j1}^L}{nk_4^u}, \frac{v_{j2}^L}{nk_4^u}, \frac{v_{j3}^L}{nk_4^u}, \frac{v_{j4}^L}{nk_4^u}; H(v^L)), (\frac{v_{j1}^U}{nk_4^u}), \frac{v_{j2}^U}{nk_4^u}, \frac{v_{j3}^U}{nk_4^u}, \frac{v_{j4}^U}{nk_4^u}; H(v^U))],\ \forall j \in \{1, 2, \ldots, n\}\} \end{aligned} \tag{18}$$

To obtain IT2TrF weight vector $\tilde{W} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$, each $\tilde{w}_i = [(w_1^L, w_2^L, w_3^L, w_4^L; H(w^L)), (w_1^U, w_2^U, w_3^U, w_4^U; H(w^U))]$ must satisfy the conditions represented in Definitions 1 to 4. Moreover, Wan, Chen, and Dong [2] presented the following definition to obtain a normalized IT2TrF weight vector.

Definition 12 ([2]). Let $\tilde{W} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$, be an IT2TrF weight vector, where $\tilde{w}_i = [(w_1^L, w_2^L, w_3^L, w_4^L; H(w^L)), (w_1^U, w_2^U, w_3^U, w_4^U; H(w^U))]$. It is said a normalized IT2TrF weight vector if and only if the following constraints hold:

$$\begin{aligned} & w_{i4}^L + \sum_{j=1, j\neq i}^{n} w_{j1}^L \leq 1;\ w_{i3}^L + \sum_{j=1, j\neq i}^{n} w_{j2}^L \leq 1 \qquad i = 1, 2, 3, ..., n \\ & w_{i2}^L + \sum_{j=1, j\neq i}^{n} w_{j3}^L \geq 1;\ w_{i1}^L + \sum_{j=1, j\neq i}^{n} w_{j4}^L \geq 1 \qquad i = 1, 2, 3, ..., n \\ & w_{i4}^U + \sum_{j=1, j\neq i}^{n} w_{j1}^U \leq 1;\ w_{i3}^U + \sum_{j=1, j\neq i}^{n} w_{j2}^U \leq 1 \qquad i = 1, 2, 3, ..., n \\ & w_{i2}^U + \sum_{j=1, j\neq i}^{n} w_{j3}^U \geq 1;\ w_{i1}^U + \sum_{j=1, j\neq i}^{n} w_{j4}^U \geq 1 \qquad i = 1, 2, 3, ..., n \end{aligned}$$

4. Interval type-2 trapezoidal fuzzy cognitive best-worst method (IT2TrF-CBWM)

This section presents an extension of BWM called IT2TrF-CBWM. The IT2FS models the subjective and imprecise evaluations that exist in decision-making. To convert the perceptions of DMs to numbers, we benefit from the concept of the interval scale [12]. Yuen [14] argued that the AHP ratio scale might produce exaggerated results. Consequently, the fuzzy interval scale was proposed and employed to calculate the criteria weights and rank alternatives [13].

The IT2TrF-CBWM is proposed in the following algorithm to calculate the criteria weights. The term "cognitive" used in the method's name denotes that the obtained results are, in many situations, more attune to human cognition than those derived from the ratio scale, a point discussed in the introduction section. The steps of the IT2TrF-CBWM are defined as follows:

Step 1. Determine a set of decision criteria $C = \{C_1, C_2, ..., C_n\}$. The criteria set can be determined through literature review and expert interviews.

Step 2. Identify the best (most important) criterion C_B and the worst (least important) criterion C_W.

Step 3. Establish the IT2TrF reference comparisons for the best criterion. The IT2TrF reference comparisons play a pivotal role in IT2TrF-CBWM. A DM can express his level of preference for C_B over criterion j using the linguistic terms listed in Table 1. The IT2TrF Best-to-Others (BO) vector is as follows:

$$\hat{B}_B = \{\hat{b}_{B1}, \hat{b}_{B2}, \ldots, \hat{b}_{Bn}\} \tag{19}$$

Where $\hat{b}_{Bj}$ is an IT2TrFN, denoting the IT2TrF preference of criterion C_B over criterion j. It is evident that $\hat{b}_{BB} = [(0,0,0,0;1),(0,0,0,0;1)]$.

Step 4. Establish the IT2TrF reference comparisons for the worst criterion. A DM can express his level of preference for criterion j over C_W using the linguistic terms listed in Table 1. The IT2TrF Others-to-Worst (OW) vector is as follows:

$$\hat{B}_W = \{\hat{b}_{W1}, \hat{b}_{W2}, \ldots, \hat{b}_{Wn}\} \tag{20}$$

Where $\hat{b}_{jW}$ is an IT2TrFN, denoting the IT2TrF preference of criterion j over the worst criterion C_W. It is evident that $\hat{b}_{WW} = [(0,0,0,0;1),(0,0,0,0;1)]$.

Table 1: Rating scale schemas for IT2TrF-CBWM

Terms[2]	Interval scale [14]	TF interval scale [13]	IT2TrF interval scale
EMI	8	(7,8,8)	[(7.5,8,8,8.5;0.9), (7,8,8,9;1)]
EI	7	(6,7,8)	[(6.5,7,7,7.5;0.9), (6,7,7,8;1)]
VSI	6	(5,6,7)	[(5.5,6,6,6.5;0.9), (5,6,6,7;1)]
SPI	5	(4,5,6)	[(4.5,5,5,5.5;0.9), (4,5,5,6;1)]
SI	4	(3,4,5)	[(3.5,4,4,4.5;0.9), (3,4,4,5;1)]
MPI	3	(2,3,4)	[(2.5,3,3,3.5;0.9), (2,3,3,4;1)]
MI	2	(1,2,3)	[(1.5,2,2,2.5;0.9), (1,2,2,3;1)]
WI	1	(0,1,2)	[(0.5,1,1,1.5;0.9), (0,1,1,2;1)]
CEI	0	(0,0,0)	[(0,0,0,0;1), (0,0,0,0;1)]

The IT2TrFNs corresponding to the linguistic terms are obtained by fuzzifying the triangular fuzzy (TF) numbers introduced by references [13, 48] and indicated in column three of Table 1. These TF numbers are converted into IT2TrFNs using the additive fuzzification detailed below.

Definition 13. Let $a = (a_1, a_2, a_3)$ be a TF number. If a_1 and a_3 are the lower and upper values of the UMF of IT2TrFN, respectively, such that $a_1^U = a_1$, $a_4^U = a_3$, $a_2^U = a_2$, $a_3^U = a_2$, and for the LMF of IT2TrF number $a_1^L = a_1^U + \delta$, $a_4^L = a_4^U - \delta$, $a_2^L = a_2^U$, $a_3^L = a_3^U$, where the parameter δ is a tuning parameter employed to convert the TF number to IT2TrF, then the IT2TrFN can be defined as:

$$\hat{a} = [(a_1^L, a_2^L, a_3^L, a_4^L; H(A^L)), (a_1^U, a_2^U, a_3^U, a_4^U; H(A^U))] \tag{21}$$

Where $H(A^L)$ and $H(A^U)$ are the heights of LMF and UMF of $\hat{a}$, respectively, which satisfies the condition $0 \leq H(A^L) \leq H(A^U) \leq 1$. In this case, we set $\delta = 0.5$, which is the variance of the modal points of fuzzy numbers within the TF interval scale. Additionally, the heights of the LMF and UMF of the IT2TrFNs are assumed to be 0.9 and 1, respectively, which corresponds to the height of the IT2TrFNs used in the third phase (prioritizing alternatives). It is important to highlight that the IT2TrFNs are characterized by repeating the modal value of the TF numbers. Furthermore, for the "EMI" term, we consider the elements of the UMF of the IT2TrF number (7,8,8,9). In this case, if degradation occurs, the number equals to 8, which corresponds to the value of "EMI" in the crisp environment. Table 1 indicates the scheme of the rating scale used in the proposed BWM.

To check the consistency of pairwise comparisons, given the challenges associated with consistency assessment via Equation (17), we propose the following definition:

Definition 14. An IT2TrF preference $\hat{b}_{kj}$ is consistent if

$$\hat{b}_{Bk} + \hat{b}_{kj} = \hat{b}_{Bj}, \; \hat{b}_{kj} + \hat{b}_{jW} = \hat{b}_{kW} \qquad \forall j, k = 1, 2, 3, ..., n. \tag{22}$$

For the calculation of normalized IT2TrF weights of criteria, step 5 is defined as follows.

Step 5. Determine the optimal IT2TrF weight vector $(\hat{w}_1^*, \hat{w}_2^*, \ldots, \hat{w}_n^*)$. Assume $\hat{V} = (\hat{v}_1, \hat{v}_2, \ldots, \hat{v}_n)$ is the IT2TrF utility vector, where $\hat{v}_B$ and $\hat{v}_W$ represent the utilities of the best and the worst criterion, respectively. Consider IT2TrF BO and IT2TrF OW vectors. To obtain reliable IT2TrF weights the relations of $\hat{v}_B - \hat{v}_j = \hat{b}_{Bj}$ and $\hat{v}_j - \hat{v}_W = \hat{b}_{jW}$ must be satisfied for all $j \in \{1, 2, \ldots, n\}$, as much as possible. Since they are not easily obtainable relations, we endeavor to minimize the maximum absolute differences between $|\hat{v}_B - \hat{v}_j - \hat{b}_{Bj}|$ and $|\hat{v}_j - \hat{v}_W - \hat{b}_{jW}|$.

The IT2TrF weight vector is the normalized or rescaled form of utility (priority) vector. This can be swiftly derived in light of Definition 11, elucidating the relationship between a criterion's utility and weight. The foundational mathematical model is model (M1), which tries to calculate the normalized IT2TrF weights. For the sake of simplicity, we consider the heights of IT2TrF deviations 0.9 and 1, respectively. The notations for the optimization model are:

Table 2: The notation used in the proposed model

Indexes	
$i, j \in \{1, 2, \ldots, n\}$	Indices of criteria
$k \in \{1, 2, 3, 4\}$	Indices of trapezoidal fuzzy number elements
B	The best criterion index
W	The worst criterion index
Parameters	
S_k^L	The k-th element of the lower bound of the maximum IT2TrF utility
S_k^U	The k-th element of the upper bound of the maximum IT2TrF utility
b_{ijk}^L	The k-th element of the lower bound of DM's IT2TrF preference of criterion i over j
b_{ijk}^U	The k-th element of the upper bound of DM's IT2TrF preference of criterion i over j
Variables	
v_{ik}^L	The k-th element of the lower bound of the IT2TrF utility of criterion i
v_{ik}^U	The k-th element of the upper bound of the IT2TrF utility of criterion i
w_{ik}^L	The k-th element of the lower bound of the IT2TrF weight of criterion i
w_{ik}^U	The k-th element of the upper bound of the IT2TrF weight of criterion i

The two important issues in modeling and extending a crisp model to a fuzzy environment are considering how to formulate the objective function and constraints. In the following model, the obtained weights would be normalized IT2TrF values, based on Definitions 11 and 12. The model is defined as follows:

$$\text{Min} \max_i \{|\hat{v}_B - \hat{v}_i - \hat{b}_{Bi}|, |\hat{v}_i - \hat{v}_W - \hat{b}_{iW}|\}$$

subject to

$$w_{ik}^L = \frac{v_{ik}^L}{nS_k^L} \qquad i = 1, 2, 3, ..., n, k = 1, 2, 3, 4$$

$$
\begin{aligned}
& w_{ik}^U = \frac{v_{ik}^U}{nS_k^U} && i=1,2,3,...,n,k=1,2,3,4 \\
& w_{i4}^L + \sum_{j=1,j\neq i}^{n} w_{j1}^L \leq 1;\ w_{i3}^L + \sum_{j=1,j\neq i}^{n} w_{j2}^L \leq 1 && i=1,2,3,...,n \\
& w_{i2}^L + \sum_{j=1,j\neq i}^{n} w_{j3}^L \geq 1;\ w_{i1}^L + \sum_{j=1,j\neq i}^{n} w_{j4}^L \geq 1 && i=1,2,3,...,n \\
& w_{i4}^U + \sum_{j=1,j\neq i}^{n} w_{j1}^U \leq 1;\ w_{i3}^U + \sum_{j=1,j\neq i}^{n} w_{j2}^U \leq 1 && i=1,2,3,...,n \\
& w_{i2}^U + \sum_{j=1,j\neq i}^{n} w_{j3}^U \geq 1;\ w_{i1}^U + \sum_{j=1,j\neq i}^{n} w_{j4}^U \geq 1 && i=1,2,3,...,n \\
& w_{i2}^L \geq (w_{i2}^U - w_{i1}^U)(\frac{H(w^L)}{H(w^U)}) + w_{i1}^U && i=1,2,3,...,n \\
& w_{i3}^L \leq (w_{i3}^U - w_{i4}^U)(\frac{H(w^L)}{H(w^U)}) + w_{i4}^U && i=1,2,3,...,n \\
& w_{i1}^L \geq w_{i1}^U;\ w_{i4}^L \leq w_{i4}^U && i=1,2,3,...,n \\
& w_{ik}^L \leq w_{ik+1}^L;\ w_{ik}^U \leq w_{ik+1}^U && i=1,2,3,...,n,k=1,2,3 \\
& w_{ik}^L \geq 0; w_{ik}^U \geq 0 && i=1,2,3,...,n,k=1,2,3,4
\end{aligned}
\tag{M1}
$$

Where $[(S_1^L, S_2^L, S_3^L, S_4^L; H(S^L)), (S_1^U, S_2^U, S_3^U, S_4^U; H(S^U))] = [(7.5,8,8,8;0.9),(7,8,8,9;1)]$ is the maximum IT2TrF utility and the constraints are discussed in Definitions 11 and 12. For simplicity we assume that $H(w^L) = H(\delta^L) = \min\{H(b_{Bi}^L), H(b_{iW}^L)\}$ and $H(w^U) = H(\delta^U) = \min\{H(b_{Bi}^U), H(b_{iW}^U)\}$. To reduce the probability of losing fuzzy information and calculating more precise solutions, model (M1) is transformed into the following model. This transformation draws inspiration from reference [49], where the trapezoidal fuzzy objective function is converted to a multi-objective function.

$$
\begin{aligned}
& \text{Min } z_1 = \epsilon_1^L;\ \text{Min } z_2 = \frac{\epsilon_2^L + \epsilon_3^L}{2};\ \text{Min } z_3 = \epsilon_3^L;\ \text{Min } z_4 = \epsilon_4^L; \\
& \text{Min } z_5 = \epsilon_1^U;\ \text{Min } z_6 = \frac{\epsilon_2^U + \epsilon_3^U}{2};\ \text{Min } z_7 = \epsilon_3^U;\ \text{Min } z_8 = \epsilon_4^U \\
& \text{subject to} \\
& |v_{Bk}^L - v_{ik}^L - b_{Bik}^L| \leq \epsilon_k^L;\ |v_{Bk}^U - v_{ik}^U - b_{Bik}^U| \leq \epsilon_k^U && i=1,2,3,...,n,k=1,2,3,4 \\
& |v_{ik}^L - v_{Wk}^L - b_{iWk}^L| \leq \epsilon_k^L;\ |v_{ik}^U - v_{Wk}^U - b_{iWk}^U| \leq \epsilon_k^U && i=1,2,3,...,n,k=1,2,3,4 \\
& \epsilon_k^L \geq 0;\ \epsilon_k^U \geq 0 && k=1,2,3,4
\end{aligned}
$$

The remaining constraints are identical to those presented in the model (M1) (M2)

The crisp single-objective model equivalent to model (M2) can be detailed as follows. Let $\epsilon = (\epsilon_k^L, \epsilon_k^U)$ $(k = 1, 2, 3, 4)$ be the variable of the function $z_m(\epsilon)$ and, for simplicity, let $z_m(\epsilon)$ be $z_m(m = 1, 2, 3, \ldots, 8)$. The values of the optimal value of m-th objective function (minimum deviation) z_m^{min} and the corresponding optimal solutions ϵ^* are obtained by solving model (M2) for each objective function. If within this solution space, the maximum value of z_m is defined as $z_m^{max} = \{z_m(\epsilon_p^*)|p = 1, 2, 3, 4, 5, 6, 7, 8\}$ $(m = 1, 2, 3, 4, 5, 6, 7, 8)$, then according to the concept of robust multi-objective optimization [50], the crisp single-objective model (M3) is:

$$\begin{aligned}
&\text{Max } \phi \\
&\text{subject to} \\
&\frac{z_m^{max} - z^{min}(\epsilon)}{z_m^{max} - z_m^{min}} \geq \phi \qquad m = 1, 2, 3, 4, 5, 6, 7, 8 \\
&\text{The remaining constraints are identical to those presented in model (M2)}
\end{aligned} \tag{M3}$$

Solving model (M3) through different software packages such as Matlab and Lingo, optimal weights, $(\hat{w}_1^*, \hat{w}_2^*, \ldots, \hat{w}_n^*)$, and inconsistency values, $(\epsilon_1^{*L}, \epsilon_2^{*L}, \epsilon_3^{*L}, \epsilon_4^{*L}, \epsilon_1^{*U}, \epsilon_2^{*U}, \epsilon_3^{*U}, \epsilon_4^{*U})$, are obtained. The associated pseudocode of IT2TrF-CBWM, referenced as Algorithm 1, is structured as follows:

Algorithm 1 The pseudocode of IT2TrF-CBWM

Input: linguistic terms and their corresponding numbers
Output: optimal IT2TrF weights
$C \leftarrow$ determine a set of evaluation criteria (Step 1)
$B \leftarrow$ specify the best criterion (Step 2)
$W \leftarrow$ specify the worst criterion (Step 2)
$n \leftarrow |C|$
for $j = 1$ to n **do**
$\hat{B}_B \leftarrow$ print("What is preference of criterion 'B' over criterion 'j' ?") (Step 3)
$\hat{B}_W \leftarrow$ print("What is preference of criterion 'j' over criterion 'W' ?") (Step 4)
end for
Return $\hat{B}_B$, $\hat{B}_W$
for $j = 1$ to n **do**
$\hat{w}_j^* \leftarrow$ Optimize weights $(\hat{B}_B, \hat{B}_W)$ (Step 5)
end for
Return $\hat{w}_1^*, \hat{w}_2^*, ..., \hat{w}_n^*$

4.1. CI for IT2TrF-CBWM

To check the consistency of pairwise comparisons, we first define the CI according to Definition 14. Regarding Definition 14, an IT2TrF POM is perfectly consistent if the following equations hold.

$$\hat{b}_{Bi} + \hat{b}_{ij} = \hat{b}_{Bj}, \qquad i, j = 1, 2, ..., n. \tag{23}$$

$$\hat{b}_{ij} + \hat{b}_{jW} = \hat{b}_{iW}, \qquad i, j = 1, 2, ..., n. \tag{24}$$

Otherwise, inconsistency occurs. We concentrate on computing the maximum inconsistency degree. In this case, $\hat{b}_{Bi} + \hat{b}_{iw} \neq \hat{b}_{BW}$ The equation below must be held in order to achieve equality.

$$(\hat{b}_{Bi} - \hat{\delta}) + (\hat{b}_{iW} - \hat{\delta}) = (\hat{b}_{BW} + \hat{\delta}) \tag{25}$$

Where $\hat{\delta}$ is a deviation variable, concerning maximum inconsistency $\hat{b}_{Bj} = \hat{b}_{jW} = \hat{b}_{BW}$, Eq. (25) is converted into:

$$(\hat{b}_{BW} - \hat{\delta}) + (\hat{b}_{BW} - \hat{\delta}) = (\hat{b}_{BW} + \hat{\delta}) \tag{26}$$

Since $b^L_{BW4} \geq b^L_{BWk}$ and $b^U_{BW4} \geq b^U_{BWk}$ for k = 1,2,3,4, $\hat{b}_{BW}$ will be represented by b^L_{BW4} and b^U_{BW4}. Similarly, δ^L and δ^U are considered crisp numbers representing the lower and upper bound of IT2TrF deviation. Therefore, the crisp equivalent equations are:

$$(b^L_{BW4} - \delta^L) + (b^L_{BW4} - \delta^L) = (b^L_{BW4} + \delta^L) \tag{27}$$

$$(b^U_{BW4} - \delta^U) + (b^U_{BW4} - \delta^U) = (b^U_{BW4} + \delta^U) \tag{28}$$

Solving Eqs. (27) and (28) for nine different values of b^L_{BW4} and b^U_{BW4}, the maximum value of δ^L and δ^U can be obtained. The values of δ^L and δ^U are considered the lower bound of the consistency index, CI^L, and the upper bound of the consistency index, CI^U, respectively, as shown in Tables 3 and 4.

Table 3: CI^L for the proposed BWM

b^L_{BW4}	0	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5
CI^L	0	0.5	0.83	1.167	1.5	1.83	2.167	2.5	2.83

Table 4: CI^U for the proposed BWM

b^U_{BW4}	0	2	3	4	5	6	7	8	9
CI^U	0	0.67	1	1.33	1.67	2	2.33	2.67	3

4.2. CR for IT2TrF-CBWM

To increase the flexibility of the model (M3), we consider all the lower and upper bounds of the optimal deviations as distinct crisp numbers. So, they are not required to follow the rules of the IT2TrFSs mentioned in sub-section 3.1. In this case, $\epsilon^{*L} = max\{\epsilon_1^{*L}, \epsilon_2^{*L}, \epsilon_3^{*L}, \epsilon_4^{*L}\}$ and $\epsilon^{*U} = max\{\epsilon_1^{*U}, \epsilon_2^{*U}, \epsilon_3^{*U}, \epsilon_4^{*U}\}$, and the corresponding CRs are CR^L and CR^U, respectively, which are calculated as follows:

$$CR^L = \frac{\epsilon^{*L}}{\delta^L} \tag{29}$$

$$CR^U = \frac{\epsilon^{*U}}{\delta^U} \tag{30}$$

Considering the heights of IT2TrFNs as weight values, we have the following equation:

$$CR = \frac{0.9\frac{\epsilon^{*L}}{\delta^L} + \frac{\epsilon^{*U}}{\delta^U}}{1.9} \tag{31}$$

Definition 15. CR value belongs to the closed interval [0,1], where $CR \to 0$, illustrates more consistent comparisons and $CR \to 1$, illustrates less consistent comparisons.

5. A new IT2TrF method for MCGDM problems

In this section, the proposed IT2TrF MCGDM method is presented. This method consists of three distinct phases. The following sub-sections delve into these phases and associated methodologies.

5.1. Determine the initial weight of each criterion

Aligned with section 4, calculating the initial criterion weights entails several steps. Initially, the criterion evaluation set is established. Subsequently, each DM identifies the best (most critical) and the worst (least critical) criteria. The third step involves determining the IT2TrF BO vector, followed by the IT2TrF OW vector. In the fifth step, the weights of the criteria are determined by applying the model (M3). Ultimately, the consistency ratio of the comparisons is calculated using Eq. (31). The structure of the proposed IT2TrF-CBWM is illustrated in Figure 3.

5.2. Determine the global weight of each criterion

This section presents an optimization model designed to calculate the normalized IT2TrF weights, aiming to meet the DMs' preferences.

For this aim, we propose an IT2TrF constraint programming model (model (M4)) to minimize the maximum deviation between the obtained IT2TrF weights by the preferences of each DM (initial IT2TrF weights) and the final optimal IT2TrF weights. The notation $D(\hat{w}_{jt}, \hat{w}_j)$ and $D(\hat{w}_{js}, \hat{w}_j)$

represent the deviations between the final optimal IT2TrF weight of criterion j, $\hat{w}_j$, and the IT2TrF weight derived by preferences of DM_t, $\hat{w}_{jt}$, and DM_s, $\hat{w}_{js}$, respectively. α, the self-dependence coefficient, acts as a trade-off coefficient. It indicates the leaning of the decision-making group toward democratic or autocratic decision-making, offering varied scenarios for a decision-making problem. γ_t represents the weight coefficient of DM_t. At the same time, η stands for the maximum deviation between the IT2TrF weight associated with to DM's preferences and the final optimal IT2TrF weight. The IT2TrF constraint programming model is as follows:

$$
\begin{aligned}
&\text{Min}\,\eta \\
&\text{subject to} \\
&(1-\alpha)\sum_{t=1}^{T}\gamma_t D(\hat{w}_{it},\hat{w}_i)+\alpha D(\hat{w}_{is},\hat{w}_i)<\eta && i=1,2,3,...,n \\
&w_{i4}^L+\sum_{j=1,j\neq i}^{n} w_{j1}^L\leq 1;\ w_{i3}^L+\sum_{j=1,j\neq i}^{n} w_{j2}^L\leq 1 && i=1,2,3,...,n \\
&w_{i2}^L+\sum_{j=1,j\neq i}^{n} w_{j3}^L\geq 1;\ w_{i3}^L+\sum_{j=1,j\neq i}^{n} w_{j4}^L\geq 1 && i=1,2,3,...,n \\
&w_{i4}^U+\sum_{j=1,j\neq i}^{n} w_{j1}^U\leq 1;\ w_{i3}^U+\sum_{j=1,j\neq i}^{n} w_{j2}^U\leq 1 && i=1,2,3,...,n \\
&w_{i2}^U+\sum_{j=1,j\neq i}^{n} w_{j3}^U\geq 1;\ w_{i3}^U+\sum_{j=1,j\neq i}^{n} w_{j4}^U\geq 1 && i=1,2,3,...,n \\
&w_{i2}^L\geq (w_{i2}^U-w_{i1}^U)(\frac{H(w^L)}{H(w^U)})+w_{i1}^U && i=1,2,3,...,n \\
&w_{i3}^L\leq (w_{i3}^U-w_{i4}^U)(\frac{H(w^L)}{H(w^U)})+w_{i4}^U && i=1,2,3,...,n \\
&w_{i1}^L\geq w_{i1}^U;\ w_{i4}^L\leq w_{i4}^U && i=1,2,3,...,n \\
&w_{ik}^L\geq w_{ik+1}^L;\ w_{ik}^U\leq w_{ik+1}^U && i=1,2,3,...,n,k=1,2,3
\end{aligned}
\tag{M4}
$$

5.3. IT2TrF-likelihood-based MCGDM method

In this sub-section, the IT2F likelihood-based MCDM method presented by Chen [17] is extended for democratic and autocratic group decision-making problems. In this method, assume there are T DMs, denoted by DM_t $(t=1,2,\ldots,T)$ construct a decision matrix by evaluating m alternatives, denoted by A_i $(i=1,2,\ldots,m)$, over n criteria, denoted by C_j $(j=1,2,\ldots,n)$. The steps of the method are as follows:

Step 1. Evaluate each alternative concerning every criterion using an appropriate linguistic term listed in Table 5 and determine a decision matrix regarding each DM viewpoint.

Step 2. Specify the benefit and cost criteria and use the following equations to transform the decision matrix into the benefit decision matrix.

$$\hat{b}^t_{ij} = \begin{cases} \hat{q}^t_{ij} & if c_j \in C_I \\ (\hat{q}^t_{ij})^c & if c_j \in C_{II} \end{cases} \tag{32}$$

and

$$\hat{b}^s_{ij} = \begin{cases} \hat{q}^s_{ij} & if c_j \in C_I \\ (\hat{q}^s_{ij})^c & if c_j \in C_{II} \end{cases} \tag{33}$$

Where $\hat{q}^t_{ij}$, and $\hat{q}^s_{ij}$ are the IT2TrF evaluation of alternative j over criterion i based on the DM_k and SDM points of view, respectively. Moreover, $(\hat{q}^t_{ij})^c$ and $(\hat{q}^s_{ij})^c$ are the complement values of $\hat{q}^t_{ij}$ and $\hat{q}^s_{ij}$, respectively, shown in Table 6.

Step 3. Aggregate IT2TrF decision matrices. All decision matrices provided by DMs are aggregated to construct an IT2TrF aggregated decision matrix by the following equation:

$$\hat{a}_{ij} = (1-\alpha)\sum_{t=1}^{T} \gamma_t \hat{b}^t_{ij} + \alpha \hat{b}^s_{ij} \tag{34}$$

Where $\hat{a}_{ij}$ is an aggregated value of the evaluation of alternative i with respect to criterion j within the IT2TrF environment.

Step 4. Compute the lower and upper likelihoods, $L^-(\hat{a}_{ij} \geq \hat{a}_{i'j})$ and $L^+(\hat{a}_{ij} \geq \hat{a}_{i'j})$ for pair of alternatives $(A_i, A_{i'})$ over criterion j, by Eqs. (15) and (16), respectively.

Step 5. Calculate the likelihood $L(\hat{a}_{ij} \geq \hat{a}_{i'j})$ of preference of $\hat{a}_{ij} \geq \hat{a}_{i'j}$, by Eq. (14). The value of $L(\hat{a}_{ij} \geq \hat{a}_{i'j})$ shows the possibility that $\hat{a}_{i'j}$ is not greater than $\hat{a}_{ij}$. The alternative A_i performs well for a positive (benefit) criterion belongs to C_I if there is a good possibility that the evaluation score $\hat{a}_{ij}$ will be higher than or equal to the evaluation score $\hat{a}_{i'j}$ ($i' = 1, 2, ..., m$, and $i' \neq i$) among the other $m-1$ alternatives. The alternative A_i, in contrast, performs well on a cost criterion belongs to C_{II} if there is a good chance that $\hat{a}_{ij}$ will be less than or equal to the evaluation value $\hat{a}_{i'j}$ ($i' = 1, 2, ..., m$, and $i' \neq i$) for the remaining $m-1$ alternatives.

Step 6. Compute the likelihood-based performance index, $P(\hat{a}_{ij})$, by Eq. (35).

$$P(\hat{a}_{ij}) = \begin{cases} \sum_{i'=1, i' \neq i}^{m} L(\hat{a}_{ij} \geq \hat{a}_{i'j}) & if c_j \in C_I \\ \sum_{i'=1, i' \neq i}^{m} L(\hat{a}_{i'j} \geq \hat{a}_{ij}) & if c_j \in C_{II} \end{cases} \tag{35}$$

Step 7. Compute the likelihood-based comprehensive evaluation value, E_i, by the following equation:

$$E_i = \oplus_{j=1}^{n} P(\hat{a}_{ij})\hat{W}_j = \\ = [(\sum_{j=1}^{n} P(\hat{a}_{ij})w_{1j}^L, \sum_{j=1}^{n} P(\hat{a}_{ij})w_{2j}^L, \sum_{j=1}^{n} P(\hat{a}_{ij})w_{3j}^L, \sum_{j=1}^{n} P(\hat{a}_{ij})w_{4j}^L; min_j\{H(w_j^L)\}), \\ (\sum_{j=1}^{n} P(\hat{a}_{ij})w_{1j}^U, \sum_{j=1}^{n} P(\hat{a}_{ij})w_{2j}^U, \sum_{j=1}^{n} P(\hat{a}_{ij})w_{3j}^U, \sum_{j=1}^{n} P(\hat{a}_{ij})w_{4j}^U; min_j\{H(w_j^U)\})] \tag{36}$$

For all $i = 1, 2, ..., m$.

To be concise, E_i is denoted as $H(E_i^L) = min_{j=1}^{n} H(w_j^L)$, $H(E_i^U) = min_{j=1}^{n} H(w_j^U)$, $e_{\zeta i}^L = \sum_{j=1}^{n} P(\hat{a}_{ij})w_{\zeta j}^L$, $e_{\zeta i}^U = \sum_{j=1}^{n} P(\hat{a}_{ij})w_{\zeta j}^U$ for all $\zeta = 1, 2, 3, 4$. Hence, the brief form of E_i of alternative A_i is as follows:

$$E_i = [E_i^L, E_i^U] = [(e_{1i}^L, e_{2i}^L, e_{3i}^L, e_{4i}^L; H(E_i^L)), (e_{1i}^U, e_{2i}^U, e_{3i}^U, e_{4i}^U; H(E_i^U))] \tag{37}$$

Step 8. Determine the value of the distance-based evaluation for each alternative, ϵ_i, using Eq. (38). Subsequently, arrange them in descending order.

$$\epsilon_i = \frac{[e_{1i}^L + e_{2i}^L + e_{3i}^L + e_{4i}^L + 4e_{1i}^U + 2e_{2i}^U + 2e_{3i}^U + 4e_{4i}^U + 3(e_{2i}^U + e_{3i}^U - e_{1i}^U - e_{4i}^U)\frac{(H(E_j^L))}{(H(E_j^U))}]}{8} \tag{38}$$

Table 5: The linguistic terms and corresponding IT2TrFNs for assessing alternatives [23].

Linguistic terms	IT2TrFNs
Low (L)	[(0, 0.1, 0.1,0.2; 0.9), (0, 0.1, 0.1,0.2; 1)]
Moderate Low (ML)	[(0.1, 0.3, 0.3,0.5; 0.9), (0.1, 0.3, 0.3,0.5; 1)]
Moderate (M)	[(0.5, 0.7, 0.7,0.9; 0.9), (0.3, 0.5, 0.6,0.7; 1)]
Moderate High (MH)	[(0.6, 0.7, 0.75,0.8; 0.9), (0.5, 0.7, 0.8,0.9; 1)]
High (H)	[(0.8, 0.85, 0.9,0.95; 0.9), (0.7, 0.9, 0.95,1; 1)]

Table 6: Linguistic terms and their complement.

Linguistic term (LT)	L	ML	M	H	MH
LT complement	H	MH	M	L	ML

The IT2TrF-likelihood-based MCGDM method's pseudocode is outlined in Algorithm 2. Moreover, a graphical depiction of the three phases of the proposed MCGDM method can be found in Figure 3. This representation also encompasses sensitivity and comparative analyses detailed in Section 8.

Algorithm 2 The pseudocode of the IT2TrF-likelihood-based MCGDM method

Input: linguistic terms and their corresponding numbers, and the final criteria weights obtained by solving model (M3)
Output: ranking of alternatives
for $t = 1$ to T **do**
    for $i = 1$ to m **do**
        for $j = 1$ to n **do**
$\hat{q}^t_{ij} \leftarrow$ print("what is evaluation grade of alternative 'i' over criterion 'j' from 'DM_t'view point?") (Step 2)
$\hat{q}^s_{ij} \leftarrow$ print("what is evaluation grade of alternative 'i' over criterion 'j' from SDM view point?") (Step 2)
$\hat{b}^t_{ij} \leftarrow$ print("what is evaluation grade of alternative 'i' over criterion 'j' from 'DM_t' view point?") (Step 2)
$\hat{b}^s_{ij} \leftarrow$ print("what is evaluation grade of alternative 'i' over criterion 'j' from SDM view point?") (Step 2)
        end for
    end for
end for
for $i = 1$ to m **do**
    for $j = 1$ to n **do**
$\hat{a}_{ij} \leftarrow$ aggregate all decision matrices provided by DMs and SDM. (Step 3)
    end for
end for
for $j = 1$ to n **do**
    for $i = 1$ to m **do**
        for $i' = 1$ to m **do**
            if $i \neq i'$ **then**
$L^-(\hat{a}_{ij} \geq \hat{a}_{i'j}) \leftarrow$ calculate the lower likelihood for pair of alternatives $(A_i, A_{i'})$ over criterion j (Step 4)
$L^+(\hat{a}_{ij} \geq \hat{a}_{i'j}) \leftarrow$ calculate the upper likelihood for pair of alternatives $(A_i, A_{i'})$ over criterion j (Step 4)
$L(\hat{a}_{ij} \geq \hat{a}_{i'j}) \leftarrow$ calculate the likelihood of preference of $\hat{a}_{ij} \geq \hat{a}_{i'j}$ (Step 5)
            end if
        end for
    end for
end for
for $i = 1$ to m **do**
    for $j = 1$ to n **do**
$P(\hat{a}_{ij}) \leftarrow$ obtain the likelihood-based performance index (Step 6)
    end for
end for
for $i = 1$ to m **do**
$E_i \leftarrow$ compute the likelihood-based comprehensive evaluation value (Step 7)
$\epsilon_i \leftarrow$ determine the value of the distance-based evaluation for each alternative (Step 8)
end for
Return $\epsilon_1, \epsilon_2, ..., \epsilon_m$

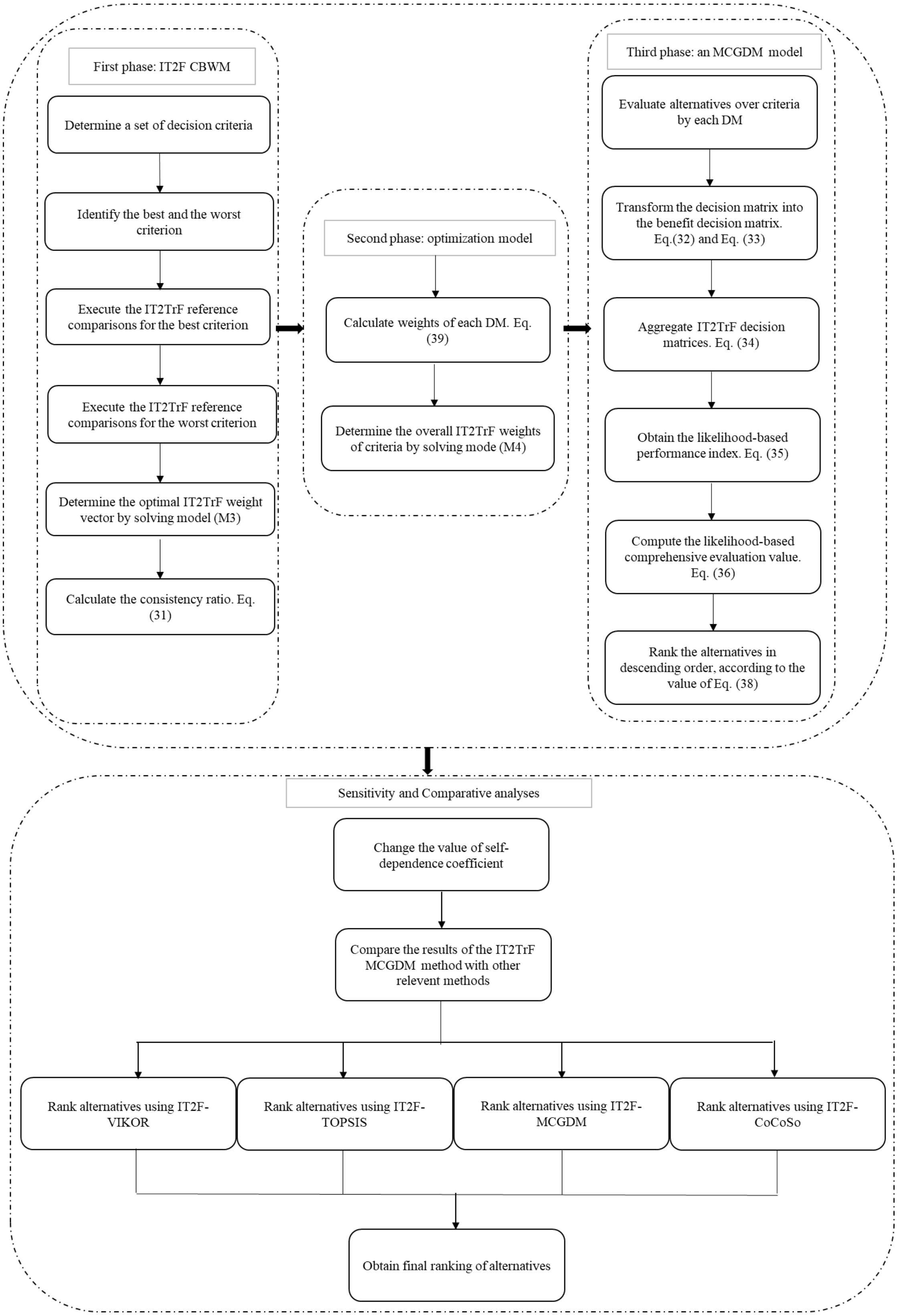


Figure 3: The scheme of the proposed MCGDM methodology

6. Case study

This section applies the proposed IT2TrF MCGDM methodology to investigate the HCW treatment technology selection problem during the COVID-19 pandemic.

6.1. Case description

HCW management is highly critical affair in hospital management, particularly in developing countries. HCWs are a distinct kind of waste that must be diligently managed due to their harmful, detrimental environmental effects [51]. Since the emergence of the COVID-19 pandemic, there has been a significant surge in waste generation attributable to the heightened use of medical supplies [52, 53]. Numerous countries have established specific management regulations to mitigate the adverse impacts of HCWs on various facets of human life. According to the Iran HCWs management guidelines, health sectors are tasked with disinfecting infectious waste. If a high-risk incident occur, penalties are levied on the health sector. Our case study focuses on a hospital in Mashhad, Iran, designated as a primary center for treating COVID-19 patients. All waste generated at this institution, particularly from COVID-19 units, is categorized as infectious and necessitates disinfection prior to disposal at sanitary landfills. Given that HCW treatment technologies are engineered for steady-state conditions, unexpected events like the recent pandemic can strain these systems and increase the risk of malfunction [54]. As a result, improper HCW management, specifically improper disinfection, may endanger the environment by hastening disease spread.

Iran's economic climate, political decisions, and sanctions restrict importing firms from procuring advanced HCW treatment technologies. As a result of these constraints and limited technological options, local manufacturers predominantly produce a specific technology: the autoclave. Consequently, healthcare administrators receive various proposals from vendors offering machines built on similar technologies. Indeed, there is a homogeneous list from which an alternative must be purchased regarding different criteria. Considering the features of the case study, we apply the proposed method to select a new autoclave for the case study during the COVID-19 pandemic. The evaluation criteria are determined by interviewing environmental health and medical engineers responsible for providing HCW treatment technology and experts in the field of HCW management. The list of potential alternatives is provided by Iran's health ministry.

6.2. Data collocation

To determine the criteria weights and rank the HCW treatment technologies, a committee was established, consisting of five DMs. This included one senior DM (SDM), and a panel of four other DMs. The specific details of these DMs are briefly outlined in Table 8. The evaluation criteria were identified through expert interviews. The list of prospective alternatives was sourced from Iran's health ministry. Committee members were tasked with completing questionnaires designed to compare criteria and assess alternatives, using the linguistic terms presented in Tables 1 and 5, respectively. These questionnaires can also be found in Appendix B.

The criteria include cost (C_1), treatment effectiveness (C_2), reliability (C_3), after-sale service (C_4), attractiveness of contract (C_5), guarantee (C_6), energy consumption (C_7), and flexibility (C_8). The definitions for each of these criteria are detailed in Table 7. Following the criteria determination, potential technologies were extracted from the verified list of HCW treatment technologies provided by Iran's health ministry (see http://imed.ir/default.aspx?pagename=Pages-Inquiry&TableID=43). Consequently, four technologies were identified: Autoclave-1 (A_1), Autoclave-2 (A_2), Autoclave-3 (A_3), and Autoclave-4 (A_4) are introduced. To adhere to research ethics guidelines, the full names of these technologies have been withheld. We assessed the quantitative criteria using linguistic terms. Since the significance of these criteria is influenced not only by the volume of waste produced by facilities but also by volatile banking inflation rates, enforced sanctions, and foreign exchange rates. Subsequent sections elucidate the procedures employed to calculate criteria weights and prioritize the alternatives. The flowchart of the proposed MCGDM methodology is depicted in Figure 3.

Table 7: selected criteria and definitions

Criteria	Concept	Description
C_1	Cost	The price of technology, installation, maintenance, etc.
C_2	Treatment effectiveness	Waste treatment effectiveness
C_3	Reliability	The reliability of technology while working
C_4	After-sale service	The availability of a certified after-sale service agency
C_5	Attractiveness of contract	The flexibility of monetary contract
C_6	Guarantee	The availability of guarantee service and its attractiveness
C_7	Energy consumption	The energy consumption while the technology is working
C_8	Flexibility	The ease of upgradability of technology

Table 8: Experts' specification

DMs	Academic Level	Positions	Years of experience
SDM	Ph.D.	Medical engineer, top manager	25
Expert panel	BS.	Supervisor of the HCW management unit of hospitals	28
	MS.	Supply specialist	12
	MS.	Chief of the HCW Management unit of the hospital	10
	MS.	Member of the HCW Management unit of the hospital	27

7. Method implementation

In this section, we are following the methodology describe in Section 5, which is visually summarized in Figure 3. Regarding Figure 3, in the first phase, each DM conducts pairwise comparisons as elaborated in Section 4. In the second phase, the final criteria weights are determined using the optimization problem presented in Section 5.2. Then, in the third phase, which is discussed in Section 5.3, alternatives are ranked.

7.1. Calculating the criteria weights

In this sub-section, the initial and final weights of criteria are determined.

7.1.1. Determine the initial weight of each criterion

The initial weight of each criterion is determined based on the DM's judgments and the steps of the IT2TrF-CBWM. According to the IT2TrF-CBWM algorithm, each DM identifies the best and worst criteria from their perspective. Then using the scale provided in Table 1, each DM conducted the BO and OW pairwise comparisons. Following these procedures, the preferences of DMs are captured, corresponding to $\hat{b}_{ij}$ ($\hat{b}_{Bj}$, $\hat{b}_{jW}$) parameter used in IT2TrF-CBWM. Table 9 illustrates both the best and the worst criteria specified by each DM. In addition, the linguistic BO and OW vectors provided by each of them are also indicated in Table 9.

Table 9: Pairwise comparison vectors conducted by DMs

DMs	vector	Criteria							
		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
1	BO, Best (C_1)	CEI	WI	VSI	MPI	SPI	SI	EMI	EI
	OW, Worst (C_7)	EI	VSI	MI	SPI	MPI	SI	CEI	WI
2	BO, Best (C_4)	MPI	MI	VSI	CEI	WI	SPI	SI	EI
	OW, Worst (C_8)	SI	SPI	WI	EI	VSI	MI	MPI	CEI
3	BO, Best (C_4)	SI	MI	VSI	CEI	MI	SPI	MPI	EI
	OW, Worst (C_8)	SI	SPI	WI	EI	VSI	MI	MPI	CEI
4	BO, Best (C_2)	SPI	CEI	EI	WI	MPI	SPI	EMI	VSI
	OW, Worst (C_7)	MPI	EMI	MI	VSI	SPI	SI	CEI	WI
SDM	BO, Best (C_2)	SI	CEI	MI	SPI	MPI	VSI	VSI	EMI
	OW, Worst (C_8)	SPI	EMI	EI	SI	VSI	MPI	WI	CEI

The linguistic terms provided in Table 9 are converted to IT2TrF numbers through the rules indicated in Table 1 and regarded as the values of $\hat{b}_{Bj}$ and $\hat{b}_{jW}$ for all $j = 1, ..., n$. Executing the model (M3) defined in step 5 of the IT2TrF-CBWM for data established by each DM, we obtained five different weight vectors. Table A.14 of Appendix A shows the five weight vectors derived from solving model (M3) for DM's pairwise evaluation data. Afterward, the obtained weights are aggregated using model (M4) through the following subsection.

7.1.2. Determine the final weight of each criterion

In this section, we adjust for imprecise evaluations using the obtained CR values from DMs' pairwise comparisons. The adjustment is intended to compensate for the effects of imprecise or inconsistent judgments. Therefore, the significance coefficient of DMs' judgments is defined by Eq. (39). The final weights of the criteria are determined by solving the model (M4).

$$\gamma_t = \frac{(1 - CR_t)}{\sum_{t=1}^{T}(1 - CR_t)} \quad t = 1, 2, \ldots, T \tag{39}$$

In line with the analysis of sub-section 8.1, the value of α is set at 0.75. It provides the closest deviation value to the average of obtained deviations regarding different values of α. Consequently, the optimal deviation value obtained by solving model (M4) with $\alpha = 0.75$ is $\eta = 0.017$, and the obtained global optimal IT2TrF weights of criteria are:

$\hat{w}_1$= [(0.116,0.131,0.131,0.134;0.9),(0.116,0.131,0.131,0.136;1)],
$\hat{w}_2$= [(0.163,0.182,0.183,0.188;0.9),(0.16,0.182,0.182,0.191;1)],
$\hat{w}_3$= [(0.139,0.152,0.153,0.154;0.9),(0.138,0.153,0.153,0.154;1)],
$\hat{w}_4$= [(0.109,0.126,0.127,0.131;0.9),(0.109,0.126,0.126,0.134;1)],
$\hat{w}_5$= [(0.135,0.151,0.152,0.153;0.9),(0.135,0.152,0.152,0.154;1)],
$\hat{w}_6$= [(0.085,0.102,0.103,0.107;0.9),(0.084,0.102,0.102,0.109;1)],
$\hat{w}_7$= [(0.073,0.091,0.092,0.097;0.9),(0.069,0.091,0.091,0.097;1)], and
$\hat{w}_8$= [(0.052,0.063,0.063,0.063;0.9),(0.052,0.063,0.063,0.064;1)].

Finally, the IT2TrF weights are defuzzified and normalized using the following Equation.

$$w_i = \frac{COA(\hat{w}_i)}{\sum_{i=1}^{n} COA(\hat{w}_i)} \tag{40}$$

From these calculations, the ranking of criteria is established as: $C_2 > C_3 > C_5 > C_1 > C_4 > C_6 > C_7 > C_8$. The crisp equivalents of IT2TrF weight, used solely to define the criteria's order of preference, are: $w_1 = 0.133$, w_2= 0.186, $w_3 = 0.16$, $w_4 = 0.12$, $w_5 = 0.149$, $w_6 = 0.101$, $w_7 = 0.086$, and $w_8 = 0.062$. All subsequent computations employ the IT2TrF weights.

7.2. IT2TrF-likelihood-based MCGDM method for ranking alternatives

After calculating the DMs and criteria weights, the expert panel and the SDM evaluate alternatives A_i (i= 1,2,3,4) with respect to each criterion, using linguistic terms listed in Table 5. Table 10 displays the linguistic evaluation of alternatives across the criteria provided by the DMs. The ranking of these alternatives follows the steps presented in sub-section 5.3:

Step 1. Evaluate the alternatives with respect to criteria to determine the decision matrix from each DM's perspective. Table 10 displays the decision matrix provided by each DM.

Step 2. Specify the benefit and cost criteria. Convert the decision matrix into the benefit decision matrix, regarding rules mentioned in Table 6 and Eqs. (32) and (33). In this problem, cost (C_1) and energy consumption (C_7) are regarded as cost criteria, and treatment effectiveness (C_2), reliability (C_3), after-sale service (C_4), attractiveness of contract (C_5), guarantee (C_6), flexibility (C_8) are considered benefit criteria.

Step 3. Aggregate the IT2TrF decision matrices. All decision matrices produced by DMs are aggregated using Eq. (34). The aggregated decisions are shown in Table A.16 of Appendix A.

Step 4. Compute the lower and upper likelihoods, $L^{-}(\hat{a}_{ij} \geq \hat{a}_{i'j})$ and $L^{+}(\hat{a}_{ij} \geq \hat{a}_{i'j})$ for each pair of evaluation $(\hat{a}_{ij}, \hat{a}_{i'j})$ using Eqs. (15) and (16), respectively.

Step 5. Calculate the likelihood $L(\hat{a}_{ij} \geq \hat{a}_{i'j})$ of preference of $\hat{a}_{ij} \geq \hat{a}_{i'j}$ with respect to criterion j, using Eq. (14). The results are displayed in Table 11.

Step 6. Obtain the likelihood-based performance index, $P(\hat{a}_{ij})$, for the performance rating of each alternative, $\hat{a}_{ij}$, using Eq. (35). The obtained results are presented in Table 12.

Step 7. Compute the likelihood-based comprehensive evaluation value, E_i, by Eq. (35). The obtained results are:
E_1= [(9.108,11.312,13.182,27.866;0.9), (8.6,11.555,11.555,27.946; 1)],
E_2= [(13.264,16.475,19.199,40.583; 0.9), (12.524,16.828,16.828,40.7; 1)],
E_3= [(9.684,12.029,14.017,29.63; 0.9), (9.144,12.286,12.286,29.716; 1)], and
E_4= [(5.448,6.766,7.885,16.667;0.9), (5.144,6.911,6.911,16.716; 1)].

Step 8. Rank the alternatives in descending order. Based on the distance-based evaluation for each alternative, ϵ_i, calculated by Eq. (38), the alternatives rank as: $A_2 > A_3 > A_1 > A_4$, which corresponds to $\epsilon_2 > \epsilon_3 > \epsilon_1 > \epsilon_4$.

Table 10: Linguistic assessment of alternatives regarding each criterion

Alternatives	DM_1								DM_2							
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	H	M	L	MH	H	H	M	H	H	M	L	H	H	H	H	M
A_2	M	H	H	H	MH	MH	H	ML	MH	H	H	H	MH	M	H	M
A_3	H	H	ML	H	M	H	MH	MH	M	MH	MH	H	H	MH	M	L
A_4	L	L	H	ML	L	L	M	L	L	L	H	ML	ML	ML	M	L
Alternatives	DM_3								DM_4							
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	MH	M	L	MH	MH	H	M	H	M	MH	L	M	H	MH	M	MH
A_2	ML	H	MH	H	M	MH	M	M	L	H	M	MH	MH	MH	H	H
A_3	H	MH	M	H	H	M	MH	L	MH	M	ML	M	MH	H	MH	ML
A_4	L	L	H	ML	L	L	H	L	L	L	MH	L	L	L	ML	L
Alternatives	SDM															
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8								
A_1	MH	ML	L	MH	H	H	MH	M								
A_2	ML	H	MH	H	MH	M	H	H								
A_3	M	M	M	M	M	MH	MH	M								
A_4	L	L	H	L	ML	L	ML	L								

Table 11: The likelihood of performance ratings

	$L(\hat{a}_{1j} \geq \hat{a}_{2j})$	$L(\hat{a}_{1j} \geq \hat{a}_{3j})$	$L(\hat{a}_{1j} \geq \hat{a}_{4j})$	$L(\hat{a}_{2j} \geq \hat{a}_{1j})$	$L(\hat{a}_{2j} \geq \hat{a}_{3j})$	$L(\hat{a}_{2j} \geq \hat{a}_{4j})$	$L(\hat{a}_{3j} \geq \hat{a}_{1j})$	$L(\hat{a}_{3j} \geq \hat{a}_{2j})$	$L(\hat{a}_{3j} \geq \hat{a}_{4j})$	$L(\hat{a}_{4j} \geq \hat{a}_{1j})$	$L(\hat{a}_{4j} \geq \hat{a}_{2j})$	$L(\hat{a}_{4j} \geq \hat{a}_{3j})$
C_1	0	0.515	0	1	1	0.079	0.485	0	0	1	0.921	1
C_2	0.017	0.272	1	0.983	0.923	1	0.728	0.077	1	0	0	0
C_3	0	0.003	0	1	0.894	0.149	0.997	0.106	0.001	1	0.851	0.999
C_4	0.132	0.196	1	0.868	0.648	1	0.804	0.352	1	0	0	0
C_5	0.739	0.412	1	0.261	0.201	1	0.588	0.799	1	0	0	0
C_6	0.893	0.707	1	0.107	0.194	0.996	0.293	0.806	1	0	0.004	0
C_7	0.142	0.203	0.429	0.858	0.61	0.827	0.797	0.39	0.748	0.571	0.173	0.252
C_8	0.886	1	1	0.114	0.976	1	0	0.024	0.867	0	0	0.133

Table 12: The likelihood-based performance indexes

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
$P(\hat{a}_{1j})$	0.52	1.29	0.00	1.33	2.15	2.60	0.77	2.89
$P(\hat{a}_{2j})$	2.08	2.91	2.04	2.45	1.46	1.48	2.30	2.09
$P(\hat{a}_{3j})$	0.48	1.80	1.10	2.16	1.80	2.10	1.94	0.89
$P(\hat{a}_{4j})$	2.92	0.00	2.85	0.00	0.00	0.00	1.00	0.13

8. Sensitivity and Comparative analyses

In this section, we discuss how changing parameters affect the final results. Subsequently, we compare our findings with outcomes from various methods to establish the validity and superiority of our method.

8.1. Sensitivity analyses

In this sub-section, we investigate the impacts of changing the value of $\alpha \in [0,1]$, the self-dependence coefficient, on η, the deviations among DMs' preferences and group preferences, and the ranking order of alternatives.

Solving model (M4) reveals that the self-dependence coefficient, α, significantly influences the decision-making approach. The great values of α show that the decisions are more influenced by the SDM's attitude than other DMs. Indeed, it means that the decisions are more autocratically made. As a result, the obtained criteria weights, and the deviation degree were changed accordingly. Given that the deviation degree or the value of η is a distance-based measure, the lower η values illustrate that the initial and final weights are closer to each other.

Figure 4 indicates that the value of η increases until $\alpha = 0.25$ and subsequently decreases. The maximum value of η is 0.0335 at $\alpha = 0.25$. Since it is less than 0.1, we consider it an acceptable deviation value [7]. Considering this figure, the deviation value associated with $\alpha = 0.75$, $\eta = 0.0171$, is close to the average of the deviation values, 0.021. This selected value was presented to the decision-making group, hinting at a collective inclination towards autocratic decision-making in this particular case.

Based on Figure 5, different ranking orders of criteria are obtained for different values of α. Accordingly, the ranking orders of criteria are categorized into three classes. Classes are divided based on the $\alpha = 0$, 0.25, and {0.5, 0.75,1}, such that each class contains individual ranking results.

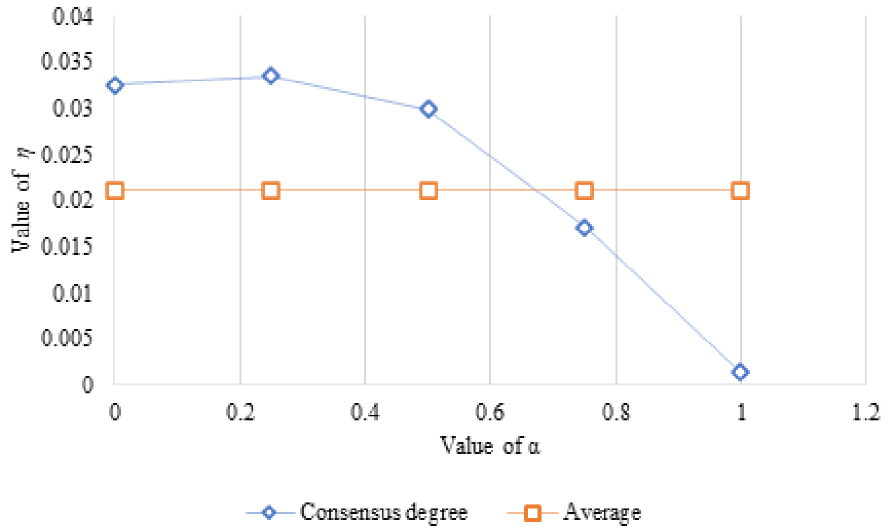


Figure 4: Deviation degree (η) versus self-dependence coefficient (α)

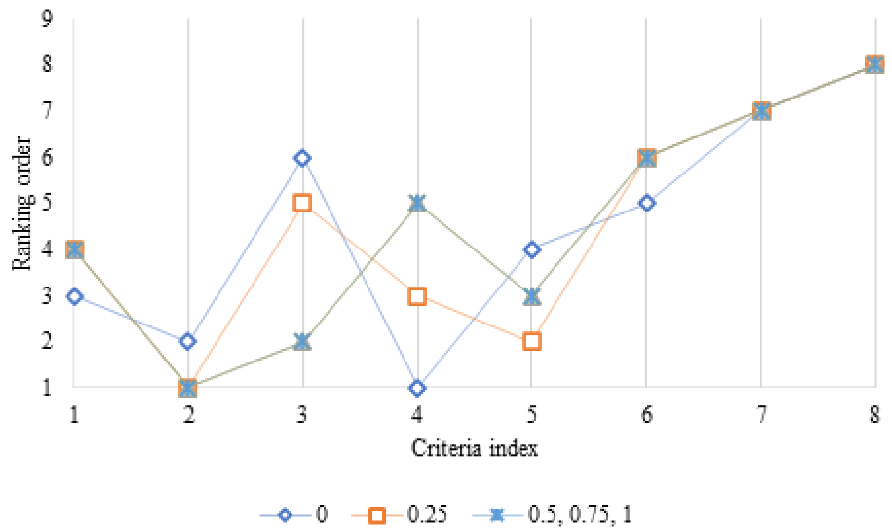


Figure 5: Ranking order of criteria for different values of α

8.2. Comparative analyses

Comparative analyses are performed to demonstrate the validity of the proposed IT2TrF MCGDM method. First, the CR values of different BWM methods [15, 38, 55] are compared with IT2TrF-CBWM. Further analyses are provided to compare the ranking results of the proposed MCGDM method with other methods [2, 19, 30, 32, 56?], under the IT2F environment in terms of distinguishing power and ranking order of alternatives. Moreover, another analysis is also conducted to evaluate the effect of fuzzy factors on the results of the case study.

8.2.1. Comparison with other BWM methods

Considering five DMs, including one senior DM and four DMs, each DM conducted pairwise comparisons distinctly to calculate the criteria weights. Indeed, their evaluations are seen as five separate problems and are investigated individually. Referring to Figure 6, the results of these five problems are compared with those of three BWMs: the classical BWM [38], CBWM [15], and FCBWM [55]. Figure 6 indicates that the CR values derived by IT2TrF-CBWM are lower than the CR values of the aforementioned methods except for CBWM [15]. Since there is no defined threshold for different values of CR, it is hard to verify them. Therefore, to mitigate the effects of inconsistent evaluations, we transform the CR values associated with each DM into a weight for their decisions.

Figure 6 also reveals that the CR values of BWM are more than the CR values of CBWM and IT2TrF-CBWM. In half of these cases, BWM outperforms FCBWM and vice versa. The CRs obtained by the proposed method are higher than those obtained by CBWM. It can be regarded as a direct result of extending CBWM to the IT2TrF environment, but it cannot be used to say that the proposed method performs poorly. Furthermore, the CRs of the FCBWM are all greater than the CRs obtained by the proposed method.

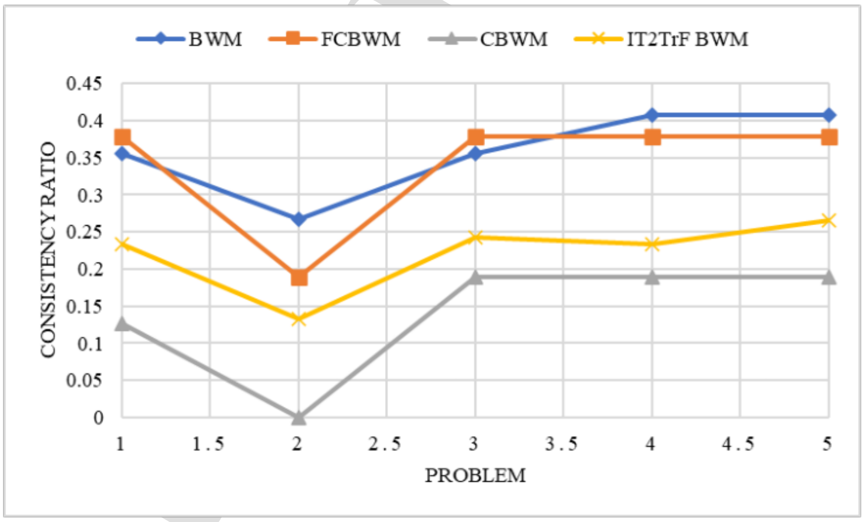


Figure 6: Consistency ratio obtained by different methods for five problems

8.2.2. Comparison with ranking order of alternatives obtained by other methods

To validate the proposed method, we compared the ranking order of alternatives derived from it to those obtained from several other methods: IT2F TOPSIS [30], IT2F VIKOR [2**?**], IT2F MCGDM [56], IT2F CoCoSo [19], and IT2F PROMETHEE [32]. The results are presented in Table 13. According to the five decision-making methods, A_2 emerges as the most favorable alternative. Consequently, this supports the validity of the proposed method. Therefore, based on the comparison analysis and the dominance theory, the final ranking order is $A_2 > A_1 > A_3 > A_4$, where ">" means "preferred to".

8.2.3. Comparison with distinguishing power of the methods

Another comparison is conducted to compare the distinguishing power (DSP) of the best alternative obtained by the presented MCGDM method with the aforementioned methods. First, the ranks of alternatives are normalized by Eq. (41). Second, The superiority of the first alternative over the second alternative is calculated by Eq. (42), named distinguishing power.

$$N(e_i) = \frac{e_i - \min_i(e_i)}{\max_i(e_i) - \min_i(e_i)} \tag{41}$$

$$DSP = |N(e_F) - N(e_S)| \tag{42}$$

where $N(e_i)$ represents the normalized overall score of e_i, $N(e_F)$, and $N(e_S)$ denote the normalized overall score of the first and second alternatives, respectively, and DSP represents the distinguishing power. For example, the overall score for e_2 (the first alternative) is 39.61, while e_1 (the second alternative) is 28.92.Their respective normalized values are 1 and 0.542. Therefore, the DSP value for the proposed method is 0.458. Table 13 displays DSP values for the proposed method and other methods [2, 19, 30, 32, 56**?**]. The results show that our method scores higher in DSP than the other methods [2, 19, 30, 32, 56**?**], meaning our method is more effective in terms of DSP.

Table 13: Ranking order of alternatives and DSP of different methods

methods	ranking order	distinguishing power
Proposed method	$A_2 > A_1 > A_3 > A_4$	0.458
IT2F TOPSIS [30]	$A_2 > A_3 > A_4 > A_1$	0.354
IT2F MCGDM [56]	$A_2 > A_3 > A_1 > A_4$	0.384
IT2F VIKOR [2]	$A_2 > A_3 > A_1 > A_4$	0.377
IT2F CoCoSo [19]	$A_2 > A_1 > A_3 > A_4$	0.181
IT2F PROMETHEE [32]	$A_4 > A_1 > A_3 > A_2$	0.367
IT2F VIKOR [?]	$A_2 > A_3 > A_1 > A_4$	0.012
T1F MCGDM method	$A_2 > A_1 > A_3 > A_4$	0.33

8.2.4. Evaluations on fuzzy factors

In this section, we delve into how fuzzy factors influence experimental results. Given that T1FS is a specific type of IT2FS, we tackled the HCW treatment selection problem using the reduced form of the method within the T1FS and assessed the results for CR values, alternative rankings, and distinguishing power.

Figure 6 reveals that the IT2TrF-CBWM method we proposed is more sensitive to inconsistencies in input parameters than its counterparts. While CBWM and FCBWM couldn't offer distinct CRs for problems 3, 4, and 5, IT2TrF-CBWM generated varied CRs for each issue, proving its enhanced precision.

Furthermore, referring to Table 13, the ranking order of alternatives in the T1F environment is equivalent to the rankings produced by our IT2TrF MCGDM method. This alignment serves as validation for solving the problem within the IT2TrF environment. Additionally, to highlight the advantages of our proposed IT2TrF MCGDM method, we compared these two methods in terms of distinguishing power. As observed in Table 13, our method attained a distinguishing power of 0.46, surpassing the MCGDM method in the T1F context, which is 0.38. This demonstrates the superiority of our approach in handling the HCW treatment selection problem.

While both T1FSs and IT2FSs serve to capture imprecision and ambiguity in human subjective evaluations, IT2FSs offer more versatility by providing greater freedom to T1F memberships. Consequently, building models based on IT2FSs leads to more precise results and solutions, as evidenced by the obtained findings.

9. Discussion

In this section, we discuss our study's theoretical, methodological, and managerial implications.

9.1. Theoretical implication

This study aims to present a method that DMs and analysts can use to assess a set of homogeneous alternatives. Our approach takes into account the distinct roles of both SDM and a panel of DMs, which are often overlooked in previous research [20, 57–61]. The proposed methodology is applicable to emergencies and is designed to be adaptable to various decision-making scenarios. Our MCGDM method integrates a new variant of BWM with the IT2TrF-likelihood MCDM method in the context of group decision-making problems.

Yuen [12–14, 48] contends that the ratio scale used in AHP leads to exaggerated results in problems with similar lists of alternatives. The BWM method yields more reliable results than AHP and requires fewer pairwise comparisons [62]. Since no research focuses on extending BWM based on the interval scale within the IT2F environment in this paper, we introduce a new BWM to calculate criteria weights. The IT2TrF-likelihood MCDM method, proposed by Chen [17], offers a straightforward means for ranking alternatives. This paper enhanced the method to align it with group decision-making problems. We employed two renowned methods for calculating criteria weights and alternative rankings. Furthermore, to tackle the imprecise decisions that might emerge during the DMs' decision-making process, the concept of IT2FSs is integrated into the framework.

In our proposed BWM, our objective is to derive normalized IT2TrF weights and minimize the risk of losing fuzzy information provided by the DMs. By not treating the deviations as IT2FNs, we simplify the model and decrease the number of constraints while striving to achieve optimal deviations by considering all solutions obtained by solving the model (M3). The integration of IT2FSs into the proposed BWM results in more precise solutions. As demonstrated in Figure 6, the IT2TrF-CBWM method proposed in this study, displays heightened sensitivity to inconsistencies in input parameters. For example, both CBWM and FCBWM could not produce varying CRs for problems 3, 4, and 5. In contrast, IT2TrF-CBWM generated distinct CRs for each issue, signifying that our proposed method surpasses other similar versions in precision.

To simplify the process of aggregating IT2TrF weights and to accommodate various decision-making situations with or without an SDM, we propose an optimization model to generate normalized IT2TrF weights. The derived weights, coupled with the evaluations of DMs for alternatives, serve as input for the proposed IT2TrF-likelihood-MCGDM method. We have demonstrated that the utilization of IT2TrF factors can effectively enhance the quality of the results. As shown in Table 13, it is evident that the proposed method outperforms its T1F counterpart in terms of the DSP value. Additionally, both methods yield the same ranking order for alternatives, which further validates the reliability of the results. Therefore, the proposed MCGDM method consistently generated reliable outcomes in terms of both DSP values and the ranking order of alternatives.

9.2. Methodological implication

Given that HCW treatment technologies are designed for steady-state environments, their likelihood of failure surged during the COVID-19 pandemic [54]. In this study, we proposed a new MCGDM methodology to select an HCW treatment technology for a hospital. Regarding the characteristics of the case study, we proposed a set of evaluation criteria based primarily on the expertise and experiences of the experts. We presented the IT2TrF-CBWM to calculate criteria weights for a homogeneous list of alternatives. This weighting method is grounded in the relative importance of criteria as determined by DMs using linguistic terms. Final criteria weights are derived from an optimization model, and the alternatives are ranked via our IT2TrF-likelihood-based MCGDM method. It is important to mention that all calculations are conducted within the IT2F environment rather than the crisp environment to preserve intrinsic linguistic information.

The criteria weights and their priority order of them were confirmed by DMs. Treatment (C_2) is identified as the most important criterion. Improper treatment of HCWs can have disastrous consequences for the producer. In case of any incident which threatens the environment and society, the HCW producer is charged with penalties by the medical institution and the department of environment. The second most important criterion is reliability (C_3). The highly reliable devices will reduce the workload for those in charge of finding alternative solutions and solving the problem. All governmental and non-governmental institutions are experiencing financial difficulties as a result of the country's economic situation. Consequently, the flexibility of financial contracts (C_5) and long-term payments, as well as presenting various offers and sales plans, is the next most appealing criterion, even more important than the cost criterion (C_1). After-sale service (C_4) is followed by the cost criterion (C_1), which reduces maintenance time by making a reliable and professional service agency available. The following significant criterion is guaranteed (C_6). Generally, there is no severe shortage of energy resources in the country. As a corollary, the seventh important criterion is energy consumption (C_7). Finally, the flexibility criterion (C_8) is determined to be the least important criterion, indicating the device's ease of upgradability to comply with the new environmental rules.

Upon determining the criteria weights, alternatives were assessed using the IT2TrF-likelihood-based MCGDM method. The evaluation revealed A_2 as the premier alternative or autoclave for procurement. Based on the results, A_2 excels, especially in C_2, which was identified as the most crucial criterion.

9.3. Managerial implication

The findings of this study offer a comprehensive approach to problem-solving, enabling managers to convene a diverse panel of experts, each with distinct proficiency levels. Furthermore, the structured framework presented in this research can be employed by managers during urgent emergencies to benefit from expert opinions.

Our case study's focus is specifically on the intricate task of selecting an HCW treatment technology within the unique context of the COVID-19 pandemic in Iran. This inquiry is framed within the operational capability of the medical institution and the regulatory framework outlined by the Iranian health ministry. Consequently, both public and private hospitals in Iran can benefit from the proposed methodology to effectively decide on procuring HCW treatment technologies, in terms of identified criteria and the proposed method.

While the proposed methodology has already been proven effective in addressing HCW management during the critical times of the COVID-19 pandemic, it is imperative to emphasize that the applicability of this approach extends beyond the immediate scenario. It can be flexibly adapted to address various decision-making problems, be the flat or hierarchy. This versatility is immensely beneficial in problems where DMs lack precedent experience and struggle with ranking different criteria and alternatives, such as transportation, energy management, supplier management, etc.

10. Conclusion

This research introduced an adaptive MCGDM method suitable for solving emergencies and regular group decision-making problems. The proposed methodology combined BWM and likelihood-based MCDM methods within the IT2F environment. A new BWM based on the interval scale and IT2FS was developed to obtain the initial criteria weights. The proposed BWM can process pairwise comparison data obtained from evaluating a homogeneous set of alternatives. Comparative analysis revealed that the proposed method outperforms other comparable BWM variants in distinguishing inconsistent judgments and CR values. An optimization model was introduced to aggregate the obtained criteria weights from the DMs' perspective to determine the final criteria weights. By employing the model, normalized IT2TrF weights were calculated based on the reliability of DMs' judgments. The IT2TrF weights of criteria, combined with the evaluation data of alternatives, were considered as inputs for the proposed IT2TrF-likelihood-based MCGDM method. The proposed methodology was applied to a real scenario: selecting an HCW treatment technology for a health center during the COVID-19 pandemic. The comparative analysis demonstrated the superior performance of the proposed MCGDM methodology over other methods [2, 19, 30, 32, 56**?**].

For future research directions, it would be beneficial to incorporate objective data into computations.While the IT2TrFNs presented in this study have been formulated based on T1F numbers, developing more precise IT2TrFNs corresponding to different linguistic terms could improve the accuracy of the results. Investigating the impact of factors such as DMs' hesitation and confidence levels in MCGDM methods could also provide valuable insights for enhancing these methods. Lastly, the proposed method is adaptable to the specific questions of decision-making problems and can be applied to other real-world scenarios, such as logistics, technology selection, and manufacturing.

Acknowledgment:

The authors would like to extend their sincere appreciation to Managing Editor Dr. Layth Sliman and the anonymous reviewers for their valuable comments, which significantly contributed to the improvement of our paper. Additionally, we acknowledge the support provided by Ferdowsi University of Mashhad, Mashhad, Iran, for this research project (No. 57863).

Appendix A.

Table A.14: The criteria weight based on the DM_1 and DM_2's preferences

The criteria weight based on the DM_1's preferences	
C_1	[(0.181,0.189,0.189,0.194;0.9),(0.176,0.189,0.189,0.194;1)]
C_2	[(0.164,0.168,0.168,0.168;0.9),(0.164,0.168,0.168,0.168;1)]
C_3	[(0.093,0.1,0.1,0.102;0.9),(0.093,0.1,0.1,0.103;1)]
C_4	[(0.146,0.146,0.146,0.146;0.9),(0.146,0.146,0.146,0.146;1)]
C_5	[(0.111,0.115,0.115,0.117;0.9),(0.111,0.116,0.116,0.117;1)]
C_6	[(0.131,0.131,0.131,0.132;0.9),(0.128,0.132,0.132,0.132;1)]
C_7	[(0.064,0.069,0.069,0.069;0.9),(0.063,0.069,0.069,0.069;1)]
C_8	[(0.075,0.08,0.08,0.087;0.9),(0.075,0.08,0.08,0.089;1)]
The criteria weight based on the DM_2's preferences	
C_1	[(0.117,0.133,0.133,0.136;0.9),(0.117,0.133,0.133,0.136;1)]
C_2	[(0.135,0.148,0.148,0.15;0.9),(0.135,0.148,0.148,0.15;1)]
C_3	[(0.065,0.086,0.086,0.091;0.9),(0.063,0.086,0.086,0.095;1)]
C_4	[(0.16,0.18,0.18,0.185;0.9),(0.159,0.18,0.18,0.187;1)]
C_5	[(0.153,0.164,0.164,0.164;0.9),(0.153,0.164,0.164,0.164;1)]
C_6	[[(0.081,0.102,0.102,0.107;0.9),(0.081,0.102,0.102,0.109;1)]
C_7	[(0.099,0.117,0.117,0.122;0.9),(0.099,0.117,0.117,0.122;1)]
C_8	[(0.057,0.07,0.07,0.071;0.9),(0.057,0.07,0.07,0.071;1)]

Table A.15: The criteria weight based on the DM_3, DM_4 and SDM's preferences

The criteria weight based on the DM_3's preferences	
C_1	[(0.119,0.124,0.124,0.129;0.9),(0.119,0.124,0.124,0.129;1)]
C_2	[(0.153,0.155,0.155,0.155;0.9),(0.153,0.155,0.155,0.155;1)]
C_3	[(0.086,0.086,0.086,0.088;0.9),(0.081,0.086,0.086,0.088;1)]
C_4	[(0.169,0.179,0.179,0.188;0.9),(0.164,0.179,0.179,0.19;1)]
C_5	[(0.155,0.155,0.155,0.157;0.9),(0.155,0.155,0.155,0.157;1)]
C_6	[(0.099,0.108,0.108,0.108;0.9),(0.099,0.108,0.108,0.108;1)]
C_7	[(0.119,0.124,0.124,0.128;0.9),(0.117,0.124,0.124,0.128;1)]
C_8	[(0.069,0.069,0.069,0.069;0.9),(0.069,0.069,0.069,0.069;1)]
The criteria weight based on the DM_4's preferences	
C_1	[(0.101,0.114,0.114,0.114;0.9),(0.101,0.114,0.114,0.114;1)]
C_2	[(0.184,0.193,0.193,0.2;0.9),(0.181,0.193,0.193,0.202;1)]
C_3	[(0.08,0.091,0.091,0.091;0.9),(0.08,0.091,0.091,0.091;1)]
C_4	[(0.168,0.169,0.169,0.169;0.9),(0.168,0.169,0.169,0.169;1)]
C_5	[(0.151,0.151,0.151,0.152;0.9),(0.151,0.151,0.151,0.152;1)]
C_6	[(0.118,0.122,0.122,0.127;0.9),(0.118,0.122,0.122,0.128;1)]
C_7	[(0.068,0.068,0.068,0.068;0.9),(0.068,0.068,0.068,0.068;1)]
C_8	[(0.084,0.091,0.091,0.097;0.9),(0.08,0.091,0.091,0.097;1)]
The criteria weight based on the SDM's preferences	
C_1	[(0.118,0.133,0.133,0.137;0.9),(0.118,0.133,0.133,0.137;1)]
C_2	[(0.168,0.188,0.188,0.196;0.9),(0.164,0.188,0.188,0.197;1)]
C_3	[(0.151,0.164,0.164,0.164;0.9),(0.151,0.164,0.164,0.164;1)]
C_4	[(0.101,0.117,0.117,0.123;0.9),(0.101,0.117,0.117,0.123;1)]
C_5	[(0.135,0.148,0.148,0.15;0.9),(0.135,0.148,0.148,0.15;1)]
C_6	[(0.085,0.102,0.102,0.109;0.9),(0.085,0.102,0.102,0.109;1)]
C_7	[(0.068,0.086,0.086,0.09;0.9),(0.063,0.086,0.086,0.09;1)]
C_8	[(0.051,0.063,0.063,0.063;0.9),(0.051,0.063,0.063,0.063;1)]

Table A.16: Aggregated decision matrix

Criteria	Alternatives	
	A_1	A_2
C_1	[(0.035,0.052,0.062,0.074;0.9),(0.019,0.052,0.072,0.097;1)]	[(0.106,0.128,0.138,0.153;0.9),(0.082,0.131,0.148,0.176;1)]
C_2	[(0.086,0.11,0.12,0.134;0.9),(0.063,0.11,0.13,0.159;1)]	[(0.154,0.17,0.18,0.197;0.9),(0.126,0.18,0.19,0.213;1)]
C_3	[(0.014,0.026,0.036,0.048;0.9),(0.003,0.026,0.046,0.071;1)]	[(0.111,0.133,0.143,0.158;0.9),(0.086,0.136,0.153,0.181;1)]
C_4	[(0.107,0.13,0.14,0.155;0.9),(0.082,0.131,0.15,0.179;1)]	[(0.144,0.163,0.173,0.189;0.9),(0.117,0.17,0.183,0.208;1)]
C_5	[(0.13,0.15,0.16,0.175;0.9),(0.105,0.155,0.17,0.196;1)]	[(0.11,0.133,0.143,0.158;0.9),(0.085,0.134,0.153,0.182;1)]
C_6	[(0.138,0.158,0.168,0.184;0.9),(0.111,0.164,0.178,0.204;1)]	[(0.096,0.12,0.13,0.144;0.9),(0.073,0.12,0.14,0.169;1)]
C_7	[(0.091,0.113,0.123,0.138;0.9),(0.067,0.115,0.133,0.161;1)]	[(0.125,0.144,0.154,0.169;0.9),(0.099,0.15,0.164,0.19;1)]
C_8	[(0.13,0.15,0.16,0.175;0.9),(0.103,0.155,0.17,0.196;1)]	[(0.092,0.113,0.123,0.138;0.9),(0.069,0.116,0.133,0.16;1)]
Criteria	A_3	A_4
C_1	[(0.034,0.051,0.061,0.074;0.9),(0.017,0.051,0.071,0.097;1)]	[(0.154,0.17,0.18,0.197;0.9),(0.126,0.18,0.19,0.213;1)]
C_2	[(0.104,0.127,0.137,0.152;0.9),(0.079,0.128,0.147,0.177;1)]	[(0.01,0.02,0.03,0.041;0.9),(0,0.02,0.04,0.064;1)]
C_3	[(0.07,0.094,0.104,0.118;0.9),(0.047,0.094,0.114,0.143;1)]	[(0.144,0.163,0.173,0.189;0.9),(0.117,0.17,0.183,0.208;1)]
C_4	[(0.134,0.153,0.163,0.179;0.9),(0.107,0.161,0.173,0.198;1)]	[(0.031,0.05,0.06,0.073;0.9),(0.013,0.05,0.07,0.096;1)]
C_5	[(0.136,0.156,0.166,0.182;0.9),(0.109,0.162,0.176,0.202;1)]	[(0.019,0.034,0.044,0.056;0.9),(0.006,0.034,0.054,0.079;1)]
C_6	[(0.123,0.143,0.153,0.169;0.9),(0.098,0.148,0.163,0.189;1)]	[(0.037,0.052,0.062,0.075;0.9),(0.022,0.053,0.072,0.097;1)]
C_7	[(0.114,0.137,0.147,0.163;0.9),(0.089,0.139,0.157,0.187;1)]	[(0.096,0.118,0.128,0.143;0.9),(0.071,0.121,0.138,0.166;1)]
C_8	[(0.039,0.055,0.065,0.077;0.9),(0.023,0.055,0.075,0.101;1)]	[(0.014,0.026,0.036,0.048;0.9),(0.003,0.026,0.046,0.071;1)]

Appendix B.

Demographic questions:

1. What is your position?

   ………………………………………………

2. How many years do you have experience in health care waste management area?

   ………………………………………………

3. What is your academic degree?

   ………………………………………………

Figure B.7: Demographic questioner

The following questionnaires are designed with the aim of determining criterion weight. Please indicate the most important and least important criteria based on your point of view.

Table 1: List of the specified criteria and the most and the least important ones

Concept	Description	The most important criterion	The least important criterion
Cost (C_1)	The price of technology, installation, maintenance, and etc.		
Treatment effectiveness (C_2)	Waste treatment effectiveness		
Reliability (C_3)	The reliability of technology while working		
After-sale service (C_4)	The availability of a certified after-sale service agency		
Attractiveness of contract (C_5)	Flexibility of monetary contract		
Guarantee (C_6)	The availability of guarantee service and its attractiveness		
Energy consumption (C_7)	The energy consumption while the technology is working		
Flexibility (C_8)	The ease of upgradability of technology		

Figure B.8: Determining the most and the least important criteria

Please indicate the superiority of the most important criterion over the other criteria using the linguistic terms provided in Table 3. For example, the term "CEI" indicates that two criteria are completely equally important.

The most important criterion is ....

Table 2: The superiority of the most important criteria to others

Pairwise comparison	Linguistic term
The superiority of the most important criterion over C_1	
The superiority of the most important criterion over C_2	
The superiority of the most important criterion over C_3	
The superiority of the most important criterion over C_4	
The superiority of the most important criterion over C_5	
The superiority of the most important criterion over C_6	
The superiority of the most important criterion over C_7	
The superiority of the most important criterion over C_8	

Figure B.9: Best to other questioner

Table 3: Linguistic terms

Extreme more importance (EMI)
Extreme importance (EI)
Very strong importance (VSI)
Strong plus importance (SPI)
Strong importance (SI)
Moderate plus importance (MPI)
Moderate importance (MI)
Weak importance (WI)
Completely equal importance (CEI)

Figure B.10: Linguistic terms

Please indicate the superiority of each criterion over the least important criterion using the linguistic terms provided in Table 3. For example, the term "CEI" indicates that two criteria are completely equally important.

The least important criterion is ....

Table 4: The superiority of each criterion to the least important one

Pairwise comparison	Linguistic term
The superiority of criterion C_1 over the worst criterion.	
The superiority of criterion C_2 over the worst criterion.	
The superiority of criterion C_3 over the worst criterion.	
The superiority of criterion C_4 over the worst criterion.	
The superiority of criterion C_5 over the worst criterion.	
The superiority of criterion C_6 over the worst criterion.	
The superiority of criterion C_7 over the worst criterion.	
The superiority of criterion C_8 over the worst criterion.	

Figure B.11: Worst to other comparisons questioner

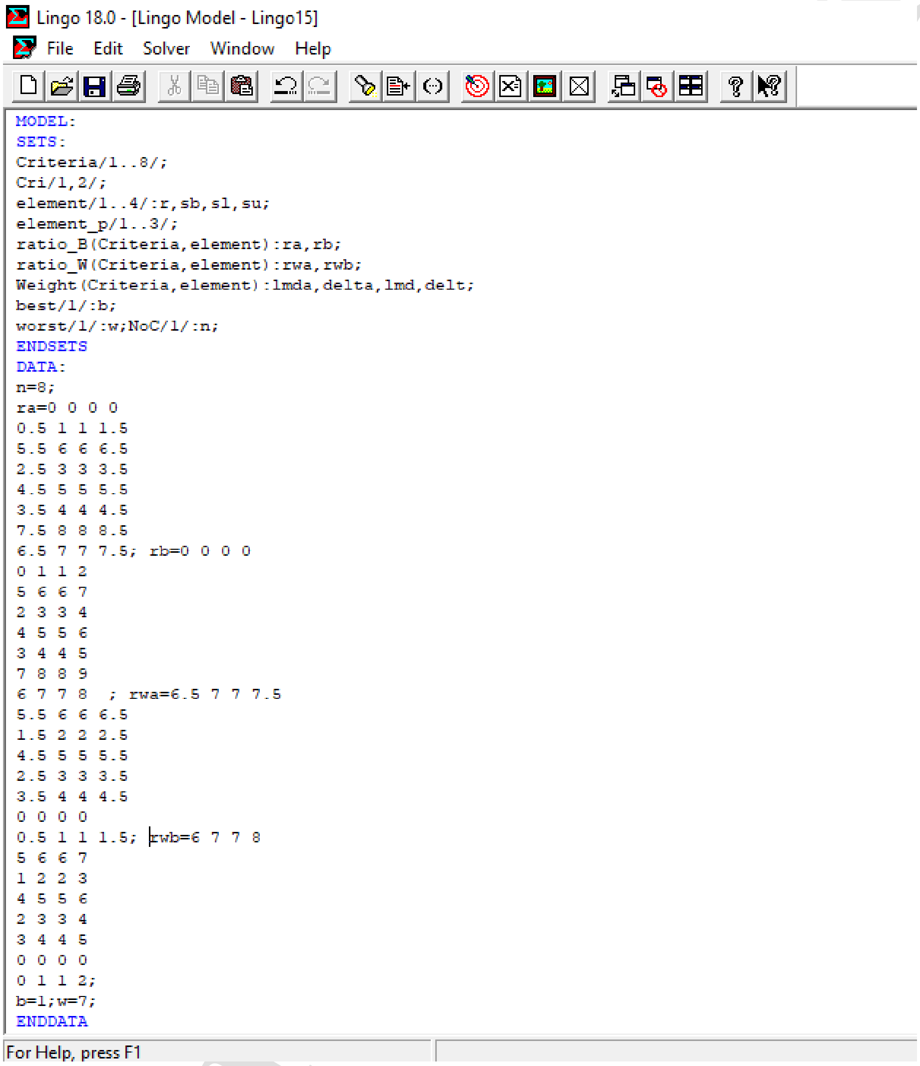
The questionnaire aims to determine alternative ranking and ranking order. Please evaluate each alternative based on each criterion using the linguistic terms: Low (L), Moderate Low (ML), Moderate (M), Moderate High (MH), High (H).

Table 5: Decision matrix

Alternatives	Criteria							
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1								
A_2								
A_3								
A_4								

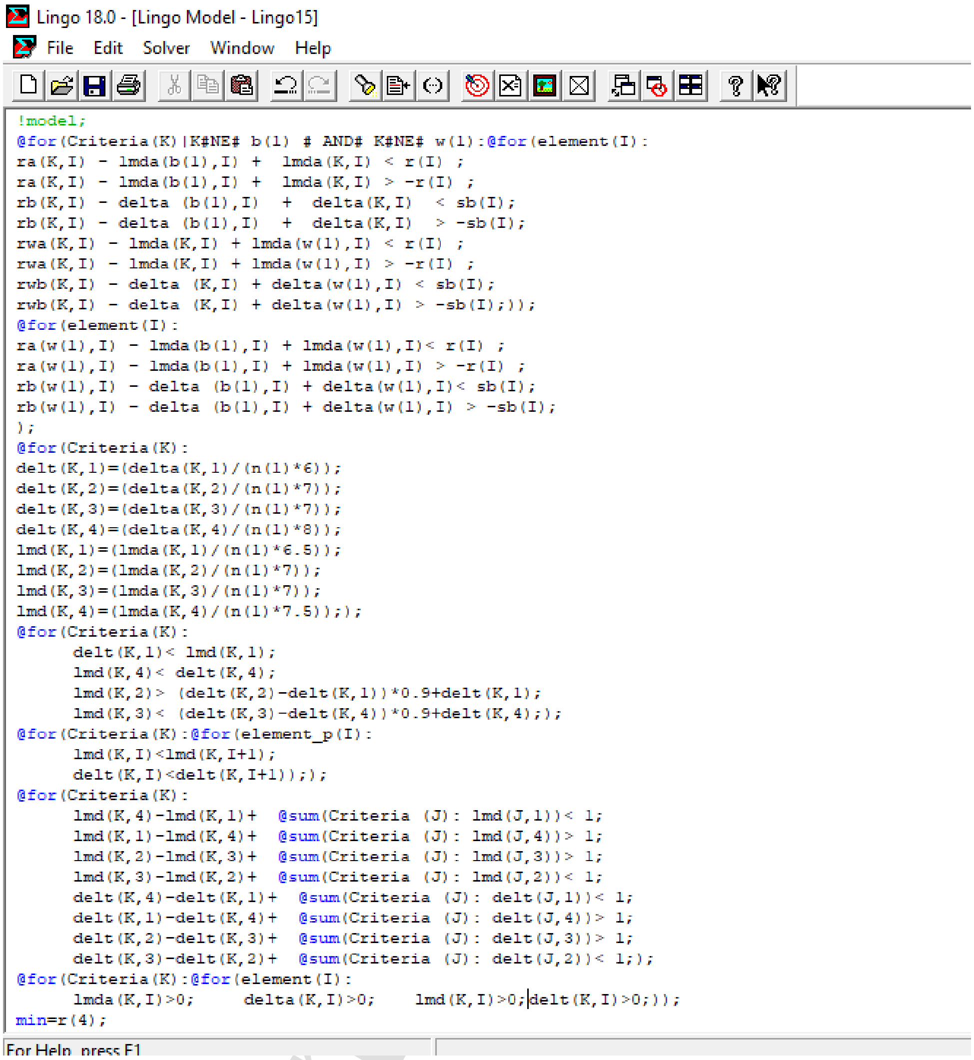
Figure B.12: Alternative evaluation questioner

Appendix C.




```
MODEL:
SETS:
Criteria/1..8/;
Cri/1,2/;
element/1..4/:r,sb,sl,su;
element_p/1..3/;
ratio_B(Criteria,element):ra,rb;
ratio_W(Criteria,element):rwa,rwb;
Weight(Criteria,element):lmda,delta,lmd,delt;
best/1/:b;
worst/1/:w;NoC/1/:n;
ENDSETS
DATA:
n=8;
ra=0 0 0 0
0.5 1 1 1.5
5.5 6 6 6.5
2.5 3 3 3.5
4.5 5 5 5.5
3.5 4 4 4.5
7.5 8 8 8.5
6.5 7 7 7.5; rb=0 0 0 0
0 1 1 2
5 6 6 7
2 3 3 4
4 5 5 6
3 4 4 5
7 8 8 9
6 7 7 8  ; rwa=6.5 7 7 7.5
5.5 6 6 6.5
1.5 2 2 2.5
4.5 5 5 5.5
2.5 3 3 3.5
3.5 4 4 4.5
0 0 0 0
0.5 1 1 1.5; rwb=6 7 7 8
5 6 6 7
1 2 2 3
4 5 5 6
2 3 3 4
3 4 4 5
0 0 0 0
0 1 1 2;
b=1;w=7;
ENDDATA
```

Figure .13: Model M2 code executed by Lingo 18.0




```
!model;
@for(Criteria(K)|K#NE# b(1) # AND# K#NE# w(1):@for(element(I):
ra(K,I) - lmda(b(1),I) +  lmda(K,I) < r(I) ;
ra(K,I) - lmda(b(1),I) +  lmda(K,I) > -r(I) ;
rb(K,I) - delta (b(1),I)  +  delta(K,I)  < sb(I);
rb(K,I) - delta (b(1),I)  +  delta(K,I)  > -sb(I);
rwa(K,I) - lmda(K,I) + lmda(w(1),I) < r(I) ;
rwa(K,I) - lmda(K,I) + lmda(w(1),I) > -r(I) ;
rwb(K,I) - delta (K,I) + delta(w(1),I) < sb(I);
rwb(K,I) - delta (K,I) + delta(w(1),I) > -sb(I);));
@for(element(I):
ra(w(1),I) - lmda(b(1),I) + lmda(w(1),I)< r(I) ;
ra(w(1),I) - lmda(b(1),I) + lmda(w(1),I) > -r(I) ;
rb(w(1),I) - delta (b(1),I) + delta(w(1),I)< sb(I);
rb(w(1),I) - delta (b(1),I) + delta(w(1),I) > -sb(I);
);
@for(Criteria(K):
delt(K,1)=(delta(K,1)/(n(1)*6));
delt(K,2)=(delta(K,2)/(n(1)*7));
delt(K,3)=(delta(K,3)/(n(1)*7));
delt(K,4)=(delta(K,4)/(n(1)*8));
lmd(K,1)=(lmda(K,1)/(n(1)*6.5));
lmd(K,2)=(lmda(K,2)/(n(1)*7));
lmd(K,3)=(lmda(K,3)/(n(1)*7));
lmd(K,4)=(lmda(K,4)/(n(1)*7.5)););
@for(Criteria(K):
      delt(K,1)< lmd(K,1);
      lmd(K,4)< delt(K,4);
      lmd(K,2)> (delt(K,2)-delt(K,1))*0.9+delt(K,1);
      lmd(K,3)< (delt(K,3)-delt(K,4))*0.9+delt(K,4););
@for(Criteria(K):@for(element_p(I):
      lmd(K,I)<lmd(K,I+1);
      delt(K,I)<delt(K,I+1)););
@for(Criteria(K):
      lmd(K,4)-lmd(K,1)+  @sum(Criteria (J): lmd(J,1))< 1;
      lmd(K,1)-lmd(K,4)+  @sum(Criteria (J): lmd(J,4))> 1;
      lmd(K,2)-lmd(K,3)+  @sum(Criteria (J): lmd(J,3))> 1;
      lmd(K,3)-lmd(K,2)+  @sum(Criteria (J): lmd(J,2))< 1;
      delt(K,4)-delt(K,1)+  @sum(Criteria (J): delt(J,1))< 1;
      delt(K,1)-delt(K,4)+  @sum(Criteria (J): delt(J,4))> 1;
      delt(K,2)-delt(K,3)+  @sum(Criteria (J): delt(J,3))> 1;
      delt(K,3)-delt(K,2)+  @sum(Criteria (J): delt(J,2))< 1;);
@for(Criteria(K):@for(element(I):
      lmda(K,I)>0;    delta(K,I)>0;    lmd(K,I)>0;delt(K,I)>0;));
min=r(4);
```

Figure .14: Model M2 code executed by Lingo 18.0

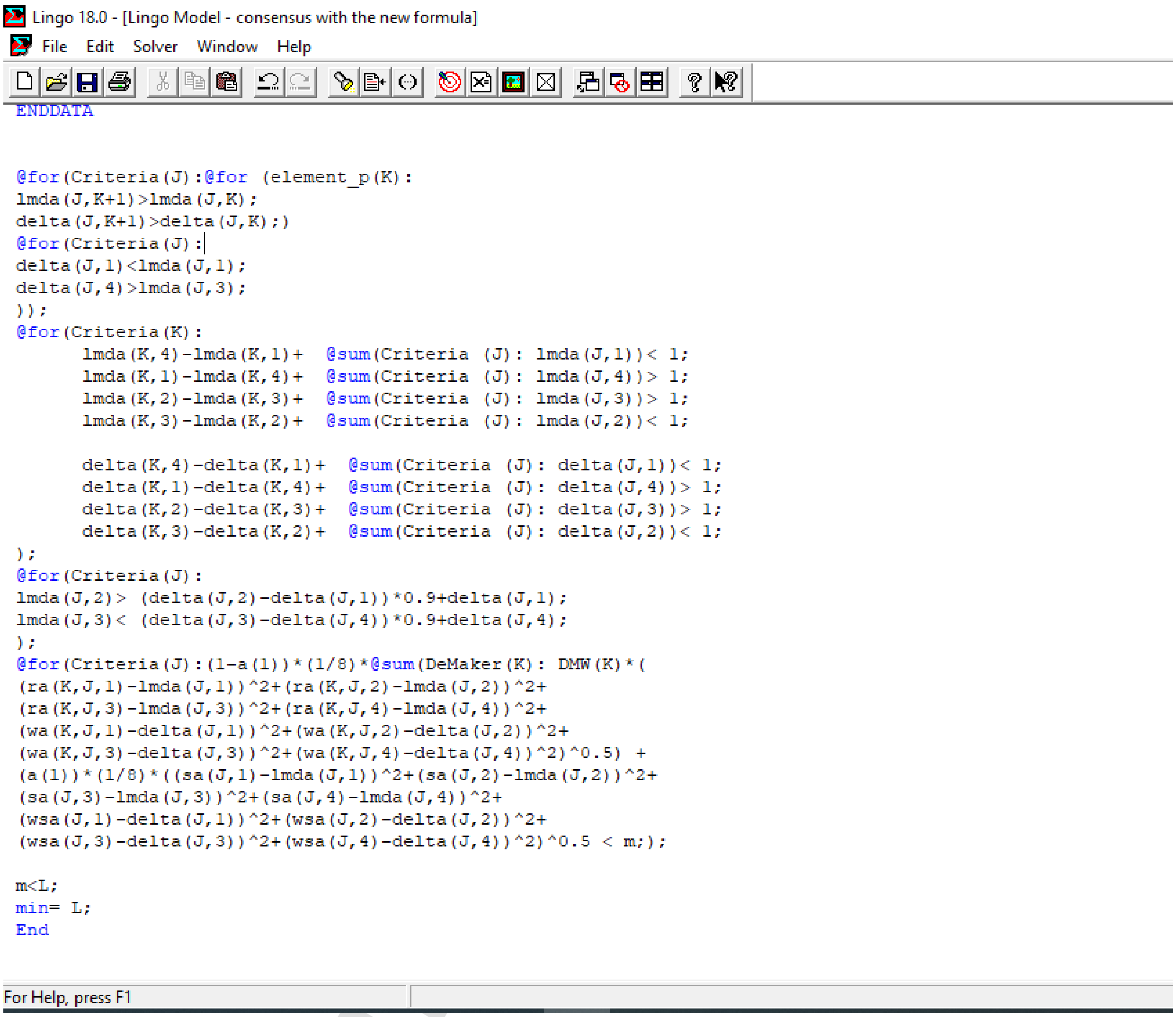
Lingo 18.0 - [Lingo Model - Lingo3]

File Edit Solver Window Help

```
!model;
@for(Criteria(K)|K#NE# b(1) # AND# K#NE# w(1):@for(element(I):
ra(K,I) - lmda(b(1),I) +  lmda(K,I) < r(I) ;ra(K,I) - lmda(b(1),I) +  lmda(K,I) > -r(I) ;
rb(K,I) - delta (b(1),I)  +  delta(K,I)  < sb(I); rb(K,I) - delta (b(1),I)  +  delta(K,I)  > -sb(I);
rwa(K,I) - lmda(K,I) + lmda(w(1),I) < r(I) ;rwa(K,I) - lmda(K,I) + lmda(w(1),I) > -r(I) ;
rwb(K,I) - delta (K,I) + delta(w(1),I) < sb(I);rwb(K,I) - delta (K,I) + delta(w(1),I) > -sb(I););
@for(element(I):
ra(w(1),I) - lmda(b(1),I) + lmda(w(1),I)< r(I) ;ra(w(1),I) - lmda(b(1),I) + lmda(w(1),I) > -r(I) ;
rb(w(1),I) - delta (b(1),I) + delta(w(1),I)< sb(I);rb(w(1),I) - delta (b(1),I) + delta(w(1),I) > -sb(I););
@for(Criteria(K):
delt(K,1)=(delta(K,1)/(n(1)*7));
delt(K,2)=(delta(K,2)/(n(1)*8));
delt(K,3)=(delta(K,3)/(n(1)*8));
delt(K,4)=(delta(K,4)/(n(1)*9));
lmd(K,1)=(lmda(K,1)/(n(1)*7.5));
lmd(K,2)=(lmda(K,2)/(n(1)*8));
lmd(K,3)=(lmda(K,3)/(n(1)*8));
lmd(K,4)=(lmda(K,4)/(n(1)*8.5)););
@for(Criteria(K):
      delt(K,1)< lmd(K,1);
      lmd(K,4)< delt(K,4);
      lmd(K,2)> (delt(K,2)-delt(K,1))*0.9+delt(K,1);lmd(K,3)< (delt(K,3)-delt(K,4))*0.9+delt(K,4););
@for(Criteria(K):@for(element_p(I):
      lmd(K,I)<lmd(K,I+1);delt(K,I)<delt(K,I+1)););
@for(Criteria(K):
      lmd(K,4)-lmd(K,1)+  @sum(Criteria (J): lmd(J,1))< 1;
      lmd(K,1)-lmd(K,4)+  @sum(Criteria (J): lmd(J,4))> 1;
      lmd(K,2)-lmd(K,3)+  @sum(Criteria (J): lmd(J,3))> 1;
      lmd(K,3)-lmd(K,2)+  @sum(Criteria (J): lmd(J,2))< 1;
      delt(K,4)-delt(K,1)+  @sum(Criteria (J): delt(J,1))< 1;
      delt(K,1)-delt(K,4)+  @sum(Criteria (J): delt(J,4))> 1;
      delt(K,2)-delt(K,3)+  @sum(Criteria (J): delt(J,3))> 1;
      delt(K,3)-delt(K,2)+  @sum(Criteria (J): delt(J,2))< 1;);
@for(Criteria(K):@for(element(I):
      lmda(K,I)>0;delta(K,I)>0;lmd(K,I)>0;   delt(K,I)>0;));
@for(element(I):
0.816239-r(I)>(0.816239-0.5)*khi;
0.923077-r(I)>(0.923077-0.333333)*khi;
0.923077-r(I)>(0.923077-0.333333)*khi;
0.978022-r(I)>(0.978022-0.25)*khi;
0.66667-sb(I)>(0.66667)*khi;
0.978022- sb(I)> (0.978022 -0.333333) *khi;
0.978022- sb(I)> (0.978022 -0.333333) *khi;
0.666667- sb(I)>(0.666667 -0.333333)*khi;);
max=(khi);
```

For Help, press F1

Figure .15: Model M3 code executed by Lingo 18.0




```
ENDDATA

@for(Criteria(J):@for (element_p(K):
lmda(J,K+1)>lmda(J,K);
delta(J,K+1)>delta(J,K);)
@for(Criteria(J):
delta(J,1)<lmda(J,1);
delta(J,4)>lmda(J,3);
));
@for(Criteria(K):
      lmda(K,4)-lmda(K,1)+  @sum(Criteria (J): lmda(J,1))< 1;
      lmda(K,1)-lmda(K,4)+  @sum(Criteria (J): lmda(J,4))> 1;
      lmda(K,2)-lmda(K,3)+  @sum(Criteria (J): lmda(J,3))> 1;
      lmda(K,3)-lmda(K,2)+  @sum(Criteria (J): lmda(J,2))< 1;

      delta(K,4)-delta(K,1)+  @sum(Criteria (J): delta(J,1))< 1;
      delta(K,1)-delta(K,4)+  @sum(Criteria (J): delta(J,4))> 1;
      delta(K,2)-delta(K,3)+  @sum(Criteria (J): delta(J,3))> 1;
      delta(K,3)-delta(K,2)+  @sum(Criteria (J): delta(J,2))< 1;
);
@for(Criteria(J):
lmda(J,2)> (delta(J,2)-delta(J,1))*0.9+delta(J,1);
lmda(J,3)< (delta(J,3)-delta(J,4))*0.9+delta(J,4);
);
@for(Criteria(J):(1-a(1))*(1/8)*@sum(DeMaker(K): DMW(K)*(
(ra(K,J,1)-lmda(J,1))^2+(ra(K,J,2)-lmda(J,2))^2+
(ra(K,J,3)-lmda(J,3))^2+(ra(K,J,4)-lmda(J,4))^2+
(wa(K,J,1)-delta(J,1))^2+(wa(K,J,2)-delta(J,2))^2+
(wa(K,J,3)-delta(J,3))^2+(wa(K,J,4)-delta(J,4))^2)^0.5) +
(a(1))*(1/8)*((sa(J,1)-lmda(J,1))^2+(sa(J,2)-lmda(J,2))^2+
(sa(J,3)-lmda(J,3))^2+(sa(J,4)-lmda(J,4))^2+
(wsa(J,1)-delta(J,1))^2+(wsa(J,2)-delta(J,2))^2+
(wsa(J,3)-delta(J,3))^2+(wsa(J,4)-delta(J,4))^2)^0.5 < m;);

m<L;
min= L;
End
```

Figure .16: Model M4 code executed by Lingo 18.0

References

[1] A. Hafezalkotob, A. Hafezalkotob, A novel approach for combination of individual and group decisions based on fuzzy best-worst method, Applied Soft Computing 59 (2017) 316–325.

[2] S.-p. Wan, Z.-h. Chen, J.-y. Dong, An integrated interval type-2 fuzzy technique for democratic–autocratic multi-criteria decision making, Knowledge-Based Systems 214 (2021) 106735.

[3] X. Pan, Y. Wang, Evaluation of renewable energy sources in china using an interval type-2 fuzzy large-scale group risk evaluation method, Applied Soft Computing 108 (2021) 107458.

[4] F. Herrera, E. Herrera-Viedma, Aggregation operators for linguistic weighted information, IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans 27 (5) (1997) 646–656.

[5] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning—i, Information sciences 8 (3) (1975) 199–249.

[6] H. Wang, X. Pan, J. Yan, J. Yao, S. He, A projection-based regret theory method for multi-attribute decision making under interval type-2 fuzzy sets environment, Information Sciences 512 (2020) 108–122.

[7] J. Qin, X. Ma, An it2fs-pt3 based emergency response plan evaluation with multimoora method in group decision making, Applied Soft Computing 122 (2022) 108812.

[8] H. Li, X. Dai, L. Zhou, Q. Wu, Encoding words into interval type-2 fuzzy sets: The retained region approach, Information Sciences 629 (2023) 760–777.

[9] F. Liu, J. M. Mendel, Encoding words into interval type-2 fuzzy sets using an interval approach, IEEE transactions on fuzzy systems 16 (6) (2008) 1503–1521.

[10] D. Wu, J. M. Mendel, Computing with words for hierarchical decision making applied to evaluating a weapon system, IEEE Transactions on Fuzzy Systems 18 (3) (2010) 441–460.

[11] J. M. Mendel, Computing with words and its relationships with fuzzistics, Information Sciences 177 (4) (2007) 988–1006.

[12] K. K. F. Yuen, et al., Cognitive network process with fuzzy soft computing technique in collective decision aiding (2009).

[13] K. K. F. Yuen, Fuzzy cognitive network process: comparisons with fuzzy analytic hierarchy process in new product development strategy, IEEE Transactions on Fuzzy Systems 22 (3) (2013) 597–610.

[14] K. K. F. Yuen, Pairwise opposite matrix and its cognitive prioritization operators: comparisons with pairwise reciprocal matrix and analytic prioritization operators, Journal of the Operational Research Society 63 (3) (2012) 322–338.

[15] H. Zhang, C. Yin, X. Qi, R. Zhang, X. Kang, Cognitive best worst method for multiattribute decision-making, Mathematical Problems in Engineering 2017 (2017).

[16] F. Liang, M. Brunelli, J. Rezaei, Best-worst tradeoff method, Information Sciences 610 (2022) 957–976. doi:https://doi.org/10.1016/j.ins.2022.07.097.

[17] T.-Y. Chen, Likelihoods of interval type-2 trapezoidal fuzzy preference relations and their application to multiple criteria decision analysis, Information Sciences 295 (2015) 303–322.

[18] J. M. Mendel, R. I. John, F. Liu, Interval type-2 fuzzy logic systems made simple, IEEE transactions on fuzzy systems 14 (6) (2006) 808–821.

[19] M. Tavana, A. Shaabani, D. Di Caprio, A. Bonyani, A novel interval type-2 fuzzy best-worst method and combined compromise solution for evaluating eco-friendly packaging alternatives, Expert Systems with Applications 200 (2022) 117188.

[20] T.-Y. Chen, An interactive method for multiple criteria group decision analysis based on interval type-2 fuzzy sets and its application to medical decision making, Fuzzy Optimization and Decision Making 12 (3) (2013) 323–356.

[21] E. Ontiveros, P. Melin, O. Castillo, Comparative study of interval type-2 and general type-2 fuzzy systems in medical diagnosis, Information Sciences 525 (2020) 37–53.

[22] W. Chen, S. Zeng, E. Zhang, Fermatean fuzzy iwp-topsis-gra multi-criteria group analysis and its application to healthcare waste treatment technology evaluation, Sustainability 15 (7) (2023) 6056.

[23] S. Hendiani, H. Liao, R. Ren, B. Lev, A likelihood-based multi-criteria sustainable supplier selection approach with complex preference information, Information sciences 536 (2020) 135–155.

[24] T. R. Ayodele, A. Ogunjuyigbe, O. Odigie, J. L. Munda, A multi-criteria gis based model for wind farm site selection using interval type-2 fuzzy analytic hierarchy process: The case study of nigeria, Applied energy 228 (2018) 1853–1869.

[25] F. Çebi, İ. Otay, Multi-criteria and multi-stage facility location selection under interval type-2 fuzzy environment: a case study for a cement factory, international Journal of computational intelligence systems 8 (2) (2015) 330–344.

[26] S. Karagöz, M. Deveci, V. Simic, N. Aydin, Interval type-2 fuzzy aras method for recycling facility location problems, Applied Soft Computing 102 (2021) 107107.

[27] B. C. Altay, E. Celik, A. Okumus, A. Balin, M. Gul, An integrated interval type-2 fuzzy bwm-marcos model for location selection of e-scooter sharing stations: The case of a university campus, Engineering Applications of Artificial Intelligence 122 (2023) 106095.

[28] T. Wu, X. Liu, F. Liu, An interval type-2 fuzzy topsis model for large scale group decision making problems with social network information, Information Sciences 432 (2018) 392–410.

[29] S.-M. Chen, J.-A. Hong, Fuzzy multiple attributes group decision-making based on ranking interval type-2 fuzzy sets and the topsis method, IEEE Transactions on Systems, Man, and Cybernetics: Systems 44 (12) (2014) 1665–1673.

[30] Q. Wu, L. Zhou, Y. Chen, H. Chen, An integrated approach to green supplier selection based on the interval type-2 fuzzy best-worst and extended vikor methods, Information Sciences 502 (2019) 394–417.

[31] J. Qin, X. Liu, W. Pedrycz, An extended vikor method based on prospect theory for multiple attribute decision making under interval type-2 fuzzy environment, Knowledge-Based Systems 86 (2015) 116–130.

[32] Q. Wei, C. Zhou, Q. Liu, W. Zhou, J. Huang, A barrier evaluation framework for forest carbon sink project implementation in china using an integrated bwm-it2f-promethee ii method, Expert Systems with Applications (2023) 120612.

[33] X.-H. Pan, Y.-M. Wang, S.-F. He, Á. Labella, L. Martínez, An interval type-2 fuzzy oreste method for waste-to-energy plant site selection: A case study in china, Applied Soft Computing 136 (2023) 110092.

[34] E. Celik, M. Yucesan, M. Gul, Green supplier selection for textile industry: a case study using bwm-todim integration under interval type-2 fuzzy sets, Environmental Science and Pollution Research 28 (45) (2021) 64793–64817.

[35] M. Alimohammadlou, S. Sharifian, Industry 4.0 implementation challenges in small-and medium-sized enterprises: an approach integrating interval type-2 fuzzy bwm and dematel, Soft Computing 27 (1) (2023) 169–186.

[36] C. Kahraman, B. Öztayşi, İ. U. Sarı, E. Turanoğlu, Fuzzy analytic hierarchy process with interval type-2 fuzzy sets, Knowledge-Based Systems 59 (2014) 48–57.

[37] E. Celik, M. Gul, N. Aydin, A. T. Gumus, A. F. Guneri, A comprehensive review of multi criteria decision making approaches based on interval type-2 fuzzy sets, Knowledge-Based Systems 85 (2015) 329–341.

[38] J. Rezaei, Best-worst multi-criteria decision-making method, Omega 53 (2015) 49–57.

[39] S. Wan, J. Dong, S.-M. Chen, Fuzzy best-worst method based on generalized interval-valued trapezoidal fuzzy numbers for multi-criteria decision-making, Information Sciences 573 (2021) 493–518.

[40] S. Guo, H. Zhao, Fuzzy best-worst multi-criteria decision-making method and its applications, Knowledge-Based Systems 121 (2017) 23–31.

[41] A. Ali, T. Rashid, Hesitant fuzzy best-worst multi-criteria decision-making method and its applications, International Journal of Intelligent Systems 34 (8) (2019) 1953–1967.

[42] H. Aboutorab, M. Saberi, M. R. Asadabadi, O. Hussain, E. Chang, Zbwm: The z-number extension of best worst method and its application for supplier development, Expert Systems with Applications 107 (2018) 115–125.

[43] P. Gulum Tas, An overview of the applications of bwm in health, in: The International Workshop on Best-Worst Method, Springer, 2022, pp. 1–18.

[44] R.-L. Huang, M.-h. Deng, Y.-y. Li, J.-q. Wang, J.-B. Li, Cloud decision support framework for treatment technology selection of health-care waste, Journal of Intelligent & Fuzzy Systems 42 (6) (2022) 5565–5590.

[45] M. Yazdani, M. Tavana, D. Pamučar, P. Chatterjee, A rough based multi-criteria evaluation method for healthcare waste disposal location decisions, Computers & Industrial Engineering 143 (2020) 106394.

[46] D. Pamučar, A. Puška, Ž. Stević, G. Ćirović, A new intelligent mcdm model for hcw management: The integrated bwm–mabac model based on d numbers, Expert Systems with Applications 175 (2021) 114862.

[47] A. E. Torkayesh, B. Malmir, M. R. Asadabadi, Sustainable waste disposal technology selection: The stratified best-worst multi-criteria decision-making method, Waste Management 122 (2021) 100–112.

[48] K. K. F. Yuen, The fuzzy cognitive pairwise comparisons for ranking and grade clustering to build a recommender system: An application of smartphone recommendation, Engineering Applications of Artificial Intelligence 61 (2017) 136–151.

[49] D.-F. Li, S.-P. Wan, Fuzzy heterogeneous multiattribute decision making method for outsourcing provider selection, Expert systems with applications 41 (6) (2014) 3047–3059.

[50] A. Ben-Tal, L. El Ghaoui, A. Nemirovski, Robust optimization, Vol. 28, Princeton university press, 2009.

[51] F. Xiao, A novel multi-criteria decision making method for assessing health-care waste treatment technologies based on d numbers, Engineering Applications of Artificial Intelligence 71 (2018) 216–225.

[52] N. J. Rowan, J. G. Laffey, Challenges and solutions for addressing critical shortage of supply chain for personal and protective equipment (ppe) arising from coronavirus disease (covid19) pandemic–case study from the republic of ireland, Science of the Total Environment 725 (2020) 138532.

[53] S. Kargar, M. Pourmehdi, M. M. Paydar, Reverse logistics network design for medical waste management in the epidemic outbreak of the novel coronavirus (covid-19), Science of the Total Environment 746 (2020) 141183.

[54] J. J. Klemeš, Y. Van Fan, R. R. Tan, P. Jiang, Minimising the present and future plastic waste, energy and environmental footprints related to covid-19, Renewable and Sustainable Energy Reviews 127 (2020) 109883.

[55] N. Goldani, M. Kazemi, A fuzzy best-worst method based on the fuzzy interval scale, in: Advances in Best-Worst Method: Proceedings of the Third International Workshop on Best-Worst Method (BWM2022), Springer, 2023, pp. 59–73.

[56] W. Wang, X. Liu, Y. Qin, Multi-attribute group decision making models under interval type-2 fuzzy environment, Knowledge-Based Systems 30 (2012) 121–128.

[57] S.-M. Chen, L.-W. Kuo, Autocratic decision making using group recommendations based on interval type-2 fuzzy sets, enhanced karnik–mendel algorithms, and the ordered weighted aggregation operator, Information Sciences 412 (2017) 174–193.

[58] S.-H. Cheng, S.-M. Chen, Z.-C. Huang, Autocratic decision making using group recommendations based on ranking interval type-2 fuzzy sets, Information Sciences 361 (2016) 135–161.

[59] S.-M. Chen, B.-H. Tsai, Autocratic decision making using group recommendations based on the owa operator and correlation coefficients, Information Sciences 290 (2015) 106–119.

[60] S.-H. Cheng, Autocratic decision making using group recommendations based on hesitant fuzzy sets for green hotels selection and bidders selection, Information Sciences 467 (2018) 604–617.

[61] J. A. Morente-Molinera, F. J. Cabrerizo, J. Mezei, C. Carlsson, E. Herrera-Viedma, A dynamic group decision making process for high number of alternatives using hesitant fuzzy ontologies and sentiment analysis, Knowledge-Based Systems 195 (2020) 105657.

[62] J. Dong, S. Wan, S.-M. Chen, Fuzzy best-worst method based on triangular fuzzy numbers for multi-criteria decision-making, Information Sciences 547 (2021) 1080–1104.

Highlights

- A novel MCGDM method under the IT2F environment is proposed.
- A novel linear best-worst method (BWM) based on the interval scale is proposed.
- A novel mathematical optimization model is introduced to obtain collective weights.
- The healthcare waste treatment selection problem during COVID-19 is investigated.

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: